Quadripartitioned Neutrosophic Pythagorean Soft Set

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Abstract

The aim of this paper is to introduce the new concept of Quadripartitioned Neutrosophic Pythagorean soft set with T, C, U, F as dependent neutrosophic components and have also discussed some of its properties.

Keywords: Neutrosophic pythagorean soft set, Quadripartioned Neutrosophic Pythagorean set and Quadripartioned Neutrosophic Pythagorean soft set .

1.Introduction

The fuzzy set was introduced by Zadeh [19] in 1965. The concept of neutrosophic set was introduced by Smarandache which is a mathematical tool for handling problems involving imprecise, indeterminancy and inconsistent data.

Smarandache introduced neutrosophic sets [14]. In neutrosophic sets, the indeterminacy membership function walks along independently of the truth membership or of the falsity membership. Neutrosophic theory has been widely explored by researchers for application purpose in handling real life situations involving uncertainty. Although the hesitation margin of neutrosophic theory is independent of the truth or falsity membership, looks more general than intuitionistic fuzzy sets yet. Recently, in Atanassov et al. [3] studied the relations between inconsistent intuitionistic fuzzy sets, picture fuzzy sets, neutrosophic sets and intuitionistic fuzzy sets; however, it remains in doubt that whether the indeterminacy associated to a particular element occurs due to the belongingness of the element or the non-belongingness. This has been pointed out by Chattejee et al. [4] while introducing a more general structure of neutrosophic set viz. quadripartitioned single valued neutrosophic set (QSVNS). The idea of QSVNS is actually stretched from Smarandache's four numerical-valued neutrosophic logic and Belnap's four valued logic, where the indeterminacy is divided into two parts, namely, "unknown" i.e., neither true nor false and "contradiction" i.e., both true and false. In the context of neutrosophic study however, the QSVNS looks quite logical. Also, in their study, Chatterjee [4] et al. analyzed a real-life example for a better understanding of a QSVNS environment and showed that such situations occur very naturally.

In 2018 Smarandache [17] generalized the soft set to the hyper soft set by transforming the classical uni-argument function F into a multi-argument function.

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In 2016, Smarandache [14] introduced for the first time the degree of dependence between the components of fuzzy set and neutrosophic sets. The main idea of Neutrosophic sets is to characterize each value statement in a 3D – Neutrosophic space, where each dimension of the space represents respectively the truth membership, falsity membership and the indeterminacy, when two components T and F are dependent and I is independent then T+I+F≤ 2.

Radha and Stanis Arul Mary [10] introduced the concept of Quadripartitioned neutrosophic pythagorean set with dependent neutrosophic components.

If T and F are dependent neutrosophic pythagorean components then $T^2 + F^2 \leq 1$. Similarly, for U and C as dependent neutrosophic pythagorean components then $C^2 + U^2 \leq 1$. When combining both we get Quadripartitioned pythagorean set with dependent components as $T^2 + F^2 + C^2 + U^2 \leq 2$

Pabitra kumar Maji [9] had combined the neutrosophic set with soft sets and introduced a new mathematical model neutrosophic soft set. Arockiarani [1] introduced the new concept of fuzzy neutrosophic soft set. Yager introduced pythagorean fuzzy sets. Radha and tanis Arul Mary [11] introduced neutrosophic pythagorean soft set with T and F as neutrosophic dependent components.

In this we have to introduce the concept of introduced the concept of quadripartitioned neutrosophic pythagorean set with dependent components and establish some of its properties.

2.Preliminaries

Definition:2.1[14]

Let X be a universe. A neutrosophic set A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where $T_A , I_A, F_A: U \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A (x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminancy and $F_A(x)$ is the degree of non-membership.

Here, $T_A(x)$ and $F_A(x)$are dependent neutrosophic components and $I_A(x)$ is an independent component.

