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Fuzzy Neutrosophic Weakly-Generalized Closed Sets in Fuzzy Neutrosophic Topological Spaces

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ABSTRACT: In this paper, we will define a new class of sets, called fuzzy neutrosophic weakly- generalized closed sets, then we proved some theorems related to this definition. After that, we studied some relations between fuzzy neutrosophic weakly-generalized closed sets and fuzzy neutrosophic α closed sets, fuzzy neutrosophic closed sets, fuzzy neutrosophic regular closed sets, fuzzy neutrosophic pre closed sets and fuzzy neutrosophic semi closed sets.

Keywords: Fuzzy Neutrosophic set, Fuzzy Neutrosophic Topology, Fuzzy Neutrosophic Weakly-Generalized closed sets.

Introduction:

The first use of the concept of fuzzy sets was introduced by Zadeh in 1965 [1]. After that, the fuzzy set theory extension by many researchers. Intutionistic fuzzy sets (IFS) was one of the extension sets and defined by K. Atanassov in 1983 [2, 3, 4], when fuzzy set gives the degree of membership of an element in the sets, whenever intuitionistic fuzzy sets give a degree of membership and a degree of non-membership. After that, several researches were conducted on the generalizations of the notion of intuitionistic fuzzy sets, one of them was Florentin Smarandache in 2010 [5] when he developed another membership in addition to the two memberships which were defined in intuitionistic fuzzy sets and called it neutrosophic set.

The term of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In the last year, (2017) Veereswari [9] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

In this paper, We introduced define a new class of sets via fuzzy neutrosophic sets and called it fuzzy neutrosophic weakly-generalized closed sets in fuzzy neutrosophic topological spaces, we discuss some new properties and examples based of this define concept.

1. Basic definitions and terms

Definition (1.1) [7, 9]: Let X be a non-empty fixed set, the fuzzy neutrosophic set (Briefly, FNS), λ_N is an object having the form $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ where the functions $\mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N} : X \to [0, 1]$. Denote the degree of membership function (namely $\mu_{\lambda N}(x)$), the degree of indeterminacy function (namely $\sigma_{\lambda N}(x)$) and the degree of non-membership (namely $\nu_{\lambda N}(x)$) respectively, of each set λ_N we have, $0 \leq \mu_{\lambda N}(x) + \sigma_{\lambda}(x) + \nu_{\lambda N}(x) \leq 3$, for each $x \in X$.

Remark (1.2) [9]: FNS $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ can be identified to an ordered triple $<x, \mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N}>$ in $[0, 1]$ on X.

Definition (1.3) [6]: Let X be a non-empty set and the FNSs λ_N and β_N on X be in the form:

$\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ and, $\beta_N = \{< x, \mu_{\beta N}(x), \sigma_{\beta N}(x), \nu_{\beta N}(x) >: x \in X\}$ then:

- **i.** $\lambda_N \subseteq \beta_N$ iff $\mu_{\lambda N}(x) \leq \mu_{\beta N}(x)$, $\sigma_{\lambda N}(x) \leq \sigma_{\beta N}(x)$ and $\nu_{\lambda N}(x) \geq \nu_{\beta N}(x)$ for all $x \in X$,
- **ii.** $\lambda_N = \beta_N$ iff $\lambda_N \subseteq \beta_N$ and $\beta_N \subseteq \lambda_N$,
- **iii.** $\underline{1}_N - \lambda_N = \{< x, \nu_{\lambda N}(x), 1 - \sigma_{\lambda N}(x), \mu_{\lambda N}(x) >: x \in X\}$,
- **iv.** $\lambda_N \cup \beta_N = \{< x, \mathrm{Max}(\mu_{\lambda N}(x), \mu_{\beta N}(x)), \mathrm{Max}(\sigma_{\lambda N}(x), \sigma_{\beta N}(x)), \mathrm{Min}(\nu_{\lambda N}(x), \nu_{\beta N}(x)) >: x \in X\}$,
- **v.** $\lambda_N \cap \beta_N = \{< x, \mathrm{Min}(\mu_{\lambda N}(x), \mu_{\beta N}(x)), \mathrm{Min}(\sigma_{\lambda N}(x), \sigma_{\beta N}(x)), \mathrm{Max}(\nu_{\lambda N}(x), \nu_{\beta N}(x)) >: x \in X\}$,
- **vi.** $\underline{0}_N = < x, 0, 0, 1>$ and $\underline{1}_N = <x, 1, 1, 0>$.

Definition (1.4) [9]: A fuzzy neutrosophic topology (Briefly, FNT) on a non-empty set X is a family τ of fuzzy neutrosophic subsets in X satisfying the following axioms:

- **i.** $\underline{0}_N, \underline{1}_N \in \tau$,
- **ii.** $O_{N1} \cap O_{N2} \in \tau$ for any $O_{N1}, O_{N2} \in \tau$,
- **iii.** $\cup O_{Ni} \in \tau, \forall \{O_{Ni}: i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space (Briefly, FNTS). The elements of τ are called fuzzy neutrosophic open sets (Briefly, F_N). The complements of F_N-open sets in the FNTS (X, τ) are called fuzzy neutrosophic closed sets (Briefly, F_N-closed sets).

