Design of $S_N^2 - NEWMA$ Control Chart for Monitoring Process having Indeterminate Production Data

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Abstract: The existing charts for monitoring the variance are designed under the assumption that all production data must consist of exact, precise, and determined observations. This paper presents the design of a new neutrosophic exponentially weighted moving average (NEWMA) combining with a neutrosophic logarithmic transformation chart for monitoring the variance having neutrosophic numbers. The computation of the neutrosophic control chart parameters is done through the neutrosophic Monte Carlo simulation (NMCS). The performance of the proposed chart is discussed with the existing charts.

Keywords: neutrosophic; numbers; neutrosophic logarithmic transformation; variance; monitoring

1. Introduction

The product manufactured according to the specified limits is known as the high-quality product. The manufacturing and service companies adopt the statistical process control (SPC) techniques to achieve a high-quality product. The process at or beyond these limits indicates the shift in the process and cause the non-conforming items. The more non-conforming items directly affect the profit and the reputation of the company. Therefore, the control charts, which are the essential part of the SPC, have been applied for controlling the non-conforming items. The control charts are used to monitor the average and variation in the normal distribution parameters. The control chart in the industry works as the indicator and guides the experimenters to solve the issue in the process. A timely shift in the process minimizes the non-conforming items. Due to several advantages of the control chart, a variety of charts have been designed for the monitoring of the process. The details about the application of the control charts can be seen in [1–5].

To make the Shewhart control chart more sensitive to detect a small shift in the process, Roberts [6] introduced the exponentially weighted moving average (EWMA) statistic in the control chart. The control charts using the EWMA statistic utilizes the current and previous subgroup information about the state of the process. Reference [7] presented an EWMA chart for monitoring the variation in the process. The application of EWMA-based control chart can be seen in References [8–17].

Fuzzy based control charts are the alternative of the traditional control charts when there is uncertainty in parameters or in the data. These control charts have the ability to detect the shift in the process under imprecise and incomplete production data. Cheng [18] presented chart for fuzzy numbers. Faraz [19] proved the efficiency of the fuzzy-based Shewhart control chart. Faraz et al. [20] presented a chart for combining fuzzy and randomness in the process. Teksen and Anagun [21] proposed variable charts for fuzzy data. Fadaei and Pooya [22] evaluated the performance of fuzzy

chart using the operating characteristic curve. Alakoc and Apaydin [23] worked on Shewhart control charts using fuzzy approach. Darestani et al. [24] presented the control limits of U-chart under uncertainty. For a detailed study on a fuzzy-based control chart, the reader may refer to References [25–28] proposed EWMA chart for monitoring fuzzy data.

Classical statistics (CS) works only when all observations in the data are determined and precise. In practice, it may not be possible that the data have determined values, especially in complex situations. When the data is interval, as in weather or in the stock exchange market, CS cannot be effectively applied. The neutrosophic logic, introduced by Reference [29] in 1998, is a generalization of fuzzy logic and of intuitionistic fuzzy logic, where the measure of indeterminacy plays an important role, see [30–32]. The neutrosophic statistics (NS), introduced by Smarandache [29] and developed in 2014 is the generalization of CS which has the ability to analyze the imprecise data, see References [33,34]. Aslam [35] introduced the NS in the area of quality control. A variety of control chart using the NS can be seen in References [36–40].

Abbas et al. [41] proposed the cumulative sum EWMA chart for monitoring process variance. Although a rich literature is available on the control chart for monitoring variation under CS, the existing charts can be only applied when the assumption of certain observations is met. By exploring the literature, we did not see work on the EWMA variance chart under NS. In this paper, we will first introduce the neutrosophic EWMA based on variance. We will also introduce the neutrosophic Monte Carlo (NMS) simulation originally in the paper. The methodology of the proposed chart is different than that in Reference [41]. The performance of the proposed chart will be compared with the control chart proposed by References [7] and [38].

2. Preliminaries and the Proposed $S_N^2 - NEWMA$ Statistic

Suppose $n_N \epsilon [n_L, n_U]$ be a neutrosophic random sample from the neutrosophic normal distribution with mean $\mu_N = \sum_{i=1}^{N_N} X_N / N_N$; $\mu_N \epsilon [\mu_L, \mu_U]$ and $\sigma_N^2 = [\{\sum_{i=1}^{n_N} (X_N - \mu_N)^2 / N_N - 1\} / n_N]$; $\sigma_N^2 \epsilon [\sigma_L^2, \sigma_U^2]$ having $X_{iN} \epsilon [X_L, X_U], i = 1,2,3,\ldots, n_N$ observations. Suppose $\bar{X}_N = \sum_{i=1}^{n_N} X_N / n_N$; $\bar{X}_N \epsilon [\bar{X}_L, \bar{X}_U]$ and $S_N^2 = \sum_{i=1}^{n_N} (X_N - \bar{X}_N)^2 / n_N - 1$; $S_N^2 \epsilon [S_L^2, S_U^2]$ be neutrosophic sample mean and variance, respectively. Let $EWMA_N = Z_{k_N}$ be the NEWMA. The modified form, say $S_N^2 - NEWMA$, of EWMA statistic under CS suggested by [42,43] is given by

$$Z_{k_N} = (1 - \lambda_N) Z_{k-1,N} + \lambda_N T_{k_N}; \ Z_{k_N} \epsilon [Z_{k_L}, Z_{k_U}], \lambda_N \epsilon [\lambda_L, \lambda_U] \tag{1}$$

where $\lambda_N \epsilon \{\lambda_L, \lambda_U\}$; $[0,0] \le \lambda_N \le [1,1]$ is neutrosophic smoothing constant and ranges from [0.05,0.05] to [0.25,0.25], see [14]. Note that $T_{k_N} = \ln S_N^2$; $T_{k_N} \epsilon [T_{k_L}, T_{k_U}]$ shows logarithmic of successive values of $S_N^2 \epsilon [S_L^2, S_U^2]$ and defined as follows

$$T_{k_N} = a_N + b_N . \ln(S_{k_N}^2 + c_N); \ a_N \epsilon [a_L, a_U], \ b_N \epsilon [b_L, b_U], c_N \epsilon [c_L, c_U] > [0,0] \tag{2}$$

Johnsson et al. [44] argued that $T_{k_N} \epsilon [T_{k_L}, T_{k_U}]$ that is modelled by log–gamma distribution tends to more neutrosophic normally distributed as compared to $S_N^2 \epsilon [S_L^2, S_U^2]$. According to Reference [7] "the main expectation of this approach is that if $a_N \epsilon [a_L, a_U]$, $b_N \epsilon [b_L, b_U]$, and $c_N \epsilon [c_L, c_U]$ are judiciously selected, then this transformation may result in approximate normality to $T_{k_N} \epsilon [T_{k_L}, T_{k_U}]$". The neutrosophic control limits (NCLs) with starting values of $Z_{0_N} = 0$ are given by

$$LCL_N = E(T_{k_N}) - k_N \sqrt{\frac{\lambda_N}{2 - \lambda_N}} \sigma(T_{k_N}); \ LCL_N \epsilon [LCL_L, LCL_U] \tag{3}$$

$$UCL_N = E(T_{k_N}) + k_N \sqrt{\frac{\lambda_N}{2 - \lambda_N}} \sigma(T_{k_N}); \ UCL_N \epsilon [UCL_L, UCL_U] \tag{4}$$

where $k_N \epsilon [k_L, k_U]$ be a neutrosophic control limits coefficient.

