Generalized Single Valued Neutrosophic Trapezoidal Numbers and their Application to Solve Transportation Problem

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Abstract - A neutrosophic set has the potentiality of being a general framework of intuitionistic fuzzy set, classical set, and fuzzy set. The idea of neutrosophic set provides a new tool to deal with uncertainty analysis in data sets. Single valued trapezoidal neutrosophic numbers is a special case of neutrosophic set that handles ill-defined and very difficult data. This work intends to introduce the weighted average ranking to transportation problem in neutrosophic environment.

Keywords - Neutrosophic set, Single Valued Neutrosophic Numbers SVNNs and Generalized Single Valued Trapezoidal Neutrosophic Numbers, Generalized Single Valued Trapezoidal Neutrosophic Transportation Problem.

1. Introduction

Classical mathematical set theory is insufficient to deal with lack of complete knowledge, vagueness, imprecision in real time situations. In 1965, Zadeh [18] modeled the concept of fuzzy set theory with membership grade to deal with uncertainties which is not caused by randomness. Generalization of fuzzy set was done by Atanassov [2] in 1986, with inclusion of non-membership grade and hesitancy margin called as intuitionistic fuzzy sets, which are capable to handle impresice data of both complete and in complete in nature. Later on several other generalizations like L-fuzzy sets, interval valued fuzzy sets, intuitionistic L-fuzzy sets have been developed and used to solve many practical problems in various fields.

In 1995 a new theory involving philosophical thought which discourses nature and scope of neutralities is being researched and introduced by Florentin Smarandache [13]. A Neutrosophic set considers truth membership, inderminancy membership and falsity membership. In 2005, Wang et. al [16] defined an illustration of neutrosophic set known as single valued neutrosophic set. A single valued neutrosophic number is a special case of neutrosophic set which can handle ill known quantity and applied to solve decision making, scientific and engineering problems effectively. Ye [17] introduced the notations of simplified neutrosophic sets and proposed a ranking method. Abdel-Basset et. al [1] discussed group decision making model based on trapezoidal neutrosophic numbers. Biswas et. al [3, 4] introduced a new concept for multi-attribute group decision making problems by extending the technique for order preference by similarity for single valued neutrosophic numbers. [11] Peng et. al developed the approach of multi-valued neutrosophic set. Majumdar et. al [10], developed a measure of entropy of single valued neutrosophic sets. In recent years researchers are working on ranking of single valued neutrosophic numbers and their applications in decision making problems [5, 6, 7, 8, 9, 14, 15].

The organization of this paper is as follows: Section 2 reveals the basics of fuzzy sets, neutrosophic sets, single valued neutrosophic numbers, and single valued trapezoidal neutrosophic numbers. In section 3, the transportation problem whose cost is generalized single valued trapezoidal neutrosophic numbers and its mathematical formulation presented. Section 4, displays the proposed

algorithm. In section 5 the numerical example is exhibited to prove the proposed ranking. Section 6, exhibits the ranking comparison and concluding remarks.

2. Basic Definition

Definition 2.1 FUZZY SET

A Fuzzy set $\tilde{A}$ is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1].(i.e.) $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \epsilon X\}$, here $\mu_{\tilde{A}}: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ called the membership function value of $x \in X$ in the fuzzy set. These membership grades are often represented by real numbers ranging from [0, 1].

Definition 2.2 FUZZY NUMBER

A fuzzy set $\widetilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

➢ $\mu_{\tilde{A}}(x): R \rightarrow [0,1]$ is continuous.

➢ $\mu_{\tilde{A}}(x) = 0$ for all x $\in (-\infty, a] \cup [d, \infty)$.

➢ $\mu_{\tilde{A}}(x)$ $strictly$ $increasing$ on $[a,\ b]$ and $strictly$ $decreasing$ on $[c,\ d]$.

➢ $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b,\ c]$, $where$ $a < b < c < d$.

Definition 2.3 GENERALIZED FUZZY NUMBER

A fuzzy set $\widetilde{A}$, defined on the universal set of real numbers R , is said to be generalized fuzzy number if its membership function has the following characteristics:

✓ $\mu_{\tilde{A}}: R \rightarrow [0, \omega]$ is continuous.

✓ $\mu_{\tilde{A}}(x) = 0$ for all x $\in (-\infty, a] \cup [d, \infty)$.

✓ $\mu_{\tilde{A}}(x)$ strictly increasing on [a, b] and strictly decreasing on [c, d].

✓ $\mu_{\tilde{A}}(x) = \omega$, for all x $\in [b,\ c]$, where $0 < \omega \leq 1$.