Definition (1.5) [9]: Let (X, τ) be FNTS and $\lambda_N = <x, \mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N}>$ be FNS in X. Then the fuzzy neutrosophic closure of λ_N (Briefly, FNCl) and fuzzy neutrosophic interior of λ_N (Briefly, FNInt) are defined by:

$\mathrm{FNCl}(\lambda_N) = \cap \{C_N: C_N$ is F_N-closed set in X and $\lambda_N \subseteq C_N\}$,

$\mathrm{FNInt}(\lambda_N) = \cup \{O_N: O_N$ is F_N-open set in X and $O_N \subseteq \lambda\}$.

Note that $\mathrm{FNCl}(\lambda_N)$ be F_N-closed set and $\mathrm{FNInt}(\lambda_N)$ be F_N-open set in X. Furthermore

i. λ_N be F_N-closed set in X iff FNCl (λ_N) = λ_N,

ii. λ_N be F_N-open set in X iff FNInt (λ_N) = λ_N.

Proposition (1.6) [9]: Let (X, τ) be FNTS and λ_N, β_N are FNSs in X. Then the following properties hold:

i. $FNInt(\lambda_N) \subseteq \lambda_N$ and $\lambda_N \subseteq FNCl(\lambda_N)$,

ii. $\lambda_N \subseteq \beta_N \Longrightarrow FNInt(\lambda_N) \subseteq FNInt(\beta_N)$ and $\lambda_N \subseteq \beta_N \Longrightarrow FNCl(\lambda_N) \subseteq FNCl(\beta_N)$,

iii. $FNInt(FNInt(\lambda_N)) = FNInt(\lambda_N)$ and $FNCl(FNCl(\lambda_N)) = FNCl(\lambda_N)$,

iv. $FNInt(\lambda_N \cap \beta_N) = FNInt(\lambda_N) \cap FNInt(\beta_N)$ and $FNCl(\lambda_N \cup \beta_N) = FNCl(\lambda_N) \cup FNCl(\beta_N)$,

v. $FNInt(\underline{1}_N) = \underline{1}_N$ and $FNCl(\underline{1}_N) = \underline{1}_N$,

vi. $FNInt(\underline{0}_N) = \underline{0}_N$ and $FNCl(\underline{0}_N) = \underline{0}_N$.

Definition (1.7) [8]: FNS λ_N in FNTS (X, τ) is called:

i. Fuzzy neutrosophic regular-open set (Briefly, FNR-open)
if $\lambda_N = FNInt(FNCl(\lambda_N))$.

ii. Fuzzy neutrosophic regular-closed set (Briefly, FNR-closed)
if $\lambda_N = FNCl(FNInt(\lambda_N))$.

iii. Fuzzy neutrosophic semi-open set (Briefly, FNS-open)
if $\lambda_N \subseteq FNCl(FNInt(\lambda_N))$.

iv. Fuzzy neutrosophic semi-closed set (Briefly, FNS-closed)
if $FNInt(FNcl(\lambda_N)) \subseteq \lambda_N$.

v. Fuzzy neutrosophic pre-open set (Briefly, FNP-open)
if $\lambda_N \subseteq FNInt(FNCl(\lambda_N))$.

vi. Fuzzy neutrosophic pre-closed set (Briefly, FNP-closed)
if $FNCl(FNInt(\lambda_N)) \subseteq \lambda_N$.

vii. Fuzzy neutrosophic α-open set (Briefly, FNα-open)
if $\lambda_N \subseteq FNInt(FNCl(FNInt(\lambda_N)))$.

viii. Fuzzy neutrosophic α-closed set (Briefly, FNα-closed)
if $FNCl(FNInt(FNCl(\lambda_N))) \subseteq \lambda_N$.

2. Characterizations and properties of Fuzzy Neutrosophic Weakly-Generalized Closed Sets in Fuzzy Neutrosophic Topological Spaces.

In this section we introduce and investigate some characterizations and several properties concerning of Fuzzy Neutrosophic Weakly-Generalized Closed Sets in Fuzzy Neuotrosophic Topological spaces.

Definition (2.1) : Fuzzy neutrosophic sub set λ_N of FNTS (X, τ) is called:

i. Fuzzy neutrosophic-generalized closed set (Briefly, FNGCS) if FNCl (λ_N) $\subseteq U_N$ wherever, $\lambda_N \subseteq U_N$ and U_N be F_N-open set in X. And λ_N is said to be fuzzy neutrosophic-generalized open set (Briefly, FNGOS) if the complement $\underline{1}_N - \lambda_N$ be FNGCS set in (X, τ).

ii. Fuzzy neutrosophic weakly-closed set (Briefly, FNWCS)
if FNCl (λ_N) $\subseteq U_N$ wherever, $\lambda_N \subseteq U_N$ and U_N be FNS-open set in X . And λ_N is said to be fuzzy neutrosophic weakly-open set (Briefly, FNWOS) if the complement $\underline{1}_N - \lambda_N$ is FNWCS in (X, τ).

iii. Fuzzy neutrosophic weakly-generalized closed set (Briefly, FNWGCS) if $FNCl(FNInt(\lambda_N)) \subseteq U_N$ wherever, $\lambda_N \subseteq U_N$ and U_N be F_N-open set in X. And λ_N is a said to be fuzzy neutrosophic weakly-generalized open set (Briefly, FNWGOS) if the complement $\underline{1}_N-\lambda_N$ is FNWGCS in (X, τ).

