

New Operations on Intuitionistic Fuzzy Soft Sets based on Second Zadeh's logical Operators

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Abstract — In this paper, three new operations have been introduced on intuitionistic fuzzy soft sets. They are based on Second Zadeh's implication, conjunction and disjunction operations on intuitionistic fuzzy sets. Some examples of these operations were given and a few important properties were also studied.

Index Terms — Second Zadeh's implication, Second Zadeh's conjunction, Second Zadeh's disjunction, Intuitionistic fuzzy soft set.

1. Introduction

The concept of the intuitionistic fuzzy (IFS, for short) was introduced in 1983 by K. Atanassov [1] as an extension of Zadeh's fuzzy set. All operations, defined over fuzzy sets were transformed for the case the IFS case. This concept is capable of capturing the information that includes some degree of hesitation and applicable in various fields of research. For example, in decision making problems, particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services, etc. Atanassov et al. [2,3] have widely applied theory of intuitionistic sets in logic programming, Szmidt and Kacprzyk [4] in group decision making, De et al [5] in medical diagnosis etc. Therefore in various engineering application, intuitionistic fuzzy sets techniques have been more popular than fuzzy sets techniques in recent years. After defining a lot of operations over intuitionistic fuzzy sets during last ten years [6], in 2011, K. Atanassov [7] constructed two new operations based on the First Zadeh's IF-implication [8] which are the First Zadeh's conjunction and disjunction, after that, in 2013, K. Atanassov [9] introduced the second type of Zadeh's conjunction and disjunction based on the

Second Zadeh's IF-implication. Later on, S. Broumi et al. [22] introduced three new operations based on first Zadeh's implication, conjunction and disjunction operations on intuitionistic fuzzy soft sets.

Another important concept that addresses uncertain information is the soft set theory originated by Molodtsov [10]. This concept is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Molodtsov has successfully applied the soft set theory in many different fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. In recent years, soft set theory has been received much attention since its appearance. There are many papers devoted to fuzzify the concept of soft set theory which leads to a series of mathematical models such as fuzzy soft set [11,12,13,14,15], generalized fuzzy soft set [16,17], possibility fuzzy soft set [18] and so on. Thereafter, P.K. Maji and his coworker [19] introduced the notion of intuitionistic fuzzy soft set which is based on a combination of the intuitionistic fuzzy sets and soft set models and studied the properties of intuitionistic fuzzy soft set. Later, a lot of extensions of intuitionistic fuzzy soft are appeared such as generalized intuitionistic fuzzy soft set [20], possibility intuitionistic fuzzy soft set [21] etc.

In this paper our aim is to extend the three new operations introduced by K.T. Atanassov to the case of intuitionistic fuzzy soft and study its properties. This paper is arranged in the following manner. In section 2, some basics related to soft set, fuzzy soft set and intuitionistic fuzzy soft set are presented. These definitions will help us in the section that will follow. In section 3, we discuss the three operations of intuitionistic fuzzy soft such as Second Zadeh's implication, Second Zadeh's intuitionistic fuzzy

conjunction and Second Zadeh's intuitionistic fuzzy disjunction. In section 4, we conclude the paper.

2. Preliminaries

In this section, some definitions and notions about soft sets and intuitionistic fuzzy soft set are given. These will be useful in later sections. For more detailed the reader can see [10, 11, 12, 13, 19].

Let U be an initial universe, and E be the set of all possible parameters under consideration with respect to U . The set of all subsets of U , i.e. the power set of U is denoted by $P(U)$ and the set of all intuitionistic fuzzy subsets of U is denoted by IF^U . Let A be a subset of E .

2.1. Definition.

A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U . For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε -approximate elements of the soft set (F, A) .

2.2. Definition

Let U be an initial universe set and E be the set of parameters. Let IFU denote the collection of all intuitionistic fuzzy subsets of U . Let $A \subseteq E$ pair (F, A) is called an intuitionistic fuzzy soft set over U where F is a mapping given by $F: A \rightarrow IFU$.

2.3. Definition

Let $F: A \rightarrow IFU$ then F is a function defined as $F(\varepsilon) = \{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U, \varepsilon \in E\}$ where μ, ν denote the degree of membership and degree of non-membership respectively.

