



On $\alpha\omega$ -closed sets and its connectedness in terms of neutrosophic topological spaces

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Abstract

The aim of this paper is to introduce the notion of neutrosophic $\alpha\omega$ -closed sets and study some of the properties of neutrosophic $\alpha\omega$ -closed sets. Further, we investigated neutrosophic $\alpha\omega$ -continuity, neutrosophic $\alpha\omega$ -irresoluteness, neutrosophic $\alpha\omega$ connectedness and neutrosophic contra $\alpha\omega$ continuity along with examples.

Keywords: neutrosophic topology, neutrosophic $\alpha\omega$ -closed set, neutrosophic $\alpha\omega$ -continuous function and neutrosophic contra $\alpha\omega$ -continuous mappings.

1 Introduction

Zadeh [19] introduced truth (t) or the degree of membership of an object in fuzzy set theory. The falsehood (f) or the degree of non-membership of an object along with membership of an object introduced by Atanassov [4,5,6] in intuitionistic fuzzy set. Neutrosophic (i) or the degree of indeterminacy of an object along with membership and non-membership of an objects for incomplete, imprecise, indeterminate information is introduced by Smarandache [16,17] in 1998. The neutrosophic triplet set consist of three components $(t, f, i) = (\text{truth}, \text{falsehood}, \text{indeterminacy})$. The neutrosophic topological spaces introduced and developed by Salama et al., [15]. This leads to many investigation among researchers in the field of neutrosophic topology and their application in decision making algorithms [8,11,12,13,14]. Arokianani et al., [3] introduced and studied α -open sets in neutrosophic topological spaces. Devi et al., [7,9,10] introduced $\alpha\omega$ -closed sets in general topology, fuzzy topology and intuitionistic fuzzy topology. In this article, we introduce neutrosophic $\alpha\omega$ -closed sets in neutrosophic topological spaces. Also, we introduce and investigate neutrosophic $\alpha\omega$ -continuous, neutrosophic $\alpha\omega$ -irresoluteness, neutrosophic $\alpha\omega$ connectedness and neutrosophic contra $\alpha\omega$ -continuous mappings.

2 Preliminaries

Let (X, τ) be the neutrosophic topological space (NTS). Each neutrosophic set (NS) in (X, τ) is called a neutrosophic open set (NOS) and its complement is called a neutrosophic closed set (NCS).

We provide some of the basic definitions in neutrosophic sets. These are very useful in the sequel.

Definition 2.1. [17] A neutrosophic set (NS) A is an object of the following form

$$U = \{ \langle u, \mu_U(u), \nu_U(u), \omega_U(u) \rangle : u \in X \}$$

where the mappings $\mu_U : X \rightarrow I$, $\nu_U : X \rightarrow I$ and $\omega_U : X \rightarrow I$ denote the degree of membership (namely $\mu_U(u)$), the degree of indeterminacy (namely $\nu_U(u)$) and the degree of nonmembership (namely $\omega_U(u)$) for

each element $u \in X$ to the set U , respectively and $0 \leq \mu_U(u) + \nu_U(u) + \omega_U(u) \leq 3$ for each $u \in X$.

Definition 2.2. [17] Let U and V be NSs of the form $U = \{\langle u, \mu_U(u), \nu_U(u), \omega_U(u) \rangle : u \in X\}$ and $V = \{\langle u, \mu_V(u), \nu_V(u), \omega_V(u) \rangle : u \in X\}$. Then

- (i) $U \subseteq V$ if and only if $\mu_U(u) \leq \mu_V(u)$, $\nu_U(u) \geq \nu_V(u)$ and $\omega_U(u) \geq \omega_V(u)$;
- (ii) $\overline{U} = \{\langle u, \nu_U(u), \mu_U(u), \omega_U(u) \rangle : u \in X\}$;
- (iii) $U \cap V = \{\langle u, \mu_U(u) \wedge \mu_V(u), \nu_U(u) \vee \nu_V(u), \omega_U(u) \vee \omega_V(u) \rangle : u \in X\}$;
- (iv) $U \cup V = \{\langle u, \mu_U(u) \vee \mu_V(u), \nu_U(u) \wedge \nu_V(u), \omega_U(u) \wedge \omega_V(u) \rangle : u \in X\}$.

