Chapter 10
Interval-Valued Neutrosophic Subgroup Based on Interval-Valued Triple T-Norm

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ABSTRACT

Presently, interval-valued neutrosophic set theory has become an important research topic. It is widely used in various pure as well as applied fields. This chapter will provide some essential scopes to study interval-valued neutrosophic subgroup. Here the notion of interval-valued triple T-norm has been introduced, and based on that, interval-valued neutrosophic subgroup has been defined. Furthermore, some homomorphic characteristics of this notion have been studied. Additionally, based on interval-valued triple T-norm, interval-valued neutrosophic normal subgroup has been introduced and some of its homomorphic characteristics have been analyzed.

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1. INTRODUCTION

In our real physical world, many uncertainties are involved. To tackle these ambiguities, crisp set (CS) theory is not always enough. As a result, researchers needed more capable set theories. Consequently, different set theories have emerged, for instance, fuzzy set (FS) (Zadeh, Fuzzy sets, 1965), intuitionistic fuzzy set (IFS) (Atanassov, 1986), neutrosophic set (NS) (Smarandache, 1999), plithogenic set (PS) (Smarandache, 2018), etc. FS theory is capable of handling real-life uncertainties very well. Still, in some ambiguous situations, researchers need sets that are more general i.e. IFSs or sometimes even more general sets like NSs or PSs, etc. Presently, NS theory has grabbed quite lot attentions of different researchers from various fields. Presently, NS theory has become an important and fruitful research field. Furthermore, Smarandache has also developed neutrosophic measure and probability (Smarandache, 2013), calculus (Smarandache & Khalid, 2015), psychology (Smarandache, 2018), etc. At present, NS theory is used in different applied fields, for instance, in pattern recognition problem (Vlachos & Sergiadis, 2007), image segmentation (Guo & Cheng, 2009), decision making problem (Majumdar, 2015;

Table 1. Some applications of IVNS in various fields

Author & Year	Applications of IVNS in various fields
(Broumi et. al., 2015)	Introduced the concept of n-valued IVNS and mentioned how it can be applied in medical diagnosing.
(Broumi et. al., 2014)	Presented the definition of parameterized soft set in IVNS environment and its application in DMPs.
(Ye, 2014)	Defined Hamming and Euclidean distances between two IVNSs and introduced similarity measures in IVNSs with an application in DMP.
(Ye, 2014)	Introduced a correlation coefficient (improved) of single-valued NSs and extended it to a correlation coefficient between IVNSs. Further, applied it in multiple attribute DMPs.
(Zhang et. al., 2014)	Proposed a technique based on IVNS to solve multi-criteria DMPs.
(Aiwu et. al., 2015)	Proposed an aggregation operation rules (improved) for IVNS and extended the generalized weighted aggregation operator.
(Zhang et. al., 2016)	Illustrated a novel outranking method for multi-criteria DMPs with IVNSs.
(Broumi et. al., 2016)	Extended the notion of neutrosophic graph-based multi-criteria decision-making approach in interval-valued neutrosophic graph theory.
(Deli, 2017)	Proposed the concept of the soft IVNS and investigated its application in DMP.
(Yuan et. al., 2019)	Applied IVNSs in image segmentation.
(Thong et. al., 2019)	Proposed dynamic IVNS for dynamic DMP.

Abdel-Basset et. al., 2017; Abdel-Basset et. al., 2019), mobile-edge computing (Abdel-Basset et. al., 2019), neutrosophic forecasting (Abdel-Basset et. al., 2019), supply chain management (Abdel-Basset et. al., 2019; Abdel-Basset et. al., 2019), supplier selection problems (Abdel-Basset et. al., 2018; Abdel-Basset et. al., 2018), goal programming problem (Abdel-Basset et. al., 2016), multi-objective programming problem (Hezam et. al., 2015), medical diagnosis (Kumar et. al., 2015; Deli et. al., 2015), shortest path problem (Kumar, et al., 2019; Kumar et. al., 2018; Kumar et. al., 2020), transportation problem (Kumar et. al., 2019) etc. Again, gradually some other set theories, like, interval-valued FS (IVFS) (Zadeh, 1975), interval-valued IFS (IVIFS) (Atanassov, 1999) and interval-valued NS (IVNS) (Wang et. al., 2005), etc. have evolved. These notions are generalizations of CS, FS, IFS, and NSs. Presently; these set theories are extensively applied in different fields, mainly in decision-making problems (DMP). In the following Table 1 some applications of IVNSs have been discussed.

Based on FS, Rosenfeld introduced the notion of fuzzy subgroup (FSG) (Rosenfeld, 1971). Gradually, various mathematicians have developed intuitionistic fuzzy subgroup (IFSG) (Biswas, 1989), neutrosophic subgroup (NSG) (Çetkin & Aygün, 2015), etc. Furthermore, they have studied effects of homomorphism on them. Some researchers have analyzed their normal forms also. Furthermore, the notions of interval-valued fuzzy subgroup (IVFSG) (Biswas, 1994), interval-valued intuitionistic fuzzy subgroup (IVIFSG) (Aygünoğlu et. al., 2012), etc. have been defined. In addition, different researchers have studied their normal forms, homomorphic image, homomorphic pre-image, etc. Still, the concept of the interval-valued neutrosophic subgroup is undefined. Also, different algebraic aspects of IVNSGs are needed to be studied.

This Chapter has been arranged as follows: In Segment 2, literature surveys of FS, IFS, INS, FSG, IFSG, and NSG are given. In Segment 3, the notions of IVFS, IVIFS, IVFSG, IVIFSG, etc. have been mentioned. In Segment 4, interval-valued triple T-norm (IVTTN), IVNSG, normal form of IVNSG, etc. are introduced and the effects of homomorphism on these notions are mentioned. Finally, in segment 5, the conclusion has been provided and some scopes of future researches are given.

2. LITERATURE SURVEY

In this segment, some essential notions, like, FS, IFS, NS, FSG, IFSG, NSG, level set, level subgroup, etc., are discussed and also, some of their basic fundamental properties are given. All these notions play vital roles in the development of IVNSG.

Definition 2.1 (Zadeh, 1965) Let J=[0,1]. A FS σ of a CS M is a mapping from M to J i.e σ: $M \to J$.

Definition 2.2 (Atanassov, 1986) A IFS **γ** of a CS M is denoted as $\gamma = \{(k, t_\gamma(k), f_\gamma(k)) : k \in M\}$, where both t_γ and f_γ are FSs of R and $\forall k \in M$ t_γ and f_γ satisfy the criteria $1 \geq t_\gamma(k) + f_\gamma(k) \geq 0$.

Definition 2.3 (Smarandache, 1999) A NS η of a CS M is denoted as $\eta = \{(s, t_\eta(s), i_\eta(s), f_\eta(s)); s \in M\}$, where $f_\eta, i_\eta, t_\eta : M \to]^-0, 1^+[$ are the respective degree of falsity, indeterminacy, and truth and of any element $k \in R$. Here $\forall s \in M$, f_η, i_η and t_η satisfy the criteria $3+ \geq f_\eta(s) + i_\eta(s) + t_\eta(s) \geq {}^-0$.

Definition 2.4 (Zadeh, 1965) Let α be a FS of M. Then $\forall t \in J$ the sets $\alpha_t = \{k \in M : \alpha(k) \geq t\}$ are called level subsets of α.

2.1. Fuzzy Subgroup, Intuitionistic Fuzzy Subgroup and Neutrosophic Subgroup

Definition 2.5 (Rosenfeld, 1971) A FS α of a crisp group M is called a FSG of R iff $\forall k, s \in M$, conditions given below are fulfilled:

1. $\alpha(ks) \geq \min\{\alpha(k), \alpha(s)\}$
2. $\alpha(s^{-1}) \geq \alpha(s)$.

Here $\alpha(s^{-1}) = \alpha(s)$ and $\alpha(s) \leq \alpha(e)$ (e is the neutral element of M). Also, in the above definition if only condition (i) is satisfied by α then we call it a fuzzy subgroupoid.

Theorem 2.1 (Rosenfeld, 1971) α is a FSG of M iff $\forall k, s \in R$ $\boldsymbol{\alpha}(ks^{-1}) \geq \min\{\alpha(k), \alpha(s)\}$.

Definition 2.6 (Das, 1981) Suppose α is a FSG of a group M. Then $\forall t \in J$ the level subgroups of α are α_t, where $\alpha(e) \geq t$.

Definition 2.7 (Biswas, 1989)An IFS $\boldsymbol{\gamma} = \{(k, t_\gamma(k), f_\gamma(k)) : k \in M\}$ of a crisp set M is called an IFSG of M iff $\forall k, s \in M$

1. $t_\gamma(ks^{-1}) \geq \min\{t_\gamma(k), t_\gamma(s)\}$
2. $f_\gamma(ks^{-1}) \leq \max\{f_\gamma(k), f_\gamma(s)\}$

The collection of all IFSG of M will be denoted as IFSG(M).

Definition 2.8 (Çetkin & Aygün, 2015) Let M be a group and δ be a NS of M. δ is called a NSG of M iff the conditions given below are fulfilled:

1. $\delta(k\bullet s) \geq \min\{\delta(k),\delta(s)\}$, i.e. $t_\delta(k\cdot s) \geq \min\{t_\delta(k),t_\delta(s)\}$, $i_\delta(k\cdot s) \geq \min\{i_\delta(k),i_\delta(s)\}$ and $f_\delta(k\cdot s) \leq \max\{f_\delta(k),f_\delta(s)\}$
2. $\delta(s^{-1}) \geq \delta(s)$ i.e. $t_\delta(s^{-1}) \geq t_\delta(s)$, $i_\delta(s^{-1}) \geq i_\delta(s)$ and $f_\delta(s^{-1}) \leq f_\delta(s)$

The collection of all NSG will be denoted as NSG(R). Here notice that t_δ and i_δ are following Definition 2.5 i.e. both of them are actually FSGs of R.

Example 2.1 (Çetkin & Aygün, 2015) Suppose $M=\{1, -1, i, -i\}$ and δ is a NS of M, where $\delta= \{(1, 0.6, 0.5, 0.4), (-1, 0.7, 0.4, 0.3), (i, 0.8, 0.4, 0.2), (-i, 0.8, 0.4, 0.2)\}$. Notice that $\delta \in$ NSG(M).

Theorem 2.2 (Çetkin & Aygün, 2015) Let M be a group and δ be a NS of M. Then $\delta \in$ NSG(M) iff $\forall k, s\in M$ $\delta(k\cdot s^{-1}) \geq \min\{\delta(k), \delta(s)\}$.

Theorem 2.3 (Çetkin & Aygün, 2015) $\delta \in$ NSG(M) iff $\forall p \in [0,1]$ the p-level sets $(t_\delta)_p$, $(i_\delta)_p$ and p-lower-level set $(\overline{f_\delta})_p$ are CSGs of M.

