

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.

Digital Object Identifier 10.1109/ACCESS.2017.DOI

Derivable Single Valued Neutrosophic Graphs Based on KM -Fuzzy Metric

M. HAMIDI¹ and F. SMARANDACHE²

¹ Department of Mathematics, University of Payame Noor, Tehran, Iran (e-mail: m.hamidi@pnu.ac.ir)

² Mathematics & Science, University of New Mexico, Gallup Campus, USA (e-mail: smarand@unm.edu)

Corresponding author: M. Hamidi (e-mail: m.hamidi@pnu.ac.ir).

ABSTRACT In this paper we consider the concept of KM -fuzzy metric spaces and we introduce a novel concept of KM -single valued neutrosophic metric graphs based on KM -fuzzy metric spaces. Then we investigate the finite KM -fuzzy metric spaces with respect to KM -fuzzy metrics and we construct the KM -fuzzy metric spaces on any given non-empty sets. We try to extend the concept of KM -fuzzy metric spaces to a larger class of KM -fuzzy metric spaces such as union and product of KM -fuzzy metric spaces and in this regard we investigate the class of products of KM -single valued neutrosophic metric graphs. In the final, we define some operations such as tensor product, Cartesian product, semi-strong product, strong product, union, semi-ring sum, suspension, and complement of KM -single valued neutrosophic metric graphs.

INDEX TERMS (Derivable) KM -single valued neutrosophic metric graph, KM -Fuzzy metric space, triangular-norm(conorm).

I. INTRODUCTION

CLASSICAL theory is a pure concept and without quality or criteria, so it is not attractive to use in our world, that's why we use the neutrosophic sets theory as one of a generalizations of set theory in order to deal with uncertainties, which is a key action in the contemporary world introduced by Smarandache for the first time in 1998 [26] and in 2005 [27]. This concept is a new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. This theory describes an important role in modeling and controlling unsure hypersystems in nature, society and industry. In addition, fuzzy topological spaces as a generalization of topological spaces, have a fundamental role in construction of fuzzy metric spaces as an extension of the concept of metric spaces. The theory of fuzzy metric spaces works on finding the distance between two points as non-negative fuzzy numbers, which have various applications. The structure of fuzzy metric spaces is equipped with mathematical tools such as triangular norms and fuzzy subsets depending on time parameter and on other variables. This theory has been proposed by different researchers with different definitions from several points of views ([5]–[7], [15]), and that this study was applied to the notion of KM -fuzzy metric space introduced in 1975 [6] by Kramosil and Michalek. Fuzzy graphs, introduced by

Rosenfeld, are finding an increasing number of applications in modelling real time systems where the level of information inherent in the system varies with respect to different levels of precision. Fuzzy models are becoming useful because of their aim in reducing the difference between the traditional numerical models used in engineering and sciences [23]. The generalization of the concept of a fuzzy graph is noticed by some researchers on more subjects, such as fuzzy graph based on t-norm, intuitionistic fuzzy threshold graphs, m-polar fuzzy graphs and single-valued neutrosophic graphs. Mordeson et al. generalized the definition of a fuzzy graph by replacing minimum in the basic definitions with an arbitrary t-norm. They developed a measure on the susceptibility of trafficking in persons for networks by using a t-norm other than minimum [21]. Recently, F. Smarandache, introduced a new concept as a generalization of hypergraphs to n-SuperHypergraph, Plithogenic n-SuperHypergraph {with super-vertices (that are groups of vertices) and hyper-edges {defined on power-set of power-set...} that is the most general form of graph as today}, and n-ary HyperAlgebra, n-ary NeutroHyperAlgebra, n-ary AntiHyperAlgebra respectively, which have several properties and are connected with the real world [28]. Further materials regarding graphs, single-valued neutrosophic metric graphs, hypergraphs, intuitionistic fuzzy set, n-SuperHypergraph and Plithogenic n-SuperHypergraph,

and NeutroAlgebras {Smarandache generalized the classical algebraic structures to neutro algebraic structures (or Neutro Algebras) [whose operations and axioms are partially true, partially indeterminate, and partially false] as extensions of PartialAlgebra, and to AntiAlgebraic structures (or Anti Algebras) [whose operations and axioms are totally false], and in general, he extended any classical structure, in no matter what field of knowledge, to a Neutro structure and an Anti structure}. All these are available in the literature too [2]–[4], [8], [10]–[13], [16], [17], [22], [24], [25], [29]–[33], [35].