We will use the notation $U = \langle u, \mu_U, \nu_U, \omega_U \rangle$ instead of $U = \{\langle u, \mu_U(u), \nu_U(u), \omega_U(u) \rangle : u \in X\}$. The NSs $0_\sim$ and $1_\sim$ are defined by $0_\sim = \{\langle u, 0, 1, 1 \rangle : u \in X\}$ and $1_\sim = \{\langle u, 1, 0, 0 \rangle : u \in X\}$.

Let $r, s, t \in [0, 1]$ such that $0 \leq r + s + t \leq 3$. A neutrosophic point (NP) $p_{(r,s,t)}$ is neutrosophic set defined by

$$p_{(r,s,t)}(u) = \begin{cases} (r, s, t)(x) & \text{if } u = p \\ (0, 1, 1) & \text{otherwise} \end{cases}$$

Let f be a mapping from an ordinary set X into an ordinary set Y , If $V = \{\langle y, \mu_V(y), \nu_V(y), \omega_V(y) \rangle : y \in Y\}$ is a NS in Y , then the inverse image of V under f is a NS defined by

$$f^{-1}(V) = \{\langle u, f^{-1}(\mu_V)(u), f^{-1}(\nu_V)(u), f^{-1}(\omega_V)(u) \rangle : u \in X\}$$

The image of NS $U = \{\langle v, \mu_U(v), \nu_U(v), \omega_U(v) \rangle : v \in Y\}$ under f is a NS defined by $f(U) = \{\langle v, f(\mu_U)(v), f(\nu_U)(v), f(\omega_U)(v) \rangle : v \in Y\}$ where

$$\begin{aligned} f(\mu_U)(v) &= \begin{cases} \sup_{u \in f^{-1}(v)} \mu_U(u), & \text{if } f^{-1}(v) \neq \emptyset \\ 0 & \text{otherwise,} \end{cases} \\ f(\nu_U)(v) &= \begin{cases} \inf_{u \in f^{-1}(v)} \nu_U(u), & \text{if } f^{-1}(v) \neq \emptyset \\ 1 & \text{otherwise,} \end{cases} \\ f(\omega_U)(v) &= \begin{cases} \inf_{u \in f^{-1}(v)} \omega_U(u), & \text{if } f^{-1}(v) \neq \emptyset \\ 1 & \text{otherwise,} \end{cases} \end{aligned}$$

for each $v \in Y$.

Definition 2.3. [15] A neutrosophic topology (NT) in a nonempty set X is a family τ of NSs in X satisfying the following axioms:

- (NT1) $0_\sim, 1_\sim \in \tau$;
- (NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
- (NT3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

Definition 2.4. [15] Let U be a NS in NTS X . Then

- $\text{Nint}(U) = \cup \{O : O \text{ is an NOS in } X \text{ and } O \subseteq U\}$ is called a neutrosophic interior of U ;
- $\text{Ncl}(U) = \cap \{O : O \text{ is an NCS in } X \text{ and } O \supseteq U\}$ is called a neutrosophic closure of U .

Definition 2.5. [15] Let $p_{(r,s,t)}$ be a NP in NTS X . A NS U in X is called a neutrosophic neighborhood (NN) of $p_{(r,s,t)}$ if there exists a NOS V in X such that $p_{(r,s,t)} \in V \subseteq U$.