Definition 2.9 (Çetkin & Aygün, 2015) Let M be a group and δ be a NS of M. Here δ is called a neutrosophic normal subgroup (NNSG) of M iff $\forall k,s\in M$ $\delta(k\cdot s\cdot k^{-1}) \leq \delta(s)$ i.e. $t_\delta(k\cdot s\cdot k^{-1}) \leq t_\delta(s)$, $i_\delta(k\cdot s\cdot k^{-1}) \leq i_\delta(s)$ and $f_\delta(k\cdot s\cdot k^{-1}) \geq f_\delta(s)$.

The collection of all NNSGs of M will be denoted as NNSG(M).

Definition 2.10 (Anthony & Sherwood, 1979) A FS α of M is said to have supremum property if for any $\alpha' \subseteq \alpha$ $\exists k_0 \in \alpha'$ such that $\alpha(k_0) = \sup_{k\in\alpha'} \alpha(k)$.

Theorem 2.4 (Anthony & Sherwood, 1979) Suppose α is a fuzzy subgroupoid of M based on a continuous TN T and l be a homomorphism on M, then the image (supremum image) of α is a fuzzy subgroupoid on $l(M)$ based on T.

Theorem 2.5 (Rosenfeld, 1971) Homomorphic image or pre-image of any FSG having supremum property is a FSG.

Theorem 2.6 (Sharma, 2011) Let M_1 and M_2 are two crisp groups. Also, suppose l is a homomorphism of M_1 into M_2 then preimage of an IFSG γ of M_2 i.e. $l^{-1}(\gamma)$ is an IFSG of M_1.

Theorem 2.7 (Sharma, 2011) Let l be a surjective homomorphism of a group M_1 to another group M_2, then the image of an IFSG γ of M_1 i.e. $l(\gamma)$ is an IFSG of M_2.

Theorem 2.8 (Çetkin & Aygün, 2015) Homomorphic image or pre-image of any NSG is a NSG.

Theorem 2.9 (Çetkin & Aygün, 2015) Let $\delta \in$ NNSG(M) and l be a homomorphism on M. Then the homomorphic pre-image of δ i.e. $l^{-1}(\delta) \in$ NNSG(M).

Theorem 2.10 (Çetkin & Aygün, 2015) Let $\delta \in$ NNSG(M) and l be a surjective homomorphism on M. Then the homomorphic image of δ i.e. $l(\delta)\in$NNSG(M).

Table 2. Significance and influences of some authors in FSG, IFSG, and NSG

Author and Year	Different contributions in FSG, IFSG, and NSG
(Rosenfeld, 1971)	Introduced FSG.
(Das, 1981)	Introduced level subgroup.
(Anthony & Sherwood, 1979)	Introduced FSG using general T-norm.
(Anthony & Sherwood, 1982)	Introduced subgroup generated and function generated FSG.
(Sherwood, 1983)	Studied product of FSGs.
(Mukherjee & Bhattacharya, 1984)	Introduced fuzzy normal subgroups and cosets.
(Biswas, 1989)	Introduced IFSG.
(Eroğlu, 1989)	Studied homomorphic image of FSG.
(Hur et. al., 2003)	Investigated some properties of IFSGs and inutionistic fuzzy subrings.
(Hur et. al., 2004)	Defined normal version of IFSG and intuitionistic fuzzy cosets.
(Sharma, 2011)	Studied homomorphism of IFSG.
(Çetkin & Aygün, 2015)	Introduced NSG and NNSG and studied some of their fundamental properties by introducing homomorphism.

In the following Table 2, some sources have been mentioned which have some major contributions in the fields of FSG, IFSG, and NSG.

2.2. A List of Abbreviations

CS signifies “crisp set”.
FS signifies “fuzzy set”.
IFS signifies “intuitionistic fuzzy set”.
NS signifies “neutrosophic set”.
FSG signifies “fuzzy subgroup”.
IFSG signifies “intuitionistic fuzzy subgroup”.
NSG signifies “neutrosophic subgroup”.
TN signifies “T-norm”.
TC signifies “T-conorm”.
IVTN signifies “interval-valued T-norm”.
IVTC signifies “interval-valued T-conorm”.
IVDTN signifies “interval-valued double T-norm”.
IVTTN signifies “interval-valued triple T-norm”.
IVFS signifies “interval-valued fuzzy set”.
IVIFS signifies “interval-valued intuitionistic fuzzy set”.
IVNS signifies “interval-valued neutrosophic set”.

IVFSG signifies "interval-valued fuzzy subgroup".
IVIFSG signifies "interval-valued intuitionistic fuzzy subgroup".
IVNSG signifies "interval-valued neutrosophic subgroup".
IVIFNSG signifies "interval-valued intuitionistic fuzzy normal subgroup".
IVNNSG signifies "interval-valued neutrosophic normal subgroup".

2.3. Motivation of the Work

So far, IVFSG and IVIFSG have grabbed a lot of attention and hence, as a result, as a result, it has yielded a lot of promising research fields. Some researchers have introduced functions in the environments of IVFSG and IVIFSG. Furthermore, they have introduced homomorphism in IVFSG and IVIFSG environments and studied some of their fundamental algebraic properties. IVNSG is relatively new and may become fruitful research field in near future. Also, the notion of IVNNSG is needed to be introduced. Furthermore, functions are needed to be introduced in the interval-valued neutrosophic environment and some homomorphic characteristics of IVNSG and IVNNSG are needed to be introduced. In this chapter, the subsequent research gaps are discussed:

- Still, the notion of IVNSG is undefined.
- Homomorphic image and preimage of IVNSG are needed to be studied.
- Still, the notion of IVNNSG is undefined.
- Also, some homomorphic characteristics of IVNNSG are needed to be analyzed.

Therefore, this inspires us to introduce and develop these notions of IVNSG and IVNNSG and analyze some of their algebraic characteristics.

2.4. Contribution of the Work

On the basis of the above gaps, the purpose of this chapter is to give some important definitions, examples and, theories in the field of IVNSG. Also, function has been introduced in interval-valued neutrosophic environment and some homomorphic properties of IVNSG and IVNNSG are discussed properly. The following are some goals that are planned and accomplished during this research work:

- To define IVNSG and study its algebraic properties.
- To define IVNNSG and study its algebraic properties.
- To introduce a function in interval-valued neutrosophic environment.
- To study some properties of homomorphic images and preimages of IVNSG and IVNNSG.

3. DESCRIPTION OF THE WORK

3.1. Research Problem

Until now, several researchers have studied different fundamental properties and algebraic structures of FSG, IFSG, as well as NSG. Again, some researchers have introduced IVFSG, IVIFSG and analyzed their fundamental properties. It is known that homomorphism preserves algebraic structures of any entity. Therefore, it is an essential tool to study some fundamental algebraic properties. Hence, several researchers have introduced and studied homomorphism in the environments of FSG, IFSG, NSG, IVFSG, IVIFSG, etc. In addition, some researchers have introduced the normal forms of FSG, IFSG, NSG, IVFSG, IVIFSG and studied their homomorphic properties. Until now, the notion of IVNSG is undefined and unexplored. Also, the normal form of IVNSG is undefined. Hence, these notions are yet to be introduced. Furthermore, the effects of homomorphism on these notions i.e. fundamental properties of homomorphic images and preimages of these notions are needed to be analyzed.

In this chapter, these essential notions of IVNSG and its normal form have been introduced and analyzed with proper examples. In the following subsection, some important notions have been discussed, which were introduced earlier.

3.1.1. Preliminaries

In this segment, the notions of interval number, IVFS, IVIFS, IVFSG, IVDTN, IVIFSG, IVTTN, etc. have been discussed. These notions are essential for introducing IVNSG.

Definition 3.1 Let J=[0,1]. An interval number of J is denoted as $\overline{g} = [g^-, g^+]$, where $0 \le g^- \le g^+ \le 1$.

The set of all the interval numbers of J will be denoted as ρ(J) where $\rho(J) = \{\overline{g} = [g^-, g^+] : g^- \le g^+, g^-, g^+ \in J\}$.

Again, $\forall g \in J$, g=[g, g] ∈ ρ(J) i.e. interval numbers are more general than ordinary numbers.

Let $\forall i$, $\overline{u}_i = [u_i^-, u_i^+] \in J$. Then supremum and infimum of $\overline{u}_i$ are defined as:

$$\sup(\overline{u}_i) = [\vee u_i^-, \vee u_i^+] \text{ and } \inf(\overline{u}_i) = [\wedge u_i^-, \wedge u_i^+].$$

Also, let

$\bar{g}=[g^-,g^+]\in\rho(J)$ and $\bar{u}=[u^-,u^+]\in\rho(J)$,

then the subsequent are true:

1. $\bar{g}\le\bar{u}$ iff $g^- \le u^-$ and $g^+ \le u^+$.
2. $\bar{g}=\bar{u}$ iff $g^-=u^-$ and $g^+=u^+$.
3. $\bar{g}<\bar{u}$ iff $g^- \le u^-$ and $g^{+1}u^+$.

Definition 3.2 (Zadeh, 1975) Let *M* be crisp set, then the mapping $\bar{\mu}:M\to\rho(J)$ is called an IVFS of *M*.

A set of all IVFS of *M* is denoted as IVFS(*M*). For each $\bar{\mu}\in \text{IVFS}(M)$, $\bar{\mu}^-(k)\le\bar{\mu}^+(k)$ for all $k\in M$. Here, $\bar{\mu}^-(k)$ and $\bar{\mu}^+(k)$ are fuzzy sets of $\bar{\mu}$. Also, Let $(\bar{g},\bar{u}_i)\in\rho(J)\times\rho(J)$, where $\bar{g}_i=[g_i^-,g_i^+]$ and $\bar{u}_i=[u_i^-,u_i^+]$ with $g_i^+ + u_i^+\le 1$, for all *i*. Then supremum and infimum $(\bar{g}_i,\bar{u}_i)$ are defined as:

(1.) $\bigwedge_{i\in\lambda}(\bar{g}_i,\bar{u}_i)=\left(\bigwedge_{i\in\lambda}\bar{g}_i,\bigvee_{i\in\lambda}\bar{u}_i\right)=\left([\bigwedge_{i\in\lambda}g_i^-,\bigwedge_{i\in\lambda}g_i^+],[\bigvee_{i\in\lambda}u_i^+,\bigvee_{i\in\lambda}u_i^+]\right)$

(2.) $\bigvee_{i\in\lambda}(\bar{g}_i,\bar{u}_i)=\left(\bigvee_{i\in\lambda}\bar{g}_{\boldsymbol{i}},\bigwedge_{i\in\lambda}\bar{u}_{\boldsymbol{i}}\right)=\left([\bigvee_{i\in\lambda}g_i^-,\bigvee_{i\in\lambda}g_i^+],[\bigwedge_{i\in\lambda}u_i^+,\bigwedge_{i\in\lambda}u_i^+]\right)$

Again, for all

$(\bar{g}_1,\bar{u}_1),(\bar{g}_2,\bar{u}_2)\in\rho(J)\times\rho(J)$,

with

$(\bar{g}_1,\bar{u}_1)=\left([g_1^-,g_1^+],[u_1^-,u_1^+]\right),(\bar{g}_2,\bar{u}_2)=\left([g_2^-,g_2^+],[u_2^-,u_2^+]\right)$,

the subsequent are true:

1. $(\bar{g}_1,\bar{u}_1)\le(\bar{g}_2,\bar{u}_2)$ iff $\bar{g}_1\le\bar{g}_2$ and $\bar{u}_1\ge\bar{u}_2$,
2. $(\bar{g}_1,\bar{u}_1)=(\bar{g}_2,\bar{u}_2)$ iff $\bar{g}_1=\bar{g}_2$ and $\bar{u}_1=\bar{u}_2$,
3. $(\bar{g}_1,\bar{u}_1)<(\bar{g}_2,\bar{u}_2)$ iff $\bar{g}_1\le\bar{g}_2,\bar{u}_1\ge\bar{u}_2$ and $\bar{g}_1\ne\bar{g}_2,\bar{u}_1\ne\bar{u}_2$.

