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Introduction to Hyper Neutrosophic Set

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Abstract: This paper is devoted to introducing a novel concept known as Hyper Neutrosophic Set (HNS) as another subclass of neutrosophic Set (NS). The purpose of introducing the notion of HNS is to give a new mathematical theory that is more promising and purposeful than the existing fuzzy-centric theories to solve the uncertainty-based real-world problems lucidly. From a decision-makers point of view, the new mathematical tool can be viewed as a direct extension of the Pythagorean neutrosophic set (PNS). The PNS has its inherent limitation for which the decision-makers can't answer a certain type of problem. For example, in a certain problem, if we consider the degree of truth-membership $T_A(x)$ =0.8, degree of indeterminate-membership $I_A(x)=0.9$, and the degree of falsity-membership $F_A(x)$=0.8, then it gives an absurd result under PNS. To remove such kind of absurdity, there is a demand to introduce another superior set-theoretical concept that provides more information for the decision-makers. This gives rise to the introduction of HNS. In HNS, we choose any value that belongs to $[0,1]$ for the three membership degrees so that their product is always limited to 1. So, the beauty of HNS is that it can accommodate more information within a small range with relaxed membership values i.e under HNS we can consider the maximum membership triplet as $(1,1,1)$, that is not allowed in PNS. Undoubtedly the HNS gives a more compact set-theoretical model to describe imprecise knowledge with ease.

Keywords: Neutrosophic set; Pythagorean neutrosophic set; Hyper neutrosophic set; Decision-making

1. Introduction

Cantor set is introduced to present the well-defined countable/uncountable objects. The characteristic function is introduced that allows the users to take only two values 0(for non-belongingness) and 1(for belongingness) of an object. To measure any degree of imprecision, the characteristic function is replaced by the membership function which is a basic constituent of a fuzzy set (FS) introduced by Zadeh [1]. Later on, to measure the incomplete information of an object, the notion of the intuitionistic fuzzy set (IFS) [2] is forwarded. In IFS, the sum of the membership and the non-membership degree is limited to 1. But, what do we do when their sum exceeds 1. Realizing the importance to defend such a situation that is inspired by nature due to computation intelligence, Yager [3] introduced the Pythagorean fuzzy set (PyFS). In PyFS, there is more liberty in the selection of membership and non-membership values with a condition that the sum of their squares does not exceed 1. So, by PyFS, we measure both the membership and non-membership degrees of an object with better accuracy and precision. For practical decision-making, PyFS provides more information than IFS. Cuong et al. [4] tried to handle uncertainty differently with an aid of a picture fuzzy set (PFS). PFS can be regarded as a direct extension of FS and IFS that is promised to address more complex phenomena found in human opinions. The decision-makers' decision was somehow restricted under the PFS because of its structural system. To enlarge the space of the membership degrees of PFS, in 2019, Ashraf et al.[5] introduced the spherical fuzzy set(SFS) as an extension of PyFS. Furthermore, the PyFS has been generalized by introducing the q-rung orthopair fuzzy set (q-ROFS) [6]. Also, Senapati et al.[7] introduced the Farmatean fuzzy set(FFS) as a special case of q-ROFS.

Indeterminacy is an important component of uncertainty that we encounter in the real-world. The concept of FS, IFS, and PyFS are not sufficient to tackle the indeterminate information. In 2005, Smarandache[8] introduced the neutrosophic set(NS) as a generalization of IFS. In NS, every object x has three neutrosophic components denoted by $x(T,I,F)$, where T for truth-component, I for indeterminate-component, and F is false-component. Here, $T,I,F \in \,]^{-}0,1^{+}[$ such that $^{-}0 \le T+I+F \le 3^{+}$. Though it is sensible to use non-standard unit interval in NS for describing the linguistic and philosophical concept but it is not useful for scientific modeling as it demands specified membership values. For this purpose, the single-valued neutrosophic set (SVNS) [9] is introduced where the neutrosophic components belong to the standard unit interval $[0,1]$. Normally in NS, the components are independent, but to reach different target problems, researchers put various restrictions on the components and utilize them to solve certain types of real problems. In 2019, Jansi et al.[10] presented the Pythagorean neutrosophic set(PNS) as a special type of NS with dependent components (T,F) and apply in medical diagnosis by using its correlation measure. Ajay et al.[11] gives an introduction of the Pythagorean neutrosophic fuzzy graph. Veerappan et al. [12] proposed the Pythagorean neutrosophic ideal in semigroups. Rajan et al.[13] have introduced the similarity measures of PNS. In [14], Jansi et al. introduced the pairwise Pythagorean neutrosophic P-spaces. So far several works associated with PNS have been successfully carried out and thus the PNS has rich potential in many practice areas and so there is a natural question that arises how to generalize it further. Because of the present discussion, it is worthy to introduce HNS as a special type of NS that is undertaken to generalize the PNS to cover up the information gap realized by the decision-makers.

The hierarchy structure of HNS is shown in Fig 1.

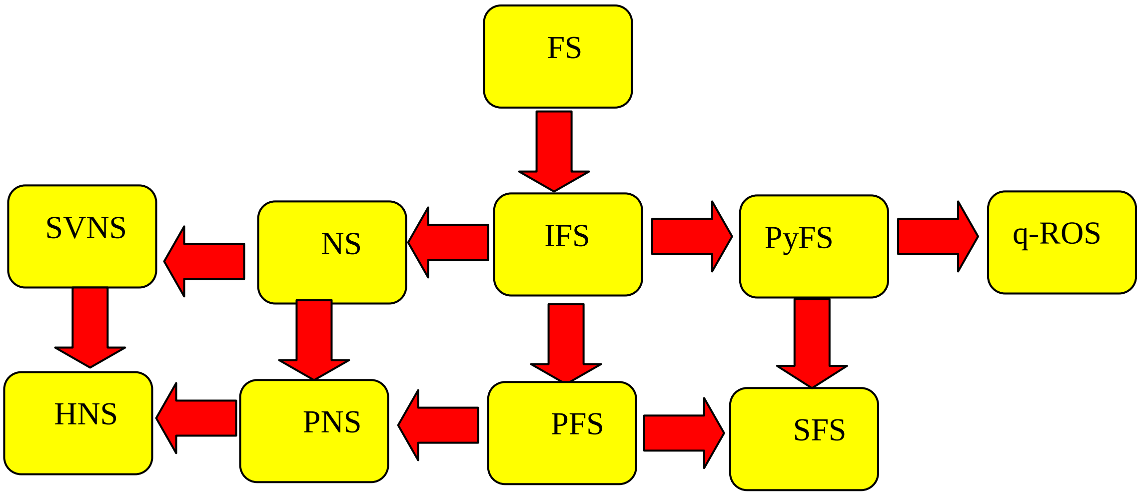

Fig. 1. Hierchy formation of HNS

To show the feasibility of the proposed study, we make a comparison of SVNS, PNS, and HNS in the following Table 1.