Note here that NCLs given in Equations (3) and (4) are approximate but widely used by the industrial practitioner, see [7]. The exact NCLs for $S_N^2 - NEWMA$ are given as

$$LCL_N = E(T_{k_N}) - k_N\sqrt{\frac{\lambda_N\{1-(1-\lambda_N)^{2k}\}}{2-\lambda_N}}\sigma(T_{k_N}); \ LCL_N \epsilon [LCL_L, LCL_U] \tag{5}$$

$$UCL_N = E(T_{k_N}) + k_N\sqrt{\frac{\lambda_N\{1-(1-\lambda_N)^{2k}\}}{2-\lambda_N}}\sigma(T_{k_N}); \ UCL_N \epsilon [UCL_L, UCL_U] \tag{6}$$

The NCLs given in Equations (3) and (4) are easy to apply, and therefore we will use them in the rest of the paper.

3. The Proposed Control Chart

Note here that although the limits of existing S^2 chart are not symmetry, the use of transformation $T_{k_N} = \ln S_N^2$ makes the limits symmetry for the proposed $S_N^2 - NEWMA$ chart. The steps to apply the proposed $S_N^2 - NEWMA$ control chart are stated as follows

Step-1: Select a random sample of size $n_N \epsilon [n_L, n_U]$ from the industrial process and compute statistic $Z_{k_N} \epsilon [Z_{k_L}, Z_{k_U}]$.

Step-2: Announce the process is shifted if $Z_{k_N} \epsilon [Z_{k_L}, Z_{k_U}] \geq UCL_N \epsilon [UCL_L, UCL_U]$ or $Z_{k_N} \epsilon [Z_{k_L}, Z_{k_U}] \leq LCU_N \epsilon [LCU_L, LCU_U]$.

Note here that the proposed $S_N^2 - NEWMA$ chart is the extension of Reference [7] and the traditional S^2 chart. It reduces to the Reference [7] chart if no uncertain observation is recorded in the production data. The probability of in-control under neutrosophic statistics is given by

$$P_{inN}^0 = P(LCL_N \leq Z_{k_N} \leq UCL_N/S_{N0}^2); \ S_{N0}^2 \epsilon [S_{L0}^2, S_{U0}^2] \tag{7}$$

where S_{N0}^2 represents the in-control variance of the process. The neutrosophic average run length (NARL), say ARL_{0N} is given by

$$ARL_{0N} = \frac{1}{1-P(LCL_N \leq Z_{k_N} \leq UCL_N/S_{N0}^2)}; \ ARL_{0N} \epsilon [ARL_{0L}, ARL_{0U}] \tag{8}$$

The probability of in-control for the shifted process under neutrosophic statistics is given by

$$P_{inN}^1 = P(LCL_N \leq Z_{k_N} \leq UCL_N/S_{N1}^2); \ S_{N1}^2 \epsilon [S_{L1}^2, S_{U1}^2] \tag{9}$$

where S_{N1}^2 represents shifted variance of the process. The neutrosophic average run length (NARL) for the shifted process, say ARL_{1N} is given by

$$ARL_{1N} = \frac{1}{1-P(LCL_N \leq Z_{k_N} \leq UCL_N/S_{N1}^2)}; \ ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}] \tag{10}$$

The values of $k_N \epsilon [k_L, k_U]$, $ARL_{0N} \epsilon [ARL_{0L}, ARL_{0U}]$, $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$, and neutrosophic standard deviation (NSD) will be determined using the following NMS.

1. Generate 10,000 random sample of size $n_N \epsilon [n_L, n_U]$ from neutrosophic normal distribution from the in-control process. Choose values of $a_N \epsilon [a_L, a_U]$, $b_N \epsilon [b_L, b_U]$, and $c_N \epsilon [c_L, c_U]$ from Table 1. Compute $Z_{k_N} \epsilon [Z_{k_L}, Z_{k_U}]$ and plot them on $LCU_N \epsilon [LCU_L, LCU_U]$ and $UCL_N \epsilon [UCL_L, UCL_U]$. Note first out-of-control and calculate their average.
2. Compute $ARL_{0N} \epsilon [ARL_{0L}, ARL_{0U}]$ and NSD and determine $k_N \epsilon [k_L, k_U]$ where $ARL_{0N} \epsilon [ARL_{0L}, ARL_{0U}] \geq r_{0N}$, where $r_{0N} \epsilon [r_{0L}, r_{0U}]$ are specified values of $ARL_{0N} \epsilon [ARL_{0L}, ARL_{0U}]$.
3. Select those $k_N \epsilon [k_L, k_U]$ when $ARL_{0N} \epsilon [ARL_{0L}, ARL_{0U}]$ very close to r_{0N}.
4. Generate random sample of size $n_N \epsilon [n_L, n_U]$ from the shifted process. Compute $Z_{k_N} \epsilon [Z_{k_L}, Z_{k_U}]$ and plot them on $LCU_N \epsilon [LCU_L, LCU_U]$ and $UCL_N \epsilon [UCL_L, UCL_U]$. Note first out-of-control and calculate their average.
5. Compute $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ and NSD for a various shift c.

Table 1. The values of constants.

n	[3,5]	[8,10]
$A(n)$	[−0.6627,−0.8969]	[−1.1647,−1.3135]
$B(n)$	[1.8136,2.3647]	[2.9992,3.3548]
$C(n)$	[0.6777,0.5979]	[0.5588,0.5465]
$E(T_k)$	[0.02472,0.00748]	[0.00243,0.00141]
$E(T_k)$	[0.9165,0.967]	[0.9864,0.9912]
Z_0	[0.276,0.211]	[0.167,0.149]

Using above-mentioned NMC simulation, we presented Tables 2–7 for various values of $n_N \epsilon [n_L, n_U]$ and c. From Tables 2–7, we note the decreasing trend in $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ and NSD for the same values of $n_N \epsilon [n_L, n_U]$ but when $\lambda_N \epsilon \{\lambda_L, \lambda_U\}$ increases. The values of $ARL_{1N} \epsilon [ARL_{1L}, ARL_{1U}]$ and NSD decrease when c increases.