Definition 3.3 (Atanassov, 1999) Let *M* be a crisp set, then a mapping $\tilde{\gamma}:M\to\rho(J)\times\rho(J)$ defined as $\tilde{\gamma}(k)=(\bar{t}_{\tilde{\gamma}}(k),\ \bar{f}_{\tilde{\gamma}}(k))$, with $\bar{t}_{\tilde{\gamma}}^+(k)+\bar{f}_{\tilde{\gamma}}^+(k)\le 1$, for all $k\in M$ is called an IVIFS of *M*.

In the above definition, both $\bar{t}_{\tilde{\gamma}}(k)$ and $\bar{f}_{\tilde{\gamma}}(k)$ are IVFSs of *M*. A set of all IVIFSs of *M* will be denoted as IVFS(*M*).

Definition 3.4 (Mondal & Samanta, 2001) Suppose M_1 and M_2 are two crisp sets and l: $M_1\to M_2$ be a function. Let $\tilde{\gamma}_1\in\text{IVIFS}(M_1)$ and $\tilde{\gamma}_2\in\text{IVIFS}(M_2)$. Then $\forall k\in M_1$ the image of $\tilde{\gamma}_1$ i.e. $l(\tilde{\gamma}_1)$ is denoted as $l(\tilde{\gamma}_1)(s)=\left(l(\bar{t}_{\tilde{\gamma}_1})(s),l(\bar{f}_{\tilde{\gamma}_1})(s)\right)$ and $\forall s\in M_2$ the preimage of $\tilde{\gamma}_2$ i.e. $l^{-1}(\tilde{\gamma}_2)$ is denoted as $l^{-1}(\tilde{\gamma}_2)(k)=\tilde{\gamma}_2(l(k))$. where

$$l(\tilde{\gamma}_1)(s) = \left[\bigvee_{k \in l^{-1}(s)} (\bar{t}_{\tilde{\gamma}_1})(k), \bigwedge_{k \in l^{-1}(s)} (\bar{f}_{\tilde{\gamma}_1})(k) \right]$$
$$= \left[[l(t^-_{\tilde{\gamma}_1})(s), l(t^+_{\tilde{\gamma}_1})(s)], [l(f^-_{\tilde{\gamma}_1})(s), l(f^+_{\tilde{\gamma}_1})(s)] \right]$$

and

$$l^{-1}(\tilde{\gamma}_2)(k) = \left[[l^{-1}(t^-_{\tilde{\gamma}_2})(k), l^{-1}(t^+_{\tilde{\gamma}_2})(k)], [l^{-1}(f^-_{\tilde{\gamma}_2})(k), l^{-1}(f^+_{\tilde{\gamma}_2})(k)] \right]$$
$$= \left[[t^-_{\tilde{\gamma}_2}(l(k)), t^+_{\tilde{\gamma}_2}(l(k))], [f^-_{\tilde{\gamma}_2}(l(k)), f^+_{\tilde{\gamma}_2}(l(k))] \right]$$

Definition 3.5 (Gupta & Qi, 1991) A function T: $J \to J$ is called a TN iff $\forall k,s,t \in J$, conditions given below are fulfilled:

(1.) $T(k, 1) = k$

(2.) $T(k, s) = T(s, k)$

(3.) $T(k, s) \leq T(t, s)$ if $k \leq t$

(4.) $T(k, T(s, t)) = T(T(k, s), t)$

Notice that, T is an idempotent TN iff T is minimum TN or $T = \wedge$.

Definition 3.6 (Klement et. al., 2013) Suppose T is a TN, then the function $\bar{T}: \rho(J) \times \rho(J) \to \rho(J)$ defined as $\bar{T}(\bar{g}, \bar{u}) = [T(g^-, u^-), T(g^+, u^+)]$ is called an IVTN.

Notice that, $\bar{T}$ is idempotent if T is idempotent.

Definition 3.7 (Gupta & Qi, 1991) A function S: $J \to J$ is called a TC iff $\forall k,s,t \in J$, subsequent conditions are fulfilled:

(1.) $S(k,0)=k$

(2.) $S(k,s)=S(s,k)$

(3.) $S(k,s) \leq S(t,s)$ if $k \leq t$

(4.) $S(k,S(s,t))=S(S(k,s),t)$

Note that, S is an idempotent TC iff S is maximum TC or $S=\vee$.

Definition 3.8 (Klement et. al., 2013) Let S be a TC, then the mapping $\bar{S}: \rho(J) \times \rho(J) \to \rho(J)$ defined as $S(\bar{g}, \bar{u}) = [\bar{S}(g^-, u^-), S(g^+, u^+)]$ is called an IVTC.

Note that, $\bar{S}$ is idempotent if S is idempotent.

Definition 3.9 (Aygünoğlu et. al., 2012) Suppose $\bar{T}$ is an IVTN and $\bar{S}$ is an IVTC. Then a mapping $\tilde{T}: (\rho(J) \times \rho(J))^2 \to \rho(J) \times \rho(J)$ denoted as $\tilde{T}((\bar{g}_1, \bar{u}_1), (\bar{g}_2, \bar{u}_2)) = (\bar{T}(\bar{g}_1, \bar{g}_2), \bar{S}(\bar{u}_1, \bar{u}_2))$ is called an IVDTN.

Note that, $\tilde{T}$ is idempotent if both $\bar{T}$ and $\bar{S}$ are idempotent.

Definition 3.10 (Aygünoğlu et. al., 2012) Let *M* be a crisp group. An IVIFS $\tilde{\gamma}=\{(s,\overline{t}_{\tilde{\gamma}}(s),\overline{f}_{\tilde{\gamma}}(s)):s\in M\}$ of *M* is called an IVIFSG of *M* with respect to IVDTN $\tilde{T}$ if the conditions given below are fullfilled:

1. $\tilde{\gamma}(k\cdot s)\geq \tilde{T}(\tilde{\gamma}(k),\tilde{\gamma}(s)),\ \forall k,s\in M,$
2. $\tilde{\gamma}(s^{-1})\geq \tilde{\gamma}(s),\ \forall s\in M.$

Where condition (1.) implies that,

$$\overline{t}_{\tilde{\gamma}}(k\cdot s)\geq \overline{T}(\overline{t}_{\tilde{\gamma}}(k),\overline{t}_{\tilde{\gamma}}(s)),\overline{f}_{\tilde{\gamma}}(k\cdot s)\leq \overline{S}(\overline{f}_{\tilde{\gamma}}(k),\overline{f}_{\tilde{\gamma}}(s))$$

and condition (2.) implies that, $\overline{t}_{\tilde{\gamma}}(s^{-1})\geq \overline{t}_{\tilde{\gamma}}(s),\ \overline{f}_{\tilde{\gamma}}(s^{-1})\leq \overline{f}_{\tilde{\gamma}}(s)$.

The set of all IVIFSG of a group *M* based on IVDTN $\tilde{T}$ will be mentioned as $\text{IVIFSG}(M,\tilde{T})$.

Theorem 3.1 (Aygünoğlu et. al., 2012) Suppose *M* is a group and $\tilde{\gamma}\in\text{IVIFS}(M)$. Then $\tilde{\gamma}\in\text{IVIFSG}(M,\tilde{T})$ iff $\forall k,s\in M$. $\tilde{\gamma}(k\cdot s^{-1})\geq \tilde{T}(\tilde{\gamma}(k),\tilde{\gamma}(s))$.

Theorem 3.2 (Aygünoğlu et. al., 2012) Let M_1 and M_2 be two crisp groups with l: $M_1\to M_2$ be a homomorphism and $\tilde{T}$ be a continuous IVDTN. If $\tilde{\gamma}\in\text{IVIFSG}(M_1,\tilde{T})$, then $l(\tilde{\gamma})\in\text{IVIFSG}(M_2,\tilde{T})$.

Theorem 3.3 (Aygünoğlu et. al., 2012) Suppose M_1 and M_2 are two crisp groups and l be a homomorphism from M_1 into M_2. If $\tilde{\gamma}'\in\text{IVIFSG}(M_2,\tilde{T})$, then $l^{-1}(\tilde{\gamma})\in\text{IVIFSG}(M_1,\tilde{T})$.

Definition 3.11 (Aygünoğlu et. al., 2012) Let *M* be a crisp group and $\tilde{\gamma}\in\text{IVIFSG}(M,\tilde{T})$. Then $\tilde{\gamma}$ is called an IVIFNSG of *M* with respect to IVDTN $\tilde{T}$ if $\forall k,s\in M$, $\tilde{\gamma}(k\cdot s)=\tilde{\gamma}(s\cdot k)$.

The set of all IVIFNSG of a crisp group *M* with respect to $\tilde{T}$ will be denoted as $\text{IVIFNSG}(M,\tilde{T})$.

Theorem 3.4 (Aygünoğlu et. al., 2012) Suppose M_1 and M_2 are two crisp groups and l be a homomorphism from M_1 into M_2. If $\tilde{\gamma}'\in\text{IVIFNSG}(M_2,\tilde{T})$, then $l^{-1}(\tilde{\gamma}')\in\text{IVIFNSG}(M_1,\tilde{T})$.

Theorem 3.5 (Aygünoğlu et. al., 2012) Let M_1 and M_2 be two crisp groups and l be a surjective homomorphism from M_1 into M_2. If $\tilde{\gamma}\in\text{IVIFNSG}(M_1,\tilde{T})$, then $l(\tilde{\gamma})\in\text{IVIFNSG}(M_1,\tilde{T})$.

In the following Table 3, some sources have been mentioned which have some major contributions in the fields of IVFS, IVIFS, IVFSG and IVIFSG.