2.4. Definition.

For two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe U , we say that (F, A) is an intuitionistic fuzzy soft subset of (G, B) if

- (1) $A \subseteq B$ and
 - (2) $F(\varepsilon) \subseteq G(\varepsilon)$ for all $\varepsilon \in A$. i.e $\mu_{F(\varepsilon)}(x) \leq \mu_{G(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \geq \nu_{G(\varepsilon)}(x)$ for all $\varepsilon \in E$ and
- We write $(F, A) \subseteq (G, B)$.

2.5. Definition.

Two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) .

2.6. Definition.

Let U be an initial universe, E be the set of parameters, and $A \subseteq E$.

- (a) (F, A) is called a null intuitionistic fuzzy soft set (with respect to the parameter set A), denoted by φ_A , if $F(\varepsilon) = \varphi$, with $\varphi_A = (\varphi, A) = \{(0, 1), \forall x \in U, \forall \varepsilon \in A\}$. (1)

- (b) (G, A) is called a absolute intuitionistic fuzzy soft set (with respect to the parameter set A), denoted by U_A , if $G(\varepsilon) = U$, with $U_A = (U, A) = \{(1, 0), \forall x \in U, \forall \varepsilon \in A\}$. (2)

2.7. Definition

Let (F, A) and (G, B) be two IFSSs over the same universe U . Then the union of (F, A) and (G, B) is denoted by ' $(F, A) \cup (G, B)$ ' and is defined by $(F, A) \cup (G, B) = (H, C)$, where $C = A \cup B$ and the truth-membership, falsity-membership of (H, C) are as follows:

$$H(\varepsilon) = \begin{cases} \{(\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)) : x \in U\}, & \text{if } \varepsilon \in A - B, \\ \{(\mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) : x \in U\}, & \text{if } \varepsilon \in B - A \\ \{\max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) : x \in U\}, & \text{if } \varepsilon \in A \cap B \end{cases} \quad (3)$$

Where $\mu_{H(\varepsilon)}(x) = \max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))$ and $\nu_{H(\varepsilon)}(x) = \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))$

2.8. Definition

Let (F, A) and (G, B) be two IFSSs over the same universe U such that $A \cap B \neq \emptyset$. Then the intersection of (F, A) and (G, B) is denoted by ' $(F, A) \cap (G, B)$ ' and is defined by $(F, A) \cap (G, B) = (K, C)$, where $C = A \cap B$ and the truth-membership, falsity-membership of (K, C) are related to those of (F, A) and (G, B) by:

$$K(\varepsilon) = \begin{cases} \{(\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)) : x \in U\}, & \text{if } \varepsilon \in A - B, \\ \{(\mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) : x \in U\}, & \text{if } \varepsilon \in B - A \\ \{\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) : x \in U\}, & \text{if } \varepsilon \in A \cap B \end{cases} \quad (4)$$

In the next section, we state and prove some new operations involving second implication, conjunction and disjunction of intuitionistic fuzzy soft set.

3. New Operations on Intuitionistic Fuzzy Soft Sets.

3.1 Second Zadeh's implication of intuitionistic fuzzy soft sets.

3.1.1. Definition:

Let (F, A) and (G, B) are two intuitionistic fuzzy soft set over (U, E) . We define the second Zadeh's intuitionistic fuzzy soft set implication $(F, A) \xrightarrow{z,2} (G, B)$ by

$$(F, A) \xrightarrow{z,2} (G, B) = \left[\max\{\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\}, \min\{\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))\} \right], \forall x \in U, \forall \varepsilon \in A. \quad (5)$$

3.1.2. Example:

Let (F, A) and (G, B) be two intuitionistic fuzzy soft set over (U, E) where $U = \{a, b, c\}$ and $E = \{e_1, e_2\}$, $A = \{e_1\} \subseteq E$, $B = \{e_1\} \subseteq E$.

$$(F, A) = \{F(e_1) = (a, 0.3, 0.2), (b, 0.2, 0.5), (c, 0.4, 0.2)\}$$

$$(G, B) = \{G(e_1) = (a, 0.4, 0.5), (b, 0.3, 0.5), (c, 0.6, 0.1)\}$$

Then

$$(F, A) \xrightarrow{z,2} (G, B) = \{(a, 0.3, 0.3), (b, 0.5, 0.2), (c, 0.4, 0.2)\}$$

3.1.3. Proposition:

Let (F, A), (G, B) and (H, C) are three intuitionistic fuzzy soft sets over (U, E).