Table 2. The neutrosophic average run length NARL when $n_N \epsilon$ [3, 5] and $\lambda_N \epsilon$ [0.08, 0.12].

c	$k_N \epsilon$ [2.58,2.65] when $ARL_{0N} \epsilon [300, 300]$		$k_N \epsilon$ [2.66,2.74] when $ARL_{0N} \epsilon [370, 370]$	
	NARL	**NSD**	**NARL**	**NSD**
1	[309.77,300.59]	[314.47,292.06]	[367.88,380.1]	[362.73,361.76]
1.05	[149.57,128.34]	[162.15,126.34]	[175.89,155.28]	[184.21,153.18]
1.1	[71.82,56.01]	[77.12,53.67]	[81.69,62.06]	[84.91,58.71]
1.15	[42.17,30.16]	[44.03,26.33]	[45.63,33.13]	[46.57,29.59]
1.2	[26.81,19.45]	[26.18,15.69]	[29.69,21.13]	[28.94,16.94]
1.25	[19.33,14.18]	[17.52,10.48]	[20.88,15.26]	[19.43,11.16]
1.3	[14.88,11.22]	[13.02,7.64]	[16.2,11.84]	[14.1,7.95]
1.4	[10.35,7.85]	[7.9,4.51]	[11.07,8.29]	[8.49,4.83]
1.5	[7.92,6.23]	[5.51,3.16]	[8.49,6.56]	[5.92,3.37]
1.6	[6.63,5.27]	[4.3,2.38]	[6.96,5.49]	[4.42,2.49]
1.7	[5.73,4.69]	[3.38,1.92]	[5.98,4.78]	[3.51,1.97]
1.8	[5.12,4.2]	[2.85,1.58]	[5.37,4.33]	[2.95,1.65]
1.8	[5.07,4.18]	[2.76,1.6]	[5.31,4.32]	[2.91,1.63]
1.9	[4.65,3.86]	[2.4,1.39]	[4.87,4]	[2.57,1.44]
2	[4.31,3.61]	[2.11,1.22]	[4.43,3.74]	[2.14,1.26]
2.25	[3.72,3.17]	[1.64,0.96]	[3.85,3.27]	[1.7,1.01]
2.5	[3.33,2.88]	[1.36,0.84]	[3.44,2.97]	[1.37,0.84]
3	[2.86,2.54]	[1.01,0.65]	[2.97,2.59]	[1.05,0.67]
4	[2.48,2.23]	[0.72,0.45]	[2.54,2.27]	[0.75,0.48]

Table 3. The neutrosophic average run length (NARL) when $n_N \epsilon$ [3, 5] and $\lambda_N \epsilon$ [0.18, 0.22].

c	$k_N \epsilon$ [2.725,2.75] when $ARL_{0N} \epsilon [300, 300]$		$k_N \epsilon$ [2.805,2.82] when $ARL_{0N} \epsilon [370, 370]$	
	NARL	**NSD**	**NARL**	**NSD**
1	[299.38,301.29]	[294.16,294.1]	[371,368.43]	[354.51,353.79]
1.05	[145.4,135.87]	[143.8,133.09]	[175.25,157.77]	[172.85,157.06]
1.1	[79.22,61.69]	[77.84,58.36]	[90.3,71.05]	[89.21,67.86]
1.15	[47.72,34.23]	[45.83,31.21]	[53.61,37.7]	[51.47,34.38]
1.2	[31.89,22.1]	[29.18,18.67]	[35.11,23.67]	[33.45,20.05]
1.25	[23.1,15.49]	[20.83,12.15]	[25.04,16.65]	[22.3,13.24]
1.3	[17.5,12.02]	[14.6,8.8]	[19.26,12.55]	[16.29,9.27]
1.4	[12.09,8.15]	[9.36,5.12]	[12.9,8.52]	[10.04,5.38]
1.5	[8.97,6.2]	[6.32,3.43]	[9.5,6.51]	[6.78,3.7]
1.6	[7.3,5.21]	[4.81,2.56]	[7.67,5.37]	[5.01,2.69]

1.7	[6.26,4.54]	[3.8,2.09]	[6.46,4.67]	[3.93,2.14]
1.8	[5.46,4.05]	[3.12,1.7]	[5.69,4.19]	[3.3,1.74]
1.8	[5.44,4.05]	[3.09,1.66]	[5.7,4.19]	[3.2,1.74]
1.9	[4.97,3.74]	[2.75,1.48]	[5.09,3.82]	[2.82,1.48]
2	[4.56,3.48]	[2.36,1.28]	[4.66,3.54]	[2.37,1.3]
2.25	[3.83,3.03]	[1.79,0.98]	[3.97,3.09]	[1.84,1.04]
2.5	[3.4,2.76]	[1.41,0.83]	[3.53,2.82]	[1.51,0.85]
3	[2.95,2.46]	[1.08,0.63]	[3.02,2.48]	[1.13,0.64]
4	[2.52,2.18]	[0.75,0.41]	[2.55,2.19]	[0.76,0.42]

Table 4. The neutrosophic average run length (NARL) when $n_N \epsilon$ [3, 5] and $\lambda_N \epsilon$ [0.28, 0.32].

c	$k_N \epsilon$ [2.805,2.77] when $ARL_{0N} \epsilon [300, 300]$		$k_N \epsilon$ [2.9,2.845] when $ARL_{0N} \epsilon [370, 370]$	
	NARL	**NSD**	**NARL**	**NSD**
1	[307.39,301.71]	[300.94,295.29]	[383.23,373.86]	[366.27,355.75]
1.05	[150.37,135.85]	[148.76,131.81]	[180.89,160.78]	[180.39,158.2]
1.1	[83.18,65.68]	[83.13,62.62]	[98.76,74.46]	[97.61,70.81]
1.15	[51.57,36.48]	[49.63,33.12]	[58.37,41.21]	[56.78,38.13]
1.2	[34.13,23.17]	[31.98,20.3]	[39.31,25.28]	[36.69,22.61]
1.25	[24.59,16.32]	[22.26,13.21]	[27.65,17.67]	[24.76,14.84]
1.3	[18.92,12.21]	[16.56,9.42]	[21.24,13.37]	[18.58,10.39]
1.4	[12.47,8.13]	[10.04,5.5]	[13.53,8.63]	[11.01,5.94]
1.5	[9.47,6.19]	[7.06,3.64]	[9.88,6.5]	[7.22,3.94]
1.6	[7.52,5.11]	[5.21,2.77]	[8.01,5.28]	[5.6,2.84]
1.7	[6.37,4.42]	[4.1,2.12]	[6.71,4.58]	[4.25,2.26]
1.8	[5.52,3.95]	[3.3,1.77]	[5.78,4.07]	[3.51,1.82]
1.8	[5.58,3.97]	[3.35,1.78]	[5.79,4.06]	[3.53,1.85]
1.9	[4.94,3.59]	[2.83,1.48]	[5.16,3.68]	[2.95,1.56]
2	[4.54,3.31]	[2.49,1.26]	[4.71,3.41]	[2.56,1.35]
2.25	[3.83,2.91]	[1.84,0.98]	[3.9,2.98]	[1.88,1.02]
2.5	[3.39,2.66]	[1.49,0.81]	[3.49,2.71]	[1.56,0.85]
3	[2.88,2.38]	[1.07,0.6]	[2.97,2.39]	[1.14,0.61]
4	[2.48,2.14]	[0.73,0.37]	[2.51,2.16]	[0.76,0.39]