Table 3. Some important contributions in the fields of IVFS, IVIFS, IVFSG, and IVIFSG

Author and Year	Different contributions in IVFS, IVIFS, IVFSG and IVIFSG
(Zadeh, 1975)	Introduced IVFS
(Biswas, 1994)	Defined IVFSG which is of Rosenfeld's nature.
(Guijun & Xiaoping, 1996)	Introduced IVSGs induced by triangular norms.
(Atanassov, 1999)	Introduced IVIFS.
(Mondal & Samanta, 1999)	Defined topology of IVFSs is and studied some of its properties.
(Davvaz, Interval-valued fuzzy subhypergroups, 1999)	Introduced the concepts of interval-valued fuzzy subhypergroup of a hypergroup.
(Li & Wang, 2000)	Introduced the notion of S_H-IVFSG.
(Mondal & Samanta, 2001)	Defined topology of IVIFSs is and studied some of its properties.
(Davvaz, 2001)	Extended the notion of fuzzy ideal of a near-ring by introducing interval-valued L-fuzzy ideal of a near-ring.
(Jun & Kim, 2002)	Introduced interval-valued fuzzy R-subgroups in near rings.
(Aygünoğlu et. al., 2012)	Defined IVDTN and using that introduced IVIFSG.

In the following section, the notion of IVNSG has been defined, which is based on IVTTN. Also, some essential homomorphic properties of IVNSG has been investigated. Furthermore, the normal form of IVNSG has been defined and its homomorphic characteristics have been studied.

4. PROPOSED NOTION OF INTERVAL-VALUED NEUTROSOPHIC SUBGROUP

Definition 4.1 Suppose $\bar{T}$ and $\bar{I}$ are two IVTNs and $\bar{F}$ be an IVTC. The function

$$\breve{T} : (\rho(J)\times\rho(J)\times\rho(J))^2 \to \rho(J)\times\rho(J)\times\rho(J)$$

denoted as

$$\breve{T}((\bar{g}_1,\bar{u}_1,\bar{t}_1),(\bar{g}_2,\bar{u}_2,\bar{t}_2)) = (\bar{T}(\bar{g}_1,\bar{g}_2),\bar{I}(\bar{u}_1,\bar{u}_2),\bar{F}(\bar{t}_1,\bar{t}_2))$$

is called an IVTTN.

Definition 4.2 Suppose M is a crisp group. An IVNS $\breve{\delta} = \{(s, \overline{t}_{\breve{\delta}}(s), \overline{i}_{\breve{\delta}}(s), \overline{f}_{\breve{\delta}}(s)) : s \in M\}$ of M is called an IVNSG of M with respect to IVTTN $\breve{T}$ if the conditions given below are fulfilled:

1. $\breve{\delta}(k \cdot s) \geq \breve{T}(\breve{\delta}(k), \breve{\delta}(s)),\ \forall\, k, s \in M,$
2. $\breve{\delta}(s^{-1}) \geq \breve{\delta}(s),\ \forall\, s \in M.$

Now, by condition (1.)

$$\overline{t}_{\breve{\delta}}(k \cdot s) \geq \overline{T}(\overline{t}_{\breve{\delta}}(k), \overline{t}_{\breve{\delta}}(s)), \overline{i}_{\breve{\delta}}(k \cdot s) \geq \overline{I}(\overline{i}_{\breve{\delta}}(k), \overline{i}_{\breve{\delta}}(s)), \overline{f}_{\breve{\delta}}(k \cdot s) \leq \overline{F}(\overline{f}_{\breve{\delta}}(k), \overline{f}_{\breve{\delta}}(s))$$

and by condition (2.) $\overline{t}_{\breve{\delta}}(s^{-1}) \geq \overline{t}_{\breve{\delta}}(s), i_{\breve{\delta}}(s^{-1}) \geq i_{\breve{\delta}}(s)$ and $\overline{f}_{\breve{\delta}}(s^{-1}) \leq \overline{f}_{\breve{\delta}}(s)$.

The set of all IVNSG of a group M with respect to an IVTTN $\breve{T}$ will be denoted as IVNSG$(M, \breve{T})$.

Example 4.1 Let $R=\{e,k,s,ks\}$ be the Klein's four group. Let

$$\breve{\delta} = \{(e,[0.1,0.3],[0.2,0.4],[0.1,0.4]),(k,[0.1,0.3],[0.2,0.3],[0.2,0.4]),\\ (s,[0.1,0.2],[0.2,0.4],[0.2,0.5]),\ (ks,[0.1,0.2],[0.2,0.3],[0.2,0.5])\}$$

be a IVNS of M. Also, let in IVTTN $\breve{T}$, the corresponding IVTNs $\overline{T}$ and $\overline{I}$ consist of minimum TN and corresponding IVTC $\overline{F}$ consists of maximum TC. In Table 4 all possible compositions of elements in $\breve{\delta}$ and their corresponding interval-valued memberships are mentioned.

Clearly, from Table 4, $\breve{\delta}$ satisfies condition (i) of Definition 4.2. Again, each element belonging to $\breve{\delta}$ is its own inverse. Hence, $\breve{\delta}$ satisfies condition (ii) of Definition 4.2. So, $\breve{\delta} \in$ IVNSG$(M, \breve{T})$.

Theorem 4.1 Let M be a group and $\breve{\delta} \in$ IVNSG$(M, \breve{T})$. Then $\forall s \in M$

1. $\breve{\delta}(s^{-1}) = \breve{\delta}(s)$ and
2. $\breve{\delta}(e) \geq \breve{\delta}(s)$, where e is the neutral element of M.

Proof:

1. From Definition 4.2, we have $\breve{\delta}(s^{-1}) \geq \breve{\delta}(s),\ \forall\, s \in M$. Again, for any $s \in M$, $\breve{\delta}(s) = \breve{\delta}((s^{-1})^{-1}) \geq \breve{\delta}(s^{-1})$. So, $\breve{\delta}(s^{-1}) = \breve{\delta}(s)$.
2. For any $k \in R$, $\breve{\delta}(e) \geq \breve{\delta}(s \cdot s^{-1}) \geq \breve{T}(\breve{\delta}(s), \breve{\delta}(s^{-1})) = \breve{T}(\breve{\delta}(s), \breve{\delta}(s)) = \breve{\delta}(s)$.

Theorem 4.2 Suppose M is a crisp group and $\breve{\delta} \in$ IVNS(M). Then $\breve{\delta} \in$ IVNSG$(M, \breve{T})$ iff $\forall k,s \in M$, $\breve{\delta}(k \cdot s^{-1}) \geq \breve{T}(\breve{\delta}(k), \breve{\delta}(s))$.

Theorem 4.3 Suppose M is a crisp group and $\breve{\delta}_1, \breve{\delta}_2 \in$ IVNSG$(M, \breve{T})$. Then $\breve{\delta}_1 \cap \breve{\delta}_2 \in$ IVNSG$(M, \breve{T})$.

Proof: Let $\breve{\delta}_1, \breve{\delta}_2 \in$ IVNSG$(M, \breve{T})$. To prove $\breve{\delta}_1 \cap \breve{\delta}_2 \in$ IVNSG$(M, \breve{T})$, it is needed to show that

Table 4. All possible compositions of elements in $\breve{\delta}$ and their interval-valued memberships