Theorem (2.2): For every FNS, the following statements satisfy:

i. Every F_N-closed set is FNGCS.
ii. Every FNα-closed set is FNWGCS.
iii. Every F_N-closed set is FNWGCS.
iv. Every FNR-closed set is FNWGCS.
v. Every FNP-closed set is FNWGCS.

Proof:

i. Let $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x)>: x \in X\}$ be F_N-closed set in FNTS (X,τ). Then, $FNCl(\lambda_N) = \lambda_N$.
Now, let β_N be F_N-open set such that, $\lambda_N \subseteq \beta_N$.
Therefore, $FNCl(\lambda_N) = \lambda_N \subseteq \beta_N$.
Hence, λ_N be FNGCS in (X,τ).

ii. Let $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x)>: x \in X\}$ be FNα-closed set in FNTS (X, τ). Then, $FNCl(FNInt(FNCl(\lambda_N))) \subseteq \lambda_N$.
Now, let β_N be F_N-open set such that, $\lambda_N \subseteq \beta_N$.
Then, $FNCl(FNInt(\lambda_N)) \subseteq FNCl(FNInt(FNCl(\lambda_N))) \subseteq \lambda_N \subseteq \beta_N$
Therefore, $FNCl(FNInt(\lambda_N)) \subseteq \beta_N$.
Hence, λ_N be FNWGCS in (X, τ).

iii. Let $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x)>: x \in X\}$ be F_N-closed set in FNTS (X, τ). Then by Definition (1.5) (i),
we get $\lambda_N = FNCl(\lambda_N)$……(1)
By Proposition (1.6) (i) we get, $FNInt(\lambda_N) \subseteq \lambda_N$……(2)
But, $FNCl(FNInt(\lambda_N)) \subseteq FNCl(\lambda_N)$.
So by (1), $FNCl(FNInt(\lambda_N) \subseteq \lambda_N$
Now, let β_N be F_N-open set such that, $\lambda_N \subseteq \beta_N$.
Then, $FNCl(FNInt(\lambda_N)) \subseteq \lambda_N \subseteq \beta_N$.
Therefore, $FNCl(FNInt(\lambda_N)) \subseteq \beta_N$.
Hence, λ_N be FNWGCS in (X, τ).

iv. Let $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x)>: x \in X\}$ be FNR-closed set in FNTS (X, τ). Then, $FNCl(FNInt(\lambda_N)) = \lambda_N$
New, let β_N be F_N-open set such that, $\lambda_N \subseteq \beta_N$
Then, $FNCl(FNInt(\lambda_N)) = \lambda_N \subseteq \beta_N$
Therefore, $FNCl(FNInt(\lambda_N)) \subseteq \beta_N$
Hence, λ_N be FNWGCS in (X, τ).

v. Let $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x)>: x \in X\}$ be FNP-closed set in FNTS (X, τ). Then, $FNCl(FNInt(\lambda_N)) \subseteq \lambda_N$
New, let β_N be F_N-open set such that, $\lambda_N \subseteq \beta_N$
Then, $FNCl(FNInt(\lambda_N)) \subseteq \lambda_N \subseteq \beta_N$
Therefore, $FNCl(FNInt(\lambda_N)) \subseteq \beta_N$
Hence, λ_N be FNWGCS in (X, τ).

Remark (2.3) : The convers of Theorem (2.2) is not true in general and omit it, it is significant to show it by the following examples:

Examples (2.4) :

i. Let X={a, b} define FNS λ_N in X as follows:

λ_N = <x, $(\frac{a}{0.5}, \frac{b}{0.2})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.5}, \frac{b}{0.7})$ >.

The family τ ={0_N, 1_N, λ_N}be FNTS.

Now if, ω_N = < x, $(\frac{a}{0.9}, \frac{b}{0.3})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.1}, \frac{b}{0.6})$ >.

And, U_N =$\underline{1}_N$, where U_N be F_N-open set such that, $\omega_N \subseteq U_N$.

Then, FNCl(ω_N) = ∩ {C_N: C_N is F_N-closed set in X and $\omega_N \subseteq C_N$ }

= < x, $(\frac{a}{0.9}, \frac{b}{0.3})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.1}, \frac{b}{0.6})$ > ⊆ <x, $(\frac{a}{1}, \frac{b}{1})$, $(\frac{a}{1}, \frac{b}{1})$, $(\frac{a}{0}, \frac{b}{0})$ > such that,

$(\frac{a}{0.9}, \frac{b}{0.3}) \leq (\frac{a}{1}, \frac{b}{1})$, $(\frac{a}{0.5}, \frac{b}{0.5}) \leq (\frac{a}{1}, \frac{b}{1})$ and $(\frac{a}{0.1}, \frac{b}{0.6}) \geq (\frac{a}{0}, \frac{b}{0})$ = 1_N.