Regarding these points, we introduce the concept of KM -single valued neutrosophic metric graphs based on the concept of KM -fuzzy metrics. One of the main motivations of KM -single valued neutrosophic metric graphs is obtained from the fuzzy graphs and so we want to use this concept to model many decision making problems in uncertain environment. We need to construct the KM -single valued neutrosophic metric graphs based on finite or infinite sets, so we develop the concept of KM -fuzzy metric on any nonempty set and prove that for every given set with respect to the concept of C -graphable sets one can construct a KM -metric space. It is a natural generalization of the fuzzy graphs to the single-valued neutrosophic metric graphs, so it shows our main motivation for introducing the notion of the KM -single valued neutrosophic metric graphs. This notion is based on one of the fundamental concepts of fuzzy mathematics, which includes tools such as t -norms, t -conorms, and fuzzy subsets. We apply the notation of KM -fuzzy metric spaces to generate the finite KM -single valued neutrosophic metric graphs. We have extended some production operations on the KM -fuzzy metric spaces to the KM -single valued neutrosophic metric graphs.

II. PRELIMINARIES

In this section, we recall some definitions and results, which we use in what follows.

Definition 1: [14], [18] Let $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ be simple graphs, $(x_1, x_2), (y_1, y_2) \in V_1 \times V_2$, where $V_1 \times V_2$ is the vertex set of the following graphs:

(i) categorical(tensor, direct, cardinal, Kronecker) product graph $G_1 \times G_2$:

$$E(G_1 \times G_2) = \{(x_1, x_2)(y_1, y_2) \mid x_1 y_1 \in E_1 \text{ and } x_2 y_2 \in E_2\};$$

(ii) Cartesian product graph $G_1 \otimes G_2$:

$$E(G_1 \otimes G_2) = \{(x_1, x_2)(y_1, y_2) \mid (x_1 = y_1 \text{ and } x_2 y_2 \in E_2) \text{ or } (x_1 y_1 \in E_1 \text{ and } x_2 = y_2)\};$$

(iii) semi-strong product graph $G_1 \bullet G_2$:

$$E(G_1 \bullet G_2) = \{(x_1, x_2)(y_1, y_2) \mid (x_1 = y_1 \text{ and } x_2 y_2 \in E_2) \text{ or } (x_1 y_1 \in E_1 \text{ and } x_2 y_2 \in E_2)\};$$

(iv) strong product (symmetric composition) graph $G_1 \odot G_2$:

$$E(G_1 \odot G_2) = E(G_1 \otimes G_2) \cup E(G_1 \times G_2);$$

(v) lexicographic product (composition) graph $G_1 \circ G_2$ ($G_1.G_2, G_1[G_2]$):

$$E(G_1 \circ G_2) = \{(x_1, x_2)(y_1, y_2) \mid (x_1 y_1 \in E_1) \text{ or } (x_1 = y_1 \text{ and } x_2 y_2 \in E_2)\};$$

(vi) union graph $G_1 \cup G_2$:

$$V(G_1 \cup G_2) = V(G_1) \cup V(G_2); \text{ and } E(G_1 \cup G_2) = E(G_1) \cup E(G_2);$$

(vii) join product graph $G_1 + G_2$:

$$E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup E', \text{ where } E' \text{ is the set of all line joining } V_1 \text{ with } V_2.$$

Definition 2: [20] A fuzzy graph $G = (V, \sigma, \mu)$ is an algebraic structure of non-empty set V together with a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ such that for all $x, y \in V$, $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$. It is called σ as fuzzy vertex set and μ as fuzzy edge set of G .

Definition 3: [1] A single valued neutrosophic graph (SVN- G) is defined to be a form $G = (V, E, A, B)$ where

(i) $V = \{v_1, v_2, \dots, v_n\}$, $T_A, I_A, F_A : V \rightarrow [0, 1]$ denote the degree of membership, degree of indeterminacy and non-membership of the element $v_i \in V$; respectively, and for every $1 \leq i \leq n$, we have $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$.

(ii) $E \subseteq V \times V$, $T_B, I_B, F_B : E \rightarrow [0, 1]$ are called degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, such that for any $1 \leq i, j \leq n$, we have $T_B(v_i, v_j) \leq \min\{T_A(v_i), T_A(v_j)\}$, $I_B(v_i, v_j) \geq \max\{I_A(v_i), I_A(v_j)\}$, $F_B(v_i, v_j) \geq \max\{F_A(v_i), F_A(v_j)\}$ and $0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$. Also A is called the single valued neutrosophic vertex set of V and B is called the single valued neutrosophic edge set of E .