e•e	$\overline{t}_{\breve{\delta}}(e\cdot e)=\overline{t}_{\breve{\delta}}(e)\geq \overline{T}(\overline{t}_{\breve{\delta}}(e),\overline{t}_{\breve{\delta}}(e))$, $\overline{i}_{\breve{\delta}}(e\cdot e)=\overline{i}_{\breve{\delta}}(e)\geq \overline{I}(\overline{i}_{\breve{\delta}}(e),\overline{i}_{\breve{\delta}}(e))$, $\overline{f}_{\breve{\delta}}(e\cdot e)=\overline{f}_{\breve{\delta}}(e)\leq \overline{F}(\overline{f}_{\breve{\delta}}(e),\overline{f}_{\breve{\delta}}(e))$
e•k	$\overline{t}_{\breve{\delta}}(e\cdot k)=\overline{t}_{\breve{\delta}}(k)=[0.1,0.3]\geq[0.1,0.3]=\overline{T}([0.1,0.3],[0.1,0.3])=\overline{T}(\overline{t}_{\breve{\delta}}(e),\overline{t}_{\breve{\delta}}(k))$, $\overline{i}_{\breve{\delta}}(e\cdot k)=\overline{i}_{\breve{\delta}}(k)=[0.2,0.3]\geq[0.2,0.3]=\overline{I}([0.2,0.4],[0.2,0.3])=\overline{I}(\overline{i}_{\breve{\delta}}(e),\overline{i}_{\breve{\delta}}(k))$, $\overline{f}_{\breve{\delta}}(e\cdot k)=\overline{f}_{\breve{\delta}}(k)=[0.2,0.4]\leq[0.2,0.4]=\overline{F}([0.1,0.4],[0.2,0.4])=\overline{F}(\overline{f}_{\breve{\delta}}(e),\overline{f}_{\breve{\delta}}(k))$
e•s	$\overline{t}_{\breve{\delta}}(e\cdot s)=\overline{t}_{\breve{\delta}}(s)=[0.1,0.2]\geq[0.1,0.2]=\overline{T}([0.1,0.3],[0.1,0.2])=\overline{T}(\overline{t}_{\breve{\delta}}(e),\overline{t}_{\breve{\delta}}(s))$, $\overline{i}_{\breve{\delta}}(e\cdot s)=\overline{i}_{\breve{\delta}}(s)=[0.2,0.4]\geq[0.2,0.4]=\overline{I}([0.2,0.4],[0.2,0.4])=\overline{I}(\overline{i}_{\breve{\delta}}(e),\overline{i}_{\breve{\delta}}(s))$, $\overline{f}_{\breve{\delta}}(e\cdot s)=\overline{f}_{\breve{\delta}}(s)=[0.2,0.5]\leq[0.2,0.5]=\overline{F}([0.1,0.4],[0.2,0.5])=\overline{F}(\overline{f}_{\breve{\delta}}(e),\overline{f}_{\breve{\delta}}(s))$
e•ks	$\overline{t}_{\breve{\delta}}(e\cdot ks)=\overline{t}_{\breve{\delta}}(ks)=[0.1,0.2]\geq[0.1,0.2]=\overline{T}([0.1,0.3],[0.1,0.2])=\overline{T}(\overline{t}_{\breve{\delta}}(e),\overline{t}_{\breve{\delta}}(ks))$, $\overline{i}_{\breve{\delta}}(e\cdot ks)=\overline{i}_{\breve{\delta}}(ks)=[0.2,0.3]\geq[0.2,0.3]=\overline{I}([0.2,0.4],[0.2,0.3])=\overline{I}(\overline{i}_{\breve{\delta}}(e),\overline{i}_{\breve{\delta}}(ks))$, $\overline{f}_{\breve{\delta}}(e\cdot ks)=\overline{f}_{\breve{\delta}}(ks)=[0.2,0.5]\leq[0.2,0.5]=\overline{F}([0.1,0.4],[0.2,0.5])=\overline{F}(\overline{f}_{\breve{\delta}}(e),\overline{f}_{\breve{\delta}}(ks))$
a•a=e	$\overline{t}_{\breve{\delta}}(a\cdot a)=\overline{t}_{\breve{\delta}}(e)=[0.1,0.3]\geq[0.1,0.3]=\overline{T}([0.1,0.3],[0.1,0.3])=\overline{T}(\overline{t}_{\breve{\delta}}(a),\overline{t}_{\breve{\delta}}(a))$, $\overline{i}_{\breve{\delta}}(a\cdot a)=\overline{i}_{\breve{\delta}}(e)=[0.2,0.4]\geq[0.2,0.3]=\overline{I}([0.2,0.3],[0.2,0.3])=\overline{I}(\overline{i}_{\breve{\delta}}(a),\overline{i}_{\breve{\delta}}(a))$, $\overline{f}_{\breve{\delta}}(a\cdot a)=\overline{f}_{\breve{\delta}}(e)=[0.1,0.4]\leq[0.2,0.4]=\overline{F}([0.1,0.4],[0.2,0.4])=\overline{F}(\overline{f}_{\breve{\delta}}(a),\overline{f}_{\breve{\delta}}(a))$
a•b=b•a	$\overline{t}_{\breve{\delta}}(a\cdot b)=\overline{t}_{\breve{\delta}}(b\cdot a)=[0.1,0.2]\geq[0.1,0.2]=\overline{T}([0.1,0.3],[0.1,0.2])=\overline{T}(\overline{t}_{\breve{\delta}}(a),\overline{t}_{\breve{\delta}}(b))$, $\overline{i}_{\breve{\delta}}(a\cdot b)=\overline{i}_{\breve{\delta}}(b\cdot a)=[0.2,0.4]\geq[0.2,0.3]=\overline{I}([0.2,0.3],[0.2,0.3])=\overline{I}(\overline{i}_{\breve{\delta}}(a),\overline{i}_{\breve{\delta}}(b))$, $\overline{f}_{\breve{\delta}}(a\cdot b)=\overline{f}_{\breve{\delta}}(b\cdot a)=[0.1,0.4]\leq[0.2,0.4]=\overline{F}([0.1,0.4],[0.2,0.4])=\overline{F}(\overline{f}_{\breve{\delta}}(a),\overline{f}_{\breve{\delta}}(b))$
a•ab=ab•a=b	$\overline{t}_{\breve{\delta}}(a\cdot ab)=\overline{t}_{\breve{\delta}}(b)=[0.1,0.2]\geq[0.1,0.2]=\overline{T}([0.1,0.3],[0.1,0.2])=\overline{T}(\overline{t}_{\breve{\delta}}(a),\overline{t}_{\breve{\delta}}(ab))$, $\overline{i}_{\breve{\delta}}(a\cdot ab)=\overline{i}_{\breve{\delta}}(b)=[0.2,0.4]\geq[0.2,0.3]=\overline{I}([0.2,0.3],[0.2,0.3])=\overline{I}(\overline{i}_{\breve{\delta}}(a),\overline{i}_{\breve{\delta}}(ab))$, $\overline{f}_{\breve{\delta}}(a\cdot ab)=\overline{f}_{\breve{\delta}}(b)=[0.2,0.5]\leq[0.2,0.5]=\overline{F}([0.2,0.4],[0.2,0.5])=\overline{F}(\overline{f}_{\breve{\delta}}(a),\overline{f}_{\breve{\delta}}(ab))$
b•b=e	$\overline{t}_{\breve{\delta}}(b\cdot b)=\overline{t}_{\breve{\delta}}(e)=[0.1,0.3]\geq[0.1,0.2]=\overline{T}([0.1,0.2],[0.1,0.2])=\overline{T}(\overline{t}_{\breve{\delta}}(b),\overline{t}_{\breve{\delta}}(b))$, $\overline{i}_{\breve{\delta}}(b\cdot b)=\overline{i}_{\breve{\delta}}(e)=[0.2,0.4]\geq[0.2,0.4]=\overline{I}([0.2,0.4],[0.2,0.4])=\overline{I}(\overline{i}_{\breve{\delta}}(b),\overline{i}_{\breve{\delta}}(b))$, $\overline{f}_{\breve{\delta}}(b\cdot b)=\overline{f}_{\breve{\delta}}(e)=[0.1,0.4]\leq[0.2,0.5]=\overline{F}([0.2,0.5],[0.2,0.5])=\overline{F}(\overline{f}_{\breve{\delta}}(b),\overline{f}_{\breve{\delta}}(b))$
b•ab=ab•b=a	$\overline{t}_{\breve{\delta}}(b\cdot ab)=\overline{t}_{\breve{\delta}}(a)=[0.1,0.3]\geq[0.1,0.2]=\overline{T}([0.1,0.2],[0.1,0.2])=\overline{T}(\overline{t}_{\breve{\delta}}(b),\overline{t}_{\breve{\delta}}(ab))$, $\overline{i}_{\breve{\delta}}(b\cdot ab)=\overline{i}_{\breve{\delta}}(a)=[0.2,0.3]\geq[0.2,0.3]=\overline{I}([0.2,0.4],[0.2,0.3])=\overline{I}(\overline{i}_{\breve{\delta}}(b),\overline{i}_{\breve{\delta}}(ab))$, $\overline{f}_{\breve{\delta}}(b\cdot ab)=\overline{f}_{\breve{\delta}}(a)=[0.2,0.4]\leq[0.2,0.5]=\overline{F}([0.2,0.5],[0.2,0.5])=\overline{F}(\overline{f}_{\breve{\delta}}(b),\overline{f}_{\breve{\delta}}(ab))$
ab•ab=e	$\overline{t}_{\breve{\delta}}(ab\cdot ab)=\overline{t}_{\breve{\delta}}(e)=[0.1,0.3]\geq[0.1,0.3]=\overline{T}([0.1,0.2],[0.1,0.2])=\overline{T}(\overline{t}_{\breve{\delta}}(ab),\overline{t}_{\breve{\delta}}(ab))$, $\overline{i}_{\breve{\delta}}(ab\cdot ab)=\overline{i}_{\breve{\delta}}(e)=[0.2,0.4]\geq[0.2,0.3]=\overline{I}([0.2,0.3],[0.2,0.3])=\overline{I}(\overline{i}_{\breve{\delta}}(ab),\overline{i}_{\breve{\delta}}(ab))$, $\overline{f}_{\breve{\delta}}(ab\cdot ab)=\overline{f}_{\breve{\delta}}(e)=[0.1,0.4]\leq[0.2,0.5]=\overline{F}([0.2,0.5],[0.2,0.5])=\overline{F}(\overline{f}_{\breve{\delta}}(ab),\overline{f}_{\breve{\delta}}(ab))$

$$(\overline{t}_{\breve{\delta}_1} \wedge \overline{t}_{\breve{\delta}_2})(k \cdot s^{-1}) \geq \overline{T}((\overline{t}_{\breve{\delta}_1} \wedge \overline{t}_{\breve{\delta}_2})(k), (\overline{t}_{\breve{\delta}_1} \wedge \overline{t}_{\breve{\delta}_2})(s)), (\overline{i}_{\breve{\delta}_1} \wedge \overline{i}_{\breve{\delta}_2})(k \cdot q^{-1}) \geq \overline{I}((\overline{i}_{\breve{\delta}_1} \wedge \overline{i}_{\breve{\delta}_2})(k), (\overline{i}_{\breve{\delta}_1} \wedge \overline{i}_{\breve{\delta}_2})(q))$$

and

$$(\overline{f}_{\breve{\delta}_1} \vee \overline{f}_{\breve{\delta}_2})(k \cdot s^{-1}) \leq \overline{F}((\overline{f}_{\breve{\delta}_1} \vee \overline{f}_{\breve{\delta}_2})(k), (\overline{f}_{\breve{\delta}_1} \vee \overline{f}_{\breve{\delta}_2})(s)).$$

As $\breve{\delta}_1, \breve{\delta}_2 \in \mathrm{IVNSG}(M, \breve{T})$, by Theorem 4.2,

$\breve{\delta}_1(k \cdot s^{-1}) \geq \breve{T}(\breve{\delta}_1(k), \breve{\delta}_1(s))$ and $\breve{\delta}_2(k \cdot s^{-1}) \geq \breve{T}(\breve{\delta}_2(k), \breve{\delta}_2(s))$.

Which implies,

$$\overline{t}_{\breve{\delta}_1}(k \cdot s^{-1}) \geq \overline{T}(\overline{t}_{\breve{\delta}_1}(k), \overline{t}_{\breve{\delta}_1}(s)), \overline{i}_{\breve{\delta}_1}(k \cdot s^{-1}) \geq \overline{I}(\overline{i}_{\breve{\delta}_1}(k), \overline{i}_{\breve{\delta}_1}(s)), \overline{f}_{\breve{\delta}_1}(k \cdot s^{-1}) \leq \overline{F}(\overline{f}_{\breve{\delta}_1}(k), \overline{f}_{\breve{\delta}_1}(s))$$

and

$$\overline{t}_{\breve{\delta}_2}(k \cdot s^{-1}) \geq \overline{T}(\overline{t}_{\breve{\delta}_2}(k), \overline{t}_{\breve{\delta}_2}(s)), \overline{i}_{\breve{\delta}_2}(k \cdot s^{-1}) \geq \overline{I}(\overline{i}_{\breve{\delta}_2}(k), \overline{i}_{\breve{\delta}_2}(s)), \overline{f}_{\breve{\delta}_2}(k \cdot s^{-1}) \leq \overline{F}(\overline{f}_{\breve{\delta}_2}(k), \overline{f}_{\breve{\delta}_2}(s)).$$

So,

$$\overline{t}_{\breve{\delta}_1}(k \cdot s^{-1}) \wedge \overline{t}_{\breve{\delta}_2}(k \cdot s^{-1}) \geq \overline{T}(\overline{t}_{\breve{\delta}_1}(k), \overline{t}_{\breve{\delta}_1}(s)) \wedge \overline{T}(\overline{t}_{\breve{\delta}_2}(k), \overline{t}_{\breve{\delta}_2}(s)) \Rightarrow (\overline{t}_{\breve{\delta}_1} \wedge \overline{t}_{\breve{\delta}_2})(k \cdot s^{-1}) \geq \overline{T}((\overline{t}_{\breve{\delta}_1} \wedge \overline{t}_{\breve{\delta}_2})(k), (\overline{t}_{\breve{\delta}_1} \wedge \overline{t}_{\breve{\delta}_2})(s)).$$

Similarly, the following can be proved:

$$(\overline{i}_{\breve{\delta}_1} \wedge \overline{i}_{\breve{\delta}_2})(k \cdot s^{-1}) \geq \overline{I}((\overline{i}_{\breve{\delta}_1} \wedge \overline{i}_{\breve{\delta}_2})(k), (\overline{i}_{\breve{\delta}_1} \wedge \overline{i}_{\breve{\delta}_2})(s))$$

and

$$(\overline{f}_{\breve{\delta}_1} \vee \overline{f}_{\breve{\delta}_2})(k \cdot s^{-1}) \leq \overline{F}((\overline{f}_{\breve{\delta}_1} \vee \overline{f}_{\breve{\delta}_2})(k), (\overline{f}_{\breve{\delta}_1} \vee \overline{f}_{\breve{\delta}_2})(s)).$$

Hence, $\breve{\delta}_1 \cap \breve{\delta}_2 \in \mathrm{IVNSG}(M, \breve{T})$.