Definition 2.6. [3] A subset U of a neutrosophic space (X, τ) is called

1. a neutrosophic pre-open set if $U \subseteq \text{Nint}(\text{Ncl}(U))$ and a neutrosophic pre-closed set if $\text{Ncl}(\text{Nint}(U)) \subseteq U$,
2. a neutrosophic semi-open set if $U \subseteq \text{Ncl}(\text{Nint}(U))$ and a neutrosophic semi-closed set if $\text{Nint}(\text{Ncl}(U)) \subseteq U$,
3. a neutrosophic α -open set if $U \subseteq \text{Nint}(\text{Ncl}(\text{Nint}(U)))$ and a neutrosophic α -closed set if $\text{Ncl}(\text{Nint}(\text{Ncl}(U))) \subseteq U$,

The pre-closure (resp. semi-closure, α -closure) of a subset U of a neutrosophic space (X, τ) is the intersection of all pre-closed (resp. semi-closed, α -closed) sets that contain U and is denoted by $Npcl(U)$ (resp. $Nscl(U)$, $N\alpha cl(U)$).

3 On neutrosophic $\alpha\omega$ -closed sets

Definition 3.1. A subset A of a neutrosophic topological space (X, τ) is called

1. a neutrosophic $N\omega$ -closed set if $Ncl(U) \subseteq G$ whenever $U \subseteq G$ and G is neutrosophic semi-open in (X, τ) .
2. a neutrosophic $\alpha\omega$ -closed ($N\alpha\omega$ -closed) set if $N\omega cl(U) \subseteq G$ whenever $U \subseteq G$ and G is an $N\alpha$ -open set in (X, τ) . Its complement is called a neutrosophic $\alpha\omega$ -open ($N\alpha\omega$ -open) set.

Definition 3.2. Let U be a NS in NTS X . Then

$N\alpha\omega int(U) = \cup\{O : O \text{ is an } N\alpha\omega\text{OS in } X \text{ and } O \subseteq U\}$ is said to be a neutrosophic $\alpha\omega$ -interior of U ;
 $N\alpha\omega cl(U) = \cap\{O : O \text{ is an } N\alpha\omega\text{CS in } X \text{ and } O \supseteq U\}$ is said to be a neutrosophic $\alpha\omega$ -closure of U .

Theorem 3.3. Every $N\alpha$ -closed set and N -closed set are $N\alpha\omega$ -closed set.

Proof. Let U be an $N\alpha$ -closed set, then $U = N\alpha cl(U)$. Let $U \subseteq G$, G is $N\alpha$ -open. Since U is $N\alpha$ -closed, $N\omega cl(U) \subseteq N\alpha cl(U) \subseteq G$. Thus U is $N\alpha\omega$ -closed.

Theorem 3.4. Every neutrosophic semi-closed set in a neutrosophic set is an $N\alpha\omega$ -closed.

Proof. Let U be a N semi-closed set in (X, τ) , then $U = Nscl(U)$. Let $U \subseteq G$, G is $N\alpha$ -open in (X, τ) . Since U is N semi-closed, $N\omega cl(U) \subseteq Nscl(U) \subseteq G$. This shows that U is $N\alpha\omega$ -closed set.

The converses of the above theorems are not true as explained in Example 3.5.

Example 3.5. Let $X = \{u, v, w\}$ and neutrosophic sets A, B, C be defined by:

$$\begin{aligned} A &= \langle (0.1, 0.4, 0.7), (0.9, 0.6, 0.3), (0.9, 0.6, 0.3) \rangle \\ B &= \langle (0.6, 0.6, 0.4), (0.2, 0.7, 0.8), (1, 0.6, 0.5) \rangle \\ C &= \langle (0.1, 0.4, 0.8), (0.2, 0.6, 0.4), (0.6, 0.5, 0.9) \rangle \end{aligned}$$

Let $\tau = \{0_\sim, A, 1_\sim\}$. Then B is $N\alpha\omega$ -closed in (X, τ) but not $N\alpha$ -closed and thus it is not N -closed and C is $N\alpha\omega$ -closed in (X, τ) but not N semi-closed.

Theorem 3.6. Let (X, τ) be a NTS and let $U \in NS(X)$. If U is $N\alpha\omega$ -closed set and $U \subseteq V \subseteq N\omega cl(U)$, then V is $N\alpha\omega$ -closed set.