SVNS	PNS	HNS
$T, I, F \in [0,1]$ such that $0 \le T + I + F \le 3$ $\Pi = 3 - (T + I + F)$	$T + F \le 1$ and $0 \le T^2 + I^2 + F^2 \le 2$ $\Pi = \sqrt{2 - (T^2 + F^2 + I^2)}$	$T, I, F \in [0,1]$ such that $0 \le T.I.F \le 1$ and $\Pi = 1 - T.I.F$

Table 1. Comparison of SVNS, PNS, and HNS

This paper is structured in the following manner: Section 2 contains some basic definitions that are relevant to the proposed topic. Section 3 includes the introduction of HNS and its various properties. Correlation measures of two HNSs are introduced in section 4. Section 5 includes the application of HNS in medical decision-making. In the last section i.e. in section 6, the conclusion and the future study of the current topic has been briefly discussed

2. Preliminaries

In the context of the proposed study, in this section, we recollect some basic notions. Throughout the section, the set of universe is denoted by U .

Definition 2.1 [1] A fuzzy set A over U is denoted by an ordered pair of the form $A = \{(x, \mu_A(x)) : x \in U\}$, where μ_A represents the membership function that is defined by the mapping $\mu_A : U \to [0,1]$.

Definition 2.2 [2] An intuitionistic fuzzy set B over U is defined in the following form:

$\mathrm{B}=\left\{\left(y,\mu_B\left(y\right),\gamma_B\left(y\right)\right):y\in U\right\}$, where μ_B and γ_B represent the membership and the non-membership degree respectively, and $\mu_B,\gamma_B:U\to[0,1]$ such that $0\le\mu_B\left(y\right)+\gamma_B\left(y\right)\le 1,\ \forall y\in U$.

Definition 2.3 [4] A picture fuzzy set X over U is an object of the form

$\mathrm{X}=\left\{\left(\alpha,\mu_X\left(\alpha\right),\eta_X\left(\alpha\right),\gamma_X\left(\alpha\right)\right):\alpha\in U\right\}$, where $\mu_X\left(\alpha\right),\eta_X\left(\alpha\right),\gamma_X\left(\alpha\right)\in[0,1]$ denote the degree of acceptance membership-degree, neutral membership-degree, and rejection membership-degrees respectively such that $0\le\mu_X\left(\alpha\right)+\eta_X\left(\alpha\right)+\gamma_X\left(\alpha\right)\le 1,\ \forall\alpha\in U$.

Definition 2.4 [5] A spherical fuzzy set E over U can be written as;

$\mathrm{E}=\left\{\left(\beta,\mu_E\left(\beta\right),\eta_E\left(\beta\right),\gamma_E\left(\beta\right)\right):\beta\in U\right\}$, where $\mu_E\left(\beta\right),\eta_E\left(\beta\right),\gamma_E\left(\beta\right)\in[0,1]$ indicate the positive, neutral, and negative membership degrees such that $0\le{\mu^2}_E\left(\beta\right)+{\eta^2}_E\left(\beta\right)+{\gamma^2}_E\left(\beta\right)\le 1,\ \forall\beta\in U$.

Definition 2.5 [3] A Pythagorean fuzzy set H over U is an object of the form given as;

$\mathrm{H}=\left\{\left(\varepsilon,\mu_H\left(\varepsilon\right),\gamma_H\left(\varepsilon\right)\right):\varepsilon\in U\right\}$, where the functions μ_H, $\gamma_H:U\to[0,1]$ defined the degree of membership $\mu_H\left(\varepsilon\right)$ and the degree of non-membership $\gamma_H\left(\varepsilon\right)$ respectively such that $0\le{\mu^2}_H\left(\varepsilon\right)+{\gamma^2}_H\left(\varepsilon\right)\le 1,\ \forall\varepsilon\in U$.

Definition 2.6 [9] A single-valued neutrosophic set J over U is an object that can be written as;

$J=\left\{\left(t,T_J\left(t\right),I_J\left(t\right),F_J\left(t\right)\right):t\in U\right\}$, where the functions $T_J,I_J,F_J:U\to[0,1]$ defined the degree of truth membership $T_J\left(t\right)$, indeterminate membership $I_J\left(t\right)$, and false membership $F_J\left(t\right)$ such that $0\le T_J\left(t\right)+I_J\left(t\right)+F_J\left(t\right)\le 3,\ \forall t\in U.$

Definition 2.7 [10] A Pythagorean neutrosophic set G over U is defined as;

$G=\left\{\left(s,T_G\left(s\right),I_G\left(s\right),F_G\left(s\right)\right):s\in U\right\}$, where $T_G\left(s\right),I_G\left(s\right),F_G\left(s\right)\in[0,1]$ denote the degrees of truth membership, indeterminate membership, and false membership respectively with the conditions $0\le T_G\left(s\right)+I_G\left(s\right)\le 1$ and $0\le{T^2}_G\left(s\right)+{I^2}_G\left(s\right)+{F^2}_G\left(s\right)\le 2.$

From the above discussion, an information gap has been identified that cannot be filled up by the existing theories. Such a research gap has been produced due to human intelligence. To make it visible to the readers, we discuss the following:

Suppose there exist a certain domain where a piece of information provided by a decision-maker is denoted by a triplet (T,I,F) where $T=0.9$, $I=0.9$, and $F=0.9$. Such information cannot be answered by the SFS and PNS. Though it can be easily defined by the SVNS, we are looking for another robust system where the same information can be easily accommodated within a small range. This lead to the introduction of NHS where even the maximum triplet $(1,1,1)$ can easily accommodate with the condition that their product is restricted up to 1. Undoubtedly, this new neutrosophic system is finer than SVNS.

3. Hyper Neutrosophic Set

Definition 3.1 Let $\Omega \neq \varnothing$ be a set of universe. A hyper neutrosophic set (HNS) Λ over Ω is characterized by the degrees of truth membership $T_\Lambda(\xi)$, indeterminate membership $I_\Lambda(\xi)$, and false membership $F_\Lambda(\xi)$ and it is defined as;

$\Lambda = \{\langle \xi, T_\Lambda(\xi), I_\Lambda(\xi), F_\Lambda(\xi)\rangle : \xi \in \Omega\}$, where T_Λ, I_Λ, $F_\Lambda : \Omega \to [0,1]$ with the condition $0 \leq T_\Lambda(\xi).I_\Lambda(\xi).F_\Lambda(\xi) \leq 1, \forall \xi \in \Omega$.

The hesitation degree is denoted by $\Pi_\Lambda(\xi)$ and it is defined as $\Pi_\Lambda(\xi) = 1 - T_\Lambda(\xi).I_\Lambda(\xi).F_\Lambda(\xi)$.