Theorem 4.4 Suppose *M* be a group and $\breve{\delta} \in$ IVNS(*M*). Then $\breve{\delta} \in \mathrm{IVNSG}(M, \breve{T})$ iff for every $[g_1, u_1]$, $[g_2, u_2]$ and $[g_3, u_3] \in \rho(J)$ with $u_1 + u_2 + u_3 \leq 1$, $(\breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])} \neq \phi)$ $\breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])}$ is a crisp subgroup of *M*.

Proof: Suppose $\breve{\delta} \in \text{IVNSG}(M, \breve{T})$ and $k, s \in \breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])}$, for arbitrary $[g_1,u_1]$, $[g_2,u_2]$ and $[g_3, u_3] \in \rho(J)$ with $u_1+u_2+u_3 \leq 1$. Then we have

$$\bar{t}_{\breve{\delta}}(k) \geq [g_1,u_1], \bar{i}_{\breve{\delta}}(k) \geq [g_2,u_2], \bar{f}_{\breve{\delta}}(k) \leq [g_3,u_3] \text{ and } \bar{t}_{\breve{\delta}}(s) \geq [g_1,u_1], \bar{i}_{\breve{\delta}}(s) \geq [g_2,u_2], \bar{f}_{\breve{\delta}}(s) \leq [g_3,u_3].$$

Now, by Theorem 4.2, we have

$$\begin{aligned}
\breve{\delta}(k \cdot s^{-1}) &\geq \breve{T}(\breve{\delta}(k), \breve{\delta}(s)) \\
&= \breve{T}((\bar{t}_{\breve{\delta}}(k), \bar{i}_{\breve{\delta}}(k), \bar{f}_{\breve{\delta}}(k)), (\bar{t}_{\breve{\delta}}(s), \bar{i}_{\breve{\delta}}(s), \bar{f}_{\breve{\delta}}(s))) \\
&= (\bar{T}(\bar{t}_{\breve{\delta}}(k), \bar{t}_{\breve{\delta}}(s)), \bar{I}(\bar{i}_{\breve{\delta}}(k), \bar{i}_{\breve{\delta}}(s)), \bar{F}(\bar{f}_{\breve{\delta}}(k), \bar{f}_{\breve{\delta}}(s))) \\
&\geq (\bar{T}([g_1,u_1],[g_1,u_1]), \bar{I}([g_2,u_2],[g_2,u_2]), \bar{F}([g_3,u_3],[g_3,u_3])) \\
&= ([g_1,u_1],[g_2,u_2],[g_3,u_3])
\end{aligned}$$

So, from $k \cdot s^{-1} \in \breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])}$. Hence, $\breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])}$ is a crisp subgroup of *M*.

Conversely, let $\exists k_0, s_0 \in M$ such that $\breve{\delta}(k_0 \cdot s_0^{-1}) \ngeq \breve{T}(\breve{\delta}(k_0), \breve{\delta}(s_0))$ i.e $\bar{t}_{\breve{\delta}}(k_0 \cdot s_0^{-1}) \ngeq \bar{T}(\bar{t}_{\breve{\delta}}(k_0), \bar{t}_{\breve{\delta}}(s_0))$ or $\bar{i}_{\breve{\delta}}(k_0 \cdot s_0^{-1}) \ngeq \bar{I}(\bar{i}_{\breve{\delta}}(k_0), \bar{i}_{\breve{\delta}}(s_0))$ or $\bar{f}_{\breve{\delta}}(k_0 \cdot s_0^{-1}) \nleq \bar{F}(\bar{f}_{\breve{\delta}}(k_0), \bar{f}_{\breve{\delta}}(s_0))$.

Without losing any generality, let $\bar{t}_{\breve{\delta}}(k_0 \cdot s_0^{-1}) \ngeq \bar{T}(\bar{t}_{\breve{\delta}}(k_0), \bar{t}_{\breve{\delta}}(s_0))$, then

$$\bar{t}^{-}_{\breve{\delta}}(k_0 \cdot s_0^{-1}) < \bar{T}(\bar{t}^{-}_{\breve{\delta}}(k_0), \bar{t}^{-}_{\breve{\delta}}(s_0)) \text{ or } \bar{t}^{+}_{\breve{\delta}}(k_0 \cdot s_0^{-1}) < \bar{T}(\bar{t}^{+}_{\breve{\delta}}(k_0), \bar{t}^{+}_{\breve{\delta}}(s_0)).$$

Let us assume $\bar{t}^{-}_{\breve{\delta}}(k_0 \cdot s_0^{-1}) < \bar{T}(\bar{t}^{-}_{\breve{\delta}}(k_0), \bar{t}^{-}_{\breve{\delta}}(s_0))$.

Again, let $\bar{t}_{\breve{\delta}}(k_0) = [n_1,t_1], \bar{t}_{\breve{\delta}}(s_0) = [n_2,t_2]$. If $[g_1,u_1] = \bar{T}([n_1,t_1],[n_2,t_2])$, then $k_0 \cdot s_0^{-1} \notin \breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])}$ for any $[g_2, u_2]$, $[g_3, u_3] \in \rho(J)$. Again,

$$\bar{t}_{\breve{\delta}}(k_0) = [n_1,t_1] \geq \bar{T}([n_1,t_1],[n_2,t_2]) = [g_1,u_1] \text{ and } \bar{t}_{\breve{\delta}}(k_0) = [n_2,t_2] \geq \bar{T}([n_1,t_1],[n_2,t_2]) = [g_1,u_1].$$

Now, by choosing $[g_2, u_2]$ and $[g_3, u_3]$, satisfying the conditions

$$\bar{i}_{\breve{\delta}}(k_0) \geq [g_2,u_2], \bar{i}_{\breve{\delta}}(k_0) \geq [g_2,u_2], \bar{f}_{\breve{\delta}}(k_0) \leq [g_3,u_3] \text{ and } \bar{f}_{\breve{\delta}}(k_0) \leq [g_3,u_3],$$

it can be proved that, $k_0, s_0^{-1} \in \breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])}$, which contradicts the fact that $\breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])}$ is a crisp subgroup of *M*.

Similarly,forthecasesof $\bar{i}_{\breve{\delta}}(k_0 \cdot s_0^{-1}) \ngeq \bar{I}(\bar{i}_{\breve{\delta}}(k_0), \bar{i}_{\breve{\delta}}(s_0))$ or $\bar{f}_{\breve{\delta}}(k_0 \cdot s_0^{-1}) \nleq \bar{F}(\bar{f}_{\breve{\delta}}(k_0), \bar{f}_{\breve{\delta}}(s_0))$ the same conclusion as above can be drawn.

4.1. Homomorphism on Interval-valued Neutrosophic Subgroup

In Definition 3.4, image and inverse image of IVNSs under any function has been introduced. Extending Definition 3.4 in neutrosophic environment, the following Definition 4.3 can be given:

Definition 4.3 Suppose M_1 and M_2 are two crisp sets and l: $M_1 \to M_2$ be a function. Let $\breve{\delta}_1 \in \text{IVNS}(M_1)$ and $\breve{\delta}_2 \in \text{IVIFS}(M_2)$. Then $\forall k \in M_1$ and $\forall s \in M_2$, the image of $\breve{\delta}_1$ i.e. $l(\breve{\delta}_1)$ is denoted as $l(\breve{\delta}_1)(s) = \left(l(\overline{t}_{\breve{\delta}_1})(s), l(\overline{f}_{\breve{\delta}_1})(s)\right)$ and the preimage of $\breve{\delta}_2$ i.e. $l^{-1}(\breve{\delta}_2)$ is denoted as $l^{-1}(\breve{\delta}_2)(k) = \breve{\delta}_2(l(k))$, where

$$\begin{aligned} l(\breve{\delta}_1)(s) &= \left[\bigvee_{k \in l^{-1}(s)} (\overline{t}_{\breve{\delta}_1})(k), \bigvee_{k \in l^{-1}(s)} (\overline{i}_{\breve{\delta}_1})(k), \bigwedge_{k \in l^{-1}(s)} (\overline{f}_{\breve{\delta}_1})(k)\right] \\ &= \left[[l(t^-_{\breve{\delta}_1})(s), l(t^+_{\breve{\delta}_1})(s)], [l(i^-_{\breve{\delta}_1})(s), l(i^+_{\breve{\delta}_1})(s)], [l(f^-_{\breve{\delta}_1})(s), l(f^+_{\breve{\delta}_1})(s)]\right] \end{aligned}$$

and

$$\begin{aligned} l^{-1}(\breve{\delta}_2)(k) &= \left[[l^{-1}(t^-_{\breve{\delta}_2})(k), l^{-1}(t^+_{\breve{\delta}_2})(k)], [l^{-1}(i^-_{\breve{\delta}_2})(k), l^{-1}(i^+_{\breve{\delta}_2})(k)], [l^{-1}(f^-_{\breve{\delta}_2})(k), l^{-1}(f^+_{\breve{\delta}_2})(k)]\right] \\ &= \left[[t^-_{\breve{\delta}_2}(l(k)), t^+_{\breve{\delta}_2}(l(k))], [i^-_{\breve{\delta}_2}(l(k)), i^+_{\breve{\delta}_2}(l(k))], [f^-_{\breve{\delta}_2}(l(k)), f^+_{\breve{\delta}_2}(l(k))]\right] \end{aligned}$$

Theorem 4.5 Let M_1 and M_2 be two crisp groups with l: $M_1 \to M_2$ be a homomorphism and $\breve{T}$ be a continuous IVTTN. If $\breve{\delta} \in \text{IVNSG}(M_1, \breve{T})$, then $l(\breve{\delta}) \in \text{IVNSG}(M_2, \breve{T})$.

