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An intuitionistic fuzzy clustering algorithm based on a new correlation coefficient with application in medical diagnosis

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Abstract. In this paper, we propose a new correlation coefficient between intuitionistic fuzzy sets. We then use this new result to compute some examples through which we find that it benefits from such an outcome with some well-known results in the literature. As in statistics with real variables, we refer to variance and covariance between two intuitionistic fuzzy sets. Then, we determined the formula for calculating the correlation coefficient based on the variance and covariance of the intuitionistic fuzzy set, the value of this correlation coefficient is in [–1,1]. Then, we develop this direction to build correlation coefficients between the interval-valued intuitionistic fuzzy sets and apply it in the pattern recognition problem. Finally, we apply this correlation coefficient in clustering problem with intuitionistic fuzzy information.

Keywords: Intuitionistic fuzzy set, interval – valued intuitionistic fuzzy set, variance, covariance, correlation coefficient

1. Introduction

In 1986, Atanassov introduced the intuitionistic fuzzy set [1], which is a generalization of Zadeh's fuzzy set [19]. An intuitionistic fuzzy set (IFS) consider the information both the membership function and non-membership function. After that, the interval-valued intuitionistic fuzzy set (IVIFS) was introduced by Atanassov and Gargov [2]. In which, the membership function and non-membership function are subintervals of [0, 1]. As opposed to fuzzy sets, intuitionistic fuzzy set also have broad applications for uncertain data processing such as decision making, medical diagnose, agriculture [3, 9–14, 20]. Along with similar measurements, distance measurements, correlation measurements of intuitionistic fuzzy sets and interval –valued intuitionistic fuzzy set are also studied and widely used in many areas and now it is a hot topic [4–8, 11, 15–17, 21].

The concept of correlation coefficient of intuitionistic fuzzy sets was first studied by Gerstenkorn, and Mańko in 1991 [6]. In this correlation coefficient, the variance and covariance are constructed directly from the scalar product of the values of the membership function and the non-membership function, respectively, of two intuitionistic fuzzy sets. Then, the correlation coefficient between the interval–valued intuitionistic fuzzy set is introduced by the Bustince and Burillo [4] in 1995. Later, many authors have studied and developed correlations in different trends and in other spaces. In the results of the correlation studies, in the 1990 s, we find that the values of the correlation coefficient are in the range [0, 1].

In the 21st century, scholars have developed new methods for building correlation coefficients for fuzzy sets. In the new results, some studies show the value of the correlation coefficient value received in

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the interval [–1, 1]. We can include some methods as Hung's method [7], and the method of Liu et al. [11]. Hung's method is based on a statistical viewpoint to calculate the correlation coefficient. In 2016, Liu et al. [11] constructed the correlation coefficient between intuitionistic fuzzy sets based on the concept of the deviation of the intuitionistic fuzzy numbers, after that they also extend this approach to the IVIFSs case.

As the correlation coefficient reflect the relationship between two objects. When the correlation coefficient has values ± 1, it shows a linear relationship. Thus it reflects the "consistency" of the two sets. That is the motivation for us to study the correlation coefficient. Moreover, the correlation coefficients found in the literature still have certain limitations, as shown in the examples in this paper.

However, there are some cases where the correlation coefficient according to Hung's method or method of (Liu et al) does not cover them. These cases can be viewed in the examples presented in this article. In those cases, if using our method, there are reasonable results. In this paper, we propose a new method to determine the correlation coefficient between the intuitionistic fuzzy sets, in which the value of the correlation coefficient computed according to our method lies in interval [–1, 1]. Then, we develop the method to construct the correlation coefficient between the interval–valued intuitionistic fuzzy sets. Finally, we also provide examples to illustrate our approach, and apply it to diagnostic problems in medicine and to the problem of pattern recognition.

The contribution of this article is to provide a new method for determining the correlation coefficient between intuitionistic fuzzy sets. It has overcome the limitations of existing methods. This method is quite simple and its applications are quite varied. As in sample identification, medicine and clustering analysis.

The rest of this paper is organized as follows. In Section 2, we recall the concept of intuitionistic fuzzy set, interval – valued intuitionistic fuzzy set, the correlation coefficients which are computed by using the methods of Gerstenkorn and Mansko [6], Hung [7], Xu [16] and Liu et al. [11]. In Section 3, we construct the new correlation coefficient between the IFSs, in this section we also give some examples that compare the results computed based on our method to other methods. In Section 4, we extend our method to determine the correlation coefficient between the IVIFSs. Finally, we present the conclusion in Section 5.

2. Preliminaries

Let X be a universal set. We have

Definition 1. [1] An intuitionistic fuzzy set on X is a defined by form

$$A = \{(x, \mu_A(x), \nu_A(x)) | \, x \in X\} \tag{1}$$

in which $\mu_A(x) \in [0, 1]$ and $\nu_A(x) \in [0, 1]$ are the membership degree and the non-membership of the element x in X to A, respectively, and $\mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$.

Definition 2. [2] An interval – value intuitionistic fuzzy set on X is a defined by form

$$A = \left\{\left(x, \left[\mu_A^L(x), \mu_A^U(x)\right], \left[\nu_A^L(x), \nu_A^U(x)\right]\right) \big| \, x \in X\right\}$$

in which $\widetilde{\mu}_A(x) = \left[\mu_A^L(x), \mu_A^U(x)\right] \subseteq [0, 1]$ and $\tilde{\nu}_A(x) = \left[\nu_A^L(x), \nu_A^U(x)\right] \subseteq [0, 1]$ are the membership degree and the non-membership of the element x in X to A, respectively, and

$$\mu_A^U(x) + \nu_A^U(x) \leq 1, \forall x \in X. \tag{2}$$

Now, we recall some existing correlation coefficient of intuitionistic fuzzy sets in literature. Given $X = \{x_1, x_2, \ldots, x_n\}$ is a universal set and

$$\begin{aligned} A &= \{(x_i, \mu_A(x_i), \nu_A(x_i)) | \, x_i \in X\}, \\ B &= \{(x_i, \mu_B(x_i), \nu_B(x_i)) | \, x_i \in X\} \end{aligned} \tag{3}$$

are two IFSs on X.