For $T_\Lambda(\xi) = I_\Lambda(\xi) = F_\Lambda(\xi) = 1, \Pi_\Lambda(\xi) = 0$.

The set of all HNSs over Ω is denoted by $I^{HNS(\Omega)}$. Also, for any $\xi \in \Omega$, the hyper neutrosophic number (HNN) is denoted by $\langle T_\Lambda(\xi), I_\Lambda(\xi), F_\Lambda(\xi)\rangle$.

Example 3.1.1 Let Ω be a HNS with $T_\Lambda(\xi) = 0.95$, $I_\Lambda(\xi) = 0.94$, and $F_\Lambda(\xi) = 0.96$. Then, $\Pi_\Lambda(\xi) = 1 - (0.95).(0.94).(0.96) = 0.142$.

Definition 3.2 For any two HNNs $\Lambda_1 = \langle T_{\Lambda_1}(\xi), I_{\Lambda_1}(\xi), F_{\Lambda_1}(\xi)\rangle$ and $\Lambda_2 = \langle T_{\Lambda_2}(\xi), I_{\Lambda_2}(\xi), F_{\Lambda_2}(\xi)\rangle$, we have the following properties:

1. $\Lambda_1 \subseteq \Lambda_2 \Leftrightarrow T_{\Lambda_1}(\xi) \leq T_{\Lambda_2}(\xi), I_{\Lambda_1}(\xi) \leq I_{\Lambda_2}(\xi)$, and $F_{\Lambda_1}(\xi) \geq F_{\Lambda_2}(\xi)$.

2. $\Lambda_1 = \Lambda_2 \Leftrightarrow \Lambda_1 \subseteq \Lambda_2$ and $\Lambda_2 \subseteq \Lambda_1$

3. If $\Lambda_1 = \langle T_{\Lambda_1}(\xi), I_{\Lambda_1}(\xi), F_{\Lambda_1}(\xi)\rangle$, then its complement is defined as $\Lambda^c{}_1 = \langle F_{\Lambda_1}(\xi), 1 - I_{\Lambda_1}(\xi), T_{\Lambda_1}(\xi)\rangle$

4. $\Lambda_1 \tilde{\bigcup} \Lambda_2 = \langle \max(T_{\Lambda_1}(\xi), T_{\Lambda_2}(\xi)), \max(I_{\Lambda_1}(\xi), I_{\Lambda_2}(\xi)), \min(F_{\Lambda_1}(\xi), F_{\Lambda_2}(\xi)) \rangle$

5. $\Lambda_1 \tilde{\bigcap} \Lambda_2 = \left\langle \min(T_{\Lambda_1}(\xi), T_{\Lambda_2}(\xi)), \min(I_{\Lambda_1}(\xi), I_{\Lambda_2}(\xi)), \max(F_{\Lambda_1}(\xi), F_{\Lambda_2}(\xi)) \right\rangle$

6. $\left(\Lambda_1 \tilde{\bigcup} \Lambda_2\right)^c = \Lambda^c{}_1 \tilde{\bigcap} \Lambda^c{}_2$; $\left(\Lambda_1 \tilde{\bigcap} \Lambda_2\right)^c = \Lambda^c{}_1 \tilde{\bigcup} \Lambda^c{}_2$

Definition 3.3 For any two HNNs $\Lambda_1 = \left\langle T_{\Lambda_1}(\xi), I_{\Lambda_1}(\xi), F_{\Lambda_1}(\xi) \right\rangle$ and $\Lambda_2 = \left\langle T_{\Lambda_2}(\xi), I_{\Lambda_2}(\xi), F_{\Lambda_2}(\xi) \right\rangle$, we define the following operators:

1. **(t-norm)**

$$\Lambda_1 \oplus \Lambda_2 = \left\langle T_{\Lambda_1}(\xi), I_{\Lambda_1}(\xi), F_{\Lambda_1}(\xi) \right\rangle \oplus \left\langle T_{\Lambda_2}(\xi), I_{\Lambda_2}(\xi), F_{\Lambda_2}(\xi) \right\rangle$$

$$= \left\langle T_{\Lambda_1}(\xi) + T_{\Lambda_2}(\xi) - T_{\Lambda_1}(\xi).T_{\Lambda_2}(\xi), I_{\Lambda_1}(\xi) + I_{\Lambda_2}(\xi) - I_{\Lambda_1}(\xi).I_{\Lambda_2}(\xi), F_{\Lambda_1}(\xi).F_{\Lambda_2}(\xi) \right\rangle$$

2. **(t-co norm)**

$$\Lambda_1 \otimes \Lambda_2 = \left\langle T_{\Lambda_1}(\xi), I_{\Lambda_1}(\xi), F_{\Lambda_1}(\xi) \right\rangle \otimes \left\langle T_{\Lambda_2}(\xi), I_{\Lambda_2}(\xi), F_{\Lambda_2}(\xi) \right\rangle$$

$$= \left\langle T_{\Lambda_1}(\xi).T_{\Lambda_2}(\xi), I_{\Lambda_1}(\xi).I_{\Lambda_2}(\xi), F_{\Lambda_1}(\xi) + F_{\Lambda_2}(\xi) - F_{\Lambda_1}(\xi).F_{\Lambda_2}(\xi) \right\rangle$$

3. **(Scalar t-norm)**

For any scalar $\varsigma > 0$, $\varsigma^{\oplus} \Lambda_1 = \left\langle 1 - \left(1 - T_{\Lambda_1}(\xi)\right)^{\varsigma}, 1 - \left(1 - I_{\Lambda_1}(\xi)\right)^{\varsigma}, \left(F_{\Lambda_1}(\xi)\right)^{\varsigma} \right\rangle$

4. **(Scalar t-co norm)**

For any scalar $\varsigma > 0$, $\varsigma^{\otimes} \Lambda_1 = \left\langle \left(T_{\Lambda_1}(\xi)\right)^{\varsigma}, \left(I_{\Lambda_1}(\xi)\right)^{\varsigma}, 1 - \left(1 - F_{\Lambda_1}(\xi)\right)^{\varsigma} \right\rangle$

Definition 3.4 (Score function) Let $\Lambda = \left\langle T_{\Lambda}(\xi), I_{\Lambda}(\xi), F_{\Lambda}(\xi) \right\rangle$ be any HNN. Then the score function Γ defined on Λ is a mapping $\Gamma : \Lambda \to [-1,1]$ such that $\Gamma(\Lambda) = 1 - T_{\Lambda}(\xi) . I_{\Lambda}(\xi) - F_{\Lambda}(\xi) . I_{\Lambda}(\xi)$.