Proof. Let G be a $N\alpha$ -open set such that $V \subseteq G$. Since $U \subseteq V$, then $U \subseteq G$. But U is $N\alpha\omega$ -closed, so $N\omega cl(U) \subseteq G$. Since $V \subseteq N\omega cl(U)$. Since $N\omega cl(V) \subseteq N\omega cl(U)$ and hence $N\omega cl(V) \subseteq G$. Therefore V is a $N\alpha\omega$ -closed set.

Theorem 3.7. Let U be a $N\alpha\omega$ -open set in X and $N\omega int(U) \subseteq V \subseteq U$, then V is $N\alpha\omega$ -open.

Proof. Suppose U is $N\alpha\omega$ -open in X and $N\omega int(U) \subseteq V \subseteq U$. Then $\overline{U}$ is $N\alpha\omega$ -closed and $\overline{U} \subseteq \overline{V} \subseteq N\omega cl(\overline{U})$. Then $\overline{U}$ is a $N\alpha\omega$ -closed set by theorem 3.5. Hence V is a $N\alpha\omega$ -open set in X .

Theorem 3.8. A NS U in a NTS (X, τ) is a $N\alpha\omega$ -open set if and only if $V \subseteq N\omega int(U)$ whenever V is a $N\alpha$ -closed set and $V \subseteq U$.

Proof. Let U be a $N\alpha\omega$ -open set and let V be a $N\alpha$ -closed set such that $V \subseteq U$. Then $\overline{U} \subseteq \overline{V}$ and hence $N\omega cl(\overline{U}) \subseteq \overline{V}$, since $\overline{U}$ is $N\alpha\omega$ -closed. But $N\omega cl(\overline{U}) = \overline{N\omega int(U)}$, thus $V \subseteq N\omega int(U)$.

Conversely, suppose that the condition is satisfied, then $\overline{N\omega int(U)} \subseteq \overline{V}$ whenever $\overline{V}$ is $N\alpha$ -open set and $\overline{U} \subseteq \overline{V}$. This implies that $N\omega cl(\overline{U}) \subseteq \overline{V} = G$ where G is $N\alpha$ -open set and $\overline{U} \subseteq G$. Therefore $\overline{U}$ is $N\alpha\omega$ -closed set and hence U is $N\alpha\omega$ -open.

Theorem 3.9. Let U be a $N\alpha\omega$ -closed subset of (X, τ) . Then $N\omega cl(U) - U$ does not contain any non-empty $N\alpha\omega$ -closed set.

Proof. Assume that U is a $N\alpha\omega$ -closed set. Let F be a non-empty $N\alpha\omega$ -closed set, such that $F \subseteq$

$N\omega cl(U) - U = N\omega cl(U) \cap \overline{U}$. i.e., $F \subseteq N\omega cl(U)$ and $F \subseteq \overline{U}$. Therefore, $U \subseteq \overline{F}$. Since $\overline{F}$ is a $N\alpha\omega$ -open set, $N\omega cl(U) \subseteq \overline{F} \Rightarrow F \subseteq (N\omega cl(U) - U) \cap (N\omega cl(U)) \subseteq N\omega cl(U) \cap \overline{N\omega cl(U)}$. i.e., $F \subseteq \phi$. Therefore F is empty.

Corollary 3.10. Let U be a $N\alpha\omega$ -closed set of (X, τ) . Then $N\omega cl(U) - U$ does not contain no non-empty N -closed set.

Proof. The proof follows from the Theorem 3.9.

Theorem 3.11. If U is both $N\omega$ -open and $N\alpha\omega$ -closed set, then U is a $N\omega$ -closed set.

Proof. Since U is both $N\omega$ -open and $N\alpha\omega$ -closed set in X , then $N\omega cl(U) \subseteq U$. Also we have $U \subseteq N\omega cl(U)$. This gives that $N\omega cl(U) = U$. Therefore U is a $N\omega$ -closed set in X .

4 On neutrosophic $\alpha\omega$ -continuity, connectedness and contra-continuity

Definition 4.1. Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces.