Definition 4: [19] A triplet (X, ρ, T) is called a KM -fuzzy metric space, if X is an arbitrary non-empty set, T is a left-continuous t -norm and $\rho : X^2 \times \mathbb{R}^{\geq 0} \rightarrow [0, 1]$ is a fuzzy set, such that for each $x, y, z \in X$ and $t, s \geq 0$, we have:

(i) $\rho(x, y, 0) = 0$,

(ii) $\rho(x, x, t) = 1$ for all $t > 0$,

(iii) $\rho(x, y, t) = \rho(y, x, t)$ (commutative property),

(iv) $T(\rho(x, y, t), \rho(y, z, s)) \leq \rho(x, z, t + s)$ (triangular inequality),

(vi) $\rho(x, y, -) : \mathbb{R}^{\geq 0} \rightarrow [0, 1]$ is a left-continuous map,

(vii) $\lim_{t \rightarrow \infty} \rho(x, y, t) = 1$,

(viii) $\rho(x, y, t) = 1, \forall t > 0$ implies that $x = y$.

If (X, ρ, T) satisfies in conditions (i)–(vii), then it is called KM -fuzzy pseudo metric space and ρ is called a KM -fuzzy pseudo metric. a fuzzy version of the triangular inequality. The value $\rho(x, y, t)$ is considered as the degree of nearness from

Theorem 1: [19] Let (X, ρ, T) be a KM -fuzzy metric space. Then $\rho(x, y, -) : \mathbb{R}^{\geq 0} \rightarrow [0, 1]$ is a non-decreasing map.

Proof 1: See [19].

III. FINITE KM -FUZZY METRIC SPACE

In this section, we apply the concept of KM -fuzzy metric spaces and construct a new class of KM -fuzzy metric spaces under operation product and union of KM -fuzzy metric

spaces. In addition, for any given non-empty set we construct KM -fuzzy metric space with respect to α -discrete metric, where $\alpha \in \mathbb{R}^+$. From now on, for all $x, y \in [0, 1]$ we consider $T_{min}(x, y) = \min\{x, y\}$, $T_{pr}(x, y) = xy$, $T_{lu}(x, y) = \max(0, x + y - 1)$, $T_{do}(x, y) = \frac{xy}{x + y - xy}$ and $\mathcal{C}_T = \{T : [0, 1] \times [0, 1] \rightarrow [0, 1] \mid T \text{ is a left-continuous t-norm}\}$.

Theorem 2: If (X, ρ, T_{min}) is a KM -fuzzy metric space and $T \in \mathcal{C}_T$. Then (X, ρ, T) is a KM -fuzzy metric space.

Proof 2: Let $x, y, z \in X$, $r, s \in \mathbb{R}^{\geq 0}$ and $T \in \mathcal{C}_T$. Since for all $x, y \in [0, 1]$, $T(x, y) \leq T_{min}(x, y)$, we get that $T(\rho(x, y, t), \rho(y, z, s)) \leq T_{min}(\rho(x, y, t), \rho(y, z, s)) \leq \rho(x, z, t + s)$. Hence (X, ρ, T) is a KM -fuzzy metric space.

Let X be an arbitrary set and $\alpha \in \mathbb{R}^+$. For all $x, y \in X$, define $d_\alpha : X \times X \rightarrow \mathbb{R}$ by $d_\alpha(x, y) = 0$, where $x = y$ and $d_\alpha(x, y) = \alpha$, where $x \neq y$ as an α -discrete metric. So we have the following theorem.

Theorem 3: Let X be an arbitrary set and $|X| \geq 2$. Then there exists a fuzzy set $\rho : X^2 \times \mathbb{R}^{\geq 0} \rightarrow [0, 1]$, such that (X, ρ, T_{min}) is a KM -fuzzy metric space.