Definition 2.4 NEUTROSOPHIC SET [13]

Let X be a non-empty set. Then a neutrosophic set $\tilde{A}^N$ of X is defined as $\widetilde{A}^N = \{\langle x, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x)\rangle | \ x \in X, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x) \in \ ]^-0, 1^+[\ \}$, where $T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x)$ and $F_{\tilde{A}^N}(x)$ are truth membership function, an indeterminancy membership function and a falsity-membership function and there is no restriction on the sum of $T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x)$ and $F_{\tilde{A}^N}(x)$, so $^-0 \leq T_{\tilde{A}^N}(x) + \ I_{\tilde{A}^N}(x) + F_{\tilde{A}^N}(x) \leq 3^+$ and $]^-0, 1^+[$ is a nonstandard unit interval.

It is difficult to apply neutrosophic set theories in real life situations directly, so Wang introduced single valued neutrosophic set as a subset of neutrosophic set and the definition is as follows:

Definition 2.5 SINGLE VALUED NEUTROSOPHIC SET [16]

Let X be a non-empty set. A single valued neutrosophic set $\tilde{A}_S^N$ is defined as $\tilde{A}_S^N = \{\langle x, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x)\rangle | \ x \in X, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x) | x \in X\}$, where $T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x)$ and $F_{\tilde{A}^N}(x) \in [0,1]$ for each $x \in X$ and $0 \le T_{\tilde{A}^N}(x) + I_{\tilde{A}^N}(x) + F_{\tilde{A}^N}(x) \le 3$.

Definition 2.6 GENERALISED SINGLE VALUED TRAPEZOIDAL NEUTROSOPHIC NUMBER

Let $w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \in [0,1]$ and $a_1, a_2, a_3, a_4 \in \mathbb{R}$ such that $a_1 \le a_2 \le a_3 \le a_4$. Then a generalized single valued trapezoidal neutrosophic number, $\tilde{a} = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ is a special neutrosophic set on the real line set $\mathbb{R}$, whose truth-membership, indeterminacy-membership and falsity membership functions are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} w_{\tilde{a}}\left(\dfrac{x - a_1}{a_2 - a_1}\right), & a_1 \le x \le a_2 \\ w_{\tilde{a}}, & a_2 \le x \le a_3 \\ w_{\tilde{a}}\left(\dfrac{a_4 - x}{a_4 - a_3}\right), & a_3 \le x \le a_4 \\ 0, & otherwise \end{cases}$$

$$\vartheta_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + u_{\tilde{a}}(x - a_1)}{a_2 - a_1}, & a_1 \le x \le a_2 \\ u_{\tilde{a}}, & a_2 \le x \le a_3 \\ \dfrac{x - a_3 + u_{\tilde{a}}(a_4 - x)}{a_4 - a_3}, & a_3 \le x \le a_4 \\ 1, & otherwise \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + y_{\tilde{a}}(x - a_1)}{a_2 - a_1}, & a_1 \le x \le a_2 \\ y_{\tilde{a}}, & a_2 \le x \le a_3 \\ \dfrac{x - a_3 + y_{\tilde{a}}(a_4 - x)}{a_4 - a_3}, & a_3 \le x \le a_4 \\ 1, & otherwise \end{cases}$$

Definition 2.7 Arithmetic Operations of generalized single valued trapezoidal neutrosophic numbers:

Let $\tilde{a} = \langle(a_1, b_1, c_1, d_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ and $\tilde{b} = \langle(a_2, b_2, c_2, d_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}}\rangle$ be two generalized single valued trapezoidal neutrosophic numbers and $k \ne 0$, then

Addition: $\tilde{a} + \tilde{b} = \langle(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle$

Difference: $\tilde{a} - \tilde{b} = \langle(a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle$

Product: $\tilde{a}\tilde{b} = \begin{cases} \langle(a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle \ (d_1 > 0, d_2 > 0) \\ \langle(a_1 d_2, b_1 c_2, c_1 b_2, d_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle \ (d_1 < 0, d_2 > 0) \\ \langle(d_1 d_2, c_1 c_2, b_1 b_2, a_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle \ (d_1 < 0, d_2 < 0) \end{cases}$

Division: $\tilde{a}/\tilde{b} = \begin{cases} \langle(a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle \ (d_1 > 0, d_2 > 0) \\ \langle(d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle \ (d_1 < 0, d_2 > 0) \\ \langle(d_1/a_2, c_1/b_2, b_1/c_2, a_1/d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}}\rangle \ (d_1 < 0, d_2 < 0) \end{cases}$

Scalar Product: $k\tilde{a} = \begin{cases} \langle(ka_1, kb_1, kc_1, kd_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle \ (k > 0) \\ \langle(kd_1, kc_1, kb_1, ka_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle \ (k < 0) \end{cases}$

Inverse: $\tilde{a}^{-1} = \langle(1/d_1, 1/c_1, 1/b_1, 1/a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle (\tilde{a} \neq 0)$