Definition:2.2[2]

Let U be the initial universe set and E be set of parameters. Consider a non-empty set A on E, Let P(U) denote the set of all neutrosophic sets of U. The collection (F, A) is termed to be neutrosophic soft set over U, where F is a mapping given by F: A → P(U).

Definition:2.3[11]

Let X be the initial universe set and E be set of parameters. Consider a non-empty set A on E, Let P(X) denote the set of all neutrosophic pythagorean sets of X. The collection (F, A) is termed to be neutrosophic pythagorean soft set over X, where F is a mapping given by F: A → P(X).

Definition:2.4[4]

Let X be a universe. A Quadripartitioned neutrosophic set A with independent neutrosophic components on X is an object of the form

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$$A = \{< x, T_A(x), C_A(x), U_A(x), F_A(x) >: x \in X\}$$

$$and\ 0 \leq T_A(x) + C_A(x) + U_A(x) + F_A\ (x) \leq 4$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

Definition:2.5[10]

Let X be a universe. A Quadripartitioned neutrosophic pythagorean set A with dependent neutrosophic components A on X is an object of the form

$$A = \{< x, T_A(x), C_A(x), U_A(x), F_A(x) >: x \in X\}$$

Where $T_A + F_A \leq 1,\ C_A + U_A \leq 1\ and\ 0 \leq (T_A(x))^2 + (C_A(x))^2 + (U_A(x))^2 + (F_A\ (x))^2 \leq 2$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

3.Quadripartitioned Neutrosophic Pythagorean Soft Set (QNPSS or QNPS Set)

Definition:3.1

Let X be the initial universe set and E be set of parameters. Consider a non-empty set A on E, Let P(X) denote the set of all Quadripartitioned neutrosophic pythagorean sets of X. The collection (F, A) is termed to be Quadripartitioned neutrosophic pythagorean soft set over X, where F is a mapping given by F: A → P(X).

Definition:3.2

A Quadripartitioned neutrosophic pythagorean soft set A is contained in another Quadripartitioned neutrosophic pythagorean soft set B (i.e) A$\subseteq B$ if $T_A(x) \leq T_B(x), C_A(x) \leq C_B(x), U_A(x) \geq U_B(x)$ and $F_A(x) \geq F_B(x)$

Definition:3.3

The complement of a Quadripartitioned neutrosophic pythagorean soft set (F, A) on X denoted by $(F, A)^c$ and is defined as

F^c(x)= $\{< x, F_A(x), U_A\ (x), C_A(x), T_A(x) >: x \in X\}$

Definition:3.4

Let X be a non-empty set, A = $< x, T_A(x), C_A(x), U_A(x), F_A(x) >$ and

B = $< x, T_B(x), C_B(x), U_B(x), F_B(x) >$ are Quadripartitioned neutrosophic pythagorean soft sets. Then

A ∪ B = $< x, max(T_A(x), T_B(x)), max(C_A(x), C_B(x)), min(U_A(x), U_{\ B}(x)), min(F_A(x), F_{\ B}(x)) >$

A ∩ B = $< x, min(T_A(x), T_B(x)), min(C_A(x), C_B(x)), max(U_A(x), U_B(x)), max(F_A(x), F_{\ B}(x)) >$

Definition:3.5

A Quadripartitioned neutrosophic pythagorean soft set (F, A) over the universe X is said to be empty neutrosophic pythagorean soft set with respect to the parameter A if

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$T_{F(e)} = 0, C_{F(e)} = 0, U_{F(e)} = 1, F_{F(e)}$=1,$\forall x \in X, \forall e \in A$. It is denoted by 0 X

Definition:3.6

A Quadripartitioned neutrosophic pythagorean soft set (F, A) over the universe X is said to be universe neutrosophic soft set with respect to the parameter A if

$T_{F(e)} = 1, C_{F(e)} = 1, U_{F(e)} = 0, F_{F(e)}$=0,$\forall x \in X, \forall e \in A$. It is denoted by 1_X