Proof 3: Let $|X| \geq 2$ and $\alpha \in \mathbb{R}^+$ be a fixed element. Clearly (X, d_α) is a metric space, now for all $x, y \in X$, $0 \neq m, s, t \in \mathbb{R}^{\geq 0}$, define $\rho : X^2 \times \mathbb{R}^{\geq 0} \rightarrow [0, 1]$ by $\rho(x, y, 0) = 0$ and $\rho(x, y, t > 0) = \frac{\varphi(t)}{\varphi(t) + md_\alpha(x, y)}$, where $\varphi : \mathbb{R}^{\geq 0} \rightarrow \mathbb{R}^{\geq 0}$ is an increasing continuous function and for all $x, y \in X$, we have $\varphi(t) + md_\alpha(x, y) \neq 0$ and $\varphi(t) \rightarrow 0$, whence $t \rightarrow 0$. Now, we show that (X, ρ, T_{min}) is a KM -fuzzy metric space. We prove only the triangular inequality and for all $x, y, z \in X$, consider the five cases $x = y = z$, $x = y \neq z$, $x = z \neq y$, $x \neq y = z$ and $x \neq y \neq z$. In all cases for $0 \in \{t, s\}$ is clear, now for $0 \notin \{t, s\}$ we investigate it. For $x = y \neq z$, since $\varphi(t + s) \geq \varphi(s)$, we have $\varphi(t + s)(\varphi(s) + m\alpha) - \varphi(s)(\varphi(t + s) + m\alpha) \geq 0$ and so $\frac{\varphi(s)}{\varphi(t) + m\alpha} \leq \frac{\varphi(t + s)}{\varphi(t + s) + m\alpha}$. If $x \neq y \neq z$, then $d_\alpha(x, y) = d_\alpha(z, y) = d_\alpha(x, z) = \alpha$. Since φ is an increasing map, we get that $m\alpha\varphi(t) \leq m\alpha\varphi(t + s)$ and it implies that $\varphi(t)(\varphi(t + s) + m\alpha) \leq \varphi(t + s)(\varphi(t) + m\alpha)$ and so $\frac{\varphi(t)}{\varphi(t) + m\alpha} \leq \frac{\varphi(t + s)}{\varphi(t + s) + m\alpha}$, which means that $\rho(x, y, t) \leq \rho(x, z, t + s)$. By a similar way, $\rho(z, y, s) \leq \rho(x, z, t + s)$ and so $T_{min}(\rho(x, y, t), \rho(z, y, s)) \leq \rho(x, z, t + s)$.

The other cases, are proved in a similar way and so (X, ρ, T_{min}) is a KM -fuzzy metric space.

Corollary 1: Let X be an arbitrary set and $|X| \geq 2$. Then there exists a fuzzy set $\rho : X^2 \times \mathbb{R}^{\geq 0} \rightarrow [0, 1]$, such that for all $T \in \mathcal{C}_T$, (X, ρ, T) is a KM -fuzzy metric space.

A. FINITE KM -FUZZY METRIC SPACE BASED ON METRIC

In this subsection, we apply the concept of finite metric for constructing of KM -fuzzy metric space on any given non-empty set.

Definition 5: Let X be a finite set. We say that X is a C-graphable set, if $G = (X, E)$ is a connected graph, where

$E \subseteq X \times X$ and $G = (X, E)$ is called an X -derived graph. Let \mathcal{G}_X be the set of all connected graphs which are constructed on X as the set of vertices, so we have the following results.

Let $G = (X, E)$ be a connected graph. For all $x, y \in X$, define $d^g(x, y) = \min\{|P_{x,y}|\}$ where $P_{x,y}$ is a path between x, y . Obviously, d^g is a metric on X .

Theorem 4: Let X be a finite set and $|X| \geq 2$. Then there exists a non-discrete metric d on X such that (X, d) is a metric space.

Proof 4: Let $|X| \geq 2$. Clearly, X is a C-graphable set and so there exists a graph $G = (X, E) \in \mathcal{G}_X$. For all $x, y \in X$, define $d(x, y) = d^g(x, y)$. Clearly (X, d^g) is a metric space.

Corollary 2: Let $n \in \mathbb{N}$, X be a set and $|X| = n$.

- (i) If $G = (X, E) \cong K_n$ is the complete graph, then for metric spaces (X, d^g) and (X, d_1) , we have $d^g = d_1$.
- (ii) If $G = (X, E) \cong C_n$ is the cycle graph, then for metric spaces (X, d^g) and (X, d_1) , we have $d_1 \leq d^g \leq d \lfloor \frac{n}{2} \rfloor$.

Theorem 5: Let X be a non-empty set. Then there exists a fuzzy subset $\rho : X^2 \times \mathbb{R}^{\geq 0} \rightarrow [0, 1]$, such that (X, ρ, T_{pr}) is a KM -fuzzy metric space.

Proof 5: Let $|X| \geq 2$. Then clearly, X is a C-graphable set and by Theorem 4, (X, d^g) is a metric space. For all $x, y \in X$ and for all $0 \neq m, t \in \mathbb{R}^{\geq 0}$, define $\rho(x, y, 0) = 0$ and $\rho(x, y, t > 0) = \frac{\varphi(t)}{\varphi(t) + md^g(x, y)}$, where $\varphi : \mathbb{R}^{\geq 0} \rightarrow \mathbb{R}^{\geq 0}$ is an increasing continuous function, $\varphi(t) + md^g(x, y) \neq 0$ and $\varphi(t) \rightarrow 0$, whence $t \rightarrow 0$. Now, we show that (X, ρ, T_{pr}) is a KM -fuzzy metric space and in this regard, only prove triangular inequality property. Let $x, y, z \in X$. For $0 \in \{t, s\}$ is clear, now for $0 \notin \{t, s\}$ we investigate it. Since for all $s, t, m \in \mathbb{R}^+$,

