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Generating Neutrosophic Random Variables Based Gamma Distribution

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Abstract

In practical life, we encounter many systems that cannot be studied directly, either due to their high cost or because some of these systems cannot be studied directly. Therefore, we resort to the simulation method, which depends on applying the study to systems similar to real ones and then projecting these results if they are suitable for the real system. The simulation process requires a good understanding of probability distributions and the methods used to transform random numbers that follow a regular distribution in the field $[0,1]$ into random variables that follow them, so that we can achieve the greatest possible benefit from the simulation process and obtain more accurate and appropriate results for all conditions that arise. In previous research, we presented a neutrosophical vision of the process of generating random numbers that follow a regular distribution in the field $[0, 1]$ and some techniques used to generate random variables, such as the inverse transformation technique that was used to generate random variables that follow a uniform distribution in the domain $[a, b]$ and the exponential distribution, the rejection and acceptance technique, which was used to generate random variables that follow the beta distribution, and the mixed technique, which was used to generate random variables that follow the Poisson distribution. In this research, we present a neutrosophic study to generate neutrosophic random variables that follow the gamma distribution, a distribution that is frequently used in engineering applications.

Keywords: Simulation, Generating Random Numbers, Neutrosophic Logic, Generating Neutrosophic Random Numbers, Rejection and Acceptance Technique.

1 | Introduction

Operations research is the applied side of mathematics, and since its inception, it has helped improve the performance of many systems that use its methods in their workflow. Among the methods of operations research is the simulation method. The importance of the simulation process in all branches of science comes from the great difficulty that we can face when studying the operation of any system. From real systems, due to the high cost or the fact that some systems cannot be studied directly, the simulation process depends on generating a series of random numbers subject to a regular probability distribution in the range $[0,1]$, then converting these random numbers into random variables subject to the law of probability distribution



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according to which the system to be simulated operates, the attention of researchers and scholars interested in the simulation method has focused on providing scientific methods and methods that help in obtaining the transformation of random numbers into random variables that follow the probability distributions most used in practical applications, and in the face of the great revolution brought about by neutrosophic logic in All fields of science, which are confirmed by the research presented in this field [1-17], are a continuation of the research we presented previously, the aim of which is to present a neutrosophical vision of Monte Carlo simulation [18-22]. In this research, we present a neutrosophic study to generate neutrosophic random variables that follow the gamma distribution, a distribution that is frequently used in engineering applications.

2 | Discussion

In this paper we have founded a new type of hybrid algebra, called Neutrosophic TwoFold Algebra and its corresponding Neutrosophic TwoFold Law, by combining two algebras:

2.1 | The Principle Behind the Simulation Process

It is the generation of a series of random numbers that follow a uniform distribution over the field $[0,1]$, and then converting these random numbers into random variables that follow the distribution according to which the system to be simulated operates. There are many techniques used to generate these random numbers, such as the mean square, the product mean., the Fibonacci method and others, and to generate neutrosophic random numbers we presented a study in the research [18] and we used the mean square method and we reached the following results:

- The mean square method for generating a series of random numbers is defined by the following relation:

$$R_{i+1} = Mid[R_i^2]; i = 0,1,2,3, \dots$$

Where *Mid* symbolizes the middle four ranks of R_i^2 , and R_i is chosen, i.e., a fractional random number consisting of four ranks (called a seed) that does not contain a zero in any of its four ranks.

- We convert these random numbers into neutrosophic random numbers and here we distinguish three states of the field $[0,1]$ with a margin of indeterminacy, in the three states we have $\varepsilon \in [0, n]$ and $0 < n < 1$

First case: $[0 + \varepsilon, 1]$ indeterminacy at the minimum of the range. We substitute in the relation:

$$R_{iN} = \frac{R_i - \varepsilon}{1 - \varepsilon}$$

Second case: $[0, 1 + \varepsilon]$ indeterminacy at the upper limit of the range. We substitute in the relation:

$$R_{iN} = \frac{R_i}{1 + \varepsilon}$$

Third case: $[0 + \varepsilon, 1 + \varepsilon]$ Indeterminacy in the upper and lower limits of the field. We substitute in the relation:

$$R_{iN} = R_i - \varepsilon$$

2.2 | Generating Neutrosophic Random Variables

To generate neutrosophic random variables that follow the probability distribution according to which the system to be simulated operates, we distinguish three cases:

First case: Neutrosophic random numbers and probability distribution in the classical form.

Second case: Classical random numbers and the probability distribution is in the neutrosophic form.

Third case: Neutrosophic random numbers and probability distribution in neutrosophic form.