Table 5. The neutrosophic average run length (NARL) when $n_N \epsilon$ [8, 10] and $\lambda_N \epsilon$ [0.08, 0.12].

c	$k_N \epsilon$ [2.56,2.67] when $ARL_{0N} \epsilon [300, 300]$		$k_N \epsilon$ [2.65,2.755] when $ARL_{0N} \epsilon [370, 370]$	
	NARL	**NSD**	**NARL**	**NSD**
1	[301.1,309.72]	[291.7,298.17]	[375.97,384.4]	[352.21,365.97]
1.05	[95.92,92.42]	[93.82,89.13]	[109.14,107.67]	[104.73,102.66]
1.1	[35.32,33.28]	[30.58,27.98]	[38.37,35.39]	[32.73,29.6]
1.15	[19.04,17.75]	[14.04,12.83]	[20.62,18.75]	[15.19,13.51]
1.2	[12.97,11.78]	[8.19,7.09]	[13.68,12.32]	[8.67,7.63]
1.25	[9.8,8.92]	[5.4,4.74]	[10.38,9.3]	[5.76,4.91]
1.3	[8.08,7.2]	[4.13,3.4]	[8.5,7.5]	[4.3,3.52]
1.4	[6.05,5.42]	[2.55,2.12]	[6.3,5.63]	[2.63,2.2]
1.5	[4.98,4.51]	[1.82,1.53]	[5.16,4.65]	[1.87,1.58]
1.6	[4.36,3.94]	[1.43,1.16]	[4.49,4.05]	[1.45,1.21]
1.7	[3.91,3.57]	[1.17,0.96]	[4.02,3.68]	[1.19,0.99]
1.8	[3.61,3.29]	[0.97,0.82]	[3.7,3.38]	[1,0.85]
1.8	[3.61,3.3]	[0.99,0.83]	[3.7,3.4]	[0.99,0.84]
1.9	[3.37,3.09]	[0.86,0.73]	[3.47,3.15]	[0.89,0.74]

2	[3.19,2.92]	[0.77,0.67]	[3.28,2.99]	[0.79,0.67]
2.25	[2.85,2.63]	[0.66,0.61]	[2.93,2.67]	[0.67,0.59]
2.5	[2.63,2.41]	[0.6,0.52]	[2.7,2.47]	[0.6,0.55]
3	[2.33,2.18]	[0.49,0.39]	[2.38,2.2]	[0.51,0.41]
4	[2.09,2.03]	[0.29,0.17]	[2.11,2.04]	[0.32,0.19]

Table 6. The neutrosophic average run length (NARL) when $n_N \epsilon$ [8, 10] and $\lambda_N \epsilon$ [0.18, 0.22].

c	$k_N \epsilon$ [2.735,2.77] when $ARL_{0N} \epsilon [300, 300]$		$k_N \epsilon$ [2.82,2.85] when $ARL_{0N} \epsilon [370, 370]$	
	NARL	**NSD**	**NARL**	**NSD**
1	[301.08,299.1]	[291.71,288.2]	[376.19,370.34]	[353.72,348.41]
1.05	[114.19,105.22]	[109.93,100.31]	[137.38,127.57]	[132.66,123.51]
1.1	[43.75,38.87]	[38.66,35]	[49.56,43.8]	[45.41,39.46]
1.15	[22.72,19.95]	[18.5,15.97]	[24.7,21.58]	[20,17.27]
1.2	[14.52,12.71]	[10.55,8.8]	[15.49,13.31]	[11.4,9.38]
1.25	[10.55,9.08]	[6.79,5.42]	[11.11,9.62]	[7.15,6.03]
1.3	[8.29,7.22]	[4.71,3.92]	[8.71,7.56]	[4.94,4.17]
1.4	[5.98,5.25]	[2.86,2.33]	[6.22,5.45]	[2.99,2.47]
1.5	[4.83,4.3]	[1.96,1.6]	[5.02,4.42]	[2.08,1.66]
1.6	[4.15,3.73]	[1.48,1.23]	[4.29,3.81]	[1.57,1.25]
1.7	[3.7,3.33]	[1.19,1]	[3.79,3.42]	[1.22,1.02]
1.8	[3.42,3.09]	[1.04,0.87]	[3.48,3.13]	[1.05,0.88]
1.8	[3.43,3.05]	[1.06,0.86]	[3.49,3.14]	[1.05,0.89]
1.9	[3.16,2.88]	[0.9,0.76]	[3.24,2.9]	[0.92,0.77]
2	[2.98,2.7]	[0.81,0.69]	[3.06,2.77]	[0.83,0.71]
2.25	[2.66,2.43]	[0.67,0.56]	[2.72,2.47]	[0.69,0.58]
2.5	[2.45,2.26]	[0.56,0.46]	[2.5,2.29]	[0.59,0.48]
3	[2.21,2.1]	[0.42,0.3]	[2.23,2.11]	[0.44,0.32]
4	[2.05,2.02]	[0.22,0.12]	[2.06,2.02]	[0.24,0.13]

Table 7. The neutrosophic average run length (NARL)   when $n_N \epsilon$ [8,10] and $\lambda_N \epsilon$ [0.28, 0.32].

c	$k_N \epsilon$ [2.785,2.81] when $ARL_{0N} \epsilon [300, 300]$		$k_N \epsilon$ [2.865,2.88] when $ARL_{0N} \epsilon [370, 370]$	
	NARL	**NSD**	**NARL**	**NSD**
1	[298.47,303.26]	[287.32,292.35]	[373.5,371.54]	[355.62,349.8]
1.05	[120.75,116.33]	[117.13,113.52]	[144.7,135.14]	[141.47,133.07]
1.1	[49.1,44.65]	[45.51,41.06]	[56.51,49.88]	[53.44,46.62]
1.15	[25.29,22.35]	[21.97,18.88]	[28.22,24.21]	[24.64,20.57]
1.2	[15.78,13.36]	[12.37,10.09]	[16.99,14.43]	[13.46,11.01]
1.25	[11.13,9.48]	[7.92,6.37]	[11.7,10.08]	[8.34,6.9]
1.3	[8.57,7.38]	[5.46,4.41]	[9.02,7.7]	[5.77,4.72]
1.4	[5.99,5.2]	[3.22,2.57]	[6.22,5.33]	[3.31,2.63]
1.5	[4.7,4.17]	[2.11,1.7]	[4.86,4.25]	[2.21,1.74]
1.6	[4,3.57]	[1.6,1.32]	[4.1,3.65]	[1.65,1.33]
1.7	[3.53,3.18]	[1.25,1.03]	[3.63,3.26]	[1.31,1.06]
1.8	[3.23,2.91]	[1.07,0.88]	[3.32,2.98]	[1.09,0.9]
1.8	[3.22,2.91]	[1.06,0.88]	[3.3,2.96]	[1.09,0.89]
1.9	[2.99,2.73]	[0.91,0.76]	[3.06,2.77]	[0.95,0.79]
2	[2.82,2.57]	[0.82,0.68]	[2.86,2.62]	[0.84,0.7]
2.25	[2.52,2.34]	[0.64,0.54]	[2.56,2.36]	[0.66,0.55]
2.5	[2.33,2.2]	[0.53,0.42]	[2.37,2.21]	[0.55,0.42]
3	[2.15,2.07]	[0.36,0.25]	[2.16,2.08]	[0.38,0.27]