Definition 3.5 (Accuracy function) Let $\Lambda = \left\langle T_{\Lambda}(\xi), I_{\Lambda}(\xi), F_{\Lambda}(\xi) \right\rangle$ be any HNN. Then the accuracy function Υ defined on Λ is a mapping $\Upsilon : \Lambda \to [0.5,1]$ such that $\Upsilon(\Lambda) = \frac{1 + T_{\Lambda}(\xi).F_{\Lambda}(\xi).I_{\Lambda}(\xi)}{2}$.

Proposition 3.6 Let $\Lambda \in I^{HNS(\Omega)}$. Then, $\forall \Lambda \in \Omega$, we have the following properties:

1. $\Gamma(\Lambda) = 0 \Rightarrow T_{\Lambda}(\xi) = I_{\Lambda}(\xi) = 1$ and $F_{\Lambda}(\xi) = 0$; OR $F_{\Lambda}(\xi) = I_{\Lambda}(\xi) = 1$ and $T_{\Lambda}(\xi) = 0$

2. $\Gamma(\Lambda) = 1 \Rightarrow I_{\Lambda}(\xi) = 0$ or $T_{\Lambda}(\xi) = F_{\Lambda}(\xi) = 0$ or $T_{\Lambda}(\xi) = F_{\Lambda}(\xi) = I_{\Lambda}(\xi) = 0$

3. $\Gamma(\Lambda) = -1 \Rightarrow T_{\Lambda}(\xi) = F_{\Lambda}(\xi) = I_{\Lambda}(\xi) = 1$

Proposition 3.7 Let $\Lambda \in I^{HNS(\Omega)}$. Then, $\forall \Lambda \in \Omega$, we have the following properties:

1. $\Upsilon(\Lambda)=1 \Rightarrow T_{\Lambda}(\xi)=F_{\Lambda}(\xi)=I_{\Lambda}(\xi)=1$

2. $\Upsilon(\Lambda)=0.5 \Rightarrow T_{\Lambda}(\xi)=F_{\Lambda}(\xi)=I_{\Lambda}(\xi)=0$

Definition 3.8 Suppose $\Lambda_1=\langle T_{\Lambda_1}(\xi), I_{\Lambda_1}(\xi), F_{\Lambda_1}(\xi)\rangle$ and $\Lambda_2=\langle T_{\Lambda_2}(\xi), I_{\Lambda_2}(\xi), F_{\Lambda_2}(\xi)\rangle$ be two HNNs over Ω. Their score and accuracy functions are denoted by $\Gamma(\Lambda_1)$, $\Gamma(\Lambda_2)$ and $\Upsilon(\Lambda_1)$, $\Upsilon(\Lambda_2)$ respectively. Then, we have the following properties:

1. If $\Gamma(\Lambda_1)<\Gamma(\Lambda_2)$, then $\Lambda_1 \prec \Lambda_2$.

2. If $\Gamma(\Lambda_1)>\Gamma(\Lambda_2)$, then $\Lambda_1 \succ \Lambda_2$.

3. If $\Gamma(\Lambda_1)=\Gamma(\Lambda_2)$, then we compare their accuracy functions as:

i. If $\Upsilon(\Lambda_1)<\Upsilon(\Lambda_2)$, then $\Lambda_1 \prec \Lambda_2$.

ii. If $\Upsilon(\Lambda_1)>\Upsilon(\Lambda_2)$, then $\Lambda_1 \succ \Lambda_2$.

iii. If $\Upsilon(\Lambda_1)=\Upsilon(\Lambda_2)$, then $\Lambda_1 \equiv \Lambda_2$.

Theorem 3.9 Let us consider the three HNNs over Ω as $\Lambda_1=\langle T_{\Lambda_1}(\xi), I_{\Lambda_1}(\xi), F_{\Lambda_1}(\xi)\rangle$, $\Lambda_2=\langle T_{\Lambda_2}(\xi), I_{\Lambda_2}(\xi), F_{\Lambda_2}(\xi)\rangle$, and $\Lambda_3=\langle T_{\Lambda_3}(\xi), I_{\Lambda_3}(\xi), F_{\Lambda_3}(\xi)\rangle$. Then, the following properties hold true:

1. $\Lambda_1 \oplus \Lambda_2=\Lambda_2 \oplus \Lambda_1; \Lambda_1 \otimes \Lambda_2=\Lambda_2 \otimes \Lambda_1$

2. $(\Lambda_1 \oplus \Lambda_2) \oplus \Lambda_3=\Lambda_1 \oplus (\Lambda_2 \oplus \Lambda_3); (\Lambda_1 \otimes \Lambda_2) \otimes \Lambda_3=\Lambda_1 \otimes (\Lambda_2 \otimes \Lambda_3)$

3. for any scalar $\varsigma>0$, $\varsigma^{\oplus}(\Lambda_1 \oplus \Lambda_2)=\varsigma^{\oplus}\Lambda_1 \oplus \varsigma^{\oplus}\Lambda_2; \quad \varsigma^{\otimes}(\Lambda_1 \otimes \Lambda_2)=\varsigma^{\otimes}\Lambda_1 \otimes \varsigma^{\otimes}\Lambda_2$

Note: It has been observed that

$\Lambda_1 \oplus (\Lambda_2 \otimes \Lambda_3) \neq (\Lambda_1 \oplus \Lambda_2) \otimes (\Lambda_1 \oplus \Lambda_3); \Lambda_1 \otimes (\Lambda_2 \oplus \Lambda_3) \neq (\Lambda_1 \otimes \Lambda_2) \oplus (\Lambda_1 \otimes \Lambda_3)$

Definition 3.10 Let $\Lambda_p=\langle T_{\Lambda_p}(\xi), I_{\Lambda_p}(\xi), F_{\Lambda_p}(\xi)\rangle$ defined over Ω represent a family of HNNs, where $p=1,2,..,n$. Then, the hyper neutrosophic weighted average (HNWA) t-norm operator with weight vector $\omega=\left(\omega_1,\omega_1,.....,\omega_n\right)^t$, where $\omega_p\geq 0$ and $\sum_{p=1}^{n}\omega_p=1$ is given by $HNWA\left(\Lambda_1,\Lambda_2,\Lambda_3,......,\Lambda_n\right)=\oplus_{p=1}^{n}\Lambda_p\omega_p$.

Definition 3.11 Let $\Lambda_p=\left\langle T_{\Lambda_p}\left(\xi\right),I_{\Lambda_p}\left(\xi\right),F_{\Lambda_p}\left(\xi\right)\right\rangle$ defined over Ω represent a family of HNNs, where $p=1,2,..,n$. Then, the hyper neutrosophic weighted geometric (HNWG) t-co norm operator with weight vector $\omega=\left(\omega_1,\omega_1,.....,\omega_n\right)^t$, where $\omega_p\geq 0$ and $\sum_{p=1}^{n}\omega_p=1$ is given by $HNWG\left(\Lambda_1,\Lambda_2,\Lambda_3,......,\Lambda_n\right)=\otimes_{p=1}^{n}\left(\Lambda_p\right)^{\omega_p}$.