Definition 3. [11] The average of A is

$$E(A) = (\overline{\mu}_A, \overline{\nu}_A) = \left(\frac{1}{n}\sum_{i=1}^{n} \mu_A(x_i), \frac{1}{n}\sum_{i=1}^{n} \nu_A(x_i)\right) \tag{4}$$

The correlation coefficients of Gerstenkorn and Mansko [6].

$$\rho(A, B) = \frac{C(A, B)}{\sqrt{T(A)T(B)}} \tag{5}$$

where

$$\begin{aligned} C(A, B) &= \sum_{i=1}^{n} [\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)], \\ T(A) &= \sum_{i=1}^{n} \left[\mu_A^2(x_i) + \nu_A^2(x_i)\right], \\ T(B) &= \sum_{i=1}^{n} \left[\mu_B^2(x_i) + \nu_B^2(x_i)\right]. \end{aligned} \tag{6}$$

The correlation coefficient of Hung [7]

$$\rho(A, B) = \frac{1}{2}(\rho_1 + \rho_2) \tag{7}$$

where

$$\rho_1 = \frac{\sum_{i=1}^{n} (\mu_A(x_i) - \overline{\mu_A})(\mu_B(x_i) - \overline{\mu_B})}{\sqrt{\sum_{i=1}^{n} (\mu_A(x_i) - \overline{\mu_A})^2}\sqrt{\sum_{i=1}^{n} (\mu_B(x_i) - \overline{\mu_B})^2}}, \tag{8}$$

$$\rho_2 = \frac{\sum_{i=1}^{n} (\nu_A(x_i) - \overline{\nu_A})(\nu_B(x_i) - \overline{\nu_B})}{\sqrt{\sum_{i=1}^{n} (\nu_A(x_i) - \overline{\nu_A})^2}\sqrt{\sum_{i=1}^{n} (\nu_B(x_i) - \overline{\nu_B})^2}}. \tag{9}$$

The correlation coefficient of Xu [16]

$$\rho(A, B) = \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{\Delta\mu_{\min} + \Delta\mu_{\max}}{\Delta\mu_i + \Delta\mu_{\max}} + \frac{\Delta\nu_{\min} + \Delta\nu_{\max}}{\Delta\nu_i + \Delta\nu_{\max}}\right] \tag{10}$$

where

$$\begin{aligned} &\Delta\mu_i = |\mu_A(x_i) - \mu_B(x_i)|, \Delta\nu_i = |\nu_A(x_i) - \nu_B(x_i)| \\ &\Delta\mu_{\max} = max\Delta\mu_i, \Delta\nu_{\max} = max\Delta\nu_i \\ &\Delta\mu_{\min} = \min\Delta\mu_i, \Delta\nu_{\min} = \min\Delta\nu_i. \end{aligned} \tag{11}$$

The correlation coefficient of Liu et al. [11]

$$\rho(A, B) = \frac{C(A, B)}{\sqrt{D(A)D(B)}} \tag{12}$$

where

$$\begin{aligned} &C(A, B) = \tfrac{1}{n-1}\sum_{i=1}^{n} d_i(A)d_i(B), \\ &D(A) = \tfrac{1}{n-1}\sum_{i=1}^{n} d_i^2(A), \;\; D(B) = \tfrac{1}{n-1}\sum_{i=1}^{n} d_i^2(B) \end{aligned} \tag{13}$$

in which

$$\begin{aligned} d_i(A) = (\mu_A(x_i) - \overline{\mu_A}) - (\nu_A(x_i) - \overline{\nu_A}) \\ d_i(B) = (\mu_B(x_i) - \overline{\mu_B}) - (\nu_B(x_i) - \overline{\nu_B}) \end{aligned} \tag{14}$$

for all $i = 1, 2, \ldots, n$.

3. A new correlation coefficient of the IFSs

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set, A and B are two arbitrary IFSs in X.

Definition 4. The variance of A can be represented as

$$D(A) = \frac{1}{n-1}\sum_{i=1}^{n}\{(\mu_A(x_i) - \overline{\mu_A})^2 + (\nu_A(x_i) - \overline{\nu_A})^2\} \tag{1}$$

Definition 5. The covariance of A and B can be defined by

$$\begin{aligned} COV(A, B) = \tfrac{1}{n-1}\{&(\mu_A(x_i) - \overline{\mu_A})(\mu_B(x_i) - \overline{\mu_B}) \\ &+ (\nu_A(x_i) - \overline{\nu_A})(\nu_B(x_i) - \overline{\nu_B})\} \end{aligned} \tag{2}$$

where $E(A) = (\bar{\mu}_A, \; \bar{\upsilon}_A)$ and $E(B) = (\bar{\mu}_B, \; \bar{\upsilon}_B)$.

Proposition 1. *Let A and B be two arbitrary IFSs in X. We have*

(1) $COV(A, B) = COV(B, A)$
(2) $COV(A, A) = D(A)$
(3) $|COV(A, B)| \leq D(A)^{0.5}D(B)^{0.5}$

Proof.
It is easily to obtain (1), (2).
(3). According to the Cauchy – Schwarz inequality, we have

$$\begin{aligned} COV(A, B)^2 &= \tfrac{1}{(n-1)^2}\{(\mu_A(x_i) - \overline{\mu_A})(\mu_B(x_i) - \overline{\mu_B}) \\ &\quad + (\nu_A(x_i) - \overline{\nu_A})(\nu_B(x_i) - \overline{\nu_B})\}^2 \\ &\leq \tfrac{1}{n-1}\sum_{i=1}^{n}\{(\mu_A(x_i) - \overline{\mu_A})^2 + (\nu_A(x_i) - \overline{\nu_A})^2\} \\ &\quad \times \tfrac{1}{n-1}\sum_{i=1}^{n}\{(\mu_B(x_i) - \overline{\mu_B})^2 + (\nu_B(x_i) - \overline{\nu_B})^2\} \\ &= D(A)D(B). \end{aligned}$$

So that $|COV(A, B)| \leq D(A)^{0.5}D(B)^{0.5}$ □

Now, we can define the correlation coefficient of the intuitionistic fuzzy sets. It is similar to the correlation coefficient of real number variables in the statistic theory.