$$\begin{aligned} & \varphi(t + s)\varphi(s)md^g(x, y) + \varphi(t + s)\varphi(t)md^g(y, z) \\ & \geq \varphi(t)\varphi(s)md^g(x, y) + \varphi(s)\varphi(t)md^g(y, z) \\ & \geq \varphi(s)\varphi(t)md^g(x, z) + m^2d^g(y, z)d^g(y, z)\varphi(t + s) > 0, \end{aligned}$$

we get that $T_{pr}(\frac{\varphi(t)}{\varphi(t) + md^g(x, y)}, \frac{\varphi(s)}{\varphi(s) + md^g(y, z)}) \leq \frac{\varphi(t + s)}{\varphi(t + s) + md^g(x, z)}$.

It follows that $T_{pr}(\rho(x, y, t), \rho(y, z, s)) \leq \rho(x, z, t + s)$ and so (X, ρ, T_{pr}) is a KM -fuzzy metric space.

Corollary 3: Let X be a non-empty set. Then there exists a fuzzy subset $\rho : X^2 \times \mathbb{R}^{\geq 0} \rightarrow [0, 1]$, such that for all left-continuous t-norm $T \leq T_{pr}$, (X, ρ, T) is a KM -fuzzy metric space.

B. OPERATIONS ON KM -FUZZY METRIC SPACES

In this subsection, we extend KM -fuzzy metric spaces to union and product of KM -fuzzy metric spaces. Let (X_1, ρ_1, T) and (X_2, ρ_2, T) be KM -fuzzy metric spaces, $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$ and $t \in \mathbb{R}^{\geq 0}$. For an arbitrary $T \in \mathcal{C}_T$, define $T(\rho) : (X_1 \times X_2)^2 \times \mathbb{R}^{\geq 0} \rightarrow [0, 1]$ by $T(\rho)((x_1, y_1), (x_2, y_2), t) = T(\rho_1(x_1, x_2, t), \rho_2(y_1, y_2, t))$. So we have the following theorem.

Theorem 6: Let (X_1, ρ_1, T) and (X_2, ρ_2, T) be KM -fuzzy metric spaces. Then $(X_1 \times X_2, T_{\min}(\rho), T)$ is a KM -fuzzy metric space.

Proof 6: Let $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in X_1 \times X_2$ and $t, s \in \mathbb{R}^{\geq 0}$.

(i) Since for all $x_1, x_2 \in X_1, y_1, y_2 \in X_2, \rho_1(x_1, x_2, 0) = 0$ and $\rho_2(y_1, y_2, 0) = 0$, have $T_{\min}(\rho)((x_1, y_1), (x_2, y_2), 0) = 0$.

(ii) $T_{\min}(\rho)((x_1, y_1), (x_2, y_2), t) = 1$ if and only if $T_{\min}(\rho_1(x_1, x_2, t), \rho_2(y_1, y_2, t)) = 1$ if and only if $\rho_1(x_1, x_2, t) = \rho_2(y_1, y_2, t) = 1$ if and only if $(x_1, y_1) = (x_2, y_2)$.

(iii) It is clear that $T_{\min}(\rho)$ is a commutative map.

(iv)

$$\begin{aligned} & T(T_{\min}(\rho)((x_1, y_1), (x_2, y_2), t), T_{\min}(\rho)((x_2, y_2), \\ & (x_3, y_3), s)) = T(T_{\min}(\rho_1(x_1, x_2, t), \rho_2(y_1, y_2, t)), \\ & T_{\min}(\rho_1(x_2, x_3, s), \rho_2(y_2, y_3, s))) \leq T_{\min}(T(\rho_1(x_1, \\ & x_2, t), \rho_1(x_2, x_3, s)), T(\rho_2(y_1, y_2, t), \rho_2(y_2, y_3, s))) \\ & \leq T_{\min}(\rho_1(x_1, x_3, t+s), \rho_2(y_1, y_3, t+s)) \\ & = T_{\min}(\rho)((x_1, y_1), (x_3, y_3), t+s). \end{aligned}$$

(v) Since ρ_1, ρ_2 are left-continuous maps, we get that ρ is a left-continuous map.