| 4 | [2.03,2.01] | [0.17,0.1] | [2.04,2.01] | [0.19,0.11] |

4. Comparative Study

We will discuss the advantages of the proposed control chart over the existing charts in terms of NARL and NSD. Note here that the proposed chart reduces to the control chart proposed by Reference [7] when $k_L = k_U = k$ and $\lambda_L = \lambda_U = \lambda$. The proposed chart reduces to the traditional Shewhart variance control chart proposed by Reference [38] when $\lambda_N \epsilon$ [1,1]. To show the efficiency of the proposed chart, we will present the comparison in NARL and NSD in Section 4.1 and the simulation study in Section 4.2.

4.1. Advantage of the Proposed Chart in Neutrosophic Average Run Length (NARL) and Neutrosophic Standard Deviation (NSD)

The NARL and NSD are important measures to show the efficiency of the control charts. The smaller the values of NARL and NSD, the better the performance of the control chart to detect the shift in the process. To show the efficiency of the proposed chart over the existing charts proposed by References [7] and [38], we set the same values of $n_N \epsilon[n_L, n_U]$, $ARL_{0N} \epsilon[ARL_{0L}, ARL_{0U}]$, and c. We present the values $ARL_{1N} \epsilon[ARL_{1L}, ARL_{1U}]$ and NSD of the three control charts when $n_N \epsilon[3,5]$, $n_N \epsilon[8,10]$, $\lambda_N \epsilon[0.08,0.12]$ and $ARL_{0N} \epsilon[370,370]$ are shown in Table 8. Similar tables can be present for any other values of $n_N \epsilon[n_L, n_U]$ and $ARL_{0N} \epsilon[ARL_{0L}, ARL_{0U}]$.

Table 8 clearly indicates the smaller values of $ARL_{1N} \epsilon[ARL_{1L}, ARL_{1U}]$ and NSD of the proposed chart over charts proposed by References [7] and [38] at all values of c. For example, at a very small shift in the process, c=1.05, when $n_N \epsilon[3,5]$, the values of NARL and NSD from $S_N^2 - NEWMA$ chart are $ARL_{1N} \epsilon[175.89,155.28]$ and NSD $\epsilon[184.21,153.18]$, respectively. The values of NARL and NSD from [38] chart are $ARL_{1N} \epsilon[227.52,200.34]$ and NSD $\epsilon[229.3,198.5]$, respectively. From these values, we observe that the proposed $S_N^2 - NEWMA$ will declare the out-of-control process between 155th and 175th samples, while Reference [38] indicates the shift in the process between 200th and 227th samples. By comparing the proposed chart with the chart proposed by Reference [7], it can be noted that this existing chart provides the values of $ARL_{1N} \epsilon[ARL_{1L}, ARL_{1U}]$ and NSD, which is close to the upper values of the indeterminacy interval of the proposed chart. For an example, when $c = 1.05$ and $n_N \epsilon[3,5]$, the values of ARL_{1N} and NSD are $ARL_{1N} \epsilon[175.89,155.28]$ and NSD $\epsilon[184.21,153.18]$ from the proposed control chart. On the other hand, the values of ARL and SD are 175.73 and 189.55 from the existing chart. By comparing both charts, it can be noted that the proposed chart ability to detect shift between the 155th sample and 175th sample, while the existing chart detects a shift only at the 175th sample. From this comparison, we show the efficiency of the proposed chart over Reference [38]'s chart. In addition, the chart proposed by Reference [7] provides the determined values of ARL and cannot be applied under an uncertain environment. Therefore, the proposed chart is more effective, flexible, and adequate to be applied in indeterminacy.

Table 8. The neutrosophic average run length (NARL) for the proposed and existing charts when $\lambda_N \epsilon$ [0.08, 0.12].