Definition 6. Let A and B be two arbitrary IFSs on X. The correlation coefficient of A and B can be defined by

$$\rho(A, B) = \frac{COV(A, B)}{\sqrt{D(A)D(B)}} \quad (3)$$

Remark 1. Our correlation coefficient estimates the value in $[-1, 1]$, while many known correlation coefficients take only values in $[0, 1]$. This is also consistent with the correlation coefficient we often see in real-world statistics. The correlation coefficient evaluates the linear relationship between the values of the two fuzzy sets on the elements of the observed set X. When two fuzzy sets have a real linear relationship then the correlation coefficient between them are ± 1.

Theorem 1. *Given two IFSs A and B, then we have*

(1) $\rho(A, B) = \rho(B, A)$
(2) $-1 \le \rho(A, B) \le 1$
(3) If $A = kB + b$ for some $k > 0$, then $\rho(A, B) = 1$. Here, $A = kB + b$ means that $\mu_A = k\mu_B + b$ and $\nu_A = k\nu_B + b$.
(4) If $A = kB + b$ for some $k < 0$, then $\rho(A, B) = -1$.

Proof.

(1) Straightforward.
(2) From proposition 1, we have $|COV(A, B)| \le D(A)^{0.5} D(B)^{0.5}$. It means that $-D(A)^{0.5} D(B)^{0.5} \le COV(A, B) \le D(A)^{0.5} D(B)^{0.5}$ Hence, we have

$$-1 \le \rho(A, B) = \frac{COV(A, B)}{\sqrt{D(A)D(B)}} \le 1.$$

(3) If $\mu_A = k\mu_B + b$ and $\nu_A = k\nu_B + b$ we have

$$\begin{aligned}
COV(A, B) &= \tfrac{1}{n-1}\{(\mu_A(x_i) - \overline{\mu_A})(\mu_B(x_i) - \overline{\mu_B}) \\
&+(\nu_A(x_i) - \overline{\nu_A})(\nu_B(x_i) - \overline{\nu_B})\} \\
&= \tfrac{1}{n-1}\{ \text{k} \times (\mu_B(x_i) - \overline{\mu_A})(\mu_B(x_i) - \overline{\mu_B}) \\
&+k \times (\nu_B(x_i) - \overline{\nu_B})(\nu_B(x_i) - \overline{\nu_B})\} \\
&= \tfrac{k}{n-1}\{(\mu_B(x_i) - \overline{\mu_A})(\mu_B(x_i) - \overline{\mu_B}) \\
&+(\nu_B(x_i) - \overline{\nu_B})(\nu_B(x_i) - \overline{\nu_B})\} \\
&= kD(B).
\end{aligned}$$

and

$$\begin{aligned}
D(A) &= \tfrac{1}{n-1} \sum_{i=1}^{n} \{(\mu_A(x_i) - \overline{\mu_A})^2 + (\nu_A(x_i) - \overline{\nu_A})^2\} \\
&= \tfrac{1}{n-1} \sum_{i=1}^{n} \{ \text{k}^2 \times (\mu_B(x_i) - \overline{\mu_B})^2 + k \times (\nu_B(x_i) - \overline{\nu_B})^2\} \\
&= \tfrac{k^2}{n-1} \sum_{i=1}^{n} \{(\mu_B(x_i) - \overline{\mu_B})^2 + (\nu_B(x_i) - \overline{\nu_B})^2\} \\
&= k^2 D(B).
\end{aligned}$$

If $k > 0$ then

$$\rho(A, B) = \tfrac{COV(A,B)}{\sqrt{D(A)D(B)}} = \tfrac{kD(B)}{\sqrt{k^2 D(B)D(B)}} = \tfrac{kD(B)}{kD(B)} = 1.$$

(4) If $k < 0$ then

$$\rho(A, B) = \frac{COV(A, B)}{\sqrt{D(A)D(B)}} = \frac{kD(B)}{-kD(B)} = -1. \qquad \square$$

Now, we consider some examples to compare our proposed correlation coefficient and some other knowledge correlation coefficient.

Example 1. In this example, let's look at Liu's example in [11]. Suppose that A and B are two IFSs in $X = \{x_1, x_2, x_3\}$ where

$$A = \{(x_1, 0.1, 0.2), (x_2, 0.2, 0.1), (x_3, 0.3, 0)\},$$
$$B = \{(x_1, 0.3, 0), (x_2, 0.2, 0.2), (x_3, 0.1, 0.4)\}.$$

(1) By our method

$$E(A) = \left(\tfrac{0.1+0.2+0.3}{3}, \tfrac{0.2+0.1+0}{3}\right) = (0.2, 0.1)$$
$$E(B) = \left(\tfrac{0.3+0.2+0.1}{3}, \tfrac{0+0.2+0.4}{3}\right) = (0.2, 0.2)$$

$COV(A, B) = -0.035$; $D(A) = 0.02$ and $D(B) = 0.075$.

So that

$$\rho(A, B) = \frac{-0.035}{\sqrt{0.02 \times 0.075}} = -0.9037.$$

(2) By the method of Liu et al. in [11]

$$\rho(A, B) = -1$$

(3) By the method in Gerstenkorn and Marko [6]

$$\rho(A, B) = 0.12$$

(4) By the method in Xu [16]

$$\rho(A, B) \approx 0.74$$

(5) By the method in Hung [7]

$$\rho(A, B) = -1.$$

These results are consistent, because A is the set of elements whose value increases, B is a set whose values are decreasing. But not exist $k \neq 0$ such that $B = kA + b$, in particular $\mu_B = -\mu_A + 0.4$ and $\nu_B = -2\nu_A + 0.4$.