Then the following results hold

- (i) $(F, A) \cap (G, B) \xrightarrow{z,2} (H, C) \supseteq [(F, A) \xrightarrow{z,2} (H, C)] \cap [(G, B) \xrightarrow{z,2} (H, C)]$
- (ii) $(F, A) \cup (G, B) \xrightarrow{z,2} (H, C) \supseteq [(F, A) \xrightarrow{z,2} (H, C)] \cup [(G, B) \xrightarrow{z,2} (H, C)]$
- (iii) $(F, A) \xrightarrow{z,2} (F, A)^c = (F, A)^c$
- (iv) $(F, A) \xrightarrow{z,2} (\varphi, A) = (F, A)^c$ where φ denote the null intuitionistic fuzzy soft
- (v) With $(\varphi, A) = \{(0, 1), \forall x \in U, \forall \varepsilon \in A\}$

Proof.

$$(i) \quad (F, A) \cap (G, B) \xrightarrow{z,2} (H, C)$$

$$= [\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))] \xrightarrow{z,2} (\mu_{H(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))$$

$$= [\max\{\max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \min(\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \mu_{H(\varepsilon)}(x))\}, \min\{\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \nu_{H(\varepsilon)}(x))\}]$$

(a)

$$[(F, A) \xrightarrow{z,2} (H, C)] \cap [(G, B) \xrightarrow{z,2} (H, C)]$$

$$= [\max\{\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))\}, \min\{\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))\}] \cap [\max\{\nu_{G(\varepsilon)}(x), \min(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))\}, \min\{\mu_{G(\varepsilon)}(x), \max(\nu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))\}]$$

=

$$[\min\{\max(\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))), \max(\nu_{G(\varepsilon)}(x), \min(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)))\}, \max\{\min(\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))), \min(\mu_{G(\varepsilon)}(x), \max(\nu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)))\}]$$

(b)

From (a) and (b) it is clear that $(F, A) \cap (G, B) \xrightarrow{z,2} (H, C)$

$$\supseteq [(F, A) \xrightarrow{z,2} (H, C)] \cap [(G, B) \xrightarrow{z,2} (H, C)]$$

$$(ii) \quad (F, A) \cup (G, B) \xrightarrow{z,2} (H, C)$$

$$= [\max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))] \xrightarrow{z,2} (\mu_{H(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))$$

$$= [\max\{\max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \min(\max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \mu_{H(\varepsilon)}(x))\}, \min\{\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \nu_{H(\varepsilon)}(x))\}]$$

(c)

$$[(F, A) \xrightarrow{z,2} (H, C)] \cup [(G, B) \xrightarrow{z,2} (H, C)]$$

$$= [\max\{\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))\}, \min\{\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))\}] \cup [\max\{\nu_{G(\varepsilon)}(x), \min(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))\}, \min\{\mu_{G(\varepsilon)}(x), \max(\nu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))\}]$$

=

$$[\max\{\max(\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))), \max(\nu_{G(\varepsilon)}(x), \min(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)))\}, \min\{\min(\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))), \min(\mu_{G(\varepsilon)}(x), \max(\nu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)))\}]$$

(d)

From (c) and (d) it is clear that $(F, A) \cup (G, B) \xrightarrow{z,2} (H, C) \supseteq [(F, A) \xrightarrow{z,2} (H, C)] \cup [(G, B) \xrightarrow{z,2} (H, C)]$

$$(iii) \quad (F, A) \xrightarrow{z,2} (F, A)^c = (F, A)^c$$

$$= \{\max\{\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x))\}, \min\{\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x))\}\}$$

$$= \{(\nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x))\}.$$

It is shown that the second Zadeh's intuitionistic fuzzy soft implication generate the complement of intuitionistic fuzzy soft set.