	[38] Control Chart [8,10]		[38] Control Chart [3,5]		[7] Control Chart n = 3		[7] Control Chart *n* = 8		Proposed Chart $n_N \epsilon$ [8,10]		Proposed Chart $n_N \epsilon$ [3,5]	
	NARL	NSD	NARL	NSD	ARL	SD	ARL	SD	NARL	NSD	NARL	NSD
1	[372.53,376.51]	[353.12,362.16]	[383.81,374.2]	[363.99,358.16]	375.03	366.15	371.14	348.86	[375.97,384.4]	[352.21,365.97]	[367.88,380.1]	[362.73,361.76]
1.05	[174.95,177.11]	[173.32,173.32]	[227.52,200.34]	[229.3,198.5]	175.73	189.55	109.31	106.95	[109.14,107.67]	[104.73,102.66]	[175.89,155.28]	[184.21,153.18]
1.1	[89.85,86.43]	[89.3,85.09]	[140.93,112.84]	[139.96,111.44]	82.72	86.61	38.47	33.1	[38.37,35.39]	[32.73,29.6]	[81.69,62.06]	[84.91,58.71]
1.15	[51.6,47.65]	[50.04,46.73]	[93.24,67.04]	[91.71,65.33]	45.71	46.17	20.48	14.76	[20.62,18.75]	[15.19,13.51]	[45.63,33.13]	[46.57,29.59]
1.2	[31.83,28.12]	[30.42,26.62]	[65.06,44.77]	[63.7,43.53]	29.81	28.95	13.65	8.7	[13.68,12.32]	[8.67,7.63]	[29.69,21.13]	[28.94,16.94]
1.25	[20.96,18.27]	[19.21,16.59]	[46.58,31.12]	[45.27,29.66]	21.01	18.95	10.27	5.87	[10.38,9.3]	[5.76,4.91]	[20.88,15.26]	[19.43,11.16]
1.3	[14.9,12.65]	[13.6,11.41]	[35.48,22.53]	[34.36,21.16]	16.15	13.68	8.38	4.14	[8.5,7.5]	[4.3,3.52]	[16.2,11.84]	[14.1,7.95]
1.4	[8.79,7.38]	[7.3,5.89]	[22.44,13.55]	[20.5,11.77]	11.07	8.56	6.27	2.6	[6.3,5.63]	[2.63,2.2]	[11.07,8.29]	[8.49,4.83]
1.5	[6.03,4.99]	[4.44,3.44]	[15.14,9.19]	[13.67,7.65]	8.52	5.89	5.17	1.88	[5.16,4.65]	[1.87,1.58]	[8.49,6.56]	[5.92,3.37]
1.6	[4.53,3.86]	[2.99,2.32]	[11.29,6.8]	[9.7,5.33]	7.01	4.45	4.51	1.46	[4.49,4.05]	[1.45,1.21]	[6.96,5.49]	[4.42,2.49]
1.7	[3.76,3.2]	[2.17,1.63]	[9.03,5.41]	[7.38,3.92]	6	3.48	4.02	1.18	[4.02,3.68]	[1.19,0.99]	[5.98,4.78]	[3.51,1.97]
1.8	[3.23,2.85]	[1.68,1.25]	[7.25,4.54]	[5.66,3]	5.32	2.87	3.72	1.03	[3.7,3.38]	[1,0.85]	[5.37,4.33]	[2.95,1.65]
1.8	[3.22,2.83]	[1.65,1.24]	[7.38,4.48]	[5.8,2.99]	5.34	2.92	3.71	1	[3.7,3.4]	[0.99,0.84]	[5.31,4.32]	[2.91,1.63]
1.9	[2.87,2.59]	[1.27,0.98]	[6.19,3.96]	[4.62,2.37]	4.87	2.52	3.47	0.88	[3.47,3.15]	[0.89,0.74]	[4.87,4]	[2.57,1.44]
2	[2.66,2.43]	[1.06,0.77]	[5.42,3.53]	[3.83,1.96]	4.49	2.19	3.26	0.79	[3.28,2.99]	[0.79,0.67]	[4.43,3.74]	[2.14,1.26]
2.25	[2.35,2.22]	[0.7,0.51]	[4.29,2.91]	[2.74,1.34]	3.8	1.66	2.93	0.67	[2.93,2.67]	[0.67,0.59]	[3.85,3.27]	[1.7,1.01]
2.5	[2.2,2.11]	[0.49,0.35]	[3.57,2.6]	[2.01,0.96]	3.47	1.39	2.7	0.6	[2.7,2.47]	[0.6,0.55]	[3.44,2.97]	[1.37,0.84]
3	[2.07,2.03]	[0.28,0.19]	[2.93,2.3]	[1.33,0.62]	2.98	1.08	2.38	0.52	[2.38,2.2]	[0.51,0.41]	[2.97,2.59]	[1.05,0.67]
4	[2.01,2]	[0.12,0.06]	[2.46,2.1]	[0.82,0.33]	2.53	0.76	2.11	0.32	[2.11,2.04]	[0.32,0.19]	[2.54,2.27]	[0.75,0.48]

4.2. Simulation Study

Suppose that $S_{N0}^2 \epsilon [1,1]$ shows the variance of the in-control process. The twenty values are generated from the in-control process and the next 20 from the shifted process when $c = 1.25$, $n_N \epsilon [3,5]$, and $\lambda_N \epsilon [0.08, 0.12]$. The statistic $Z_{k_N} \epsilon [Z_{k_L}, Z_{k_U}]$ is computed for the proposed chart and charts proposed by References [7] and [38]. We plotted the values of $Z_{k_N} \epsilon [Z_{k_L}, Z_{k_U}]$ for the proposed chart in Figure 1, the chart proposed by Reference [38] in Figure 2 and the chart proposed by Reference [7] in Figure 3.
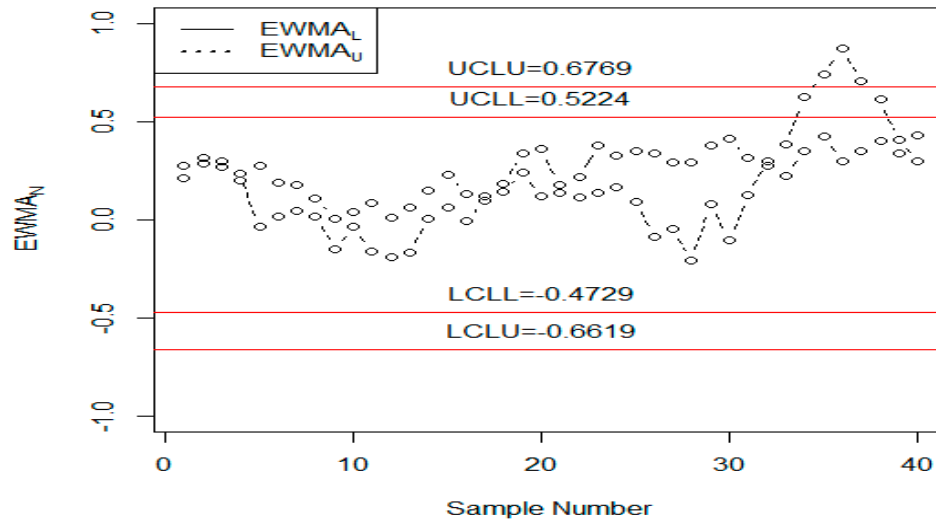
Figure 1. The $S_N^2 - NEWMA$ chart for the simulated data.
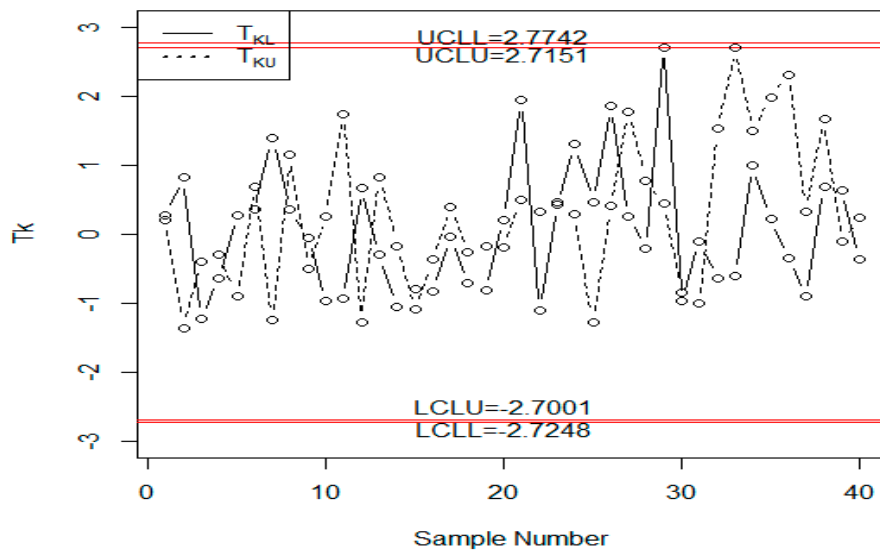
Figure 2. The control chart proposed by Reference [38] for simulated data.
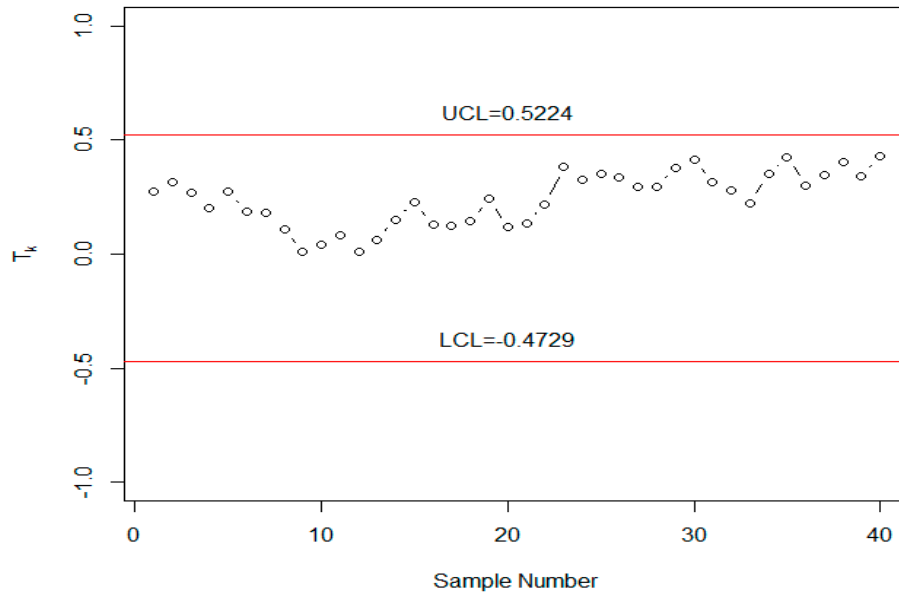
Figure 3. The existing chart proposed by Reference [7] for simulated data.