Example 2. Suppose that A and B are two IFSs in $X = \{x_1, x_2, x_3\}$ where

$$A = \{(x_1, 0.16, 0.18), (x_2, 0.16, 0.18), (x_3, 0.19, 0.21)\}$$

$$B = \left\{ \begin{array}{l} (x_1, 0.284, 0.282), (x_2, 0.284, 0.282), \\ (x_3, 0.281, 0.279) \end{array} \right\}$$

(1) By our method
$E(A) = (0.17, 0.19)$; $E(B) = (0.283, 0.281)$;
$COV(A, B) = -0.00012$; $D(A) = 0.00012$;
$D(B) = 0.00012$ and $\rho(A, B) = -1$.
(2) By the method of Liu et al. in [11]
We cannot determine the correlation coefficient according to this method.
(3) By the method in Hung [7]

$$\rho(A, B) = -1.$$

In this example, the results of our method match those of the Hung's method.In this example, the results calculated according to our method coincide with the results calculated by the Hung's method, both methods yield have correlation coefficient $\rho(A, B) = -1$. This result is reasonable, because two intuitionistic fuzzy sets A, B have a linear relation $B = kA + b$ with $k = -0.1$ and $b = 0.3$. But the method of Liu et al. in [11] does not tell us anything about this data.

Example 3. Suppose that A and B are two IFSs in $X = \{x_1, x_2, x_3\}$ where

$$A = \{(x_1, 0.16, 0.18), (x_2, 0.16, 0.18), (x_3, 0.19, 0.21)\}$$

$$B = \left\{ \begin{array}{l} (x_1, 0.432, 0.436), (x_2, 0.432, 0.436), \\ (x_3, 0.438, 0.442) \end{array} \right\}$$

(1) By our method
$E(A) = (0.17, 0.19)$; $E(B) = (0.434, 0.438)$;
$COV(A, B) = 0.00024$; $D(A) = 0.00012$;
$D(B) = 0.000048$ and $\rho(A, B) = 1$.
(2) By the method of Liu et al. in [11]
We cannot determine the correlation coefficient according to this method.
(3) By the method in Hung [7]

$$\rho(A, B) = 1.$$

In this example, the results of our method match those of the Hung's method.In this example, the results calculated according to our method also coincide with the results calculated by the Hung's method, both methods yield have correlation coefficient $\rho(A, B) = 1$. This result is reasonable, because two intuitionistic fuzzy sets A, B have a linear relation $B = kA + b$ with $k = 0.2$ and $b = 0.4$. But the method of Liu et al. in [11] does not tell us anything about this data.

Example 4. Suppose that A and B are two IFSs in $X = \{x_1, x_2, x_3\}$ where

$$A = \{(x_1, 0.1, 0.3), (x_2, 0.1, 0.4), (x_3, 0.1, 0.5)\}$$

$$B = \left\{ \begin{array}{l} (x_1, 0.41, 0.43), (x_2, 0.41, 0.44), \\ (x_3, 0.41, 0.45) \end{array} \right\}$$

(1) By our method
$E(A) = (0.1, 0.4)$; $E(B) = (0.41, 0.44)$;
$COV(A, B) = 0.002$; $D(A) = 0.02$;
$D(B) = 0.0002$ and $\rho(A, B) = 1$.
(2) By the method of Liu et al. in [11]

$$\rho(A, B) = 1.$$

(3) By the method in Hung [7]
We cannot determine the correlation coefficient according to this method.
In this example, the results of our method match those of Liu et al. [11]. In this example, the results calculated according to our method also coincide with the results calculated by using the method of Liu et al. both methods yield a correlation coefficient $\rho(A, B) = 1$. This result is reasonable, because two intuitionistic fuzzy sets A, B have a linear relation $B = kA + b$ with $k = 0.1$ and $b = 0.4$. But the method of Hung in [7] does not tell us anything about this data.

Example 5. Suppose that A and B are two IFSs in $X = \{x_1, x_2, x_3\}$ where

$$A = \{(x_1, 0.4, 0.2), (x_2, 0.4, 0.2), (x_3, 0.1, 0.2)\}$$

$$B = \left\{ \begin{array}{l} (x_1, 0.36, 0.38), (x_2, 0.36, 0.38), \\ (x_3, 0.39, 0.38) \end{array} \right\}$$

(1) By our method
$E(A) = (0.3, 0.2)$; $E(B) = (0.37, 0.38)$;

$COV(A, B) = -0.006; D(A) = 0.06;$ $D(B) = 0.0006$ and $\rho(A, B) = -1$.

(2) By the method of Liu et al. in [11]

$$\rho(A, B) = -1.$$

(3) By the method in Hung [7]

We cannot determine the correlation coefficient according to this method.

In this example, the results of our method match those of Liu et al. In this example, the results calculated according to our method also coincide with the results calculated by using the method of Liu et al, both methods yield the correlation coefficient $\rho(A, B) = -1$. This result is reasonable, because two intuitionistic fuzzy sets A, B have a linear relation $B = kA + b$ with $k = -0.1$ and $b = 0.4$. But the method of Hung in [7] does not tell us anything about this data.

Example 6. In this example, we use our method to the medical diagnosis, and data we used are quoted in Szmidt and Kacprzyk [15]. Usage of diagnostic methods D = {Viral fever (V), Malaria (M), Typhoid (T), Stomach problem (S), Chest problem} for patients with given values of symptoms S = {temperature, headache, stomach pain, cough, chest pain}. In this case, the intuitionistic fuzzy set is useful to handle them. Here, for each $d_k \in D(k = 1, 2, \ldots, 5)$ is expressed in form that is an intuitionistic fuzzy set on the universal set $S = \{s_1, s_2, s_3, s_4, s_5\}$, see Table 1. The information of symptoms characteristic for the considered patients is given in Table 2. In which, for each patient $p_i(i = 1, 2, 3, 4)$ is an intuitionistic fuzzy set in the universal set $S = \{s_1, s_2, s_3, s_4, s_5\}$.