Remark: $0^c_X = 1_X$ and $1^c_X = 0_X$

Definition:3.7

Let A and B be two Quadripartitioned neutrosophic pythagorean soft sets on X then A\B may be defined as

A\B = $< x, min(T_A(x), F_B(x)), min(C_A(x), U_B(x)), max(U_A(x), C_B(x)), max(F_A(x), T_B(x)) >$

Definition:3.8

F_E is said to be absolute Quadripartitioned neutrosophic pythagorean soft set over X if F(e) = 1_X for any

$e \in E$. We denote it by X_E

Definition:3.9

F_E is said to be relative null Quadripartitioned neutrosophic pythagorean soft set over X if F(e) = 0_X for any

$e \in E$. We denote it by $\emptyset_E$

Obviously $\emptyset_E = X_E^c$ and $X_E = \emptyset_E^c$

Definition:3.10

The complement of a Quadripartitioned neutrosophic pythagorean soft set (F, A) over X can also be defined as $(F, A)^c = U_E \backslash F(e)$ for all $e \in A$.

Note: We denote X_E by X in the proofs of proposition.

Definition:3.11

If (F, A) and (G, B) be two Quadripartitioned neutrosophic pythagorean soft set then "(F, A) AND (G, B)" is a denoted by

(F, A) ∧(G, B) and is defined by (F, A)∧ (G, B) = (H, A ×B)

where H (a, b) = F(a) ∩ G(b) $\forall a \in A$ and $\forall b \in B$, where ∩ is the operation intersection of Quadripartitioned neutrosophic pythagorean soft set.

Definition:3.12

If (F, A) and j(G, B) be two Quadripartitioned neutrosophic pythagorean soft set then "(F, A) OR (G, B)" is a denoted by (F, A) ∨(G, B) and is defined by (F, A)∨ (G, B) = (K, A ×B)

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where K (a, b) = F(a) ∪ G(b) $\forall a \in A$ and $\forall b \in B,$ where ∪ is the operation union of Quadripartitioned neutrosophic pythagorean soft set.

Theorem :3.13

Let (F, A) and (G, A) be Quadripartitioned neutrosophic pythagorean soft set over the universe X. Then the following are true.

(i) (F, A)⊆ (G, A) iff (F, A) ∩ (G, A) = (F, A)
(ii) (F, A)⊆ (G, A) iff (F, A) ∪ (G, A) = (F, A)

Proof:

(i)Suppose that (F, A)⊆ (G, A), then F(e) ⊆G(e) for all e ∈ A. Let (F, A) ∩ (G, A) = (H, A).

Since H(e) = F(e) ∩ G(e) =F(e) for all e ∈ A, by definition (H, A) = (F, A).

Suppose that (F, A) ∩ (G, A) = (F, A). Let (F, A) ∩ (G, A) = (H, A).

Since H(e) = F(e) ∩ G(e) =F(e) for all e ∈ A, we know that F(e) ⊆ G(e) for all e ∈ A.

Hence (F, A)⊆ (G, A).

(ii)The proof is similar to (i).

Theorem :3.14

Let (F, A), (G, A), (H, A), and (S, A) are Quadripartitioned neutrosophic pythagorean soft set over the universe X. Then the following are true.

(i) If (F, A) ∩ (G, A) =$\emptyset_A$, then (F, A)⊆ (G, A)^c
(ii) If (F, A)⊆ (G, A) and (G, A)⊆ (H, A) then (F, A)⊆ (H, A)
(iii) If (F, A)⊆ (G, A) and (H, A)⊆ (S, A) then (F, A) ∩ (H, A) ⊆ (G, A) ∩ (S, A)
(iv) (F, A) ⊆ (G, A) iff (G, A)^c ⊆ (F, A)^c

Proof:

(i)Suppose that (F, A) ∩ (G, A) =$\emptyset_A$.Then F(e)∩ G(e) = $\emptyset$.

So, F(e) ⊆ U\G(e)= $G^c(e)$ for all e ∈ A.