(iv) the proof is straightforward.

3.1.4. Example:

Let (F, A), (G, B) and (H, C) be three intuitionistic fuzzy soft set over (U, E) where $U = \{a, b, c\}$ and $E = \{e_1, e_2\}$, $A = \{e_1\} \subseteq E$, $B = \{e_1\} \subseteq E$ and $C = \{e_1\} \subseteq E$.

$$(F, A) = \{F(e_1) = (a, 0.3, 0.2), (b, 0.2, 0.5), (c, 0.4, 0.2)\}$$

$$(G, B) = \{G(e_1) = (a, 0.4, 0.5), (b, 0.3, 0.5), (c, 0.6, 0.1)\}$$

$$(H, C) = \{H(e_1) = (a, 0.3, 0.6), (b, 0.4, 0.5), (c, 0.4, 0.1)\}$$

Firstly, we have $(F, A) \cap (G, B) = \{(a, 0.3, 0.5), (b, 0.2, 0.5), (c, 0.4, 0.2)\}$

Then $(F, A) \cap (G, B) \xrightarrow{z,2} (H, C) = [\max\{\max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \min(\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \mu_{H(\varepsilon)}(x))\}, \min\{\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \nu_{H(\varepsilon)}(x))\}]$

$$= \{(a, 0.5, 0.3), (b, 0.5, 0.2), (c, 0.4, 0.4)\}$$

3.2. Second Zadeh's Intuitionistic Fuzzy Conjunction of intuitionistic fuzzy soft sets.

3.2.1. Definition:

Let (F, A) and (G, B) are two intuitionistic fuzzy soft sets over (U, E) . We define the second Zadeh's intuitionistic fuzzy conjunction of (F, A) and (G, B) as the intuitionistic fuzzy soft set (H, C) over (U, E) , written as $(F, A) \tilde{\wedge}_{z,2} (G, B) = (H, C)$ Where $C = A \cap B \neq \emptyset$ and $\forall \varepsilon \in C, x \in U$,

$$\begin{aligned} \mu_{H(\varepsilon)}(x) &= \min \{ \mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)) \} \\ \nu_{H(\varepsilon)}(x) &= \max \{ \nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) \}, \forall x \in U, \forall \varepsilon \in A \end{aligned} \quad (6)$$

3.2. 2.Example:

Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_4\} \subseteq E$, $B = \{e_1, e_2, e_3\} \subseteq E$

$$\begin{aligned} (F, A) &= \{F(e_1) = \{(a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)\}, F(e_2) = \{(a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)\}, \\ F(e_4) &= \{(a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)\}\} \end{aligned}$$

$$\begin{aligned} (G, B) &= \{G(e_1) = \{(a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)\}, G(e_2) = \{(a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)\}, \\ G(e_3) &= \{(a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)\}\} \end{aligned}$$

Let $(F, A) \tilde{\wedge}_{z,2} (G, B) = (H, C)$, where $C = A \cap B = \{e_1, e_2\}$

$$(H, C) = \{H(e_1) = \{(a, \min(0.5, \max(0.1, 0.2)), \max(0.1, \min(0.5, 0.6))), (b, \min(0.1, \max(0.8, 0.7)), \max(0.8, \min(0.1, 0.1))), (c, \min(0.2, \max(0.5, 0.8)), \max(0.5, \min(0.2, 0.1)))\},$$

$$H(e_2) = \{(a, \min(0.7, \max(0.1, 0.4)), \max(0.1, \min(0.7, 0.1))), (b, \min(0, \max(0.8, 0.5)), \max(0.8, \min(0, 0.3))), (c, \min(0.3, \max(0.5, 0.4)), \max(0.5, \min(0.3, 0.5)))\}$$

$$\begin{aligned} \text{Then } (H, C) &= \{H(e_1) = \{(a, \min(0.5, 0.2), \max(0.1, 0.5)), (b, \min(0.1, 0.8), \max(0.8, 0.1)), (c, \min(0.2, 0.8), \max(0.5, 0.1))\}, \\ H(e_2) &= \{(a, \min(0.7, 0.4), \max(0.1, 0.1)), (b, \min(0, 0.8), \max(0, 0.8)), (c, \min(0.3, 0.5), \max(0.5, 0.3))\}\} \end{aligned}$$

$$\text{Hence, } (H, C) = \{H(e_1) = \{(a, 0.2, 0.5), (b, 0.1, 0.8), (c, 0.2, 0.5)\}, H(e_2) = \{(a, 0.4, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)\}\}$$

