True-False Set is a particular case of the Refined Neutrosophic Set

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Abstract

Borzooei, Mohseni Takallo, and Jun recently proposed a new type of set, called True-False Set [1], and they claimed it is a generalization of Neutrosophic Set [2]. We prove that this assertion is untrue. Actually it's the opposite, the True-False Set is a particular case of the Refined Neutrosophic Set..

1. Definition of True-False Set [1]

A True-False set (TF-set), on a none-empty set X, is a structure of the form:

$A_{TFS} = \{x; t_A(x), T_A(x), f_A(x), F_A(x) | x \in X\}$; the index "TFS" stands for True-False Set;

where $t_A: X \to [0, 1]$; t_A represents the single-valued truth function;

$T_A: X \to I([0, 1])$, where $I([0, 1])$ is the set of all subintervals of $[0, 1]$; T_A represents the interval-valued truth function;

$f_A: X \to [0, 1]$; f_A represents the single-valued falsehood function;

$F_A: X \to I([0, 1])$; F_A represents the interval-valued falsehood function.

It is not clear why two truth-functions and two falsehood-functions are needed for the same element x. There is no justification.

2. Definition of neutrosophic set [2]

We try to use similar notations and language in order to make easy comparison between the two types of sets.

Let X be a non-empty universe of discourse.

A Neutrosophic Set on X is a structure of the form:

$A_{NS} = \{x; T_A(x), I_A(x), F_A(x) | x \in X\},$

where $T_A, I_A, F_A: X \rightarrow \mathcal{P}(]^-0, 1^+[)$, where $\mathcal{P}(]^-0, 1^+[)$ is the set of all standard or nonstandard subsets of the non-standard interval $]^-0, 1^+[$.

3. Distinctions between True-False Set and Neutrosophic Set

1) Clearly $\mathcal{P}(]^-0, 1^+[) \supsetneq I([0, 1])$. From this point of view, the neutrosophic set is larger than the True-False Set.

$\mathcal{P}(]^-0, 1^+[)$ includes not only strandard subintervals of $[0, 1]$ as $I([0, 1])$, but any standard subsets of $[0, 1]$.

2) $\mathcal{P}(]^-0, 1^+[)$ also includes non-standard subsets of $]^-0, 1^+[$, left and right monads, binads from non-standard analysis, that help make a distinction between absolute truth (truth in all possible worlds, according to Leibniz), whose truth-value is $T_A(x) = 1^+$, where $1^+ = 1 + \varepsilon > 1$ and ε is a positive infinitesimal number.

Similarly for absolute / relative indeterminacy and respectively falsehood.

The True-False Set cannot make distinctions between absolute and relative truth/falsehood.

3) Neutrosophic Set is much more complex as structure than the True-False Set; Neutrosophic Set has been further extended Neutrosophic Overset (where the neutrosophic components could be > 1), Neutrosophic Underset (where the neutrosophic components could be < 0), and Neutrosophic Offset (where the neutrosophic components could be > 1 and < 0) in 2007 & 2016 ([3], [4]).

4. Transformation of a Single-Valued Neutrosophic Set to a True-False Set [1]

The authors of [1] considered only the simplest form of the Neutrosophic Set, i.e. when the neutrosophic components T, I, F are single (crisp) numbers in $[0, 1]$, while the general definition [2] of neutrosophic set stated since 1998 that T, I, F can be any subsets of $[0, 1]$, or any nonstandard subsets of the non-standard unit interval $]^-0, 1^+[$.

They considered the single-valued neutrosophic set:

$$A_{NS} = \{x;\ T_{NS}(x), I_{NS}(x), F_{NS}(x) | x \in X\},$$

where $T_{NS}, I_{NS}, F_{NS}: X \rightarrow [0, 1]$ are single-valued truth, indeterminacy, and falsehood functions respectively. The index "NS" stands for Neutrosophic Set (we adjusted their Greek letter notations to Latin ones, in order to exactly match the common use notations of the neutrosophic set).

They transformed it to a True-False Set in the following way:

$$t(x) = T_{NS}(x);$$

$$f(x) = F_{NS}(x);$$

$$T_{TFS}(x) = \begin{cases} [T_{NS}(x), I_{NS}(x)], \text{if } T_{NS}(x) \leq I_{NS}(x); \\ [I_{NS}(x), T_{NS}(x)], \text{if } I_{NS}(x) \leq T_{NS}(x); \end{cases}$$

$$F_{TFS}(x) = \begin{cases} [F_{NS}(x), I_{NS}(x)], \text{if } F_{NS}(x) \leq I_{NS}(x); \\ [I_{NS}(x), F_{NS}(x)], \text{if } I_{NS}(x) \leq F_{NS}(x). \end{cases}$$

And they formed the following True-False Set:

$$A_{TFS} = \{x;\ t(x), T_{TFS}(x), f(x), F_{TFS}(x) | x \in X\} = \{\langle x;\ T_{NS}(x), T_{TFS}(x), F_{NS}(x), F_{TFS}(x)\rangle | x \in X\}.$$

This True-False Set, A_{TFS}, has two truth-functions and two-falsehood functions, but no indeterminacy (neutrality) function (they removed it).