Therefore, FNCl(ω_N) ⊆ U_N.

Hence, ω_N is FNGCS but, not F_N-closed set.

ii. Let X={a, b} define FNS λ_N in X as follows:

λ_N = < x, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.4}, \frac{b}{0.5})$ >

The family τ ={0_N, 1_N, λ_N } be FNTS,

Now if, ω_N = < x, $(\frac{a}{0.5}, \frac{b}{0.4})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.6}, \frac{b}{0.5})$ >.

And, U_N = λ_N be F_N-open set such that, $\omega_N \subseteq U_N$.

Then, FNInt (ω_N) = ∪ {O_N: O_N is F_N-open set in X and $O_N \subseteq \omega_N$ }

= <x, $(\frac{a}{0}, \frac{b}{0})$, $(\frac{a}{0}, \frac{b}{0})$, $(\frac{a}{1}, \frac{b}{1})$ > ⊆ < x, $(\frac{a}{0.5}, \frac{b}{0.4})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.6}, \frac{b}{0.5})$ >.

Such that, $(\frac{a}{0}, \frac{b}{0}) \leq (\frac{a}{0.5}, \frac{b}{0.4})$, $(\frac{a}{0}, \frac{b}{0}) \leq (\frac{a}{0.5}, \frac{b}{0.5})$ and $(\frac{a}{1}, \frac{b}{1}) \geq (\frac{a}{0.6}, \frac{b}{0.5})$ = 0_N. And FNCl (FNInt(ω_N)) = 0_N.

Therefore, FNCl (FNInt (ω_N)) ⊆ U_N.

Since, <x, $(\frac{a}{0}, \frac{b}{0})$, $(\frac{a}{0}, \frac{b}{0})$, $(\frac{a}{1}, \frac{b}{1})$ > ⊆ <x, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.4}, \frac{b}{0.5})$ > such that, $(\frac{a}{0}, \frac{b}{0}) \leq (\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0}, \frac{b}{0}) \leq (\frac{a}{0.5}, \frac{b}{0.5})$ and $(\frac{a}{1}, \frac{b}{1}) \geq (\frac{a}{0.4}, \frac{b}{0.5})$.

Hence, ω_N is FNWGCS.

But, FNCl (ω_N) = ∩ {C_N: C_N is F_N-closed set in X and ω_N ⊆ C }

= < x, $(\frac{a}{0.5}, \frac{b}{0.4})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.6}, \frac{b}{0.5})$ > ⊆ < x, $(\frac{a}{1}, \frac{b}{1})$, $(\frac{a}{1}, \frac{b}{1})$, $(\frac{a}{0}, \frac{b}{0})$ > such that,

$(\frac{a}{0.5}, \frac{b}{0.4}) \leq (\frac{a}{1}, \frac{b}{1})$, $(\frac{a}{0.5}, \frac{b}{0.5}) \leq (\frac{a}{1}, \frac{b}{1})$ and $(\frac{a}{0.6}, \frac{b}{0.5}) \geq (\frac{a}{0}, \frac{b}{0})$ = $\underline{1}_N$. So,

FNInt(FNCl(ω_N)) = $\underline{1}_N$ and FNCl(FNInt(FNCl(ω_N))) = $\underline{1}_N$.

Therefore, FNCl(FNInt(FNCl(ω_N))) ⊈ ω_N.

Since, < x, $(\frac{a}{1}, \frac{b}{1})$, $(\frac{a}{1}, \frac{b}{1})$, $(\frac{a}{0}, \frac{b}{0})$ > ⊈ <x, $(\frac{a}{0.5}, \frac{b}{0.4})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.6}, \frac{b}{0.5})$ > such that, $(\frac{a}{1}, \frac{b}{1}) \nleq (\frac{a}{0.5}, \frac{b}{0.4})$, $(\frac{a}{1}, \frac{b}{1}) \nleq (\frac{a}{0.5}, \frac{b}{0.5})$ and $(\frac{a}{0}, \frac{b}{0}) \ngeq (\frac{a}{0.6}, \frac{b}{0.5})$.

Hence, ω_N be not FNα-closed set.

iii. Take the example which defined in (ii). Then, we can see ω_N be FNWGCS but, not F_N-closed set.

iv. Take again, the example which defined in (ii). Then, ω_N be FNWGCS but not FNR-closed set.

v. Take, the example which defined in (i). Then, FNInt $(\omega_N) = \lambda_N$ and FNCl(FNInt(ω_N)) = 1_N-λ_N.

Therefore, FNCl(FNInt (ω_N)) $\subseteq U_N$.

Hence, ω_N is FNWGCS.

But, FNCl(FNInt (ω_N)) $\nsubseteq \omega_N$.

Hence, ω_N is not FNP-closed set.