2.3 | Generating Neutrosophic Random Variables Based Gamma Distribution

The gamma distribution is one of the important distributions used in engineering applications. Given this importance and so that we can provide more accurate simulation results suitable for all conditions, we present in this research a neutrosophic study of the process of generating random variables that follow the gamma distribution, based on the classical and neutrosophic studies presented in the references [18- 25].

2.3.1 | Classic formula for gamma distribution [23, 24]

The probability density function is given by the following classical formula:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad ; \quad x \geq 0, \alpha > 0, \beta > 0$$

$$\Gamma(x) = \int_0^\infty x^{c-1} e^{-x} dx$$

where $x \geq 0, \alpha > 0, \beta > 0$ are non-negative values,

Special cases of gamma distribution:

- As the value of α increases, the gamma distribution approaches the normal distribution.
- When $\alpha = 1$, the gamma distribution matches the exponential distribution.

2.3.2 | Neutrosophic Formula for Gamma Distribution [25]

If we take α or β or both, a neutrosophic number, where the neutrosophic number is written in the following standard form, $N = a + bI$, where a and b are real or complex coefficients. a represents the definite part and bI represents the indefinite part (indeterminacy) of the number N , and it is possible that $[\lambda_1, \lambda_2]$ or $\{\lambda_1, \lambda_1\}$ or...otherwise it is any set close to the real value a , we obtain the neutrosophic gamma distribution. Here we distinguish the following cases:

1. We take α a neutrosophic value, β classic value, then the probability density function is written in the following form:

$$f_N(x) = \frac{\beta^{\alpha_N}}{\Gamma(\alpha_N)} x^{\alpha_N-1} e^{-\beta x}$$

$$x \geq 0, \alpha_N > 0, \beta > 0$$

2. We take β a neutrosophic value, α classic value, then the probability density function is written in the following form:

$$f_N(x) = \frac{\beta_N^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta_N x}$$

$$x \geq 0, \alpha > 0, \beta_N > 0$$

Thus, we obtain the following special cases of the neutrosophic gamma distribution:

- As the value of α increases, the gamma distribution approaches the neutrosophic normal distribution.
 - When $\alpha = 1$, the gamma distribution matches the neutrosophic exponential distribution.
3. We take α and β are neutrosophic values, that is, we take them in the form. Then the probability density function is written in the following form:

$$f_N(x) = \frac{\beta_N^{\alpha_N}}{\Gamma(\alpha_N)} x^{\alpha_N-1} e^{-\beta_N x}$$

$$x \geq 0, \alpha_N > 0, \beta_N > 0$$

Where $\beta_N = \beta \pm \gamma$ and $\alpha_N = \alpha \pm \delta$ and γ, δ is the indeterminacy of the mediators β, α and it can be any set close to the real values β, α .

2.3.3 | Generating Neutrosophic Random Variables Based Gamma Distribution

From the classical study presented in references [23,24] and the neutrosophic study presented in the references [18,21,22], the general formulation of the technique used to generate neutrosophic random variables following a gamma distribution is as follows:

1. We search for the largest value of the function $f_N(x)$ on the domain of its definition by calculating the first derivative and making it equal to zero. We obtain the following value:

$$x_N = \frac{\alpha_N - 1}{\beta_N} = \text{mod}$$

2. We replace this value with the function $f_N(x)$. We obtain M , the largest value of the function, which corresponds to the mode. Then we follow the following steps:

- a. We generate two random numbers from the range $[0,1]$, such as R_1, R_2 .
- b. We convert these two numbers into neutrosophic random numbers R_{N1}, R_{N2} .
- c. We form a new variable from one of the two numbers, for example $x_N = \frac{1}{R_{N1}}$, then the transformation field of x becomes $[0, \infty[$, which is identical to the distribution field $f_N(x)$.
- d. We compare R_{N2} with $\frac{f_N\left(\frac{1}{R_{N1}}\right)}{M}$.
- e. If $R_{N2} > \frac{f_N\left(\frac{1}{R_{N1}}\right)}{M}$, we consider the variable $x_N = \frac{1}{R_{N1}}$ to be a neutrosophic random variable subject to the gamma distribution.
- f. If $R_2 > x = \frac{1}{R_{N1}}$, we reject the numbers R_{N1}, R_{N2} and return to step (a).

3 | Example

We explain the above through an example that corresponds to the third case of neutrosophic random numbers, and the gamma function is a neutrosophic function.