Definition 2.7 Let $\tilde{A}^N$ and $\tilde{B}$ be two SVNNs. The ranking of $\tilde{A}^N$ and $\tilde{B}^N$ by the R (.) on the set of SVNNs is defined as follows:

    i. $R(\tilde{A}^N) > R(\tilde{B}^N)$ iff $\tilde{A}^N > \tilde{B}^N$
    ii. $R(\tilde{A}^N) < R(\tilde{B}^N)$ iff $\tilde{A}^N < \tilde{B}^N$
    iii. $R(\tilde{A}^N) = R(\tilde{B}^N)$ iff $\tilde{A}^N = \tilde{B}^N$

Definition 2.8 The ordering $\geq$ and $\leq$ betwenn any two SVNNs $\tilde{A}^N$ and $\tilde{B}^N$ are defined as follows:

    i. $\tilde{A}^N \geq \tilde{B}^N$ iff $\tilde{A}^N > \tilde{B}^N$ or $\tilde{A}^N = \tilde{B}^N$ and
    ii. $\tilde{A}^N \leq \tilde{B}^N$ iff $\tilde{A}^N < \tilde{B}^N$ or $\tilde{A}^N = \tilde{B}^N$

Definition 2.9 Let $\{\tilde{A}_i^N, i = 1,2, \dots, n\}$ be a set of SVNNs. If $R(\tilde{A}_k^N) \leq R(\tilde{A}_i^N)$ for all i, then the SVNNs $\tilde{A}_k^N$ is the minimum of $\{\tilde{A}_i^N, i = 1,2, \dots, n\}$.

Definition 2.10 Let Let $\{\tilde{A}_i^N, i = 1,2, \dots, n\}$ be a set of SVNNs. If $R(\tilde{A}_k^N) \geq R(\tilde{A}_i^N)$ for all i, then the SVNNs $\tilde{A}_t^N$ is the minimum of $\{\tilde{A}_i^N, i = 1,2, \dots, n\}$.

Definition 2.11 RANKING TECHNIQUE

Let $\tilde{A}^N = \langle(a, b, c, d); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ a generalized single valued trapezoidal neutrosophic number. The ranking R (.) of $\tilde{A}^N$ on the set of single valued trapezoidal neutrosophic number is defined as follows:

$$R(\tilde{A}^N) = \left(\frac{w_{\tilde{a}} + 1 - u_{\tilde{a}} + 1 - y_{\tilde{a}}}{3}\right)\left(\frac{a + b + c + d}{4}\right)$$

3. Mathematical Formulation of Generalized Single Valued Trapezoidal Neutrosophic Transportation Problem

The generalized single valued trapezoidal neutrosophic transportation problem can be represented in the form of n x n generalized single valued trapezoidal neutrosophic cost table $[\tilde{C}_{ij}]$ after defuzzifizcation as given below.

Table 1. Generalized Single Valued Trapezoidal Neutrosophic Fuzzy Transportation Cost Table

	D_1	D_2	...	D_j	...	D_n	*Supply*
O_1	$\tilde{C}_{11}$	$\tilde{C}_{12}$	...	$\tilde{C}_{1j}$	...	$\tilde{C}_{1n}$	$\tilde{a}_1$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
O_i	$\tilde{C}_{i1}$	$\tilde{C}_{i2}$	...	$\tilde{C}_{ij}$	...	$\tilde{C}_{in}$	$\tilde{a}_2$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

O_m	$\tilde{C}_{m1}$	$\tilde{C}_{m2}$	...	$\tilde{C}_{mj}$	...	$\tilde{C}_{mn}$	$\tilde{a}_m$
Demand	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_j$		$\tilde{b}_n$	

The costs $\tilde{C}_{ij}$ are generalized single valued trapezoidal neutrosophic numbers,

$\tilde{C}_{ij}^N = \left((\tilde{C}_{ij}^1, \tilde{C}_{ij}^2, \tilde{C}_{ij}^3, \tilde{C}_{ij}^4); w_{ij}, u_{ij}, y_{ij} \right)$. The goal is to minimize the generalized single valued trapezoidal neutrosophic cost incurred in transportation effectively.

Let us assume that there are m sources and n destinations. Let $\tilde{a}_i$ be the generalized single valued trapezoidal neutrosophic supply at i, $\tilde{b}_j$ be the generalized single valued trapezoidal neutrosophic demand at destination j, $\tilde{C}_{ij}$ be the unit generalized single valued trapezoidal neutrosophic transportation cost from source i to destination j and $\tilde{X}_{ij}$ be the number of units shifted from source i to destination j.