3.2. 3 Proposition:

Let (F, A) , (G, B) and (H, C) are three intuitionistic fuzzy soft sets over (U, E)

Then the following result hold

$$\begin{aligned} (F, A) \tilde{\wedge}_{z,2} (G, B) \rightarrow_{z,2} (H, C) &\supseteq [(F, A) \rightarrow_{z,2} (H, C)] \tilde{\wedge}_{z,2} \\ [(G, B) \rightarrow_{z,2} (H, C)] &\end{aligned} \quad (7)$$

Proof: let (F, A) , (G, B) and (H, C) are three intuitionistic fuzzy soft set, then

$$(F, A) \tilde{\wedge}_{z,2} (G, B) \rightarrow_{z,2} (H, C)$$

$$\begin{aligned} &= (\max [\max \{ \nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) \}, \min \{ \min \{ \mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)) \}, \mu_{H(\varepsilon)}(x) \}], \\ &\min [\min \{ \mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)) \}, \max[\max \{ \nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) \}, \nu_{H(\varepsilon)}(x) \}]] \end{aligned}$$

$$\text{Let } [(F, A) \rightarrow_{z,2} (H, C)] \tilde{\wedge}_{z,2} [(G, B) \rightarrow_{z,2} (H, C)]$$

$$(F, A) \rightarrow_{z,2} (H, C) = (\max [\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))], \min[\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))])$$

$$\begin{aligned} [(G, B) \rightarrow_{z,2} (H, C)] &= (\max [\nu_{G(\varepsilon)}(x), \min(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))], \min[\mu_{G(\varepsilon)}(x), \max(\nu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))]) \end{aligned}$$

$$\text{Then } [(F, A) \rightarrow_{z,2} (H, C)] \tilde{\wedge}_{z,2} [(G, B) \rightarrow_{z,2} (H, C)]$$

$$= (\min[\max \{ \nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)) \}, \max \{ \nu_{G(\varepsilon)}(x), \min(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)) \}],$$

$$\max [\min[\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))], \min \{ \max [\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x))], \min[\mu_{G(\varepsilon)}(x), \max(\nu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))] \}])$$

From (e) and (f) it is clear that

$$\begin{aligned} (F, A) \tilde{\wedge}_{z,2} (G, B) \rightarrow_{z,2} (H, C) &\supseteq [(F, A) \rightarrow_{z,2} (H, C)] \tilde{\wedge}_{z,2} \\ [(G, B) \rightarrow_{z,2} (H, C)] &\end{aligned}$$

3.3. The Second Zadeh's Intuitionistic Fuzzy Disjunction of Intuitionistic Fuzzy Soft Sets.

3.3.1. Definition:

Let (F, A) and (G, B) are two intuitionistic fuzzy soft sets over (U, E) . We define the second Zadeh's intuitionistic fuzzy disjunction of (F, A) and (G, B) as the intuitionistic fuzzy soft set (H, C) over (U, E) , written as $(F, A) \tilde{\vee}_{z,2} (G, B) = (H, C)$ Where $C = A \cap B \neq \emptyset$ and $\forall \varepsilon \in A, x \in U$

$$\begin{aligned} \mu_{H(\varepsilon)}(x) &= \max(\mu_{F(\varepsilon)}(x), \min(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))) \\ \nu_{H(\varepsilon)}(x) &= \min \{ \nu_{F(\varepsilon)}(x), \max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) \}, \forall x \in U, \forall \varepsilon \in A \end{aligned} \quad (8)$$

3.3.2. Example:

Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_4\} \subseteq E$, $B = \{e_1, e_2, e_3\} \subseteq E$