Therefore, we have (F, A)⊆ (G, A)^c

Proof of (ii) and (iii) are obvious.

(iv) (F, A)⊆ (G, A)⟺ F(e) ⊆ G(e) for all e ∈ A.

⟺ (G(e))^c ⊆ (F(e))^c for all e ∈ A.
⟺ $G^c(e) \subseteq F^c(e)\ for\ all\ e \in A.$
⟺(G, A)^c ⊆ (F, A)^c

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Definition:3.15

Let I be an arbitrary index $\{(F_i, A)\}_{i\in I}$ be a subfamily of Quadripartitioned neutrosophic pythagorean soft set over the universe X.

(i)The union of these Quadripartitioned neutrosophic pythagorean soft set is the Quadripartitioned neutrosophic pythagorean soft set (H, A) where H(e) = $\bigcup_{i\in I} F_i\,(e)$ for each e $\in A$.

$We\ write\ \bigcup_{i\in I}(F_i\,, A)$= (H, A)

(ii)The intersection of these Quadripartitioned neutrosophic pythagorean soft set is the Quadripartitioned neutrosophic pythagorean soft set (M, A) where M(e) = $\bigcap_{i\in I}\ F_i(e)$ for each e $\in A$.

$We\ write\ \bigcap_{i\in I}(F_i, A)$ = (M, A)

Theorem:3.16

Let I be an arbitrary index set and $\{(F_i, A)\}_{i\in I}$be a subfamily of Quadripartitioned neutrosophic pythagorean soft set over the universe X. Then

(i) $(\bigcup_{i\in I}(F_i\,, A))^C = \bigcap_{i\in I}(F_i, A)^C$
(ii) $(\bigcap_{i\in I}(F_i, A))^C = \bigcup_{i\in I}(F_i\,, A)^C$

Proof:

(i) $(\bigcup_{i\in I}(F_i\,, A))^C$ = (H, A) ^{C}, By definition $H^C(e) = X_E \backslash H(e) = X_E \backslash \bigcup_{i\in I} F_i\,(e) = \bigcap_{i\in I}(X_E \backslash F_i(e))$ for all e $\in A$.
On the other hand, $(\bigcap_{i\in I}(F_i, A))^C$(K, A).
By definition, K(e)= $\bigcap_{i\in I}\ F_i^C(e) = \bigcap_{i\in I}(X - F_i(e))$ for all e $\in A$.
(ii) It is obvious.

Note: We denote $\emptyset_E\ by\ \emptyset$ and $X_E\ by\ X.$

Theorem:3.17

Let (F, A) be Quadripartitioned neutrosophic pythagorean soft set over the universe X. Then the following are true.

(i) $(\emptyset, A)^c$ = (X, A)
(ii) $(X, A)^c = (\emptyset, A)$

Proof:

(i) Let $(\emptyset,\ A)$ = (F, A)

Then $\forall$e $\in$ A,

F(e) = $\{< x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x) >: x \in X\}$

= {(x,0,0,1,1): $x \in X\}$

$(\emptyset, A)^c = (F, A)^c$

Then $\forall$e $\in$ A,

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$(F(e))^c = \{< x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x) >: x \in X\}^c$

$= \{< x, F_{F(e)}(x), U_{F(e)}(x), C_{F(e)}(x), T_{F(e)}(x) >: x \in X\}$

$= \{(x1,1,0,0): x \in X\} = X$

Thus $(\emptyset, A)^c = (X, A)$

(ii) Proof is similar to (i)

Theorem:3.18

Let (F, A) be Quadripartitioned neutrosophic pythagorean soft set over the universe X. Then the following are true.