Find a random variable that follows the gamma distribution defined by the following probability density function:

$$f_N(x) = \frac{\beta_N^{\alpha_N}}{\Gamma(\alpha_N)} x^{\alpha_N-1} e^{-\beta_N x}$$

Where $x \geq 0, \beta_N \in [0.2, 0.25], \alpha_N \in \{3, 5\}$

Based on the data in the example, the study is divided into two parts:

3.1 | First Part

Data $x \geq 0, \alpha_N = 3, \beta_N \in [0.02, 0.025]$

First, we calculate M from the following relation:

$$M_N = \frac{\alpha_N - 1}{\beta_N}$$

$$M_N = \frac{\alpha_N - 1}{\beta_N} = \frac{3 - 1}{[0.2, 0.25]} = \frac{2}{[0.2, 0.25]} \in [8, 10]$$

- a. We generate two random numbers that follow a uniform distribution in the range $[0, 1]$ using the mean square method given by the following relation:

$$R_{i+1} = \text{Mid}[R_i^2]; i = 0, 1, 2, 3, \dots$$

We take the seed $R_0 = 0.3176$ and from it we get $R_1 = 0.0869$, $R_2 = 0.7551$.

- b. We convert the two numbers into two neutrosophic random numbers on the range $[0 + \varepsilon, 1 + \varepsilon]$ using the relation:

$$R_{iN} = R_i - \varepsilon$$

We take in this example $\varepsilon \in [0, 0.03]$, we get the following two neutrosophic random numbers:

$$R_{1N} = 0.0869 - [0, 0.03] \in [0.0569, 0.0869]$$

$$R_{2N} = 0.7551 - [0, 0.03] \in [0.7251, 0.7551]$$

- c. We form a new variable from one of the two numbers, for example

$$x_N = \frac{1}{R_{N1}} = \frac{1}{[0.0569, 0.0869]} \in [11.5075, 17.5746]$$

- d. To compare $R_{2N} \in [0.7251, 0.7551]$ with $\frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N}$.

- We calculate $f_N\left(\frac{1}{R_{N1}}\right)$ we find: $f_N(x) = \frac{\beta_N^{\alpha_N}}{\Gamma(\alpha_N)} x^{\alpha_N-1} e^{-\beta_N x}$

$$f_N([11.5075, 17.5746]) = \frac{([0.2, 0.25])^3}{\Gamma(3)} ([11.5075, 17.5746])^2 e^{-[0.2, 0.25] \cdot [11.5075, 17.5746]}$$

$$\in [0.1766, 0.8031]$$

- We calculate $\frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N}$ we find:

$$\frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N} = \frac{[0.1766, 0.8031]}{[8, 10]} \in [0.0221, 0.0803]$$

We have, $R_{2N} \in [0.7251, 0.7551]$ and $\frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N} \in [0.0221, 0.0803]$

We note that: $[0.7251, 0.7551] > [0.0221, 0.0803] \Rightarrow R_{2N} > \frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N}$

Check the acceptance criterion, so we consider $x_N = \frac{1}{R_{N1}} \in [11.5075, 17.5746]$

To be a neutrosophic random variable subject to the gamma distribution.

3.2 | Second Part

Based on the data $x \geq 0, \alpha_N = 5, \beta_N \in [0.02, 0.025]$

We calculate M from the following relation:

$$M_N = \frac{\alpha_N - 1}{\beta_N}$$

$$M_N = \frac{\alpha_N - 1}{\beta_N} = \frac{5 - 1}{[0.2, 0.25]} = \frac{4}{[0.2, 0.25]} \in [16, 20]$$

- a. We generate two random numbers that follow a uniform distribution in the range $[0, 1]$ using the mean square method given by the following relation:

$$R_{i+1} = \text{Mid}[R_i^2]; i = 0, 1, 2, 3, \dots$$

We take the seed $R_0 = 0.3176$ and from it we get $R_1 = 0.0869$, $R_2 = 0.7551$.

- b. We convert the two numbers into two neutrosophic random numbers on the range $[0 + \varepsilon, 1 + \varepsilon]$ using the relation:

$$R_{iN} = R_i - \varepsilon$$

We take in this example $\varepsilon \in [0, 0.03]$, we get the following two neutrosophic random numbers:

$$R_{1N} = 0.0869 - [0, 0.03] \in [0.0569, 0.0869]$$

$$R_{2N} = 0.7551 - [0, 0.03] \in [0.7251, 0.7551]$$

- c. We form a new variable from one of the two numbers, for example:

$$x_N = \frac{1}{R_{N1}} = \frac{1}{[0.0569, 0.0869]} \in [11.5075, 17.5746]$$

- d. To compare $R_{2N} \in [0.7251, 0.7551]$ with $\frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N}$.