$(F, A) = \{ \mathbf{F}(e_1) = \{(a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)\}, \mathbf{F}(e_2) = \{(a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)\}, \mathbf{F}(e_4) = \{(a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)\} \}$

$(G, A) = \{ \mathbf{G}(e_1) = \{(a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)\}, \mathbf{G}(e_2) = \{(a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)\}, \mathbf{G}(e_3) = \{(a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)\} \}$

Let $(F, A) \tilde{V}_{z,2} (G, B) = (H, C)$, where $C = A \cap B = \{e_1, e_2\}$

$(H, C) = \{ \mathbf{H}(e_1) = \{(a, \max(0.5, \min(0.1, 0.2)), \min(0.1, \max(0.5, 0.6))), (b, \max(0.1, \min(0.8, 0.7)), \min(0.8, \max(0.1, 0.1)))\}, \mathbf{H}(e_2) = \{(a, \max(0.7, \min(0.1, 0.4)), \min(0.1, \max(0.7, 0.1))), (b, \max(0, \min(0.8, 0.5)), \min(0.8, \max(0, 0.3))), (c, \max(0.3, \min(0.5, 0.4)), \min(0.5, \max(0.3, 0.5)))\} \}$

Then, $(H, C) = \{ \mathbf{H}(e_1) = \{(a, \max(0.5, 0.1), \min(0.1, 0.6)), (b, \max(0.1, 0.7), \min(0.8, 0.1)), (c, \max(0.2, 0.5), \min(0.5, 0.2))\}, \mathbf{H}(e_2) = \{(a, \max(0.7, 0.1), \min(0.1, 0.7)), (b, \max(0, 0.5), \min(0.8, 0.3)), (c, \max(0.3, 0.4), \min(0.5, 0.5))\} \}$.

hence, $(H, C) = \{ \mathbf{H}(e_1) = \{(a, 0.5, 0.1), (b, 0.7, 0.1), (c, 0.5, 0.2)\}, \mathbf{H}(e_2) = \{(a, 0.7, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)\} \}$

3.3.3. Proposition:

- (i) $(\varphi, A) \tilde{\Lambda}_{z,2} (U, A) = (\varphi, A)$
- (ii) $(\varphi, A) \tilde{V}_{z,2} (U, A) = (U, A)$, where $(U, A) = \{(1, 0), \forall x \in U, \forall \varepsilon \in A\}$
- (iii) $(F, A) \tilde{V}_{z,2} (\varphi, A) = (F, A)$

Proof

- (i) Let $(\varphi, A) \tilde{\Lambda}_{z,2} (U, A) = (H, A)$, where for all $\varepsilon \in A$, $x \in U$, we have

$$\mu_{H(\varepsilon)}(x) = \min(0, \max(1, 1)) = \min(0, 1) = 0$$

$$\nu_{H(\varepsilon)}(x) = \max(1, \min(0, 0)) = \max(1, 0) = 1$$

Therefore $(H, A) = (0, 1)$, For all $\varepsilon \in A$, $x \in U$

It follows that $((\varphi, A) \tilde{\Lambda}_{z,2} (U, A) = (\varphi, A)$

- (ii) Let $(\varphi, A) \tilde{V}_{z,2} (U, A) = (H, A)$, where For all $\varepsilon \in A$, $x \in U$, we have

$$\mu_{H(\varepsilon)}(x) = \max(0, \min(1, 1)) = \max(0, 1) = 1$$

$$\nu_{H(\varepsilon)}(x) = \min(1, \max(0, 0)) = \min(1, 0) = 0$$

Therefore $(H, A) = (1, 0)$, For all $\varepsilon \in A$, $x \in U$

It follows that $((\varphi, A) \tilde{V}_{z,2} (U, A) = (U, A)$

- (iii) Let $(F, A) \tilde{V}_{z,2} (\varphi, A) = (H, A)$, where For all $\varepsilon \in A$, $x \in U$, we have

$$\mu_{H(\varepsilon)}(x) = \max(\mu_{F(\varepsilon)}(x), \min(\nu_{F(\varepsilon)}(x), 0)) = \max(\mu_{F(\varepsilon)}(x), 0) = \mu_{F(\varepsilon)}(x)$$

$$\nu_{H(\varepsilon)}(x) = \min(\nu_{F(\varepsilon)}(x), \max(\mu_{F(\varepsilon)}(x), 1)) = \min(\nu_{F(\varepsilon)}(x), 1) = \nu_{F(\varepsilon)}(x)$$