(i) $(F, A) \cup (\emptyset, A) = (F, A)$

(ii) $(F, A) \cup (X, A) = (X, A)$

Proof:

(i) $(F, A) = \{e, (x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x)): x \in X\}\ \forall e \in A$

$(\emptyset, A) = \{e, (x,0,0,1,1): x \in X\}\ \forall e \in A$

$(F, A) \cup (\emptyset, A) = \{e, (x, max(T_{F(e)}(x),0), max(C_{F(e)}(x),0), min(U_{F(e)}(x),1) min(F_{F(e)}(x),1)): x \in X\}\ \forall e \in A$

$= \{e, (x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x)): x \in X\}\ \forall e \in A$

$= (F, A)$

(ii) Proof is similar to (i).

Theorem:3.19

Let (F, A) be Quadripartitioned neutrosophic pythagorean soft set over the universe X. Then the following are true.

(i) $(F, A) \cap (\emptyset, A) = (\emptyset, A)$

(ii) $(F, A) \cap (X, A) = (F, A)$

Proof:

(i) $(F, A) = \{e, (x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x)): x \in X\}\ \forall e \in A$

$(\emptyset, A) = \{e, (x,0,0,1,1): x \in X\}\ \forall e \in A$

$(F,A) \cap (\emptyset,A) = \{e, (x, min(T_{F(e)}(x),0), min(C_{F(e)}(x),0), max(U_{F(e)}(x),1) max(F_{F(e)}(x),1)): x \in X\}\ \forall e \in A$

$= \{e, , (x, 0,0,1,1): x \in X\}\ \forall e \in A$

$= (\emptyset, A)$

(ii) Proof is similar to (i).

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Note: We denote T_F(x), $C_F(x), U_F(x)$ and $F_F(x)$ by T_F $C_F, U_F and$ F_F

Theorem:3.20

Let (F, A) and (G, A) be Quadripartitioned neutrosophic pythagorean soft set over the universe X. Then the following are true.

(i) (F, A)∪ (∅, B) = (F, A) iff B⊆A

(ii) (F, A)∪ $(X$, B) = (X, A) iff A⊆ B

Proof:

(i) We have for (F, A), F(e) = $\{(x, T_F, C_F, U_F, F_F): x \in U\}$ ∀e ∈ A

Also let (∅, B) = (G, B) then

G(e) = {(x, 0,0,1,1): x ∈ X} ∀e ∈ B

Let (F, A)∪ (∅, B) = (F, A)∪ $(G$, B) = (H, C) where C = A ∪B and for all e ∈ C

H(e) may be defined as

$$= \begin{cases} \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\}\, if \text{ e } \in \text{ A } - \text{ B} \\ \{(\text{x}, 0,0,1,1): x \in \text{X}\}\, if \text{ e } \in \text{ B } - \text{ A} \\ \{(x, max(T_{F(e)}, 0), max(\text{C}_{F(e)}, 0), min(U_{F(e)}, 1) min(F_{F(e)}, 1)): x \in X\}\, if\ e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\}\, if \text{ e } \in \text{ A } - \text{ B} \\ \{(\text{x}, 0,0,1,1): x \in \text{X}\}\, if \text{ e } \in \text{ B } - \text{ A} \\ \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\}\, if \text{ e } \in \text{ A } \cap \text{ B} \end{cases}$$

Let B⊆A

$$\text{Then H (e)} = \begin{cases} \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}(x)): x \in X\}\, if \text{ e } \in \text{ A } - \text{ B} \\ \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\}\, if \text{ e } \in \text{ A } \cap \text{ B} \end{cases}$$

= F(e) $\forall e \in A$

Conversely Let $(F, A) \cup (\emptyset, B) = (F, A)$

Then A = A ∪B⟹ $B \subseteq A$

(ii) Proof is similar to (i)

Theorem:3.21

Let (F, A) and (G, B) be Quadripartitioned neutrosophic pythagorean soft set over the universe X. Then the following are true.