We note that the tabulated $ARL_{1N}\epsilon[20.88,15.26]$ which means that the control chart should detect a shift in the process between the 15th and 20th sample. From Figure 1, we note that the proposed chart detects a shift at the 35th sample, as expected. On the other hand, the existing charts proposed by References [7] and [38] do not detect any shift in the process. From this simulation study, it is quite clear that the proposed chart has the ability to detect a shift in the process, while existing charts do not. Therefore, the use of the proposed chart in the industry will be helpful in minimizing the non-conforming items.

5. Application

For the application of the proposed chart, the data is obtained from the automobile industry. The inside diameter of engine piston rings is a variable of interest here. This variable is continuous and obtained from measurement. According to Reference [45], "observations include human judgments, and evaluations and decisions, a continuous random variable of a production process should include the variability caused by human subjectivity or measurement devices, or environmental conditions. These variability causes create vagueness in the measurement system". The data having less or more than the exact values is shown in Table 9. Table 9 clearly shows the measurement data in intervals where classical statistics cannot apply. Similar data was used by Reference [38]. The calculations statistic of $T_{k_N}\epsilon\left[T_{k_L}, T_{k_U}\right]$, $Z_{k_N}\epsilon\left[Z_{k_L}, Z_{k_U}\right]$ when $n_N\epsilon[5,5]$ and $\lambda_N\epsilon[0.12,0.12]$ and shown in Table 10.

Table 9. Real example data taken from Reference [38].

			Sample Observation		
1	[74.03,74.03]	[74.002,73.991]	[74.019,74.019]	[73.992,73.992]	[74.008,74.001]
2	[73.995,73.995]	[73.992,74.003]	[74.001,74.001]	[74.011,74.011]	[74.004,74.004]
3	[73.988,74.017]	[74.024,74.024]	[74.021,74.021]	[74.005,74.005]	[74.002,73.995]
4	[74.002,74.002]	[73.996,73.996]	[73.993,73.993]	[74.015,74.015]	[74.009,74.009]
5	[73.992,73.992]	[74.007,74.007]	[74.015,74.015]	[73.989,73.989]	[74.014,73.998]
6	[74.009,74.009]	[73.994,74.001]	[73.997,73.997]	[73.985,73.985]	[73.993,73.993]
7	[73.995,73.998]	[74.006,74.006]	[73.994,73.994]	[74,74]	[74.005,74.005]
8	[73.985,73.985]	[74.003,74.01]	[73.993,73.993]	[74.015,74.015]	[73.988,73.988]

9	[74.008,74.005]	[73.995,73.995]	[74.009,74.009]	[74.005,74.005]	[74.004,74.004]
10	[73.998,73.998]	[74,74]	[73.99,73.99]	[74.007,74.007]	[73.995,73.995]
11	[73.994,73.998]	[73.998,73.998]	[73.994,73.994]	[73.995,73.995]	[73.99,74.001]
12	[74.004,74.004]	[74,74.002]	[74.007,74.005]	[74,74.001]	[73.996,73.996]
13	[73.983,73.993]	[74.002,74.002]	[73.998,73.998]	[73.997,73.997]	[74.012,74.005]
14	[74.006,74.006]	[73.967,73.985]	[73.994,73.994]	[74,74]	[73.984,73.996]
15	[74.012,74.012]	[74.014,74.012]	[73.998,73.998]	[73.999,73.999]	[74.007,74.007]
16	[74,74]	[73.984,73.984]	[74.005,74.005]	[73.998,73.998]	[73.996,73.996]
17	[73.994,73.994]	[74.012,74.012]	[73.986,73.986]	[74.005,74.005]	[74.007,74.007]
18	[74.006,74.006]	[74.01,74.011]	[74.018,74.018]	[74.003,74.003]	[74,74.001]
19	[73.984,73.984]	[74.002,74.002]	[74.003,74.003]	[74.005,74.005]	[73.997,73.997]
20	[74,74]	[74.01,74.01]	[74.013,74.009]	[74.02,74.015]	[74.003,74.003]
21	[73.982,73.982]	[74.001,74.001]	[74.015,74.015]	[74.005,74.005]	[73.996,73.996]
22	[74.004,74.004]	[73.999,73.999]	[73.99,73.99]	[74.006,74.006]	[74.009,74.002]
23	[74.01,74.01]	[73.989,73.989]	[73.99,73.99]	[74.009,74.005]	[74.014,74.011]
24	[74.015,74.011]	[74.008,74.008]	[73.993,73.993]	[74,74]	[74.01,74.011]
25	[73.982,73.982]	[73.984,73.989]	[73.995,73.995]	[74.017,74.012]	[74.013,74.01]

Table 10. The values of statistic for the proposed chart.