1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be a neutrosophic $\alpha\omega$ -continuous (briefly, $N\alpha\omega$ -continuous) function if the inverse image of every open set in Y is a $N\alpha\omega$ -open set in X .
Equivalently, if the inverse image of every open set in (Y, σ) is $N\alpha\omega$ -open in (X, τ) ;
2. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be a neutrosophic $\alpha\omega$ -irresolute (briefly, $N\alpha\omega$ -irresolute) function if the inverse image of every $N\alpha\omega$ -open set in Y is a $N\alpha\omega$ -open set in X .
Equivalently, if the inverse image of every $N\alpha\omega$ -open set in (Y, σ) is $N\alpha\omega$ -open in (X, τ) ;

Definition 4.2. A NTS (X, τ) is said to be neutrosophic- $\alpha\omega T_{1/2}$ ($N\alpha\omega T_{1/2}$ in short) space if every $N\alpha\omega C$ in X is an NC in X .

Definition 4.3. Let (X, τ) be any neutrosophic topological space. (X, τ) is said to be neutrosophic $\alpha\omega$ -disconnected (in shortly $N\alpha\omega$ -disconnected) if there exists a $N\alpha\omega$ -open and $N\alpha\omega$ -closed set $\overline{F}$ such that $\overline{F} \neq 0_{\sim}$ and $\overline{F} \neq 1_{\sim}$. (X, τ) is said to be neutrosophic $\alpha\omega$ -connected if it is not neutrosophic $\alpha\omega$ -disconnected.

Theorem 4.4. Every $N\alpha\omega$ -connected space is neutrosophic connected.

Proof. For a $N\alpha\omega$ -connected (X, τ) space and let (X, τ) not be neutrosophic connected. Hence, there exists a proper neutrosophic set, $\overline{F} = \langle \mu_{\overline{F}(x)}, \sigma_{\overline{F}(x)}, \nu_{\overline{F}(x)} \rangle$, $\overline{F} \neq 0_{\sim}$ and $\overline{F} \neq 1_{\sim}$, such that $\overline{F}$ is both neutrosophic open and neutrosophic closed in (X, τ) . Since every neutrosophic open set is $N\alpha\omega$ -open and neutrosophic closed set is $N\alpha\omega$ -closed, X is not $N\alpha\omega$ -connected. Therefore, (X, τ) is neutrosophic connected. However, the converse is not true.

Example 4.5. Let $X = \{u, v, w\}$ and neutrosophic sets A, B and C be defined by:

$$\begin{aligned} A &= \langle (0.4, 0.5, 0.5), (0.4, 0.5, 0.5), (0.5, 0.5, 0.5) \rangle \\ B &= \langle (0.7, 0.6, 0.5), (0.7, 0.6, 0.5), (0.3, 0.4, 0.5) \rangle \\ C &= \langle (0.5, 0.6, 0.5), (0.5, 0.6, 0.5), (0.5, 0.6, 0.5) \rangle \end{aligned}$$

Let $\tau = \{0_{\sim}, A, B, 1_{\sim}\}$. It is obvious that (X, τ) is NTS. Now, (X, τ) is neutrosophic connected. However, it is not a $N\alpha\omega$ -connected.

Theorem 4.6. Let (X, τ) be a neutrosophic $\alpha\omega T_{1/2}$ space. (X, τ) is neutrosophic connected iff (X, τ) is $N\alpha\omega$ -connected.

Proof. Let (X, τ) is neutrosophic connected. Suppose that (X, τ) is not $N\alpha\omega$ -connected, and there exists a neutrosophic set $\overline{F}$ which is both $N\alpha\omega$ -open and $N\alpha\omega$ -closed. Since (X, τ) is neutrosophic $\alpha\omega T_{1/2}$, $\overline{F}$ is both neutrosophic open and neutrosophic closed. Therefore, (X, τ) is not a neutrosophic connected which is contradiction to our hypothesis. Hence, (X, τ) is $N\alpha\omega$ -connected.