Theorem 3.12 If $\Lambda\in I^{HNS(\Omega)}$, then the score function defined by $\Gamma\left(\Lambda\right)=1-T.I-I.F$ is always monotonically decreasing.

Proof. We have, $\Gamma\left(\Lambda\right)=1-T.I-I.F$

Then, $\dfrac{\partial\Gamma}{\partial I}=-(T+F)<0,\ \forall T,F\in\left[0,1\right]$

Also, $\dfrac{\partial\Gamma}{\partial T}=\dfrac{\partial\Gamma}{\partial F}=-I<0,\ \forall I\in\left[0,1\right]$

This proves the statement.

Theorem 3.13 If $\Lambda\in I^{HNS(\Omega)}$, then the accuracy function defined by $\Upsilon\left(\Lambda\right)=\dfrac{1+T.F.I}{2}$.

is always monotonically increasing.

Proof. It is left for the readers.

Proposition 3.14 Let $\Lambda_1,\Lambda_2\in I^{HNS(\Omega)}$.If $\Lambda_1\subseteq\Lambda_2$ then $\Gamma\left(\Lambda_1^c\right)\leq\Gamma\left(\Lambda_2^c\right)$.

Example 3.14.1 Let $\Lambda_1=\left\langle 0.5,0.6,0.4\right\rangle$ and $\Lambda_2=\left\langle 0.6,0.7,0.3\right\rangle$ such that $\Lambda_1\subseteq\Lambda_2$. Then, ${\Lambda^c}_1=\left\langle 0.4,0.4,0.5\right\rangle$ and ${\Lambda^c}_2=\left\langle 0.3,0.3,0.6\right\rangle$. Therefore, $\Gamma\left(\Lambda_1^c\right)=0.64\leq\Gamma\left(\Lambda_2^c\right)=0.73$.

Proposition 3.15 Let $\Lambda_1,\Lambda_2\in I^{HNS(\Omega)}$.If $\Lambda_1\subseteq\Lambda_2$ then $\Upsilon\left(\Lambda_1^c\right)\geq\Upsilon\left(\Lambda_2^c\right)$.

4. Correlation Measures of Two HNSs

Correlation measures between two variables describe how the two variables are close to each other or relate to each other. That is, it measures the degree of closeness between two variables. In a practical scenario, there are many instances where we are eager to know about the relationship between two variables. We classify the relationship between two variables as positive correlation (when both increase/decrease), negative correlation (when one increase/decrease due to the decrease/increase to other), zero correlation (when there is no relation between the variables). For example, during the pandemic situation, the price of an item will increase with the crease of its demand, so price and demand are positively correlated. An increase in the lockdown period will reduce the viral infection is an example of a negative correlation between the lockdown period and the number of infections. But there is no correlation between the ages of a player with the run scored by him/her. In this section, we attempt to define the correlation measures between two HNNs and establish some properties related to these measures. This topic will surely give an idea about the closeness between two HNNs. Motivating from the article [10], we discuss the following definitions and results that are appropriate for the proposed study:

Definition 4.1 Let Ω be an initial universe and $X,Y \subseteq \Omega$. We defined the two HNSs given by $X = \left\{ \left\langle \alpha, T_X(\alpha), I_X(\alpha), F_X(\alpha) \right\rangle : \alpha \in X \right\}$ and $Y = \left\{ \left\langle \beta, T_Y(\beta), I_Y(\beta), F_Y(\beta) \right\rangle : \beta \in Y \right\}$. Then, we defined the correlation coefficient between X and Y as;

$$\wp(X,Y) = \frac{\Re(X,Y)}{\sqrt{\Re(X,X).\Re(Y,Y)}} ....(i)$$, where

$$\Re(X,Y) = \sum_{r=1}^{n} \left(T_X(\alpha_r).T_Y(\beta_r) + I_X(\alpha_r).I_Y(\beta_r) + F_X(\alpha_r).F_Y(\beta_r) \right)$$

By putting $X = Y$ in $\Re(X,Y)$, we can easily get $\Re(X,X)$ and $\Re(Y,Y)$.

Proposition 4.2 Let $\wp(X,Y)$ denotes the correlation coefficient between two HNSs X and Y. Then, we have the following properties:

(1). $\wp(X,Y) = 0 \Leftrightarrow \Re(X,Y) = 0$

(2). $\wp(X,Y) = 1 \Leftrightarrow X = Y$

(3). $0 \le \wp(X,Y) \le 1$

(4). $\wp(X,Y) = \wp(Y,X)$

Proof. (1), (2), and (4) are obvious. We only discuss (3).

(4) $0 \le \wp(X,Y) \le 1$

$$\begin{aligned} \Re(X,Y) &= \sum_{r=1}^{n} \left(T_X(\alpha_r).T_Y(\beta_r) + I_X(\alpha_r).I_Y(\beta_r) + F_X(\alpha_r).F_Y(\beta_r) \right) \\ &= \left(T_X(\alpha_1).T_Y(\beta_1) + I_X(\alpha_1).I_Y(\beta_1) + F_X(\alpha_1).F_Y(\beta_1) \right) + \\ &\left(T_X(\alpha_2).T_Y(\beta_2) + I_X(\alpha_2).I_Y(\beta_2) + F_X(\alpha_2).F_Y(\beta_2) \right) + \\ &............ + \left(T_X(\alpha_n).T_Y(\beta_n) + I_X(\alpha_n).I_Y(\beta_n) + F_X(\alpha_n).F_Y(\beta_n) \right) \end{aligned}$$

Using the Cauchy-Schwarz inequality,

$\left(a_1b_1+a_2b_2+.......+a_nb_n\right)^2\le\left(a_1^2+a_2^2+.....+a_n^2\right).\left(b_1^2+b_2^2+.....+b_n^2\right)$ by squaring the above expression, we have

$$\left(\Re\left(X,Y\right)\right)^2\le\left(\left(T_X\left(\alpha_1\right)\right)^2+\left(I_X\left(\alpha_1\right)\right)^2+\left(F_X\left(\alpha_1\right)\right)^2\right)+$$
$$\left(\left(T_X\left(\alpha_2\right)\right)^2+\left(I_X\left(\alpha_2\right)\right)^2+\left(F_X\left(\alpha_2\right)\right)^2\right)+........+\left(\left(T_X\left(\alpha_n\right)\right)^2+\left(I_X\left(\alpha_n\right)\right)^2+\left(F_X\left(\alpha_n\right)\right)^2\right)$$
$$\times\left(\left(T_Y\left(\beta_1\right)\right)^2+\left(I_Y\left(\beta_1\right)\right)^2+\left(F_Y\left(\beta_1\right)\right)^2\right)+\left(\left(T_Y\left(\beta_2\right)\right)^2+\left(I_Y\left(\beta_2\right)\right)^2+\left(F_Y\left(\beta_2\right)\right)^2\right)+........+$$
$$\left(\left(T_Y\left(\beta_n\right)\right)^2+\left(I_Y\left(\beta_n\right)\right)^2+\left(F_Y\left(\beta_n\right)\right)^2\right)$$
$$=\Re\left(X,X\right)\times\Re\left(Y,Y\right)$$

Thus, $\left(\Re\left(X,Y\right)\right)^2=\Re\left(X,X\right)\times\Re\left(Y,Y\right)$

Putting it in (i), we obtain the result $0\le\wp\left(X,Y\right)\le1$.