Remark (2.5) : The relation between FNS-closed sets and FNWGCSs is independent, it is important to show it by the following examples:

Example (2.6) :

(1) Let X={a, b} define FNS λ_N in X as follows:

$\lambda_N = < x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.7}) >$.

The family $\tau = \{0_N, 1_N, \lambda_N\}$ be FNTS.

Now if, $\omega_N = < x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.7}) >$,

And, $U_N = \lambda_N$ where U_N be F_N-open set such that, $\omega_N \subseteq U_N$.

Then, FNCl $(\omega_N) = \underline{1}_N$-$\lambda_N$ and FNInt(FNCl(ω_N)) = λ_N.

Therefore, FNInt(FNCl (ω_N)) $\subseteq \omega_N$.

Hence, ω_N is FNS-closed set.

But, FNInt $(\omega_N) = \lambda_N$ and FNCl(FNInt(ω_N)) = $\underline{1}_N$-λ_N.

Therefore, FNCl(FNInt (ω_N)) $\nsubseteq U_N$.

Hence, ω_N is not FNWGCS.

(2) Take Example (2.4) (v) then, ω_N is FNWGCS.

But, FNCl $(\omega_N) = \underline{1}_N$ and FNInt(FNCl(ω_N)) = $\underline{1}_N$.

Therefore, FNInt(FNCl (ω_N)) $\nsubseteq \omega_N$.

Hence, ω_N is not FNS-closed set.

Proposition (2.7) : Let λ_N be F_N-closed set in (X, τ) such that, FNInt(λ_N) $\subseteq \beta_N \subseteq \lambda_N$. Then, β_N is FNWGCS on FNTS (X, τ).

Proof: Let $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ be FNS in FNTS (X, τ) such that, FNInt(λ_N) $\subseteq \beta_N \subseteq \lambda_N$.

So, there exists F_N-closed set η_N such that, η_N (FNInt(λ_N)) $\subseteq \beta_N \subseteq \lambda_N \subseteq \eta_N$.

Then, $\beta_N \subseteq \eta_N$ and also FNInt(β_N) $\subseteq \beta_N \subseteq \eta_N$.

Thus, FNCl(FNInt(β_N) $\subseteq \beta_N$.

Now, let Ψ_N be F_N-open set such that, $\beta_N \subseteq \Psi_N$.

Then, FNCl(FNInt(β_N) $\subseteq \beta_N \subseteq \Psi_N$.

Therefore, FNCl(FNInt (β_N)) $\subseteq \Psi_N$.

Hence, β_N is FNWGCS in (X, τ).

Theorem (2.8) : Let (X, τ) be FNTS, then the intersection of two FNWGCSs is also FNWGCS.

Proof: Let λ_N and β_N are FNP-closed sets on FNTS (X, τ).

Then, $FNCl(FNInt(\lambda_N)) \subseteq \lambda_N$ and $FNCl(FNInt(\beta_N)) \subseteq \beta_N$.

Consider $\lambda_N \cap \beta_N \supseteq FNCl(FNInt(\lambda_N)) \cap FNCl(FNInt(\beta_N))$

$\supseteq FNCl(FNInt(\lambda_N) \cap FNInt(\beta_N))$

$\supseteq FNCl(FNInt(\lambda_N \cap \beta_N))$

This means $FNCl(FNInt(\lambda_N \cap \beta_N)) \subseteq \lambda_N \cap \beta_N$.

Now, let η_N be F_N-open set such that, $\lambda_N \cap \beta_N \subseteq \eta_N$

Then, $FNCl(FNInt(\lambda_N \cap \beta_N)) \subseteq \lambda_N \cap \beta_N \subseteq \eta_N$.

Therefore, $FNCl(FNInt(\lambda_N \cap \beta_N)) \subseteq \eta_N$.

Hence, $\lambda_N \cap \beta_N$ be FNWGCS in (X, τ).

Remark (2.9) : The union of any FNWGCSs is not necessary to be FNWGCS see the following example:

Example (2.10) : Let $X=\{x\}$ define FNSs λ_N and β_N in X as follows:

$\lambda_N =\{<x, 0.5, 0.6, 0.7> : x\in X \}$ and $\beta_N =\{< x, 0.6, 0.7, 0.5 >: x \in X \}$.

The family $\tau = \{\underline{0}_N, \underline{1}_N, \lambda_N, \beta_N \}$ be FNTS.

Now if, $\omega_{N1} = \{< x, 0.6, 0.5, 0.5 > : x\in X \}$, $\omega_{N2} = \{< x, 0.6, 0.7, 0.8 > : x\in X \}$and

$U_N =\{< x, 0.6, 0.7, 0.5 >: x\in X \}$ where, U_N be F_N-open set such that, $\omega_{N1} \subseteq U_N$ and $\omega_{N2} \subseteq U_N$.

Then, $FNInt(\omega_{N1}) = \underline{0}_N$ and $FNCl(FNInt(\omega_{N1}) = \underline{0}_N$

Therefore, $FNcl(FNint(\omega_{N1}) \subseteq U_N$. Hence, ω_{N1} be FNWGCS.