Conversely, let (X, τ) is $N\alpha\omega$ -connected. Suppose that (X, τ) is not neutrosophic connected, and there exists a neutrosophic set $\overline{F}$ such that $\overline{F}$ is both NCs and NOs $\in (X, \tau)$. Since the neutrosophic open set is $N\alpha\omega$ -open and the neutrosophic closed set is $N\alpha\omega$ -closed, (X, τ) is not $N\alpha\omega$ -connected. Hence, (X, τ) is neutrosophic connected.

Theorem 4.7. Suppose (X, τ) and (Y, σ) are any two NTSs. If $g : (X, \tau) \rightarrow (Y, \sigma)$ is $N\alpha\omega$ -continuous surjection and (X, τ) is $N\alpha\omega$ -connected, then (Y, σ) is neutrosophic connected.

Proof. Suppose that (Y, σ) is not neutrosophic connected, such that the neutrosophic set $\bar{F}$ is both neutrosophic open and neutrosophic closed in (Y, σ) . Since g is $N\alpha\omega$ -continuous, $g^{-1}(\bar{F})$ is $N\alpha\omega$ -open and $N\alpha\omega$ -closed in (X, τ) . Thus, (X, τ) is not $N\alpha\omega$ -connected. Hence, (Y, σ) is neutrosophic connected.

Theorem 4.8. Let $g : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then the following conditions are equivalent.

- (i) g is $N\alpha\omega$ -continuous;
- (ii) The inverse $f^{-1}(U)$ of each N -open set U in Y is $N\alpha\omega$ -open set in X .

Proof. It is clear, since $g^{-1}(\bar{U}) = \overline{g^{-1}(U)}$ for each N -open set U of Y .

Theorem 4.9. If $g : (X, \tau) \rightarrow (Y, \sigma)$ be a $N\alpha\omega$ -continuous mapping, then the following statements holds:

- (i) $g(N\alpha\omega Ncl(U)) \subseteq Ncl(g(U))$, for all neutrosophic set U in X ;
- (ii) $N\alpha\omega Ncl(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))$, for all neutrosophic set V in Y .

Proof.

- (i) Since $Ncl(g(U))$ is neutrosophic closed set in Y and g is $N\alpha\omega$ -continuous, then $g^{-1}(Ncl(g(U)))$ is $N\alpha\omega$ -closed in X . Now, since $U \subseteq g^{-1}(Ncl(g(U)))$. So, $N\alpha\omega cl(U) \subseteq g^{-1}(Ncl(g(U)))$. Therefore, $g(N\alpha\omega Ncl(U)) \subseteq Ncl(g(U))$.
- (ii) By replacing U with V in (i), we obtain $g(N\alpha\omega cl(g^{-1}(V))) \subseteq Ncl(g(g^{-1}(V))) \subseteq Ncl(V)$. Hence $N\alpha\omega cl(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))$.

Theorem 4.10. Let g be a function from a NTS (X, τ) to a NTS (Y, σ) . Then the following statements are equivalent.

- (i) g is a neutrosophic $\alpha\omega$ -continuous function.
- (ii) For every NP $p_{(r,s,t)} \in X$ and each NN U of $g(p_{(r,s,t)})$, there exists a $N\alpha\omega$ -open set V such that $p_{(r,s,t)} \in V \subseteq g^{-1}(U)$.
- (iii) For every NP $p_{(r,s,t)} \in X$ and each NN U of $g(p_{(r,s,t)})$, there exists a $N\alpha\omega$ -open set V such that $p_{(r,s,t)} \in V$ and $g(V) \subseteq U$.

Proof. (i) $\Rightarrow$ (ii). If $p_{(r,s,t)}$ is a NP in X and also if U be a NN of $g(p_{(r,s,t)})$, then there exists a NOS W in Y such that $g(p_{(r,s,t)}) \in W \subset U$. we have g is neutrosophic $\alpha\omega$ -continuous, $V = g^{-1}(W)$ is an $N\alpha\omega OS$ and

$$p_{(r,s,t)} \in g^{-1}(g(p_{(r,s,t)})) \subseteq g^{-1}(W) = V \subseteq g^{-1}(U).$$

Thus (ii) is a valid statement.