To select the appropriate diagnostic method we calculate the correlation of each patient with the diagnostic methods. For each patient, the appropriate diagnostic method will have the highest correlation coefficient.

The correlation coefficients of a diagnosis $d_k \in D(k = 1, 2, \ldots, 5)$ for each patient $p_i(i = 1, 2, 3, 4)$ is

$$\rho(p_i, d_k) = \frac{COV(p_i, d_k)}{\sqrt{D(p_i)D(d_k)}}.$$

The computed results of correlation coefficients are listed in Table 3. From the results, we see that Al should use diagnostic method corresponding to Malaria, Bob uses Stomach problem, Joe uses Typhoid and Ted uses Malaria. We also cite the results listed in [11]. These results together with the results calculated according to our method are listed in Table 4. All six methods point to Bob in accordance with our S diagnosis. Our method and the other four methods indicate that Joe should use T diagnostics. Patient Al should use V, and Ted use diagnosis V.

4. The correlation coefficient of the IVIFSs

In this section, we extend our method to the IVIFSs.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universal set. Given an IVIFSs

$$\tilde{A} = \left\{\left(x_i, \left[\mu^L_{\tilde{A}}(x_i), \mu^U_{\tilde{A}}(x_i)\right], \left[\nu^L_{\tilde{A}}(x_i), \nu^U_{\tilde{A}}(x_i)\right]\right) \middle| \, x_i \in X\right\}$$

Definition 7. The average of A is

$$E(A) = \begin{pmatrix} \left[\frac{1}{n}\sum_{i=1}^{n} \mu^L_{\tilde{A}}(x_i), \frac{1}{n}\sum_{i=1}^{n} \mu^U_{\tilde{A}}(x_i)\right], \\ \left[\frac{1}{n}\sum_{i=1}^{n} \nu^L_{\tilde{A}}(x_i), \frac{1}{n}\sum_{i=1}^{n} \nu^U_{\tilde{A}}(x_i)\right] \end{pmatrix} \quad (4)$$

We denote $\mu_{\tilde{A}}(x_i) = \frac{\mu^L_{\tilde{A}}(x_i)+\mu^U_{\tilde{A}}(x_i)}{2}$, $\nu_{\tilde{A}}(x_i) = \frac{\nu^L_{\tilde{A}}(x_i)+\nu^U_{\tilde{A}}(x_i)}{2}$,

$$\bar{\mu}_{\tilde{A}} = \frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n} \mu^L_{\tilde{A}}(x_i) + \frac{1}{n}\sum_{i=1}^{n} \mu^U_{\tilde{A}}(x_i)\right) \quad (5)$$

and

$$\bar{\nu}_{\tilde{A}} = \frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n} \nu^L_{\tilde{A}}(x_i) + \frac{1}{n}\sum_{i=1}^{n} \nu^U_{\tilde{A}}(x_i)\right). \quad (6)$$

We can determine variance, covariance and correlation coefficient between IVIFSs.

Definition 8. Let $\tilde{A}$ be a IFS on X. The variance of $\tilde{A}$ can be represented as

$$D(\tilde{A}) = \frac{1}{n-1}\sum_{i=1}^{n}\{(\mu_{\tilde{A}}(x_i) - \overline{\mu_{\tilde{A}}})^2 + (\nu_{\tilde{A}}(x_i) - \overline{\nu_{\tilde{A}}})^2\} \quad (7)$$

Definition 9. Let $\tilde{A}$ and $\tilde{B}$ be two IVIFSs on X

$$\tilde{A} = \left\{ \left(x_i, \left[\mu^L_{\tilde{A}}(x_i)\, \mu^U_{\tilde{A}}(x_i)\right], \left[\nu^L_{\tilde{A}}(x_i), \nu^U_{\tilde{A}}(x_i)\right]\right) \middle| \, x_i \in X\right\}$$
$$\tilde{B} = \left\{ \left(x_i, \left[\mu^L_{\tilde{B}}(x_i)\, \mu^U_{\tilde{B}}(x_i)\right], \left[\nu^L_{\tilde{B}}(x_i), \nu^U_{\tilde{B}}(x_i)\right]\right) \middle| \, x_i \in X\right\}$$

The covariance of $\tilde{A}$ and $\tilde{B}$ can be defined by

$$COV(\tilde{A}, \tilde{B}) = \frac{1}{n-1}\{(\mu_{\tilde{A}}(x_i) - \overline{\mu_{\tilde{A}}})(\mu_{\tilde{B}}(x_i) - \overline{\mu_{\tilde{B}}}) + (\nu_{\tilde{A}}(x_i) - \overline{\nu_{\tilde{A}}})(\nu_{\tilde{B}}(x_i) - \overline{\nu_{\tilde{B}}})\} \quad (8)$$

Table 1
Symptoms characteristic for the considered diagnoses

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problems
Temperature	(0.4,0)	(0.7,0)	(0.3,0.3)	(0.1,0.7)	(0.1,0.8)
Headache	(0.3,0.5)	(0.2,0.6)	(0.6,0.1)	(0.2,0.4)	(0,0.8)
Stomach pain	(0.1,0.7)	(0,0.9)	(0.2,0.7)	(0.8,0)	(0.2,0.8)
Cough	(0.4,0.3)	(0.7,0)	(0.2,0.6)	(0.2,0.7)	(0.2,0.8)
Chest pain	(0.1,0.7)	(0.1,0.8)	(0.1,0.9)	(0.2,0.7)	(0.8,0.1)