Therefore $(H, A) = (\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x))$, for all $\varepsilon \in A$, $x \in U$

It follows that $(F, A) \tilde{V}_{z,2} (\varphi, A) = (F, A)$

3.3.4. Proposition:

$$(F, A) \tilde{V}_{z,1} (G, B) \xrightarrow{z,2} (H, C) \supseteq [(F, A) \xrightarrow{z,2} (H, C)] \tilde{V}_{z,2} [(G, B) \xrightarrow{z,2} (H, C)] \quad (9)$$

Proof, the proof is similar as in proposition 3.2.3

3.3.5. Proposition:

- (i) $[(F, A) \tilde{\Lambda}_{z,2} (G, B)]^c = (F, A)^c \tilde{V}_{z,2} (G, B)^c$
- (ii) $[(F, A) \tilde{V}_{z,2} (G, B)]^c = (F, A)^c \tilde{\Lambda}_{z,2} (G, B)^c$
- (iii) $[(F, A)^c \tilde{\Lambda}_{z,2} (G, B)^c]^c = (F, A) \tilde{V}_{z,2} (G, B)$

Proof:

- (i) Let $[(F, A) \tilde{\Lambda}_{z,2} (G, B)]^c = (H, C)$, where For all $\varepsilon \in C$, $x \in U$, we have

$$[(F, A) \tilde{\Lambda}_{z,2} (G, B)]^c = [\min\{\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\}, \max\{\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))\}]^c$$

$$= [\max\{\nu_{F(\varepsilon)}(x), \min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))\}, \min\{\mu_{F(\varepsilon)}(x), \max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\}]$$

$$= (F, A)^c \tilde{V}_{z,2} (G, B)^c$$

- (ii) Let $[(F, A) \tilde{V}_{z,2} (G, B)]^c = (H, C)$, where For all $\varepsilon \in C$, $x \in U$, we have

$$[(F, A) \tilde{V}_{z,2} (G, B)]^c = [\max\{\nu_{F(\varepsilon)}(x), \min(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\}, \min\{\nu_{F(\varepsilon)}(x), \max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))\}]$$

$$= [\min\{\nu_{F(\varepsilon)}(x), \max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))\}, \max\{\mu_{F(\varepsilon)}(x), \min(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\}]^c$$

$$= (F, A)^c \tilde{\Lambda}_{z,2} (G, B)^c$$

- (iii) The proof is straightforward.

3.3.6. Proposition:

The following equalities are not valid

- I. $(F, A) \tilde{V}_{z,2} (G, B) = (G, B) \tilde{V}_{z,2} (F, A)$
- II. $(F, A) \tilde{\Lambda}_{z,2} (G, B) = (G, B) \tilde{\Lambda}_{z,2} (F, A)$

- III. $[(F, A) \tilde{\lambda}_{z,2}(G, B)] \tilde{\lambda}_{z,2}(K, C) = (F, A) \tilde{\lambda}_{z,2} [(G, B) \tilde{\lambda}_{z,2}(K, C)]$
 IV. $[(F, A) \tilde{\nu}_{z,2}(G, B)] \tilde{\nu}_{z,2}(K, C) = (F, A) \tilde{\nu}_{z,2} [(G, B) \tilde{\nu}_{z,2}(K, C)]$
 V. $[(F, A) \tilde{\lambda}_{z,2}(G, B)] \tilde{\nu}_{z,2}(K, C) = [(F, A) \tilde{\nu}_{z,2}(G, B)] \tilde{\lambda}_{z,2} [(G, B) \tilde{\nu}_{z,2}(K, C)]$
 VI. $[(F, A) \tilde{\nu}_{z,2}(G, B)] \tilde{\lambda}_{z,2}(K, C) = [(F, A) \tilde{\lambda}_{z,2}(G, B)] \tilde{\nu}_{z,2} [(G, B) \tilde{\lambda}_{z,2}(K, C)]$