Definition 4.3 Let $\varpi=\left(\varpi_1,\varpi_2,....,\varpi_n\right)$ denotes the weight vector of elements $\alpha_r\left(r=1,2,..,n\right)$ with $\varpi_r\ge0$ and $\sum_{r=1}^{n}\varpi_r=1$, then we defined the weighted correlation coefficient as follows:

$$\wp_\varpi\left(X,Y\right)=\frac{\Re_\varpi\left(X,Y\right)}{\sqrt{\Re_\varpi\left(X,X\right).\Re_\varpi\left(Y,Y\right)}}\quad.......(ii)$$

Where

$$\Re_\varpi\left(X,Y\right)=\sum_{r=1}^{n}\varpi_r\left(T_X\left(\alpha_r\right).T_Y\left(\alpha_r\right)+I_X\left(\alpha_r\right).I_Y\left(\alpha_r\right)+F_X\left(\alpha_r\right).F_Y\left(\alpha_r\right)\right)$$

Putting $X=Y$, we can easily obtain $\Re_\varpi\left(X,X\right)$ and $\Re_\varpi\left(Y,Y\right)$

Note. If $\varpi=\left(\frac{1}{n},\frac{1}{n},......,\frac{1}{n}\right)^t$, then equation (ii) reduces to equation (i).

Proposition 4.4 Let $\wp_\varpi\left(X,Y\right)$ be the weighted correlation coefficient. Then, we consider the following properties:

(1) $0\le\wp_\varpi\left(X,Y\right)\le1$

(2) $\wp_\varpi\left(X,Y\right)=1\Leftrightarrow X=Y$

(3) $\wp_\varpi\left(X,Y\right)=\wp_\varpi\left(Y,X\right)$

Proof. (2) and (3) are straightforward. We only concentrate on $\wp_{\varpi}(X,Y) \le 1$.

(1) We have,

$$\Re_{\varpi}(X,Y) = \sum_{r=1}^{n} \varpi_r \left(T_X(\alpha_r).T_Y(\alpha_r) + I_X(\alpha_r).I_Y(\alpha_r) + F_X(\alpha_r).F_Y(\alpha_r)\right)$$
$$= \varpi_1\left(T_X(\alpha_1).T_Y(\alpha_1) + I_X(\alpha_1).I_Y(\alpha_1) + F_X(\alpha_1).F_Y(\alpha_1)\right) +$$
$$\varpi_2\left(T_X(\alpha_2).T_Y(\alpha_2) + I_X(\alpha_2).I_Y(\alpha_2) + F_X(\alpha_2).F_Y(\alpha_2)\right) +$$
$$............ + \varpi_n\left(T_X(\alpha_n).T_Y(\alpha_n) + I_X(\alpha_n).I_Y(\alpha_n) + F_X(\alpha_n).F_Y(\alpha_n)\right)$$
$$= \sqrt{\varpi_1}\left(T_X(\alpha_1).T_Y(\alpha_1) + I_X(\alpha_1).I_Y(\alpha_1) + F_X(\alpha_1).F_Y(\alpha_1)\right) +$$
$$\sqrt{\varpi_2}\left(T_X(\alpha_2).T_Y(\alpha_2) + I_X(\alpha_2).I_Y(\alpha_2) + F_X(\alpha_2).F_Y(\alpha_2)\right) +$$
$$............ + \sqrt{\varpi_n}\left(T_X(\alpha_n).T_Y(\alpha_n) + I_X(\alpha_n).I_Y(\alpha_n) + F_X(\alpha_n).F_Y(\alpha_n)\right)$$

By using Cauchy-Schwarz inequality, we obtain

$$\left(\Re_{\varpi}(X,Y)\right)^2 \le \left(\varpi_1\left(T_X(\alpha_1)\right)^2 + \left(I_X(\alpha_1)\right)^2 + \left(F_X(\alpha_1)\right)^2\right) +$$
$$\left(\varpi_2\left(T_X(\alpha_2)\right)^2 + \left(I_X(\alpha_2)\right)^2 + \left(F_X(\alpha_2)\right)^2\right) + ... + \left(\varpi_n\left(T_X(\alpha_n)\right)^2 + \left(I_X(\alpha_n)\right)^2 + \left(F_X(\alpha_n)\right)^2\right)$$
$$\times\left(\varpi_1\left(T_Y(\alpha_1)\right)^2 + \left(I_Y(\alpha_1)\right)^2 + \left(F_Y(\alpha_1)\right)^2\right) + \left(\varpi_2\left(T_Y(\alpha_2)\right)^2 + \left(I_Y(\alpha_2)\right)^2 + \left(F_Y(\alpha_2)\right)^2\right) + ... +$$
$$\left(\varpi_n\left(T_Y(\alpha_n)\right)^2 + \left(I_Y(\alpha_n)\right)^2 + \left(F_Y(\alpha_n)\right)^2\right)$$

Therefore, $\left(\Re_{\varpi}(X,Y)\right)^2 = \Re_{\varpi}(X,X) \times \Re_{\varpi}(Y,Y)$

This proves the result $0 \le \wp_{\varpi}(X,Y) \le 1$.

5. Application of HNS in Medical Decision Making

In this section, we construct an algorithm based on score function under the HNS environment and apply them in medical decision-making.

Algorithm 1

Step 1- Input the patient-symptom and the symptom-disease data set provided by the expert in the form of hyper neutrosophic triplet. And formulate their corresponding decision matrices $X_{P/S}$ and $Y_{S/D}$.

Step 2- Next we normalize $X_{P/S}$ and $Y_{S/D}$ in the following process:

Let $X_{P/S} = \left[\left\langle T_{X_{ij}}, I_{X_{ij}}, F_{X_{ij}} \right\rangle\right]$ and $Y_{S/D} = \left[\left\langle T_{Y_{ij}}, I_{Y_{ij}}, F_{Y_{ij}} \right\rangle\right]$, where $i, j \in N$.