And, $FNint(\omega_{N2}) = \underline{0}_N$ and $FNcl(FNint(\omega_{N2})) = \underline{0}_N$.

Therefore, $FNCl(FNInt(\omega_{N2})) \subseteq U_N$.

Hence, ω_{N2} be FNWGCS.

So, $\omega_{N1} \cup \omega_{N2} =\{< x, 0.6, 0.7, 0.5 > : x\in X \}$.

But, $FNInt(\omega_{N1} \cup \omega_{N2}) =\{<x, 0.6, 0.7, 0.5>:x\in X\}$and $FNCl(FNInt(\omega_{N1} \cup \omega_{N2})) =\underline{1}_N$

Therefore, $FNCl(FNInt(\omega_{N1} \cup \omega_{N2})) \not\subseteq U_N$

Hence, $\omega_{N1} \cup \omega_{N2}$ is not FNWGCS.

Definition (2.11) : Let (X, τ) be FNTS and $\lambda_N = \{<x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X \}$ be FNS. Then, the fuzzy neutrosophic is weakly generalized closure of λ_N (Briefly, FNWGCl) and fuzzy neutrosophic weakly generalized interior of λ_N (Briefly, FNWGInt) are defined by:

- **i.** $FNWGCl(\lambda_N) = \cap\{\beta_N: \beta_N$ is FNWGCS in X and $\lambda_N \subseteq \beta_N \}$
- **ii.** $FNWGInt(\lambda_N) = \cup\{\beta_N: \beta_N$ is FNWGOS in X and $\beta_N \subseteq \lambda_N\}$,

Proposition (2.12): Let (X, τ) be FNTS and λ_N, β_N are FNSs in X. Then the following properties hold:

- **i.** $FNWGCl(0_N) = \underline{0}_N$ and $FNWGCl(1_N) =\underline{1}_N$,
- **ii.** $\lambda_N \subseteq FNWGCl(\lambda_N)$,
- **iii.** If $\lambda_N \subseteq \beta_N$, then $FNWGCl(\lambda_N) \subseteq FNWGCl(\beta_N)$,
- **iv.** λ_N is FNWGCS iff $\lambda_N = FNWGCl(\lambda_N)$

v. FNWGCl(FNWGCl(λ_N)) = FNWGCl(λ_N).

Proof:

i. By Definition (2.11) (i) it is important to focus on:

FNWGCl(0_N) = ∩ {β_N: β_N is FNWGCS in X and $0_N \subseteq \beta_N$ } = $\underline{0}_N$,

And, FNWGCl(1_N) = ∩ {β_N: β_N is FNWGCS in X and $1_N \subseteq \beta_N$ }=$\underline{1}_N$.

ii. $\lambda_N \subseteq$ ∩{β_N: β_N is FNWGCS in X and $\lambda_N \subseteq \beta_N$ } = FNWGCl (λ_N).

iii. Suppose that $\lambda_N \subseteq \beta_N$ then, ∩{β_N: β_N is FNWGCS in X and $\lambda_N \subseteq \beta_N$ }

$\subseteq$ ∩{ η_N: η_N is FNWGCS in X and $\beta_N \subseteq \eta_N$ }

Therefore, FNWGCl(λ_N) $\subseteq$ FNWGCl(β_N).

iv. $\Longrightarrow$ If, λ_N is FNWGCS then,

FNWGCl (λ_N) = ∩{β_N: β_N is FNWGCS in X and $\lambda_N \subseteq \beta_N$ }……(1)

And by (ii), $\lambda_N \subseteq$ FNWGCl (λ_N)……(2)

But λ_N is necessarily to be the smallest set.

Thus, λ_N = ∩{β_N: β_N is FNWGCS in X and $\lambda_N \subseteq \beta_N$ },

Therefore, λ_N = FNWGCl (λ_N)

$\Longleftarrow$ Let λ_N = FNWGCl (λ_N) = ∩{β_N: β_N is FNWGCS in X and $\lambda_N \subseteq \beta_N$} and by using Definition 2.11 (i), we get λ_N is FNWGCS in X.

v. Since, λ_N = FNWGCl (λ_N) so we get, FNWGCl (λ_N) = FNWGCl(FNWGCl (λ_N)).

Proposition (2.13): Let (X, τ) be FNTS and λ_N, β_N are FNSs in X. Then the following properties hold:

i. FNWGInt(0_N) = $\underline{0}_N$ and FNWGInt (1_N) =$\underline{1}_N$,

ii. FNWGInt(λ_N) $\subseteq \lambda_N$,

iii. If $\lambda_N \subseteq \beta_N$, then FNWGInt($\lambda_N$) $\subseteq$ FNWGInt(β_N),

iv. λ_N is a FNWGOS iff λ_N = FNWGInt (λ_N),

v. FNWGInt (λ_N) = FNWGInt (FNWGInt (λ_N)).

Proof:

i. By Definition (2.11) (ii) we have FNWGInt(0_N) = ∪{β_N: β_N is FNWGOS in X and $\beta_N \subseteq 0_N$} = $\underline{0}_N$.