(vi) Clearly $\lim_{t \rightarrow \infty} T_{\min}(\rho_1(x_1, x_2, t), \rho_2(y_1, y_2, t)) = T_{\min}(\lim_{t \rightarrow \infty} \rho_1(x_1, x_2, t), \lim_{t \rightarrow \infty} \rho_2(y_1, y_2, t)) = T_{\min}(1, 1) = 1$. Thus $(X_1 \times X_2, T_{\min}(\rho), T)$ is a KM -fuzzy metric space.

is easy to check that (X_1, ρ_1, T_{lu}) and (X_2, ρ_2, T_{lu}) are KM -fuzzy metric spaces and by Theorem 6, $(X_1 \times X_2, T_{\min}(\rho), T_{lu})$ is a KM -fuzzy metric space.

where $\rho_1(x, y, t) = \frac{\min(x, y) + t}{\max(x, y) + t}$ and $\rho_2(x, y, t) = \frac{\min(x, y)}{\max(x, y)}$. Applying Theorem 6, $(\mathbb{R}^{\geq 0} \times \mathbb{N}, \rho, T_{pr})$ is a KM -fuzzy metric space, where $\rho((x_1, y_1), (x_2, y_2), t) = \min\{\frac{\min(x_1, x_2) + t}{\max(x_1, x_2) + t}, \frac{\min(y_1, y_2)}{\max(y_1, y_2)}\}$.

Let $X_1 \cap X_2 = \emptyset$, (X_1, ρ_1, T) and (X_2, ρ_2, T) be KM -fuzzy metric spaces, $x, y \in X_1 \cup X_2$ and $t \in \mathbb{R}^{\geq 0}$. Consider $\epsilon(x, y, t) = \bigwedge_{x, u \in X_1, y, v \in X_2} (\rho_1(x, u, t) \wedge \rho_2(y, v, t))$, define $\rho_1 \cup \rho_2 : (X_1 \cup X_2)^2 \times \mathbb{R}^{\geq 0} \rightarrow [0, 1]$ by

$$(\rho_1 \cup \rho_2)(x, y, t) = \begin{cases} \rho_1(x, y, t) & \text{if } x, y \in X_1, \\ \rho_2(x, y, t) & \text{if } x, y \in X_2, \\ \epsilon(x, y, t) & \text{if } x \in X_1, y \in X_2, \end{cases}.$$

So we have the following theorem.

Theorem 7: Let (X_1, ρ_1, T) and (X_2, ρ_2, T) be KM -fuzzy metric spaces. Then $(X_1 \cup X_2, \rho_1 \cup \rho_2, T)$ is a KM -fuzzy metric space, where $X_1 \cap X_2 = \emptyset$.

Proof 7: Let $x, y, z \in X_1 \cup X_2$ and $t, s \in \mathbb{R}^{\geq 0}$. We only prove the triangular inequality property and other cases are immediate. Let $x, y \in X_1$ (for $x, y \in X_2$, one can prove in a similar way), then $T((\rho_1 \cup \rho_2)(x, y, t), (\rho_1 \cup \rho_2)(y, z, s)) = T(\rho_1(x, y, t), \rho_1(y, z, s))$. If $z \in$

X_1 , then $T((\rho_1 \cup \rho_2)(x, y, t), (\rho_1 \cup \rho_2)(y, z, s)) = T(\rho_1(x, y, t), \rho_1(y, z, s)) \leq \rho_1(x, z, t+s) = (\rho_1 \cup \rho_2)(x, z, t+s)$. If $z \in X_2$, then $T((\rho_1 \cup \rho_2)(x, y, t), (\rho_1 \cup \rho_2)(y, z, s)) = T(\rho_1(x, y, t), \epsilon) \leq \epsilon = (\rho_1 \cup \rho_2)(x, z, t+s)$. Let $x \in X_1, y \in X_2$. Then $T((\rho_1 \cup \rho_2)(x, y, t), (\rho_1 \cup \rho_2)(y, z, s)) = T(\epsilon, (\rho_1 \cup \rho_2)(y, z, s))$. If $z \in X_2$, since $x \in X_1$ and $y \in X_2$, we get that $(\rho_1 \cup \rho_2)(x, z, t+s) = \epsilon$ and so $T(\epsilon, (\rho_1 \cup \rho_2)(y, z, s)) = T(\epsilon, \rho_2(y, z, s)) \leq \epsilon = (\rho_1 \cup \rho_2)(x, z, t+s)$. If $z \in X_1$, since $x \in X_1$ and $y \in X_2$, we get that $(\rho_1 \cup \rho_2)(x, z, t+s) \neq \epsilon$ and so $T(\epsilon, (\rho_1 \cup \rho_2)(y, z, s)) = T(\epsilon, \epsilon) \leq \epsilon \leq (\rho_1 \cup \rho_2)(x, z, t+s) = (\rho_1 \cup \rho_2)(x, z, t+s)$. It follows that $(X_1 \cup X_2, \rho_1 \cup \rho_2, T)$ is a KM -fuzzy metric space.