Then their corresponding normalized matrices are represented by

$$X'_{P/S} = \left[\left\langle \frac{T_{X_{ij}}}{\sqrt{\sum_{l=1}^{n} T^2_{X_{lj}}}}, \frac{I_{X_{ij}}}{\sqrt{\sum_{l=1}^{n} I^2_{X_{lj}}}}, \frac{F_{X_{ij}}}{\sqrt{\sum_{l=1}^{n} F^2_{X_{lj}}}} \right\rangle \right]$$ And

$$Y'_{S/D} = \left[\left\langle \frac{T_{Y_{ij}}}{\sqrt{\sum_{l=1}^{n} T^2_{Y_{lj}}}}, \frac{I_{Y_{ij}}}{\sqrt{\sum_{l=1}^{n} I^2_{Y_{lj}}}}, \frac{F_{Y_{ij}}}{\sqrt{\sum_{l=1}^{n} F^2_{Y_{lj}}}} \right\rangle \right]$$

Step 3- Perform the maxmin-maxmin-minmax composition between the two normalized matrices and obtain the Patient-disease matrix $Z_{P/D}$.

Step 4- Using the definition **3.4**, obtained the score value of each entry in $Z_{P/D}$.

Step 5- Finally, the patient with the largest score value in each row is likely to suffer from that disease. In case of a tie, there is a chance that a patient will suffer from more than one disease.

5.1 Example

Let us consider a set of five patients denoted by $P = \{p_1, p_2, p_3, p_4, p_5\}$ with symptoms Red Skin, Headache, Cough, Joint Pain, and Breathing Difficulty. Let the possible diseases relating to the above symptoms be Dengue, Pneumonia, Asthma, cholera, Viral Fever. A medical investigator is being appointed for different types of medical tests according to the symptoms are concerned. Though, it is a complicated procedure due to the information provided by the patients is ill-defined. So, the entire procedure contains a lot of uncertainty. To minimize the level of uncertainty in the detection of the diseases, the decision-maker or expert follows up the HNS environment. For this we use the above algorithm in the following manner:

Step 1-

Input the patient-symptom relation matrix in the form of **Table 2**

Patient/ Symptom	**Red Skin**	**Headache**	**Cough**	**Joint Pain**	**Breathing Difficulty**
p_1	$\langle 0.6, 0.7, 0.5 \rangle$	$\langle 0.5, 0.4, 0.6 \rangle$	$\langle 0.7, 0.4, 0.6 \rangle$	$\langle 0.8, 0.4, 0.6 \rangle$	$\langle 0.3, 0.6, 0.7 \rangle$
p_2	$\langle 0.3, 0.4, 0.5 \rangle$	$\langle 0.4, 0.6, 0.4 \rangle$	$\langle 0.45, 0.5, 0.4 \rangle$	$\langle 0.42, 0.3, 0.5 \rangle$	$\langle 0.3, 0.4, 0.5 \rangle$
p_3	$\langle 0.6, 0.5, 0.3 \rangle$	$\langle 0.55, 0.35, 0.4 \rangle$	$\langle 0.8, 0.4, 0.3 \rangle$	$\langle 0.35, 0.5, 0.5 \rangle$	$\langle 0.6, 0.7, 0.4 \rangle$

p_4	$\langle 0.3, 0.4, 0.25 \rangle$	$\langle 0.5, 0.4, 0.6 \rangle$	$\langle 0.35, 0.6, 0.53$	$\langle 0.26, 0.5, 0.2 \rangle$	$\langle 0.5, 0.4, 0.7 \rangle$
p_5	$\langle 0.65, 0.72, 0.35$	$\langle 0.35, 0.65, 0.45 \rangle$	$\langle 0.7, 0.65, 0.4 \rangle$	$\langle 0.4, 0.63, 0.6 \rangle$	$\langle 0.3, 0.4, 0.3 \rangle$

Table 2. The patient-symptom relation matrix $X_{P/S}$

Input the symptom-disease relation matrix in the form of Table 3

Symptom/ Disease	**Dengue**	**Pneumonia**	**Asthma**	**Cholera**	**Viral Fever**
Red Skin	$\langle 0.3, 0.43, 0.5 \rangle$	$\langle 0.6, 0.2, 0.43$	$\langle 0.24, 0.15, 0.6$	$\langle 0.3, 0.5, 0.6 \rangle$	$\langle 0.44, 0.7, 0.4 \rangle$
Headache	$\langle 0.22, 0.35, 0.65$	$\langle 0.4, 0.25, 0.5$	$\langle 0.85, 0.6, 0.7 \rangle$	$\langle 0.35, 0.6, 0.4$	$\langle 0.75, 0.6, 0.3 \rangle$
Cough	$\langle 0.35, 0.6, 0.4 \rangle$	$\langle 0.4, 0.64, 0.3$	$\langle 0.6, 0.47, 0.5 \rangle$	$\langle 0.5, 0.4, 0.3 \rangle$	$\langle 0.36, 0.8, 0.4 \rangle$
Joint Pain	$\langle 0.42, 0.35, 0.2 \rangle$	$\langle 0.72, 0.6, 0.4$	$\langle 0.43, 0.1, 0.3 \rangle$	$\langle 0.43, 0.4, 0.5$	$\langle 0.18, 0.2, 0.4 \rangle$
Breathing Difficulty	$\langle 0.32, 0.4, 0.7 \rangle$	$\langle 0.3, 0.43, 0.5$	$\langle 0.26, 0.3, 0.4 \rangle$	$\langle 0.4, 0.6, 0.3 \rangle$	$\langle 0.6, 0.4, 0.35 \rangle$

Table 3. The symptom-disease relation matrix $Y_{S/D}$

Step2-

The normalized patient-symptom matrix given in Table 4

Patient/ Symptom	**Red Skin**	**Headache**	**Cough**	**Joint Pain**	**Breathing Difficulty**
p_1	$\langle 0.52, 0.55, 0.56 \rangle$	$\langle 0.48, 0.36, 0.53 \rangle$	$\langle 0.5, 0.34, 0.58 \rangle$	$\langle 0.74, 0.37, 0.53 \rangle$	$\langle 0.31, 0.52, 0.57 \rangle$
p_2	$\langle 0.26, 0.31, 0.56 \rangle$	$\langle 0.38, 0.54, 0.35 \rangle$	$\langle 0.32, 0.42, 0.39 \rangle$	$\langle 0.38, 0.28, 0.44 \rangle$	$\langle 0.31, 0.34, 0.41 \rangle$

p_3	$\langle 0.52, 0.39, 0.34\rangle$	$\langle 0.52, 0.3, 0.35\rangle$	$\langle 0.57, 0.34, 0.29\rangle$	$\langle 0.32, 0.46, 0.44\rangle$	$\langle 0.63, 0.6, 0.32\rangle$
p_4	$\langle 0.26, 0.31, 0.28\rangle$	$\langle 0.48, 0.36, 0.53\rangle$	$\langle 0.25, 0.51, 0.51\rangle$	$\langle 0.24, 0.46, 0.17\rangle$	$\langle 0.53, 0.34, 0.57\rangle$
p_5	$\langle 0.56, 0.57, 0.39\rangle$	$\langle 0.33, 0.58, 0.4\rangle$	$\langle 0.5, 0.55, 0.39\rangle$	$\langle 0.37, 0.58, 0.53\rangle$	$\langle 0.31, 0.34, 0.24\rangle$