(i) (F, A)∩ (∅, B) = (∅, A∩B)

(ii) (F, A)∩ $(X$, B) = (F, A∩B)

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Proof:

(i) We have for (F, A)

F(e) = $\{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\}$ ∀e ∈ A

Also let (∅, B) = (G, B) then

G(e) = {(x, 0,0,1,1): x ∈ U} ∀e ∈ B

Let (F, A) ∩ (∅,B) = (F, A) ∩(G, B) = (H, C) where C = A ∩B and ∀e ∈ C

H(e) = $\{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}), max(U_{F(e)}, U_{G(e)}) max(F_{F(e)}, F_{G(e)})): x \in X\}$

= $\{ (x, min(T_{F(e)}, 0), min(C_{F(e)}, 0), max(U_{F(e)}, 1) max(F_{F(e)}, 1)): x \in X\}$

= $\{(x, 0,0,1,1): x \in X\}$

= (G, B) = (∅, B)

Thus (F, A)∩ (∅, B) = (∅, B) =(∅, A∩B)

(ii) Proof is similar to (i).

Theorem:3.22

Let (F, A) and (G, B) be Quadripartitioned neutrosophic pythagorean soft set over the universe X. Then the following are true.

(i) $((F, A) \cup (G, B))^C \subseteq (F, A)^C \cup (G, B)^C$

(ii) $(F, A)^C \cap (G, B)^C \subseteq ((F, A) \cap (G, B))^C$

Proof:

Let (F, A)∪(G, B) = (H, C) Where C = A∪B and∀ e∈ C

H(e) may be defined as

$$\begin{cases} \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\} \; if \; e \in A - B \\ \{(x, T_{G(e)}, C_{G(e)}, U_{G(e)}, F_{G(e)}): x \in X\} \; if \; e \in B - A \\ \{(x, max(T_{F(e)}, T_{G(e)}), max(C_{F(e)}, C_{G(e)}), min(U_{F(e)}, U_{G(e)}), min(F_{F(e)}, F_{G(e)})): x \in X\} \; if \; e \in A \cap B \end{cases}$$

Thus $(F,A)\cup(G,B))^C = (H,C)^C$ Where C = A∪B and∀ e∈ C

$$(H(e))^C = \begin{cases} (F(e))^C \; if \; e \in A - B \\ (G(e))^C \; if \; e \in B - A \\ (F(e) \cup G(e))^C if \; e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, F_{F(e)}, U_{F(e)}, C_{F(e)}, T_{F(e)}): x \in X\} \; if \; e \in A - B \\ \{(x, F_{G(e)}, U_{G(e)}, C_{G(e)}, T_{G(e)}): x \in X\} \; if \; e \in B - A \\ \{(x, min(F_{F(e)}, F_{G(e)}), min(U_{F(e)}, U_{G(e)}), max(C_{F(e)}, C_{G(e)}), max(T_{F(e)}, T_{G(e)}): x \in X\} \; if e \in A \cap B \end{cases}$$

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Again (F, A) ^{C} ∪ (G, B) ^{C} = (I, J) say J = A∪B and ∀ e∈ J

$$I(e) = \begin{cases} (F(e))^C \ if\ e \in A - B \\ (G(e))^C\ if\ e \in B - A \\ (F(e) \cup G(e))^C if\ e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, F_{F(e)}, U_{F(e)}, C_{F(e)}, T_{F(e)}) : x \in X\} if e \in A - B \\ \{(x, F_{G(e)}, U_{G(e)}, C_{G(e)}, T_{G(e)}) : x \in X\} if e \in B - A \\ \{(x, min(F_{F(e)}, F_{G(e)}), min(U_{F(e)}, U_{G(e)}), max(C_{F(e)}, C_{G(e)}), max(T_{F(e)}, T_{G(e)}), max(F_{F(e)}, F_{G(e)})) \\ \qquad : x \in X\}\ if\ e \in A \cap B \end{cases}$$

So, C⊆ J ∀ e∈ J, (H(e))^C ⊆I(e)