(ii) $\Rightarrow$ (iii). Let $p_{(r,s,t)}$ be a NP in X and take U be a NN of $g(p_{(r,s,t)})$. Then there exists a $N\alpha\omega OS$ V such that $p_{(r,s,t)} \in V \subseteq g^{-1}(U)$ by (ii). Thus, we have $p_{(r,s,t)} \in V$ and $g(V) \subseteq g(g^{-1}(U)) \subseteq U$. Hence (iii) is valid.

(iii) $\Rightarrow$ (i). Let V be a NOS in Y and let $p_{(r,s,t)} \in g^{-1}(V)$. Then $g(p_{(r,s,t)}) \in g(g^{-1}(V)) \subset V$. Since V is a NOS, it follows that V is a NN of $g(p_{(r,s,t)})$ so from (iii), there exists a $N\alpha\omega OS$ U such that $p_{(r,s,t)} \in U$ and $g(U) \subseteq V$. This implies that

$$p_{(r,s,t)} \in U \subseteq g^{-1}(g(U)) \subseteq g^{-1}(V).$$

Then, we know that $g^{-1}(V)$ is a $N\alpha\omega OS$ in X . Thus g is neutrosophic $\alpha\omega$ -continuous.

Definition 4.11. A function is said to be a neutrosophic contra $\alpha\omega$ -continuous function if the inverse image of each NOS V in Y is a $N\alpha\omega CS$ in X .

Theorem 4.12. Let $g : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then, the following assertions are equivalent:

- (i) g is a neutrosophic contra $\alpha\omega$ -continuous function;

(ii) $g^{-1}(V)$ is a $N\alpha\omega$ CS in X , for each NOS V in Y .

Proof. (i) $\Rightarrow$ (ii) Let g be any neutrosophic contra $\alpha\omega$ -continuous function and let V be any NOS in Y . Then, $\overline{V}$ is a NCS in Y . By the assumption $g^{-1}(\overline{V})$ is a $N\alpha\omega OS$ in X . Hence, we get that $g^{-1}(V)$ is a $N\alpha\omega CS$ in X .

The converse of the theorem can be done in the same sense.

Theorem 4.13. Let $g : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping from an NTS X into an NTS Y . The mapping g is neutrosophic contra $\alpha\omega$ -continuous if $Ncl(g(U)) \subseteq g(N\alpha\omega int(U))$, for each NS U in X .

Proof. Let V be any NCS in X . Then, $Ncl(V) = V$, and also g is onto, by assumption, it shows that $g(N\alpha\omega int(g^{-1}(V))) \supseteq Ncl(g(g^{-1}(V))) = Ncl(V) = V$. Hence $g^{-1}(g(N\alpha\omega int(g^{-1}(V)))) \supseteq g^{-1}(V)$. Since g is an into mapping, we have $N\alpha\omega int(g^{-1}(V)) = g^{-1}(g(N\alpha\omega int(g^{-1}(V)))) \supseteq g^{-1}(V)$. Therefore $N\alpha\omega int(g^{-1}(V)) = g^{-1}(V)$, so $g^{-1}(V)$ is a $N\alpha\omega OS$ in X . Hence g is a neutrosophic contra $\alpha\omega$ -continuous mapping.

Theorem 4.14. Let $g : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. Then the following statements are equivalent:

- (i) g is a neutrosophic contra $\alpha\omega$ -continuous mapping;
- (ii) for each NP $p_{(r,s,t)}$ in X and NCS V containing $g(p_{(r,s,t)})$ there exists $N\alpha\omega OS$ U in X containing $p_{(r,s,t)}$ such that $A \subseteq f^{-1}(B)$;
- (iii) for each NP $p_{(r,s,t)}$ in X and NCS V containing $p_{(r,s,t)}$ there exists $N\alpha\omega OS$ U in X containing $p_{(r,s,t)}$ such that $g(U) \subseteq V$.