Table 4. The normalized patient-symptom matrix

The normalized symptom-disease matrix given in Table 5

Symptom/ Disease	**Dengue**	**Pneumonia**	**Asthma**	**Cholera**	**Viral Fever**
Red Skin	$\langle 0.4, 0.44, 0.42\rangle$	$\langle 0.52, 0.19, 0.44\rangle$	$\langle 0.2, 0.17, 0.51\rangle$	$\langle 0.3, 0.44, 0.61\rangle$	$\langle 0.38, 0.53, 0.48\rangle$
Headache	$\langle 0.29, 0.35, 0.55\rangle$	$\langle 0.35, 0.24, 0.51\rangle$	$\langle 0.72, 0.71, 0.6\rangle$	$\langle 0.38, 0.52, 0.41\rangle$	$\langle 0.66, 0.46, 0.36\rangle$
Cough	$\langle 0.47, 0.61, 0.34\rangle$	$\langle 0.35, 0.62, 0.31\rangle$	$\langle 0.5, 0.56, 0.43\rangle$	$\langle 0.55, 0.35, 0.3\rangle$	$\langle 0.31, 0.61, 0.48\rangle$
Joint Pain	$\langle 0.57, 0.35, 0.17\rangle$	$\langle 0.63, 0.58, 0.41\rangle$	$\langle 0.36, 0.11, 0.25\rangle$	$\langle 0.47, 0.35, 0.51\rangle$	$\langle 0.15, 0.15, 0.48\rangle$
Breathing Difficulty	$\langle 0.43, 0.41, 0.59\rangle$	$\langle 0.26, 0.41, 0.51\rangle$	$\langle 0.22, 0.35, 0.34\rangle$	$\langle 0.44, 0.52, 0.3\rangle$	$\langle 0.53, 0.3, 0.42\rangle$

Table 5. The normalized symptom-disease matrix

Step3-

The resultant patient-disease matrix formed by combining the two normalized matrices obtained in step 2 is given in Table6

Patient / Disease	**Dengue**	**Pneumonia**	**Asthma**	**Cholera**	**Viral Fever**
p_1	$\langle 0.57, 0.44, 0.53 \rangle$	$\langle 0.63, 0.41, 0.53 \rangle$	$\langle 0.48, 0.36, 0.53 \rangle$	$\langle 0.47, 0.44, 0.53 \rangle$	$\langle 0.48, 0.53, 0.53 \rangle$
p_2	$\langle 0.38, 0.42, 0.39 \rangle$	$\langle 0.38, 0.42, 0.39 \rangle$	$\langle 0.38, 0.54, 0.41 \rangle$	$\langle 0.38, 0.52, 0.39 \rangle$	$\langle 0.38, 0.47, 0.36 \rangle$
p_3	$\langle 0.47, 0.41, 0.34 \rangle$	$\langle 0.52, 0.46, 0.31 \rangle$	$\langle 0.52, 0.35, 0.34 \rangle$	$\langle 0.55, 0.52, 0.3 \rangle$	$\langle 0.53, 0.39, 0.36 \rangle$
p_4	$\langle 0.43, 0.51, 0.17 \rangle$	$\langle 0.35, 0.51, 0.41 \rangle$	$\langle 0.48, 0.51, 0.25 \rangle$	$\langle 0.44, 0.36, 0.51 \rangle$	$\langle 0.53, 0.51, 0.48 \rangle$
p_5	$\langle 0.47, 0.55, 0.39 \rangle$	$\langle 0.52, 0.58, 0.39 \rangle$	$\langle 0.5, 0.58, 0.34 \rangle$	$\langle 0.5, 0.52, 0.3 \rangle$	$\langle 0.35, 0.55, 0.4 \rangle$

Table 6. The resultant patient-disease matrix

Step4-The score values of all the entries of the resultant patient-disease matrix obtained in the following Table 7

Score Values	**Dengue**	**Pneumonia**	**Asthma**	**Cholera**	**Viral Fever**
p_1	0.516	0.524	**0.636**	0.56	0.46
p_2	0.676	0.676	0.573	0.599	0.652
p_3	0.667	0.618	0.699	0.558	0.652
p_4	0.694	0.612	0.627	0.658	0.484
p_5	0.527	0.472	0.512	0.584	0.587

Table 7. The score values of the resultant patient-disease matrix

Step5-

In the above table, the maximum score values along each row are highlighted with a yellow mark and from which we make a decision that p_1 is suffering from Asthma, p_2 is suffering from both Dengue and

Pneumonia, p_3 is suffering from Asthma, p_4 is suffering from Dengue, and p_5 is suffering from Viral Fever. No patient is suffering from the disease Cholera.

Note. For the sake of comparison, we may obtain the accuracy values of the patient-disease matrix. Also, we use correlation measures to detect the possible disease/s of a patient having certain symptoms. This part is left for the readers.

6. Conclusion and Scope

In the present paper, we have introduced the HNS, as a new type of neutrosophic oriented set in a sense that we have not seen such a topic in any research paper until now. Like SVNS and PNS, the HNS also a subclass of NS. The main feature of the HNS is the modification of the restricted condition which makes it a robust model. So, HNS is an extension of SVNS and PNS. There is no such model ever introduced that is capable to regulate uncertainty perfectly. That's why the researchers are toiling hard with a belief that they will be capable to produce a more advanced new model in near future and thus reduce the level of uncertainty with more precision. This is actually the main reason to introduce HNS. Then we discuss some basic set-theoretic properties on HNSs. We also defined their t-norm, t-co norm, HNWA, and HNWG- operators, score and accuracy function. Some important properties of score, and accuracy functions are addressed. Furthermore, we discuss the correlation measures of HNSs and establish some useful results. An algorithm is introduced and applies it in solving a medical decision-making problem.

In the future, the present topic has a rich potential to utilize it in various types of MCDM, MADM, MCGDM problems in different fields such as risk management, weather forecasting, linear programming, game theory, green supplier selection, robotics by using TOPSIS, MULTIMOORA, ELECTRE, PROMETHEE, AHP, VIKOR, DEA, ANP methods.

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