Table 2
Symptoms characteristic for the considered patients

	Temperature	Headache	Stomach pain	Cough	Chest pain
Al	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Bob	(0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Joe	(0.8,0.1)	(0.8,0.1)	(0,0.6)	(0.2,0.7)	(0,0.5)
Ted	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

Table 3
Correlation coefficients of symptoms for each patient to the possible diagnose sets

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problems
Al	0.8101	**0.8192**	0.6611	–09228	–0.5794
Bob	–0.5206	–0.653	0.1494	**0.9230**	–0.3717
Joe	0.4852	–0.3001	**0.7352**	–0.3720	–0.4740
Ted	0.8753	**0.9072**	0.2588	–0.5766	–0.4804

Table 4
The most possible diagnosis for each patient under different methods

	Our method	Method in Ref. [11]	Method in Ref. [6]	Method in Ref. [16]	Method in Ref. [7]	Method in Ref. [15]
Al	M	V	M	M	V	V
Bob	S	S	S	S	S	S
Joe	T	T	T	V	T	T
Ted	M	M	V	V	V	M

Proof similar to proposition, we have some properties of the covariance of two interval – valued intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ as follows.

Proposition 2. *Let $\tilde{A}$ and $\tilde{B}$ be two arbitrary IVIFSs on X. We have*

(1) $COV(\tilde{A}, \tilde{B}) = COV(\tilde{B}, \tilde{A})$

(2) $COV(\tilde{A}, \tilde{A}) = D(\tilde{A})$

(3) $\left|COV(\tilde{A}, \tilde{B})\right| \leq D(\tilde{A})^{0.5} D(\tilde{B})^{0.5}$

Proof. Similarity to proof of Proposition 1. □

Now, we can define the correlation coefficient of the interval - valued intuitionistic fuzzy sets (IVIFSs). It is similar to the correlation coefficient of real number variables in statistic.

Definition 10. Let $\tilde{A}$ and $\tilde{B}$ be two arbitrary IVIFSs on X. The correlation coefficient of $\tilde{A}$ and $\tilde{B}$ can be defined by

$$\rho(\tilde{A}, \tilde{B}) = \frac{COV(\tilde{A}, \tilde{B})}{\sqrt{D(\tilde{A})D(\tilde{B})}} \tag{9}$$

Theorem 2. *Given two IVIFSs $\tilde{A}$ and $\tilde{B}$, then we have*

(1) $\rho(\tilde{A}, \tilde{B}) = \rho(\tilde{B}, \tilde{A})$
(2) $-1 \leq \rho(\tilde{A}, \tilde{B}) \leq 1$.
(3) If $\tilde{A} = k\tilde{B} + b$ for some $k > 0$, then $\rho(\tilde{A}, \tilde{B}) = 1$. Here, $\tilde{A} = k\tilde{B} + b$ means that $\mu_{\tilde{A}} = k\mu_{\tilde{B}} + b$ and $\nu_{\tilde{A}} = k\nu_{\tilde{B}} + b$.
(4) If $\tilde{A} = k\tilde{B} + b$ for some $k < 0$, then $\rho(\tilde{A}, \tilde{B}) = -1$.

Proof.

(1), (2) is easy to verify.

(3). If $\tilde{A} = k\tilde{B} + b$ means that $\mu^L_{\tilde{A}} = k\mu^L_{\tilde{B}} + b$, $\mu^U_{\tilde{A}} = k\mu^U_{\tilde{B}} + b$ and $\nu^L_{\tilde{A}} = k\nu^L_{\tilde{B}} + b$, $\nu^U_{\tilde{A}} = k\nu^U_{\tilde{B}} + b$.

We have

$$\mu_{\tilde{A}}(x_i) = \frac{\mu^L_{\tilde{A}}(x_i)+\mu^U_{\tilde{A}}(x_i)}{2} = \frac{k\mu^L_{\tilde{B}}(x_i)+b+k\mu^U_{\tilde{B}}(x_i)+b}{2}$$
$$= \frac{k(\mu^L_{\tilde{B}}(x_i)+\mu^U_{\tilde{B}}(x_i))+2b}{2} = k\mu_{\tilde{B}}(x_i)+b, \quad (10)$$

$$\nu_{\tilde{A}}(x_i) = \frac{\nu^L_{\tilde{A}}(x_i)+\nu^U_{\tilde{A}}(x_i)}{2} = \frac{k\nu^L_{\tilde{B}}(x_i)+b+k\nu^U_{\tilde{B}}(x_i)+b}{2}$$
$$= \frac{k(\nu^L_{\tilde{B}}(x_i)+\nu^U_{\tilde{B}}(x_i))+2b}{2} = k\nu_{\tilde{B}}(x_i)+b.$$

So that

$$\bar{\mu}_{\tilde{A}} = \frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\mu^L_{\tilde{A}}(x_i) + \frac{1}{n}\sum_{i=1}^{n}\mu^U_{\tilde{A}}(x_i)\right)$$
$$= k\bar{\mu}_{\tilde{B}} + b,$$

$$\bar{\nu}_{\tilde{A}} = \frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\nu^L_{\tilde{A}}(x_i) + \frac{1}{n}\sum_{i=1}^{n}\nu^U_{\tilde{A}}(x_i)\right)$$
$$= k\bar{\nu}_{\tilde{B}} + b.$$