Proof: Let for some $k_1, k_2 \in M_1$, $l(k_1) = s_1$ and $l(k_2) = s_2$. Then

$$\begin{aligned} l(\breve{\delta})(s_1 \cdot s_2^{-1}) &= \left(l(\overline{t}_{\breve{\delta}})(s_1 \cdot s_2^{-1}), l(\overline{i}_{\breve{\delta}})(s_1 \cdot s_2^{-1}), l(\overline{f}_{\breve{\delta}})(s_1 \cdot s_2^{-1})\right) \\ &= \left(\bigvee_{l(p)=s_1 \cdot s_2^{-1}} \overline{t}_{\breve{\delta}}(p), \bigvee_{l(p)=s_1 \cdot s_2^{-1}} \overline{i}_{\breve{\delta}}(p), \bigwedge_{l(p)=s_1 \cdot s_2^{-1}} \overline{f}_{\breve{\delta}}(p)\right) \\ &\geq \left(\overline{t}_{\breve{\delta}}(k_1 \cdot k_2^{-1}), \overline{i}_{\breve{\delta}}(k_1 \cdot k_2^{-1}), \overline{f}_{\breve{\delta}}(k_1 \cdot k_2^{-1})\right) \end{aligned}$$

Here,

$$\overline{t}_{\breve{\delta}}(k_1 \cdot k_2^{-1}) \geq \overline{T}(\overline{t}_{\breve{\delta}}(k_1), \overline{t}_{\breve{\delta}}(k_2)), \overline{i}_{\breve{\delta}}(k_1 \cdot k_2^{-1}) \geq \overline{I}(\overline{i}_{\breve{\delta}}(k_1), \overline{i}_{\breve{\delta}}(k_2)), \overline{f}_{\breve{\delta}}(k_1 \cdot k_2^{-1}) \leq \overline{F}(\overline{f}_{\breve{\delta}}(k_1), \overline{f}_{\breve{\delta}}(k_2)).$$

Again, for each $k_1, k_2 \in M_1$ with $l(k_1) = s_1$ and $l(k_2) = s_2$, the following can be obtained:

$$l(\overline{t}_{\breve{\delta}})(s_1 \cdot s_2^{-1}) \geq \overline{T}\left(\bigvee_{l(p)=s_1} \overline{t}_{\breve{\delta}}(p), \bigvee_{l(p)=s_2} \overline{t}_{\breve{\delta}}(p)\right) = \overline{T}\left(l(\overline{t}_{\breve{\delta}})(s_1), l(\overline{t}_{\breve{\delta}})(s_2)\right),$$

$$l(\bar{i}_{\breve{\delta}})(s_1 \cdot s_2^{-1}) \geq \bar{I}\big(\bigvee_{l(p)=s_1} \bar{i}_{\breve{\delta}}(p), \bigvee_{l(p)=s_2} \bar{i}_{\breve{\delta}}(p)\big) = \bar{I}\big(l(\bar{i}_{\breve{\delta}})(s_1), l(\bar{i}_{\breve{\delta}})(s_2)\big)$$

and

$$l(\bar{f}_{\breve{\delta}})(s_1 \cdot s_2^{-1}) \leq \bar{F}\big(\bigwedge_{l(p)=s_1} \bar{f}_{\breve{\delta}}(p), \bigwedge_{l(p)=s_2} \bar{f}_{\breve{\delta}}(p)\big) = \bar{F}\big(l(\bar{f}_{\breve{\delta}})(s_1), l(\bar{f}_{\breve{\delta}})(s_2)\big).$$

Hence, $l(\breve{\delta})(s_1 \cdot s_2^{-1}) \geq \breve{T}\big(l(\breve{\delta})(s_1), l(\breve{\delta})(s_2)\big)$.

Theorem 4.6 Suppose M_1 and M_2 are two crisp groups and l be a homomorphism from M_1 into M_2. If $\breve{\delta}' \in \text{IVSNG}(M_2, \breve{T})$, then $l^{-1}(\breve{\delta}') \in \text{IVNSG}(M_1, \breve{T})$.

Proof: Let $\breve{\delta}' \in \text{IVNSG}(M_2, \breve{T})$ and $k,s \in M_1$. Then

$$\begin{aligned} l^{-1}(\bar{t}_{\breve{\delta}'})(k \cdot s^{-1}) &= \bar{t}_{\breve{\delta}'}(l(k \cdot s^{-1})) \\ &= \bar{t}_{\breve{\delta}'}(l(k) \cdot l(s)^{-1}) \geq \bar{T}(\bar{t}_{\breve{\delta}'}(l(k)), \bar{t}_{\breve{\delta}'}(l(s))) \\ &= \bar{T}(l^{-1}(\bar{t}_{\breve{\delta}'}(k)), l^{-1}(\bar{t}_{\breve{\delta}'}(s))) \end{aligned}$$

In a similar way, the followings can be proven:

$l^{-1}(\bar{i}_{\breve{\delta}'})(k \cdot s^{-1}) \geq \bar{I}(l^{-1}(\bar{i}_{\breve{\delta}'}(k)), l^{-1}(\bar{i}_{\breve{\delta}'}(s)))$ and $l^{-1}(\bar{f}_{\breve{\delta}'})(k \cdot s^{-1}) \leq \bar{F}(l^{-1}(\bar{f}_{\breve{\delta}'}(k)), l^{-1}(\bar{f}_{\breve{\delta}'}(s)))$.

So, $l^{-1}(\breve{\delta}')(k \cdot s^{-1}) \geq \breve{T}(l^{-1}(\breve{\delta}')(k), l^{-1}(\breve{\delta}')(s))$.

Corollary 4.1 Suppose M_1 and M_2 are two crisp groups and l: $M_1 \rightarrow M_2$ be an isomorphism. If $\breve{\delta} \in \text{IVNSG}(M_1, \breve{T})$, then $l^{-1}(l(\breve{\delta})) = \breve{\delta}$.

Corollary 4.2 Let M be a crisp group and l: $M \rightarrow M$ be an isomorphism. If $\breve{\delta} \in \text{IVNSG}(M, \breve{T})$, then $l(\breve{\delta}) = \breve{\delta}$ iff $l^{-1}(\breve{\delta}) = \breve{\delta}$.

4.2. Interval-Valued Neutrosophic Normal Subgroup

Definition 4.3 Let M be a crisp group and $\breve{\delta} \in \text{IVNSG}(M, \breve{T})$. Then $\breve{\delta}$ is called an IVNNSG of M with respect to IVTTN $\breve{T}$ if $\forall k,s \in M$, $\breve{\delta}(k \cdot s) = \breve{\delta}(s \cdot k)$.

The set of all IVNNSG of a crisp group U with respect to $\breve{T}$ will be denoted as $\text{IVNNSG}(U, \breve{T})$.

Theorem 4.7 Let M be a group and $\breve{\delta}_1, \breve{\delta}_2 \in \text{IVNNSG}(M, \breve{T})$. Then $\breve{\delta}_1 \cap \breve{\delta}_2 \in \text{IVNNSG}(M, \breve{T})$.

Proof: Let $\breve{\delta}_1, \breve{\delta}_2 \in \text{IVNNSG}(M, \breve{T})$. Then $\forall k$, $s \in M$, $\breve{\delta}_1(k \cdot s) = \breve{\delta}_1(s \cdot k)$ and $\breve{\delta}_2(k \cdot s) = \breve{\delta}_2(s \cdot k)$. So,

$\overline{t}_{\breve{\delta}_1}(k\cdot s)=\overline{t}_{\breve{\delta}_1}(s\cdot k), \overline{i}_{\breve{\delta}_1}(k\cdot s)=\overline{i}_{\breve{\delta}_1}(s\cdot k), \overline{f}_{\breve{\delta}_1}(k\cdot s)=\overline{f}_{\breve{\delta}_1}(s\cdot k)$

and

$\overline{t}_{\breve{\delta}_2}(k\cdot s)=\overline{t}_{\breve{\delta}_2}(s\cdot k), \overline{i}_{\breve{\delta}_2}(k\cdot s)=\overline{i}_{\breve{\delta}_2}(s\cdot k), \overline{f}_{\breve{\delta}_2}(k\cdot s)=\overline{f}_{\breve{\delta}_2}(s\cdot k)$.

Hence,

$$\begin{aligned}(\breve{\delta}_1\cap\breve{\delta}_2)(k\cdot s)&=(\overline{t}_{\breve{\delta}_1\cap\breve{\delta}_2}(k\cdot s), \overline{i}_{\breve{\delta}_1\cap\breve{\delta}_2}(k\cdot s), \overline{f}_{\breve{\delta}_1\cap\breve{\delta}_2}(k\cdot s))\\ &=(\overline{t}_{\breve{\delta}_1}(k\cdot s)\wedge\overline{t}_{\breve{\delta}_2}(k\cdot s), \overline{i}_{\breve{\delta}_1}(k\cdot s)\wedge\overline{i}_{\breve{\delta}_2}(k\cdot s), \overline{f}_{\breve{\delta}_1}(k\cdot s)\vee\overline{f}_{\breve{\delta}_2}(k\cdot s))\\ &=(\overline{t}_{\breve{\delta}_1}(s\cdot k)\wedge\overline{t}_{\breve{\delta}_2}(s\cdot k), \overline{i}_{\breve{\delta}_1}(s\cdot k)\wedge\overline{i}_{\breve{\delta}_2}(s\cdot k), \overline{f}_{\breve{\delta}_1}(s\cdot k)\vee\overline{f}_{\breve{\delta}_2}(s\cdot k))\\ &=(\overline{t}_{\breve{\delta}_1\cap\breve{\delta}_2}(s\cdot k), \overline{i}_{\breve{\delta}_1\cap\breve{\delta}_2}(s\cdot k), \overline{f}_{\breve{\delta}_1\cap\breve{\delta}_2}(s\cdot k))=(\breve{\delta}_1\cap\breve{\delta}_2)(s\cdot k)\end{aligned}$$

So, $\breve{\delta}_1\cap\breve{\delta}_2\in\text{IVNNSG}(M,\breve{T})$.

Proposition 4.1 Suppose M is a crisp group and $\breve{\delta}\in\text{IVNSG}(M,\breve{T})$. Then $\forall k,s\in M$, the subsequent conditions are identical:

1. $\breve{\delta}(s\cdot k\cdot s^{-1})\geq\breve{\delta}(k)$
2. $\breve{\delta}(s\cdot k\cdot s^{-1})=\breve{\delta}(k)$
3. $\breve{\delta}\in\text{IVNNSG}(M,\breve{T})$

Proof: (1) $\Rightarrow$ (2): Let $k,s\in M$. As $\breve{\delta}(s\cdot k\cdot s^{-1})\geq\breve{\delta}(k)$, it can be shown that

$\overline{t}_{\breve{\delta}}(s\cdot k\cdot s^{-1})\geq\overline{t}_{\breve{\delta}}(k), \overline{i}_{\breve{\delta}}(s\cdot k\cdot s^{-1})\geq\overline{i}_{\breve{\delta}}(k)$ and $\overline{f}_{\breve{\delta}}(s\cdot k\cdot s^{-1})\leq\overline{f}_{\breve{\delta}}(k)$.

Now, replacing s with s^{-1}, $\overline{t}_{\breve{\delta}}(s^{-1}\cdot k\cdot s)=\overline{t}_{\breve{\delta}}(s^{-1}\cdot k\cdot(s^{-1})^{-1})\geq\overline{t}_{\breve{\delta}}(k)$.
So, $\overline{t}_{\breve{\delta}}(k)=\overline{t}_{\breve{\delta}}(s^{-1}\cdot(s\cdot k\cdot s^{-1})\cdot s)\geq\overline{t}_{\breve{\delta}}(s\cdot k\cdot s^{-1})\geq\overline{t}_{\breve{\delta}}(k)$ i.e. $\overline{t}_{\breve{\delta}}(s\cdot k\cdot s^{-1})=\overline{t}_{\breve{\delta}}(k)$.
In a similar way, $\overline{i}_{\breve{\delta}}(s\cdot k\cdot s^{-1})=\overline{i}_{\breve{\delta}}(k)$ and $\overline{f}_{\breve{\delta}}(s\cdot k\cdot s^{-1})=\overline{f}_{\breve{\delta}}(k)$. So, $\forall k,s\in M$, $\breve{\delta}(s\cdot k\cdot s^{-1})=\breve{\delta}(k)$.