Thus (F, A)∪(G, B)) ^{C} ⊆ (F, A) ^{C} ∪ (G, B) ^{C}

(ii) Let (F, A)∩(G, B) = (H, C) Where C = A∩B and∀ e∈ C

H(e) = F(e)∩G(e)

$$=\{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}), max(U_{F(e)}(x), U_{G(e)}(x)), max(F_{F(e)}, F_{G(e)})\}$$

Thus ((F, A)∩(G, B)) ^{C} = (H, C)^C Where C = A∩B and∀ e∈ C

$$(H(e))^C = \{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}), max(U_{F(e)}, U_{G(e)}), max(F_{F(e)}, F_{G(e)})\}^C$$

$$= \{(x, max(F_{F(e)}, F_{G(e)}), max(U_{F(e)}, U_{G(e)}), min(C_{F(e)}, C_{G(e)}), min(T_{F(e)}, T_{G(e)})\}$$

Again (F, A) ^{C}∩ (G, B) ^{C} = (I, J) say where J= A∩ B and ∀ e∈ J

I(e) = (F(e)) ^{C}∩ (G(e)) ^{C}

$$=\{(x, min(F_{F(e)}, F_{G(e)}), min(U_{F(e)}, U_{G(e)}), max(C_{F(e)}, C_{G(e)}), max(T_{F(e)}, T_{G(e)})\}$$

We see that C = J and ∀ e∈ J, I(e) ⊆(H(e)) ^{C}

Thus (F, A) ^{C} ∩ (G, B) ^{C} ⊆((F, A)∩ $(G$, B)) ^{C}

Theorem :3.23

Let (F, A) and (G, A) are two Quadripartitioned neutrosophic pythagorean soft sets over the same universe X. We have the following

(i) ((F, A)∪ $(G$, A)) ^{C} = (F, A) ^{C} ∩ (G, A) ^{C}

(ii) ((F, A)∩ $(G$, A)) ^{C} = (F, A) ^{C}∪ (G, A) ^{C}

Proof:

(i) Let (F, A) ∪ (G, A) = (H, A) ∀ e∈ A

H(e) = F(e)∪G(e)

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$=\{(x, max(T_{F(e)}, T_{G(e)}), max(C_{F(e)}, C_{G(e)}), min(U_{F(e)}, U_{G(e)}), min(F_{F(e)}, F_{G(e)})\}$

Thus (F, A) ∪ (G, A))^C = (H, A)^C ∀ e∈ A

(H(e)) ^{C} = (F(e)∪G(e)) ^{C}

$=\{(x, max(T_{F(e)}, T_{G(e)}), max(C_{F(e)}, C_{G(e)}), min(U_{F(e)}, U_{G(e)}), min(F_{F(e)}, F_{G(e)})\}^C$

$=\{(x, min(F_{F(e)}, F_{G(e)}), min(U_{F(e)}, U_{G(e)}), max(C_{F(e)}, C_{G(e)}), max(T_{F(e)}, T_{G(e)})\}$

Again (F, A) ^{C}∩ $(G$, A) ^{C} = (I, A) where$\forall\ e \in\ A$

I(e) = (F(e)) ^{C}∩ (G(e)) ^{C}

$=\{(x, \mathrm{min}(F_{F(e)}, F_{G(e)}), \mathrm{min}(U_{F(e)}, U_{G(e)}), \mathrm{max}(C_{F(e)}, C_{G(e)}), \mathrm{max}(T_{F(e)}, T_{G(e)})\}$

Thus ((F, A)∪ $(G$, A)) ^{C} = (F, A) ^{C} ∩ (G, A) ^{C}

(ii) Let (F, A) ∩ (G, A) = (H, A) ∀ e∈ A

H(e) = F(e)∩G(e)

$=\{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}), \mathrm{max}(U_{F(e)}, U_{G(e)}) max(F_{F(e)}, F_{G(e)})\}$ ∀ e∈ A

Thus (F, A) ∩ (G, A)) ^{C} = (H, A) ^{C}

(H(e)) ^{C} = (F(e)∩G(e)) ^{C}

$=\{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}), \mathrm{max}(U_{F(e)}, U_{G(e)}), max(F_{F(e)}, F_{G(e)})\}^C$