Hence, we have

$$COV(\tilde{A},\tilde{B}) = \frac{1}{n-1}\{(\mu_{\tilde{A}}(x_i)-\overline{\mu_{\tilde{A}}})(\mu_{\tilde{B}}(x_i)-\overline{\mu_{\tilde{B}}})$$
$$+(\nu_{\tilde{A}}(x_i)-\overline{\nu_{\tilde{A}}})(\nu_{\tilde{B}}(x_i)-\overline{\nu_{\tilde{B}}})\}$$
$$= \frac{1}{n-1}\{(k(\mu_{\tilde{B}}(x_i)-\overline{\mu_{\tilde{B}}})(\mu_{\tilde{B}}(x_i)-\overline{\mu_{\tilde{B}}})$$
$$+k(\nu_{\tilde{B}}(x_i)-\overline{\nu_{\tilde{B}}})(\nu_{\tilde{B}}(x_i)-\overline{\nu_{\tilde{B}}})\}$$
$$= \frac{k}{n-1}\{((\mu_{\tilde{B}}(x_i)-\overline{\mu_{\tilde{B}}})(\mu_{\tilde{B}}(x_i)-\overline{\mu_{\tilde{B}}})$$
$$+(\nu_{\tilde{B}}(x_i)-\overline{\nu_{\tilde{B}}})(\nu_{\tilde{B}}(x_i)-\overline{\nu_{\tilde{B}}})\}$$
$$= kD(\tilde{B}).$$

and

$$D(\tilde{A}) = \frac{1}{n-1}\sum_{i=1}^{n}\{(\mu_{\tilde{A}}(x_i)-\overline{\mu_{\tilde{A}}})^2+(\nu_{\tilde{A}}(x_i)-\overline{\nu_{\tilde{A}}})^2\}$$
$$= \frac{1}{n}\sum_{i=1}^{n}[k^2(\mu_{\tilde{B}}(x_i)-\overline{\mu_{\tilde{B}}})^2+k^2(\nu_{\tilde{B}}(x_i)-\overline{\nu_{\tilde{B}}})^2]$$
$$= k^2D(\tilde{B})$$

If $k > 0$ then

$$\rho(\tilde{A},\tilde{B}) = \frac{COV(\tilde{A},\tilde{B})}{\sqrt{D(\tilde{A})D(\tilde{B})}} = 1.$$

(4) If $k < 0$ then

$$\rho(\tilde{A},\tilde{B}) = \frac{COV(\tilde{A},\tilde{B})}{\sqrt{D(\tilde{A})D(\tilde{B})}}$$
$$= \frac{kD(\tilde{B})}{\sqrt{k^2D(\tilde{B})D(\tilde{B})}} = \frac{kD(\tilde{B})}{-kD(\tilde{B})} = -1. \qquad \square$$

Now, we apply the new correlation coefficient of intuitionistic fuzzy sets in a pattern recognition problem as follows.

Given m pattern $\{A_1, A_2, \ldots, A_m\}$ in the form of the interval valued intuitionistic fuzzy sets on the universal set X.

There is a new sample $A \in IVIFS(X)$.

Question: What pattern does B belong to?

To answer this question, we consider the correlation coefficient of intuitionistic fuzzy sets $\rho(A_i, A)$ of sample A to each pattern A_i for all $i = 1, 2, \ldots, m$. If $\rho(A_i, A) > \rho(A_k, A)$ then we put A belongs to the class of pattern A_i for $i, k = 1, 2, \ldots, m$.

Example 7. We consider a pattern recognition problem about the classification of minerals; the data was quoted from Liu et al. [11]. There are three classes of given minerals, which are expressed by the IVIFSs $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3$ in the feature space $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. Now, if there is a new mineral $\tilde{A}$ along with its known attribute values. Our aim is to determine which class that $\tilde{A}$ belongs to. The descriptive data information is given in Table 5.

From the result in Table 6, we can comment that the correlation coefficients between $\tilde{A}_i (i = 1, 2, 3)$ and $\tilde{A}$ as follow $\rho(\tilde{A}_3, A) > \rho(\tilde{A}_2, A) > \rho(\tilde{A}_1, A)$, so that we can put $\tilde{A}$ belongs to class $\tilde{A}_3$. We also compare the result of our method to the results obtained by other methods in Refs. [4, 7, 11, 16, 19], the result using our method is identical with the results using methods in Refs. [4, 7, 11, 16, 19].

5. Application of correlation coefficient in clustering intuitionistic sets

In this section, we use the correlation coefficient in clustering intuitionistic fuzzy sets. This ideal is based on Xu et al. [18].

Definition 11. [18] Let $R = [R_{ij}]_{m\times m}$ and $S = [S_{ij}]_{m\times m}$ be two matrices which $0 \le R_{ij}, S_{ij} \le 1$ for all $i, j = 1, 2, \ldots, m$. The composition of two matrices R and S denoted $T = R \circ S = [T_{ij}]_{m\times m}$, which is defined by $T_{ij} = max_{k=1,2,\ldots,m}\{\min\{R_{ik}, S_{kj}\}\}$ for all $i, j = 1, 2, \ldots, m$.

The clustering algorithm based on the correlation coefficient of intuitionistic fuzzy sets as follows.

Algorithm

Input: Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of intuitionistic fuzzy set on $X = \{x_1, x_2, \ldots, x_n\}$.

Output: Clustering for A.

Step 1. We construct a correlation matrix $C = [c_{ij}]_{m\times m}$ where $c_{ij} = |\rho(A_i, A_j)|$, $i, j = 1, 2, \ldots, m$.

Table 5
Data information of minerals

	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$	$\tilde{A}_4$
x_1	([0.72,0.74],[0.1,0.12])	([0.42,0.45],[0.38,0.40])	([0.30,0.32],[0.45,0.47])	([0.60,0.63],[0.30,0.35])
x_2	([0.00,0.05],[0.80,0.82])	([0.65,0.67],[0.28,0.30])	([0.90,1.00],[0.00,0.00])	([0.50,0.53],[0.34,0.36])
x_3	([0.18,0.20],[0.62,0.63])	([0.00,1.00],[0.00,0.00])	([0.18,0.20],[0.70,0.73])	([0.20,0.21],[0.68,0.70])
x_4	([0.49,0.50],[0.35,0.37])	([0.70,0.90],[0.00,0.10])	([0.15,0.16],[0.75,0.78])	([0.20,0.22],[0.75,0.77])
x_5	([0.01,0.02],[0.60,0.63])	([0.80,1.00],[0.00,0.00])	([0.00,0.05],[0.88,0.90])	([0.05,0.07],[0.87,0.90])
x_6	([0.72,0.74],[0.12,0.13])	([0.90,1.00],[0.00,0.00])	([0.65,0.68],[0.25,0.30])	([0.65,0.70],[0.25,0.30])