Sr#	S_N^2	T_{k_N}	Z_{k_N}
1	[0.000218,0.000297]	[1.521080,2.581987]	[0.211,0.211]
2	[0.000056,0.000033]	[−0.544091,−0.856070]	[−0.544091,−0.856070]
3	[0.000217,0.000146]	[1.515046,1.224781]	[1.515046,1.224781]
4	[0.000082,0.000082]	[−0.062734,0.273432]	[−0.062734,0.273432]
5	[0.000149,0.000115]	[0.847492,0.811799]	[0.847492,0.811799]
6	[0.000075,0.000079]	[−0.176903,0.227342]	[−0.176903,0.227342]
7	[0.000030,0.000024]	[−1.138271,−1.116612]	[−1.138271,−1.116612]
8	[0.000150,0.000181]	[0.857712,1.615885]	[0.857712,1.615885]
9	[0.000030,0.000026]	[−1.133061,−1.051897]	[−1.133061,−1.051897]
10	[0.000039,0.000039]	[−0.913632,−0.677915]	[−0.913632,−0.677915]
11	[0.000008,0.000007]	[−1.809110,−1.756588]	[−1.809110,−1.756588]
12	[0.000017,0.000012]	[−1.496849,−1.566774]	[−1.496849,−1.566774]
13	[0.000109,0.000021]	[0.345425,−1.227323]	[0.345425,−1.227323]
14	[0.000234,0.000060]	[1.653410,−0.172225]	[1.653410,−0.172225]
15	[0.000053,0.000046]	[−0.60183,−0.499771]	[−0.601839,−0.499771]
16	[0.000060,0.000060]	[−0.454187,−0.159260]	[−0.454187,−0.159260]
17	[0.000111,0.000111]	[0.37870829,0.7532313]	[0.378708,0.753231]
18	[0.000048,0.000046]	[−0.701836,−0.489458]	[−0.701836,−0.489458]
19	[0.000071,0.000071]	[−0.249291,0.068160]	[−0.249291,0.068160]
20	[0.000063,0.000035]	[−0.397927,−0.795335]	[−0.397927,−0.795335]
21	[0.000147,0.000147]	[0.829230,1.235694]	[0.829230,1.235694]
22	[0.000055,0.000039]	[−0.564569,−0.686131]	[−0.564569,−0.686131]
23	[0.000142,0.000115]	[0.766936,0.808990]	[0.766936,0.808990]
24	[0.000075,0.000062]	[−0.178422,−0.126542]	[−0.178422,−0.126542]
25	[0.000261,0.000171]	[1.864733,1.505833]	[1.864733,1.505833]

The proposed control chart with neutrosophic control limits is shown in Figure 4. The charts proposed by References [38] and [7] are shown in Figures 5 and 6, respectively. By comparing these figures, it can be seen that the proposed control chart indicates some points near the lower neutrosophic control limits. On the other hand, the existing charts in Figures 5 and 6 show the process is in control. We conclude here that the proposed chart indicates some issues in the process, while

the existing chart indicates noting in the process. Therefore, the use of the proposed control chart will provide strict monitoring of the automobile manufacturing process.
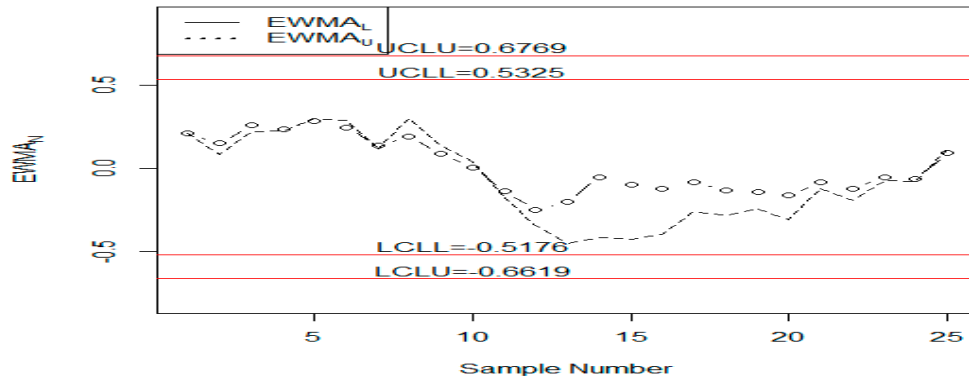
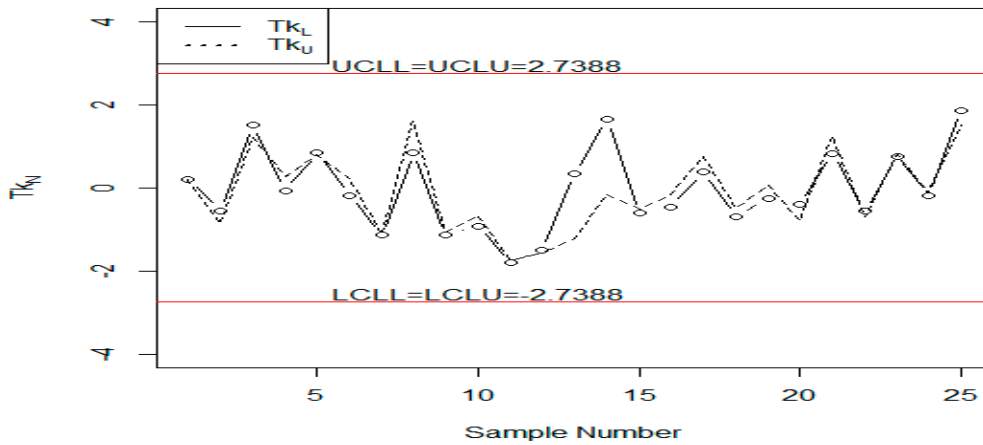**Figure 4.** $S_N^2 - NEWMA$ chart for real data.

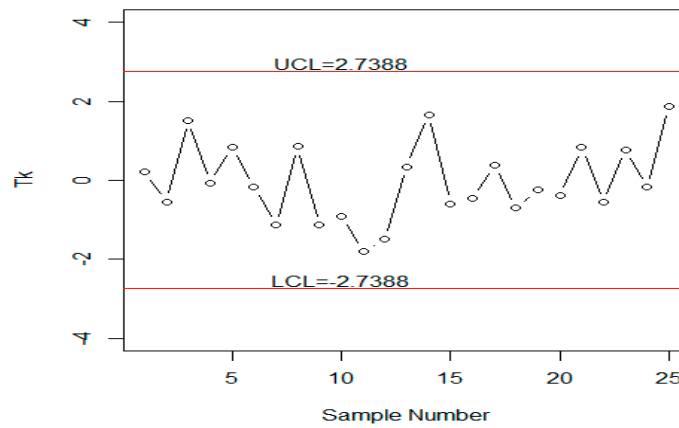**Figure 5.** The existing chart proposed by Reference [38] for real data.

Figure 6. The existing chart proposed by Reference [7] for real data.

6. Concluding Remarks

We presented a $S_N^2 - NEWMA$ chart for monitoring the variation in the process. We also presented a new NMC simulation method. The necessary tables are presented for various values of specified parameters. The simulation study and a real example are presented. From the simulation study and real example, it is concluded that the proposed $S_N^2 - NEWMA$ chart performs better than Reference [38]'s chart under uncertainty. We recommended applying the proposed control chart in the industry where the data is obtained from the complex measurement. The proposed control chart using some other sampling schemes and big data can be considered as future research. The proposed chart using the methodology of Reference [41] will be considered as future research.

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