$= \{(x, min(T_{F(e)}, T_{G(e)}), max(U_{F(e)}, U_{G(e)}) min(I_{F(e)}(x), I_{G(e)}(x)), max(F_{F(e)}(x), F_{G(e)}(x))\ \}^C$
$=\{(x, max(F_{F(e)}, F_{G(e)}), max(U_{F(e)}, U_{G(e)}), min(C_{F(e)}, C_{G(e)}), min(T_{F(e)}, T_{G(e)})\} \forall\ e \in\ A$

Again (F, A) ^{C}∪ $(G$, A) ^{C} = (I, A) where$\forall\ e \in\ A$

I(e) = (F(e)) ^{C}∪ (G(e)) ^{C}

$=\{(x, max(F_{F(e)}(x), F_{G(e)}(x)), max(1 -\ I_{F(e)}(x), 1 -\ I_{G(e)}(x)), min(T_{F(e)}(x), T_{G(e)}(x))\}$

$=\{(x, max(F_{F(e)}, F_{G(e)}), 1 - min(I_{F(e)}, I_{G(e)}), min(T_{F(e)}, T_{G(e)})$

Thus ((F, A)∩ $(G$, A)) ^{C} = (F, A) ^{C} ∪ (G, A) ^{C}

Theorem:3.24

Let (F, A) and (G, A) are two neutrosophic pythagorean soft sets over the same universe X. We have the following

(i) ((F, A)∧ $(G$, A)) ^{C} = (F, A) ^{C} ∨ (G, A) ^{C}

(ii) ((F, A)∨ $(G$, A)) ^{C} = (F, A) ^{C}∧ (G, A) ^{C}

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Proof:

Let (F, A) ∧ (G, B) = (H, A ×B) where H (a, b) =F(a) ∩G(b) ∀ $a \in A$ and ∀ $b \in B$ where ∩ is the operation intersection of QNPS.

Thus H (a, b) = F(a) ∩G(b)

$$= \{(x, min(T_{F(a)}, T_{G(b)}, min(C_{F(a)}, C_{G(b)}), max(U_{F(a)}, U_{G(b)}), max(F_{F(a)}, F_{G(b)})\}$$

((F, A) ∧ (G, B)) ^{C} = (H, A ×B) ^{C} $\forall (a, b) \in A \times B$

Thus (H (a, b)) ^{C}=$\{(x, min(T_{F(a)}, T_{G(b)}), min(C_{F(a)}, C_{G(b)}), max(U_{F(a)}, U_{G(b)}), max(F_{F(a)}, F_{G(b)})\}^C$

$$=\{(x, max(F_{F(a)}, F_{G(b)}), max(U_{F(a)}, U_{G(b)}), min(C_{F(a)}, C_{G(b)}), min(T_{F(a)}, T_{G(b)})\}$$

Let (F, A) ^{C} ∨ (G, A) ^{C} = (R, A ×B) where R (a, b) =(F(a)) ^{C} ∪ (G(b)) ^{C} ∀ $a \in A$ and ∀ $b \in B$ where ∪ is the operation union of NPSS.

R (a, b) = $\{(x, max(F_{F(a)}, F_{G(b)}), max(U_{F(a)}, U_{G(b)}), min(C_{F(a)}, C_{G(b)}), min(T_{F(a)}, T_{G(b)})\}$

Hence ((F, A)∧ (G, A)) ^{C} = (F, A) ^{C} ∨ (G, A) ^{C}. Similarly we can prove (ii).

5. Conclusion

In this paper, we have introduced the idea of Quadripartitioned neutrosophic pythagorean soft set with dependent neutrosophic compnents and discussed some of its properties.We have put forward some theorems based on this new notion. In future, this paper will leads us to develop QNPS topological space. Further, we can study on QNPSS to carry out a general framework for this application in day today life.

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