(2) $\Rightarrow$ (3): In (2), replacing k with $k{\cdot}s$ (3) can be obtained easily.

(3) $\Rightarrow$ (1): Let $k,s\in M$. As, $\breve{\delta}\in\text{IVNNSG}(M,\breve{T})$, $\breve{\delta}(k\cdot s)=\breve{\delta}(s\cdot k)$. Replacing k with $k{\cdot}s^{-1}$ the following can be obtained: $\breve{\delta}(s\cdot k\cdot s^{-1})=\breve{\delta}(k\cdot s^{-1}\cdot s)=\breve{\delta}(k)\geq\breve{\delta}(k)$.

Theorem 4.8 Let M be a group and $\breve{\delta}\in\text{IVNS}(M)$. Then $\breve{\delta}\in\text{IVNNSG}(M,\breve{T})$ iff for every $[g_1,u_1]$, $[g_2,u_2]$ and $[g_3,u_3]\in\rho(J)$ with $u_1+u_2+u_3\leq 1$, $(\breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])}\neq\phi)$ $\breve{\delta}_{([g_1,u_1],[g_2,u_2],[g_3,u_3])}$ is a crisp normal subgroup of M.

Proof: This can be proved using Theorem 4.4.

Theorem 4.9 Let M be a group and $\breve{\delta} \in \text{IVNNSG}(M, \breve{T})$ with respect to an idempotent IVTTN $\breve{T}$. Let $M|_{\breve{\delta}} = \{k \in M : \breve{\delta}(k) = \breve{\delta}(e)\}$, ($e$ is the neutral element of M). Then the crisp set $M|_{\breve{\delta}}$ is a normal subgroup of M.

Proof: Let $\breve{\delta} \in \text{IVNNSG}(M, \breve{T})$ and $k, s \in M|_{\breve{\delta}}$. So, $\breve{\delta}(k) = \breve{\delta}(e) = \breve{\delta}(s)$.

Now, $\breve{\delta}(k \cdot s^{-1}) \geq \breve{T}(\breve{\delta}(k), \breve{\delta}(s)) = \breve{T}(\breve{\delta}(e), \breve{\delta}(e)) = \breve{\delta}(e)$. Again, $\breve{\delta}(e) \geq \breve{\delta}(k \cdot s^{-1})$ and hence $\breve{\delta}(k \cdot s^{-1}) = \breve{\delta}(e)$. So, $k \cdot s^{-1} \in M|_{\breve{\delta}}$ i.e. $M|_{\breve{\delta}}$ is a subgroup of M.

Again, let $k \in M|_{\breve{\delta}}$ and $s \in M$. Since, $\breve{\delta} \in \text{IVNNSG}(M, \breve{T})$ it can be shown that $\breve{\delta}(s \cdot k \cdot s^{-1}) = \breve{\delta}(k) = \breve{\delta}(e)$. Hence, $s \cdot k \cdot s^{-1} \in M|_{\breve{\delta}}$ i.e. $M|_{\breve{\delta}}$ is a normal subgroup of M.

Note that, Theorem 4.9 is true only when $\breve{T}$ is an idempotent IVTTN. The following (Example 4.2) is a counterexample which will justify current claim.

Example 4.2 Let $M=\{1,i,-1,-i\}$ be a cyclic group and

$$\breve{\delta} = \{(1,[0.8,0.8],[0.5,0.5],[0.2,0.2]),(-1,[0.7,0.7],[0.5,0.5],[0.3,0.3]),$$
$$(i,[0.8,0.8],[0.5,0.5],[0.2,0.2]),(-i,[0.8,0.8],[0.5,0.5],[0.2,0.2])\}.$$

Also, let the corresponding IVTTN $\breve{T}$ is formed by product TNs i.e. $T(k,s)= k \bullet s$, $I(k,s)=k \bullet s$ and product TC i.e. $F(k,s)=k+s-k \bullet s$. Then, $\breve{\delta} \in \text{IVNNSG}(M, \breve{T})$. However, $M|_{\breve{\delta}} = \{1, i, -i\}$ is not a subgroup of M and hence $M|_{\breve{\delta}}$ is not a normal subgroup of M.

4.2.1. Homomorphism on Interval-Valued Neutrosophic Normal Subgroup

Theorem 4.10 Let M_1 and M_2 be two crisp groups and l be a homomorphism from M_1 into M_2. If $\breve{\delta}' \in \text{IVNNSG}(M_2, \breve{T})$, then $l^{-1}(\breve{\delta}') \in \text{IVNNSG}(M_1, \breve{T})$.

Proof: Let $\breve{\delta}' \in \text{IVNNSG}(M_2, \breve{T})$, then $\breve{\delta}' \in \text{IVNSG}(M_2, \breve{T})$ and hence from Theorem 4.6, $l^{-1}(\breve{\delta}') \in \text{IVNSG}(M_1, \breve{T})$. So, only the normality of $\breve{\delta}'$ is needed to be proved.

Let $k, s \in M_1$, then

$$\begin{aligned} l^{-1}(\breve{\delta}')(k \cdot s) &= \breve{\delta}'(l(k \cdot s)) = \breve{\delta}'(l(k) \cdot l(s)) \\ &= \breve{\delta}'(l(s) \cdot l(k))[\text{AS } \breve{\delta}' \in \text{IVNNSG}(M_2, \breve{T})] \\ &= \breve{\delta}'(l(s \cdot k)) = l^{-1}(\breve{\delta}')(s \cdot k) \end{aligned}$$

So, $l^{-1}(\breve{\delta}') \in \text{IVNNSG}(M_1, \breve{T})$.

Theorem 4.11 Suppose M_1 and M_2 be two crisp groups and l be a surjective homomorphism from M_1 into M_2. If $\breve{\delta} \in \text{IVNNSG}(M_1, \breve{T})$, then $l(\breve{\delta}) \in \text{IVNNSG}(M_2, \breve{T})$.

Proof: Let $\breve{\delta} \in \text{IVNNSG}(M_1, \breve{T})$, then $\breve{\delta} \in \text{IVNSG}(M_1, \breve{T})$ and hence by Theorem 4.5, $l(\breve{\delta}) \in \text{IVNSG}(M_2, \breve{T})$. So, only the normality of $\breve{\delta}'$ is needed to be proved.

Now, $\forall k,s \in M_2$, as l is a surjective homomorphism, $l^{-1}(k)^1\ \phi$, $l^{-1}(s)^1\phi$ and $l^{-1}(k \cdot s \cdot k^{-1})^1\phi$. So, $\forall k,s \in M_2$, $l(\overline{t}_{\breve{\delta}})(k \cdot s \cdot k^{-1}) = \bigvee_{r \in l^{-1}(k \cdot s \cdot k^{-1})} (\overline{t}_{\breve{\delta}}(r))$ and $l(\overline{t}_{\breve{\delta}})(s) = \bigvee_{r \in l^{-1}(s)} (\overline{t}_{\breve{\delta}}(r))$.

Let $n \in l^{-1}(k)$, $q \in l^{-1}(s)$ and $n^{-1} \in l^{-1}(k^{-1})$. Now as $\breve{\delta} \in \text{IVNNSG}(M_1, \breve{T})$, the followings can be drawn:

$$\overline{t}_{\breve{\delta}}(n \cdot q \cdot n^{-1}) \geq \overline{t}_{\breve{\delta}}(q),\ \overline{i}_{\breve{\delta}}(n \cdot q \cdot n^{-1}) \geq \overline{i}_{\breve{\delta}}(q) \text{ and } \overline{f}_{\breve{\delta}}(n \cdot q \cdot n^{-1}) \leq \overline{f}_{\breve{\delta}}(q).$$

Since, l is a homomorphism, $l(n \cdot q \cdot n^{-1}) = l(n) \bullet l(q) \bullet l(n^{-1}) = k \cdot s \cdot k^{-1}$ and hence, $n \cdot q \cdot n^{-1} \in l^{-1}(k \cdot s \cdot k^{-1})$. So,

$$\begin{aligned} l(\overline{t}_{\breve{\delta}})(k \cdot s \cdot k^{-1}) &= \bigvee_{r \in l^{-1}(k \cdot s \cdot k^{-1})} (\overline{t}_{\breve{\delta}}(r)) \\ &\geq \bigvee_{n \in l^{-1}(k),\ q \in l^{-1}(s),\ n^{-1} \in l^{-1}(k^{-1})} (\overline{t}_{\breve{\delta}}(n \cdot q \cdot n^{-1})) \\ &\geq \bigvee_{q \in l^{-1}(s)} (\overline{t}_{\breve{\delta}}(q)) = l(\overline{t}_{\breve{\delta}})(s) \end{aligned}$$

Hence, $\forall k,s \in M_2$, $l(\overline{t}_{\breve{\delta}})(k \cdot s \cdot k^{-1}) \geq l(\overline{t}_{\breve{\delta}})(s)$ and similarly,

$$l(\overline{i}_{\breve{\delta}})(k \cdot s \cdot k^{-1}) \geq l(\overline{i}_{\breve{\delta}})(s), l(\overline{f}_{\breve{\delta}})(k \cdot s \cdot k^{-1}) \leq l(\overline{f}_{\breve{\delta}})(s).$$

So, $l(\breve{\delta})(k \cdot s \cdot k^{-1}) \geq l(\breve{\delta})(s)$ and hence, by Proposition 4.1, $l(\breve{\delta}) \in \text{IVNNSG}(M_2, \breve{T})$.

Corollary 4.3 Let M_1 and M_2 be two crisp groups and l: $M_1 \rightarrow M_2$ be an isomorphism. If $\breve{\delta} \in \text{IVNNSG}(M_1, \breve{T})$, then $l^{-1}(l(\breve{\delta})) = \breve{\delta}$.

Corollary 4.4 Let M be a crisp group and l: $M \rightarrow M$ be an isomorphism on M. If $\breve{\delta} \in \text{IVNNSG}(M_1, \breve{T})$, then $l(\breve{\delta}) = \breve{\delta}$ iff $l^{-1}(\breve{\delta}) = \breve{\delta}$.

5. CONCLUSION

The notion of an IVNSG is nothing but generalization of FSG, IFSG, NSG, IVFSG and IVIFSG. It is known that, to study some fundamental algebraic characteristics of any entity one needs to understand functions, which preserve their algebraic characteristics i.e. one needs to study the effects of homomorphism on them. Hence, in this chapter, IVTTN has been introduced and based on that IVNSG has been introduced. Also, some effects of homomorphism on it have been studied. Furthermore, based on IVTTN, IVNNSG has been defined and some of its homomorphic characteristics have been studied. In future, one can introduce soft set theory in IVNSG and further generalize it.

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