Table 6
Correlation coefficients of $\tilde{A}$ and $\tilde{A}_i$ ($i = 1, 2, 3$) under different methods

	Our method	Method in Ref. [11]	Method in Ref. [4]	Method in Ref. [16]	Method in Ref. [7]	Method in Ref. [19]
$\rho(\tilde{A}_1, A)$	0.562	0.53	0.86	0.78	0.52	0.85
$\rho(\tilde{A}_2, A)$	–0.308	–0.52	0.52	0.77	–0.53	0.51
$\rho(\tilde{A}_3, A)$	**0.817**	**0.81**	**0.94**	**0.84**	**0.81**	**0.94**

Step 2. Compute $C^{2^{k+1}} = C^{2^k} \circ C^{2^k}, k = 0, 1, 2, \ldots$. We construct the sequence

$$C \to C^2 \to C^4 \to \ldots \to C^{2^k} \to \ldots$$

until $C^{2^{k+1}} = C^{2^k}$.

Step 3. For a confidence level $\lambda \in [0, 1]$, we construct a $\lambda-$ cutting matrix C_λ of matrix C^{2^k}. If the j-th column of C_λ is equal to the i-th column, then we put A_j, A_i in the same class.

Illustrative example

Example 8. We assume that $\{A_1, A_2, A_3, A_4\}$ is a set of intuitionistic fuzzy set on $\{x_1, x_2, x_3\}$ as follow:

$$A_1 = \{(x_1, 0.4, 0.2), (x_2, 0.1, 0.2), (x_3, 0.1, 0.2)\}$$
$$A_2 = \{(x_1, 0.5, 0.3), (x_2, 0.1, 0.1), (x_3, 0.3, 0.3)\},$$
$$A_3 = \{(x_1, 0.1, 0.5), (x_2, 0.1, 0.1), (x_3, 0.3, 0.3)\},$$
$$A_4 = \{(x_1, 0.1, 0.5), (x_2, 0.1, 0.3), (x_3, 0.3, 0.1)\}.$$

Step 1. Using Equation (3) and $c_{ij} = \left|\rho(A_i, A_j)\right|$, for all $i, j = 1, 2, \ldots, m$ we have the correlation matrix C:

$$C = \begin{bmatrix} 1 & 0.75 & 0.25 & 0.25 \\ 0.75 & 1 & 0.375 & 0 \\ 0.25 & 0.375 & 1 & 0.625 \\ 0.25 & 0 & 0.625 & 1 \end{bmatrix}$$

Step 2. Construction equivalence matrix:

$$C^2 = C \circ C = \begin{bmatrix} 1 & 0.75 & 0.375 & 0.25 \\ 0.75 & 1 & 0.375 & 0.375 \\ 0.375 & 0.375 & 1 & 0.625 \\ 0.25 & 0.375 & 0.625 & 1 \end{bmatrix}$$

$$C^4 = C^2 \circ C^2 = \begin{bmatrix} 1 & 0.75 & 0.375 & 0.375 \\ 0.75 & 1 & 0.375 & 0.375 \\ 0.375 & 0.375 & 1 & 0.625 \\ 0.375 & 0.375 & 0.625 & 1 \end{bmatrix}$$

and

$$C^8 = C^4 \circ C^4 = \begin{bmatrix} 1 & 0.75 & 0.375 & 0.375 \\ 0.75 & 1 & 0.375 & 0.375 \\ 0.375 & 0.375 & 1 & 0.625 \\ 0.375 & 0.375 & 0.625 & 1 \end{bmatrix}$$

Here, we see that $C^8 = C^4$. So that C^4 is equivalent matrix.

Step 3. Based on $\lambda-$cutting matrix C_λ of matrix C^4 we get all possible classifications of $\{A_1, A_2, A_3, A_4\}$:

+ If $0 \le \lambda \le 0.375$, then we have one cluster:

$$\{A_1, A_2, A_3, A_4\}.$$

+ If $0.375 < \lambda \le 0.625$, then we have two clusters: $\{A_1, A_2\}$ and $\{A_3, A_4\}$

+If $0.625 < \lambda \le 0.75$, then we have three clusters: $\{A_1, A_2\}$, $\{A_3\}$ and $\{A_4\}$.

+If $0.75 < \lambda \le 1$, then we have three clusters: $\{A_1\}$, $\{A_2\}$, $\{A_3\}$ and $\{A_4\}$.

6. Conclusion

From a statistical standpoint, the domain of the correlation coefficient is [–1,1]. Not only does our research indicate that, but many other studies point out. This value domain is more significant than correlation coefficients for only [0,1], because, besides pointing out the linear relationship between two sets of data in a space of observation objects, the correlation coefficient also indicates the variability of the two sets of data. Two datasets may have the same tendency, or tend to decrease (in case of positive correlation). It is also possible that the first data set is incremented, the second data set is reduced; in contrast, the first data set is reduced, the second data set is incremented (in the case of a negative correlation). The correlation coefficient of two intuitionistic fuzzy sets should also reflect this. In addition, because of the characteristics of intuitionistic fuzzy sets, there are two functions: a membership function, a non-membership function of an intuitionistic fuzzy set by the feature (elements) of the sample space. Thus, the correlation between intuitionistic fuzzy sets has its own characteristics. As many of the authors have previously studied, we consider the correlation coefficients of intuitionistic fuzzy sets based on both membership functions and non-member functions. Our method can effectively solve some cases where a previous method was difficult. This is demonstrated by the examples we present in this article. In this article, we also apply the methods we propose in determining the appropriate diagnostic methods in medicine, and in the problem of pattern recognition, clustering problems.

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