IV. KM -SINGLE VALUED NEUTROSOPHIC METRIC GRAPH

In this section, we introduce a novel concept as KM -single valued neutrosophic metric graphs and analyse some their properties.

Definition 6: Let (V, ρ, T) be a fuzzy metric space and $G^* = (V, E)$ be a simple graph. Then $G = (X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho, T, S)$ is called a KM -single valued neutrosophic metric graph (a strong KM -single valued neutrosophic metric graph) on G^* , if there exists some time $t \in \mathbb{R}^{\geq 0}$ (for $t = 0$, we call starting time) such that for all $xy \in E$, we have

- (i) the functions $T_V : V \rightarrow [0, 1], I_V : V \rightarrow [0, 1]$ and $F_V : V \rightarrow [0, 1]$ represent the degree of truth-membership, indeterminacy-membership and falsity-membership of the element $x \in V$, respectively. There is no restriction on the sum of $T_V(x), I_V(x)$ and $F_V(x)$, therefore $0 \leq T_V(x) + I_V(x) + F_V(x) \leq 3$ for all $x \in V$.
- (ii) the functions $T_E : E \subseteq V \times V \rightarrow [0, 1], I_E : E \subseteq V \times V \rightarrow [0, 1]$ and $F_E : E \subseteq V \times V \rightarrow [0, 1]$ are defined by $T(T_E(xy), T(T_V(x), T_V(y))) \leq \rho(x, y, t)$ ($T(T_E(xy), T(T_V(x), T_V(y))) = \rho(x, y, t)$), $S(I_E(xy), S(I_V(x), I_V(y))) \geq \rho(x, y, t)$ ($S(I_E(xy), S(I_V(x), I_V(y))) = \rho(x, y, t)$) and $S(F_E(xy), S(F_V(x), F_V(y))) \geq \rho(x, y, t)$ ($S(F_E(xy), S(F_V(x), F_V(y))) = \rho(x, y, t)$), where S is a triangular conorm as a dual of triangular norm T , via a negation η .

We call X as a KM -single valued neutrosophic metric vertex set of G and Y is KM -single valued neutrosophic edge set of G .

In definition of KM -single valued neutrosophic metric graph, if $t \rightarrow \infty$, then for all $x, y \in V, \rho(x, y, t) \rightarrow 1$ and so it follows that $F_E(xy) = S(F_V(x), F_V(y)) = I_E(xy) = S(I_V(x), I_V(y)) = \rho(x, y, t)$ and $T_E(xy), T(T_V(x), T_V(y))$ can be any given fuzzy values. The concept of KM -single valued neutrosophic metric graph is a generalization of KM -fuzzy metric graph, where is introduced by M. Hamidi et.al [9].

Theorem 8: Let (V, ρ, T) be a fuzzy metric space and $G = (X, Y, \rho, T, S)$ be a KM -single valued neutrosophic metric

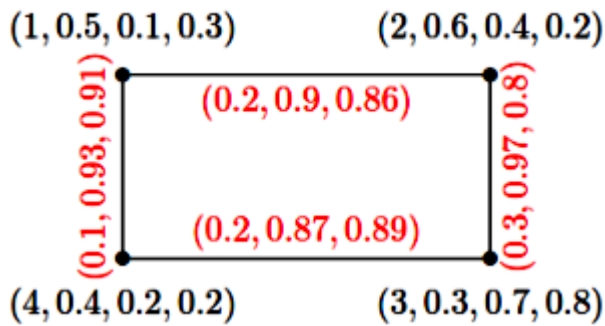


FIGURE 1. KM -single valued neutrosophic metric graph $G = (X, Y, \rho, T_{min}, S_{max})$.

graph on $G^* = (V, E)$. Then for starting time:

- (i) for all $xy \in E$, $T_E(xy) = 0$ or $T_V(x) = 0$ or $T_V(y) = 0$.
- (ii) $|Range(I_E)| = |Range(I_V)| = |Range(F_E)| = |Range(F_V)| = |[0, 1]|$.

Proof 8: (i) Let $xy \in E$. Since $G = (X, Y, \rho, T, S)$ is a KM -single valued neutrosophic metric graph $G^* = (V, E)$, we get that $T(T_E(xy), T(T_V(x), T_V(y))) \leq \rho(x, y, 0)$. Hence $T(T_E(xy), T(T_V(x), T_V(y))) = 0$ and so $T_E(xy) = 0$ or $T_V(x) = 0$ or $T_V(y) = 0$.

(ii) It is immediate by Definition.