And, FNWGInt(1_N) = ∪{β_N: β_N is FNWGOS in X and $\beta_N \subseteq 1_N$} =$\underline{1}_N$.

ii. Follows from Definition (2.11) (ii).

iii. FNWGInt(λ_N) = ∪{β_N: β_N is FNWGOS in X and $\beta_N \subseteq \lambda_N$}.

Since, $\lambda_N \subseteq \beta_N$ then, ∪{β_N: β_N is FNWGOS in X and $\beta_N \subseteq \lambda_N$.

$\subseteq$ ∪{ η_N: η_N is FNWGOS in X and $\eta_N \subseteq \beta_N$}.

Therefore, FNWGInt(λ_N) $\subseteq$ FNWG Int(β_N).

iv. $\Longrightarrow$ Omit it, there must be a proof that FNWGInt(λ_N) $\subseteq \lambda_N$ and $\lambda_N \subseteq$ FNWGInt(λ_N). Suppose that λ_N is FNWGOS in X.

Then, FNWGInt(λ_N) = ∪{β_N: β_N is FNWGOS in X and $\beta_N \subseteq \lambda_N$} by using **ii** we get, FNWGInt(λ_N) $\subseteq \lambda_N$……(1)

Now to proof, $\lambda_N \subseteq$ FNWGInt(λ_N), we have, for all $\lambda_N \subseteq \lambda_N$,
The FNWGInt(λ_N) $\subseteq \lambda_N$. So, we get

$\lambda_N \subseteq \cup\{\beta_N$: β_N is FNWGOS in X and $\beta_N \subseteq \lambda_N$ }= FNWGInt(λ_N)......(2)

From (1) and (2) we have, λ_N = FNWGInt (λ_N).

$\Leftarrow$ Suppose that λ_N = FNWGInt (λ_N) and by using Definition 2.11 (ii), we

get λ_N is a FNWGOS in X.

v. Since, λ_N = FNWGInt(λ_N) by (iv) so we get,

FNWGInt(λ_N) = FNWGInt (FNWGInt (λ_N)).

Theorem (2.14): Let (X, τ) be FNTS. Then for every fuzzy neutrosophic subsets λ_N of X, we have:

i. $\underline{1}_N$- FNWGInt (λ_N) = FNWGCl ($\underline{1}_N$ - λ_N),
ii. $\underline{1}_N$- FNWGCl (λ_N) = FNWGInt ($\underline{1}_N$ - λ_N).

Proof:

i. FNWGInt(λ_N) = $\cup\{\beta_N$: β_N is FNWGOS in X and $\beta_N \subseteq \lambda_N\}$, by the complement, we get
$\underline{1}_N$- FNWGInt (λ_N) = $\underline{1}_N$ - ($\cup\{\beta_N$: β_N is FNWGOS in X and $\beta_N \subseteq \lambda_N\}$).
So, $\underline{1}_N$ - FNWGInt(λ_N) = $\cap\{$ (1_N-β_N): (1_N-β_N) is FNWGCS in X and ($\underline{1}_N$-λ_N) $\subseteq$ ($\underline{1}_N$-β_N)$\}$.
Now, replacing ($\underline{1}_N$-β_N) by η_N we get,
$\underline{1}_N$ - FNWGInt(λ_N) = $\cap\{$ η_N: η_N is FNWGCS in X and ($\underline{1}_N$-λ_N) $\subseteq \eta_N\}$
= FNWGCl($\underline{1}_N$-λ_N).

ii. FNWGCl(λ_N) = $\cap\{\beta_N$: β_N is FNWGCS in X and $\lambda_N \subseteq \beta_N$ }, by the complement, we get
$\underline{1}_N$- FNWGCl(λ_N) = $\underline{1}_N$ - ($\cap\{\beta_N$: β_N is FNWGCS in X and $\lambda_N \subseteq \beta_N$ })
So, $\underline{1}_N$- FNWGCl (λ_N) = $\cup\{$($\underline{1}_N$-β_N) : ($\underline{1}_N$-β_N) is FNWGOS in X and ($\underline{1}_N$-β_N) $\subseteq$ ($\underline{1}_N$- λ_N)$\}$. Again replacing ($\underline{1}_N$-β_N) by η_N we get,
$\underline{1}_N$ - FNWGCl(λ_N) = $\cup\{$ η_N: η_N is FNWGOS in X and $\eta_N \subseteq$($\underline{1}_N$- λ_N)
= FNWGInt($\underline{1}_N$-λ_N).

Theorem (2.15) : If (X, τ) be FNTS, then for every fuzzy neutrosophic subsets λ_N and β_N of X we have

i. λ_N $\cup$ FNWGCl (FNWGInt (λ_N)) $\subseteq$ FNWGCl (λ_N),
ii. FNWGInt (λ_N) $\subseteq$ λ_N $\cap$ FNWGInt (FNWGCl (λ_N)).