3.3.7 .Example :

Let $U=\{a, b, c\}$ and $E=\{e_1, e_2, e_3, e_4\}$, $A=\{e_1, e_2, e_4\} \subseteq E$, $B=\{e_1, e_2, e_3\} \subseteq E$

$(F, A) = \{ \mathbf{F}(e_1) = \{(a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)\}, \mathbf{F}(e_2) = \{(a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)\}, \mathbf{F}(e_4) = \{(a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)\} \}$, $(G, B) = \{ \mathbf{G}(e_1) = \{(a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)\}, \mathbf{G}(e_2) = \{(a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)\}, \mathbf{G}(e_3) = \{(a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)\} \}$

Let $(F, A) \tilde{\lambda}_{z,2}(G, B) = (H, C)$, where $C = A \cap B = \{e_1, e_2\}$

Then $(F, A) \tilde{\lambda}_{z,2}(G, B) = (H, C) = \{ \mathbf{H}(e_1) = \{(a, 0.2, 0.5), (b, 0.1, 0.8), (c, 0.2, 0.5)\}, \mathbf{H}(e_2) = \{(a, 0.4, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)\} \}$

For $(G, B) \tilde{\lambda}_{z,2}(F, A) = (K, C)$, where $K = A \cap B = \{e_1, e_2\}$

$(K, C) = \{ \mathbf{K}(e_1) = \{(a, \min(0.2, \max(0.6, 0.5)), \max(0.6, \min(0.2, 0.1))), (b, \min(0.7, \max(0.1, 0.1)), \max(0.1, \min(0.7, 0.8))), (c, \min(0.8, \max(0.1, 0.2)), \max(0.1, \min(0.8, 0.5)))\}, \mathbf{K}(e_2) = \{(a, \min(0.7, \max(0.1, 0.4)), \max(0.1, \min(0.4, 0.1))), (b, \min(0.5, \max(0.3, 0.)), \max(0.3, \min(0.5, 0.8)))\}, (c, \min(0.5, \max(0.3, 0.)), \max(0.3, \min(0.5, 0.8))) \}$

Then, $(K, C) = \{ \mathbf{K}(e_1) = \{(a, \min(0.2, 0.6), \max(0.6, 0.1)), (b, \min(0.7, 0.1), \max(0.1, 0.7)), (c, \min(0.8, 0.2), \max(0.1, 0.5))\}, \mathbf{K}(e_2) = \{(a, \min(0.4, 0.4), \max(0.1, 0.1)), (b, \min(0.5, 0.3), \max(0.3, 0.5)), (c, \min(0.4, 0.5), \max(0.5, 0.4))\} \}$

Hence, $(K, C) = \{ \mathbf{K}(e_1) = \{(a, 0.2, 0.6), (b, 0.1, 0.7), (c, 0.2, 0.5)\}, \mathbf{K}(e_2) = \{(a, 0.4, 0.1), (b, 0, 0.5), (c, 0.3, 0.5)\} \}$
 Then $(G, B) \tilde{\lambda}_{z,2}(F, A) = (K, C) = \{ \mathbf{K}(e_1) = \{(a, 0.2, 0.6), (b, 0.1, 0.7), (c, 0.2, 0.5)\}, \mathbf{K}(e_2) = \{(a, 0.4, 0.1), (b, 0.3, 0.5), (c, 0.4, 0.5)\} \}$

It is obviously that $(F, A) \tilde{\lambda}_{z,2}(G, B) \neq (G, B) \tilde{\lambda}_{z,2}(F, A)$

4. Conclusion

In this paper, We have introduced and extended the

operations of second Zadeh's implication, second Zadeh's intuitionistic fuzzy disjunction and second Zadeh's intuitionistic fuzzy conjunction of intuitionistic fuzzy set that was introduced by Krassimir Atanasov in relation to the intuitionistic fuzzy soft set and other related properties with examples are presented. We hope that the findings, in this paper will help researchers enhance the study on the intuitionistic fuzzy soft set theory.

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