Proof. (i) $\Rightarrow$ (ii) Let g be a neutrosophic contra $\alpha\omega$ -continuous mapping, let V be any NCS in Y and let $p_{(r,s,t)}$ be a NP in X and such that $g(p_{(r,s,t)}) \in V$. Then $p_{(r,s,t)} \in g^{-1}(V) = N\alpha\omega int(g^{-1}(V))$. Let $U = N\alpha\omega int(g^{-1}(V))$. Then U is an $N\alpha\omega OS$ and $U = N\alpha\omega int(g^{-1}(V)) \subseteq g^{-1}(V)$.

(ii) $\Rightarrow$ (iii) The results follows from the evident relations $g(U) \subseteq g(g^{-1}(V)) \subseteq V$.

(iii) $\Rightarrow$ (i) Let V be any NCS in Y and let $p_{(r,s,t)}$ be a NP in X such that $p_{(r,s,t)} \in g^{-1}(V)$. Then $g(p_{(r,s,t)}) \in V$. According to the assumption, there exists an $N\alpha\omega OS$ U in X such that $p_{(r,s,t)} \in U$ and $g(U) \subseteq V$. Hence $p_{(r,s,t)} \in U \subseteq g^{-1}(g(U)) \subseteq g^{-1}(V)$. Therefore $p_{(r,s,t)} \in U = \alpha\omega int(U) \subseteq N\alpha\omega int(g^{-1}(V))$. Since, $p_{(r,s,t)}$ is an arbitrary NP and $g^{-1}(V)$ is the union of all NPs in $g^{-1}(V)$, we obtain that $g^{-1}(V) \subseteq N\alpha\omega int(g^{-1}(V))$. Thus g is a neutrosophic contra $N\alpha\omega$ -continuous mapping.

Corollary 4.15. Let X, X_1 and X_2 be NTSs, $p_1 : X \rightarrow X_1 \times X_2$ ($i = 1, 2$) and $p_2 : X \rightarrow X_1 \times X_2$ are the projections of $X_1 \times X_2$ onto X_i , ($i = 1, 2$). If $g : X \rightarrow X_1 \times X_2$ is a neutrosophic contra $\alpha\omega$ -continuous, then $p_i g$ are also neutrosophic contra $\alpha\omega$ -continuous mapping.

Proof. The proof follows from the fact that the projections are all neutrosophic continuous functions.

Theorem 4.16. Let $g : (X_1, \tau) \rightarrow (Y_1, \sigma)$ be a function. If the graph $h : X_1 \rightarrow X_1 \times Y_1$ of g is neutrosophic contra $\alpha\omega$ -continuous, then g is neutrosophic contra $\alpha\omega$ -continuous.

Proof. For every NOS V in Y_1 holds $g^{-1}(V) = 1 \wedge g^{-1}(V) = h^{-1}(1 \times V)$. Since h is a neutrosophic contra $\alpha\omega$ -continuous mapping and $1 \times V$ is a NOS in $X_1 \times Y_1$, $g^{-1}(V)$ is a $N\alpha\omega CS$ in X_1 , so g is a neutrosophic contra $\alpha\omega$ -continuous mapping.

5 Conclusions

In this paper, we introduced and investigated the neutrosophic $\alpha\omega$ closed sets and its properties. Also, we investigated the continuity, irresolute, connectedness and contra-continuity in terms of neutrosophic $\alpha\omega$ closed sets.

Acknowledgments: Authors would like to thank referees for their valuable suggestions and helpfull comments.

Author Contributions: All authors have contributed equally to this paper. The individual responsibilities and contribution of all authors can be described as follows: the idea of this paper was put forward by M. Parimala and M. Karthika completed the preparatory work of the paper. Florentin Smarandache and Said Broumi analyzed the existing work. The revision and submission of this paper was completed by M. Parimala and M.

Karthika.

Conflicts of Interest: The authors declare no conflict of interest.

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