- We calculate $f_N\left(\frac{1}{R_{N1}}\right)$ we find: $f_N(x) = \frac{\beta_N^{\alpha_N}}{\Gamma(\alpha_N)} x^{\alpha_N-1} e^{-\beta_N x}$

$$f_N([11.5075, 17.5746]) = \frac{([0.2, 0.25])^5}{\Gamma(5)} ([11.5075, 17.5746])^4 e^{-[0.2, 0.25] \cdot [11.5075, 17.5746]}$$

$$\in [0.1766, 0.8031]$$

- We calculate $\frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N}$ we find: $\frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N} = \frac{[0.1766, 0.8031]}{[16, 20]} \in [0.011, 0.0401]$

$$\text{We have, } R_{2N} \in [0.7251, 0.7551] \text{ and } \frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N} \in [0.011, 0.0401]$$

$$\text{We note that: } [0.7251, 0.7551] > [0.011, 0.0401] \Rightarrow R_{2N} > \frac{f_N\left(\frac{1}{R_{N1}}\right)}{M_N}$$

$$\text{Check the acceptance criterion, so we consider } x_N = \frac{1}{R_{N1}} \in [11.5075, 17.5746]$$

To be a neutrosophic random variable subject to the gamma distribution. From the results of the study in the first and second sections, we note that: $x_N = \frac{1}{R_{N1}} \in [11.5075, 17.5746]$

Can be considered a neutrosophic random variable that follows a gamma distribution defined by the following probability density function:

$$f_N(x) = \frac{\beta_N^{\alpha_N}}{\Gamma(\alpha_N)} x^{\alpha_N-1} e^{-\beta_N x}, \text{ where } x \geq 0, \alpha_N \in \{3, 5\}, \beta_N \in [0.2, 0.25].$$

3.3 | Classic Wording of the Example

Find a random variable that follows the gamma distribution defined by the following probability density function:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

Where $x \geq 0, \alpha = 3, \beta = 0.2$

Data $x \geq 0, \alpha = 3, \beta = 0.2$

First, we calculate M from the following relation:

$$M = \frac{\alpha - 1}{\beta}$$

$$M = \frac{\alpha - 1}{\beta} = \frac{3 - 1}{0.2} = \frac{2}{0.2} = 8$$

- a. We generate two random numbers that follow a uniform distribution in the range $[0,1]$ using the mean square method given by the following relation:

$$R_{i+1} = \text{Mid}[R_i^2]; i = 0, 1, 2, 3, \dots$$

We take the seed $R_0 = 0.3176$ and from it we get $R_1 = 0.0869, R_2 = 0.7551$.

- b. We form a new variable from one of the two numbers, for example R_1 , we find:

$$x = \frac{1}{R_1} = \frac{1}{0.0869} = 11.5075$$

- c. To compare R_2 with $\frac{f(\frac{1}{R_1})}{M}$.

- We calculate $f\left(\frac{1}{R_1}\right)$ we find: $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$

$$f_N(11.5075) = \frac{(0.2)^3}{\Gamma(3)} (11.5075)^2 e^{-0.2 \cdot 11.5075} = 0.1766$$

- We calculate $\frac{f(\frac{1}{R_1})}{M}$ we find: $\frac{f(\frac{1}{R_1})}{M} = \frac{0.1766}{8} = 0.0221$

$$\text{We have, } R_2 = 0.7551 \text{ and } \frac{f(\frac{1}{R_1})}{M} = 0.0221$$

$$\text{We note that: } 0.7551 > 0.0221 \Rightarrow R_2 > \frac{f(\frac{1}{R_1})}{M}$$

Check the acceptance criterion, so we consider: $x_N = \frac{1}{R_1} = 11.5075$

To be a neutrosophic random variable subject to the gamma distribution.

From the results of the study in the first and second sections, we note that:

$$x_N = \frac{1}{R_1} = 11.5075$$

Can be considered a neutrosophic random variable that follows a gamma distribution defined by the following probability density function:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

Where $x \geq 0, \alpha = 3, \beta = 0.2$.

The classic example is treated in the same way as before if the data are:

Find a random variable that follows the gamma distribution defined by the following probability density function:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad ; \quad x \geq 0, \alpha > 0, \beta > 0, \text{ where } x \geq 0, \alpha = 5, \beta = 0.2.$$

4 | Conclusion and Results

In this research, we presented a neutrosophic study, the purpose of which is to obtain neutrosophic random variables that follow the gamma distribution, a distribution that has many uses in engineering applications. We found through this study that the neutrosophic values of the required random variables have a margin of freedom and take into account all the conditions that the system's operating environment passes through. What is intended to be simulated works according to the gamma distribution, while the classical study gives a specific value for the random variable that we need for the simulation process, a value appropriate to conditions similar to the conditions in which such data were adopted, and the results that we obtained through the neutrosophic presentation of the example and the classical presentation confirm these results, and therefore to obtain more accurate simulation results that take into account all conditions, we recommend using the neutrosophic study.

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Author Contribution

All authors contributed equally to this work.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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