Proof:

i. Since, by Proposition (2.12) (ii) $\lambda_N \subseteq$ FNWGCl(λ_N)……(1)
and, by Proposition (2.13) (ii) we have, FNWGInt (λ_N) $\subseteq \lambda_N$
Then, FNWGCl (FNWGInt (λ_N)) $\subseteq$ FNWGCl (λ_N)……(2)
So, from (1) and (2) we get,
λ_N $\cup$ FNWGCl (FNWGInt(λ_N)) $\subseteq$ FNWGCl(λ_N).

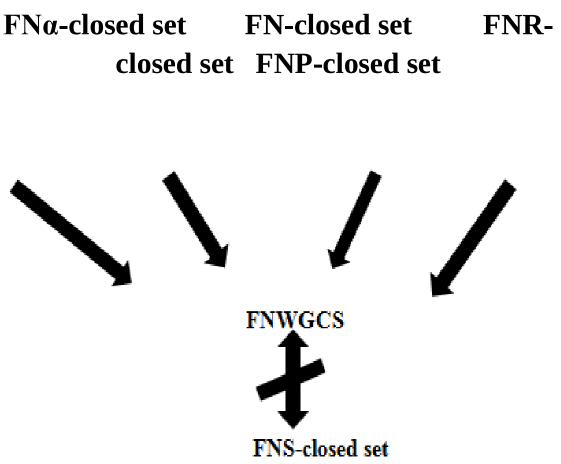
ii. Since, by Proposition (2.13) (ii) we have, FNWGInt (λ_N) $\subseteq \lambda_N$……(*) and, by Proposition (2.12) (ii) we have, $\lambda_N \subseteq$ FNWGCl (λ_N)

Then, FNWGInt(λ_N) $\subseteq$ FNWGInt (FNWGCl (λ_N))……(**)

So, from (*) and (**) we get,

FNWGInt $(\lambda_N) \subseteq \lambda_N \cap$ FNWGInt (FNWGCl (λ_N)).

Remark (2.16): The relationship between different sets in FNTS see the next Figure-1 and the convers is not true in general.



(Figure-1) The relationship between different sets of Fuzzy Neutrosophic Topological Space

Conclusion

In this paper, we defined new class of sets, is the fuzzy Neutrosophic weakly-generalized closed sets, then we proved some theorems related to this definition. We introduced defined for the new class of sets by fuzzy Neutrosophic sets and called it the fuzzy Neutrosophic weakly-generalized closed sets in fuzzy Neutrosophic topological spaces, we discuss some new properties, theorems and examples based of this define concept.

References:

[1] L. A. Zadeh. (1965). *Fuzzy Sets*. Inform and Control, Vol.8, 338:353.

[2] K. Atanassov and S. Stoeva. (August 1983). *Intuitionistic Fuzzy Sets*. in: Polish Syrup. On Interval & Fuzzy Mathematics, Poznan, 23:26.

[3] K. Atanassov. (1986). *Intuitionistic Fuzzy Sets*, Fuzzy Sets and Systems, Vol. 20, 87:96.

[4] K. Atanassov. (1988). *Review and new results on Intuitionistic Fuzzy Sets*, Preprint IM- MFAIS, Sofia. 1:88.

[5] F. Smarandache. (2010). *Neutrosophic Set : A Generalization of Intuitionistic Fuzzy Set*. Journal of Defense Resourses Management. Vol. 1, 1:10.

[6] A. A. Salama and S. A. Alblowi. (2012). *Neutrosophic Set and Neutrosophic Topological Spaces*. IOSR Journal of Mathematics,Vol. 3, Iss. 4. 31:35.

[7] I.Arockiarani and J.Martina Jency. (2014). *More on Fuzzy Neutrosophic Sets and Fuzzy Neutrosophic Topological Spaces*. IJIRS, Vol. 3. 642:652.

[8] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala. (2017). *On Some Notions and Functions in Neutrosophic Topological Spaces*. Neutrosophic Sets and System.Vol. 16, 16:19.

[9] Y.Veereswari , *An Introduction To Fuzzy Neutrosophic Topological Spaces,* IJMA,Vol.8(3), (2017). 144-149.

المجموعات المغلقة المعممة ـ الضعيفة النايتروسوفيك المضببة في الفضاءات التبولوجية النايتروسوفيك المضببة

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الخلاصة:

في هذا البحث، قمنا بتعريف فئة جديدة من المجموعات تم تسميتها بالمجموعات المغلقة المعممة – الضعيفة النايتروسوفيك المضببة، ثم أثبتنا بعض النظريات المتعلقة بهذا التعريف. بعد ذلك درسنا بعض العلاقات بين المجموعات المغلقة المعممة – الضعيفة النايتروسوفيك المضببة من جهة والمجموعات المغلقة α نيوتروسوفيك المضببة، المجموعات المغلقة النايتروسوفيك المضببة، المجموعات المغلقة المنتظمة نيوتروسوفيك المضببة، المجموعات المغلقة القبلية نيوتروسوفيك المضببة والمجموعات شبه المغلقة نيوتروسوفيك المضببة من جهة أخرى في الفضاءات التوبولوجية نيوتروسوفيك المضببة مع بعض الخصائص.