Transforming the neutrosophic set A_{NS} into a true-false set A_{TFS} is just a mathematical artifact. It is not proven that A_{NS} is equivalent to A_{TFS}. Actually, we'll prove below that they are not.

Other mathematical transformations can be designed as well, constructing new intervals, or combining the neutrosophic functions in other ways, etc. But the equivalence, if any, should be proven.

5. Indeterminacy (Neutrality)

The indeterminacy (neutrality) is the quintessence (the flavor) of neutrosophic set, that stringently distinguishes it from previous types of sets.

By eliminating the indeterminacy (or neutrality) from the neutrosophic set A_{NS}, when constructing a true-false set A_{TFS}, the true-false set A_{TFS} becomes defficient, incapable of characterizing the neutrosophic triads of the form $(\langle A \rangle, \langle neutA \rangle, \langle antiA \rangle)$, where $\langle A \rangle$ is an item (idea, proposition, attribute, concept, etc.), $\langle antiA \rangle$ is its opposite, and $\langle neutA \rangle$ is the neutral between these opposites.

For example, in games we have such triads (where $\langle A \rangle$ = winning): <winning, tie game, loosing>.

6. Numerical Counter-Example of Transforming a Single-Valued Neutrosophic Set to a True-False Set

Let's take only one element from a single valued neutrosophic set (for the other elements it will be similar):

$x_{NS}(0.3, 0.4, 0.2)$, hence $T_{NS}(x) = 0.3, I_{NS}(x) = 0.4, F_{NS}(x) = 0.2$.

Let's transform it into a true-false set's element according to [1]:

$x_{TFS}(0.3, [0.3, 0.4], 0.2, [0.2, 0.4])$, hence $t_{TFS}(x) = 0.3, T_{TFS}(x) = [0.3, 0.4], f_{TFS}(x) = 0.2, F_{TFS}(x) = [0.2, 0.4]$.

The indeterminacy $I_{NS}(x) = 0.4$ into the neutrosophic set has been replaced into the true-false set by an interval-value truth $T_{TFS}(x) = [0.3, 0.4]$ and an interval-value falsehood $F_{TFS}(x) = [0.2, 0.4]$. But these are a totally different results.

If, with respect to an element, the indeterminacy-membership is 0.4, this is not equivalent with element's truth-membership be equal to [0.3, 0.4] and its false-membership be equal to [0.2, 0.4].

7. Other Counter-Examples

Let $x_{NS}(0.3, 0.4, 0.2)$ represent, with respect to the player x in a game where he plays against others, that his degree of winning ($T_{NS} = 0.3$), his degree of tie game ($I_{NS} = 0.4$), and his degree of loosing ($F_{NS} = 0.2$).

By transforming x_{NS} to $x_{TFS}(0.3, [0.3, 0.4], 0.2, [0.2, 0.4])$, we get that with respect to the same player x, his degree of winning is 0.3 or [0.3, 0.4], and his degree of loosing is 0.2 or [0.2, 0.4].

7.1. Therefore, the true-false set does not provide any degree of "tie game", so this type of set is incomplete. The true-false set does not catch the middle side (neutrality, or indeterminacy) in between opposites.

7.2. Another drawback is that TFS increases the imprecision of the truth function: for $T_{NS} = 0.3$, it gets $T_{TFS} = 0.3$ or $[0.3, 0.4]$, so the truth value becomes vaguer after the TFS transformation.

TFS increases the imprecision of the falsehood function as well: for $FNS = 0.2$, it gets $FTFS = 0.2$ or $[0.2, 0.4]$, so the falsehood value becomes vaguer after the TFS transformation.

8. The True-False Set is a particular case of the Refined Neutrosophic Set

In the Refined Neutrosophic Set (Logic, Probability), T can be split into subcomponents $T_1, T_2, ..., T_p$, and I into $I_1, I_2, ..., I_r$, and F into $F_1, F_2, ...,F_s$, where $p, r, s \in \{0, 1, 2, ..., \infty\}$ and $p + r + s = n \in \{0, 1, 2, ..., \infty\}$. By index = 0, of a neutrosophic component T, I, or F, or of any of their subcomponents, we denote the empty set, i.e. $T_0 = \phi$, $I_0 = \phi$, $F_0 = \phi$. The case (T_0, I_0, F_0) is the most degenerated one. See [4].

From (T, I, F), where T, I, F are any subsets of [0, 1], we replace $I_0 = \phi$ (empty set), and refine/split T into T_1 (single-valued truth component) and T_2 (as an interval-valued truth component), while F is similarly refined/split into F_1 (as a single-valued falsehood component) and F_2 (as an interval-valued falsehood component). Therefore, we replaced p = 2, r = 0, and s = 2 into the general form of the Refined Neutrosophic Set, and we found the True-False Set $(T_1, T_2, I_0 = \phi, F_1, F_2)$.

9. Conclusion

We proved that the transformation of the Neutrosophic Set into a True-False Set does not give equivalent results by using several coounter-examples. Also, we proved that the True-False Set is a particular case of Refined Neutrosophic Set.

References

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