The generalized single valued trapezoidal neutrosophic transportation problem can be mathematically expressed as

$$min \ \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} \tilde{X}_{ij}$$

Subject to the constraints

$\sum_{j=1}^{n} \tilde{X}_{ij} = \tilde{a}_i, i = 1,2,3,\dots,m$, $\sum_{i=1}^{m} \tilde{X}_{ij} = \tilde{b}_j, j = 1,2,3,\dots,n$ and $\tilde{X}_{ij} \geq 0 \ for \ all \ i \ and \ j.$

A generalized single valued trapezoidal neutrosophic transportation problem is said to be balanced if the total generalized single valued trapezoidal neutrosophic supply from all sources equal to the total generalized single valued trapezoidal neutrosophic demand in all destination $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$, otherwise is called unbalanced.

4. Proposed approach of Generalized Single Valued Trapezoidal Neutrosophic Transportation Problem

Step 1: Test whether the given generalized single valued trapezoidal neutrosophic transportation problem is balanced or not.

(i)   If it is a balanced one (i.e., the total supply is equal to the total demand) then go to step 3.

(ii)   If it is an unbalanced one (i.e., the total supply is not equal to the total demand) then go to step 2.

Step 2: Introduce dummy rows and /or dummy columns with zero generalized single valued trapezoidal neutrosophic costs to form a balanced one.

Step 3: Find the rank of each cell $\tilde{C}_{ij}$ of the chosen generalized single valued trapezoidal neutrosophic cost matrix by using the ranking function as mentioned in section 2.

Step 4: Proceed by the VAM method to find the initial basic feasible solution and if m+n-1 = number of allocations, then proceeds by MODI method to obtain the optimal solution.

Step 5: Add the optimal generalized single valued trapezoidal neutrosophic cost using generalized single valued trapezoidal neutrosophic addition mentioned in section 2, to optimize the cost.

5. Numerical example

Consider a transportation problem with three origins A, B, C and five destinations P, Q, R, S and T, whose costs are considered to be generalized single valued trapezoidal neutrosophic numbers in lakhs. The problem is to find the optimal transportation in an efficient way. [Real time application in Shree Vengateswara Pharmaceuticals - Ambattur, Chennai].

Table 2. Generalized Single Valued Trapezoidal Neutrosophic Fuzzy Cost Table

Source / Destinations	P	Q	R	S	T
A	(0.3,0.4,0.5,0.7);0.5,0.4,0.3)	(0.2,0.5,0.6,0.9);0.8,0.2,0.4)	(0.3,0.4,0.7,0.8);0.6,0.3,0.2)	(0.3,0.5,0.8,0.8);0.7,0.2,0.5)	(0.4,0.6,0.7,0.8);0.3,0.5,0.6)
B	(0.2,0.3,0.5,0.6);0.5,0.3,0.7)	(0.2,0.4,0.6,0.8);0.1,0.2,0.3)	(0.3,0.5,0.8,0.9);0.2,0.5,0.8)	(0.2,0.3,0.7,0.8);0.9,0.8,0.7)	(0.1,0.3,0.5,0.7);0.9,0.7,0.5)
C	(0.1,0.2,0.7,0.8);0.9,0.1,0.5)	(0.2,0.3,0.6,0.7);0.5,0.3,0.8)	(0.3,0.4,0.5,0.6);0.8,0.2,0.6)	(0.4,0.6,0.7,0.8);0.5,0.4,0.2)	(0.4,0.5,0.6,0.7); 0.9,0.3,0.6)

Solution: Using step 4, the rank of generalized single valued trapezoidal neutrosophic cost matrix is:

Table 3. Rank Table

Source / Destinations	P	Q	R	S	T
A	0.1425	0.1425	0.1425	0.1425	0.1425
B	0.1425	0.1425	0.1425	0.1425	0.1425
C	0.1425	0.1425	0.1425	0.1425	0.1425

Proceeding by VAM and MODI method, the optimal generalized single valued trapezoidal neutrosophic minimum transportation cost is 12.63.

Table 4. Comparison Table

Ranking Methods	Example	Ranking Results:k=0.5
Score function	(4, 8, 10, 16); 0.5, 0.3, 0.6)	3.8
Accuracy function	(4, 8, 10, 16); 0.5, 0.3, 0.6)	6.65
Weighted Value	(4, 8, 10, 16); 0.5, 0.3, 0.6)	4.999185
Proposed ranking	(4, 8, 10, 16); 0.5, 0.3, 0.6)	2.53175

6. Conclusion

This paper proposes a new ranking for generalized single valued trapezoidal neutrosophic numbers. The proposed weighted average ranking is applied to explicate generalized single valued trapezoidal neutrosophic transportation problem. Further, a numerical example is exemplified whose costs are taken as generalized single valued trapezoidal neutrosophic numbers .The proficiency of the ranking technique is shown in the comparison table. As a future extension, the proposed algorithm may be used to solve, generalized single valued octagonal

neutrosophic fuzzy assignment and fuzzy transportation problems problem and interval valued neutrosophic fuzzy assignment and fuzzy transportation problems.

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