Example 1: Let $V = \{1, 2, 3, 4\}$ and $x, y \in X$. Consider a fuzzy subset $\rho(x, y, 0) = 0$ and $\rho(x, y, t) > 0) = \frac{\min\{x, y\} + t}{\max\{x, y\} + t}$. We take the negation $\eta(m) = 1 - m$ ($m \in [0, 1]$) and obtain a KM -single valued neutrosophic metric graph $G = (V, (X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho, T_{min}, S_{max}))$ on the cycle graph C_4 for $t = 1$, in Figure 1.

Let (V, ρ, T) be a fuzzy metric space and $G = (X, Y, \rho, T, S)$ be a KM -single valued neutrosophic metric graph on $G^* = (V, E)$ and $\alpha, \beta, \gamma \in [0, 1]$. Define $T_V^\alpha = \{x \in V \mid T_V(x) \geq \alpha\}$, $I_V^\beta = \{x \in V \mid I_V(x) \leq \beta\}$, $F_V^\gamma = \{x \in V \mid F_V(x) \leq \gamma\}$, $T_E^\alpha = \{xy \in E \mid T_E(x) \geq \alpha\}$, $I_E^\beta = \{xy \in E \mid I_E(x) \leq \beta\}$, $F_E^\gamma = \{xy \in E \mid F_E(x) \leq \gamma\}$, $X^{(\alpha, \beta, \gamma)} = \{x \in V \mid T_V(x) \geq \alpha, I_V(x) \leq \beta, F_V(x) \leq \gamma\}$ and $Y^{(\alpha, \beta, \gamma)} = \{xy \in E \mid T_E(x) \geq \alpha, I_E(x) \leq \beta, F_E(x) \leq \gamma\}$.

Theorem 9: Let (V, ρ, T) be a fuzzy metric space and $G = (X, Y, \rho, T, S)$ be a KM -single valued neutrosophic metric graph on $G^* = (V, E)$ and $\alpha, \beta, \gamma \in [0, 1]$. Then $X^{(\alpha, \beta, \gamma)} = T_V^\alpha \cap I_V^\beta \cap F_V^\gamma$ and $Y^{(\alpha, \beta, \gamma)} = T_E^\alpha \cap I_E^\beta \cap F_E^\gamma$.

Proof 9: Let $x \in X^{(\alpha, \beta, \gamma)}$. Then $T_V(x) \geq \alpha$, $I_V(x) \leq \beta$ and $F_V(x) \leq \gamma$ implies that $x \in T_V^\alpha \cap I_V^\beta \cap F_V^\gamma$ and conversely. In similar a way, one can see that $Y^{(\alpha, \beta, \gamma)} = T_E^\alpha \cap I_E^\beta \cap F_E^\gamma$. Let $G = (X, Y, \rho, T, S)$ be a KM -single valued neutrosophic metric graph on $G^* = (V, E)$. Consider $\alpha_{min} = \bigwedge_{xy \in E} T(T_E(xy), T(T_V(x), T_V(y)))$,

$$\beta_{max} = \bigvee_{xy \in E} S(I_E(xy), S(I_V(x), I_V(y))), \quad \gamma_{max} =$$

$\bigvee_{xy \in E} S(F_E(xy), S(F_V(x), F_V(y)))$. Thus we have the following theorem.

Theorem 10: Let (V, ρ, T) be a fuzzy metric space and $G = (X, Y, \rho, T, S)$ be a KM -single valued neutrosophic metric graph on $G^* = (V, E)$. Then For any $\alpha \leq \alpha_{min}, \beta \geq \beta_{max}, \gamma \geq \gamma_{max}$, $G^{(\alpha, \beta, \gamma)} = (X^{(\alpha, \beta, \gamma)}, Y^{(\alpha, \beta, \gamma)})$ is a subgraph of $G^* = (X, Y)$. parameters of \mathbb{R}^+ .

Proof 10: Let $xy \in E$. Since $T(T_E(xy), T(T_V(x), T_V(y))) \leq T_{min}(T_E(xy), T(T_V(x), T_V(y)))$, we get that $T_E(xy) \geq \alpha_{min} \geq \alpha$. So for any $\alpha \leq \alpha_{min}$, $T_E^\alpha \subseteq E$. Also since $S(I_E(xy), S(I_V(x), I_V(y))) \geq S_{max}(I_E(xy), S(I_V(x), I_V(y)))$, we get that $I_E(xy) \leq \beta_{max} \leq \beta$. So for any $\beta \geq \beta_{max}$, $I_E^\beta \subseteq E$. In a similar way, can see that $F_E^\gamma \subseteq E$. Using Theorem 9, $Y^{(\alpha, \beta, \gamma)} \subseteq E$ and so $G^{(\alpha, \beta, \gamma)} = (X^{(\alpha, \beta, \gamma)}, Y^{(\alpha, \beta, \gamma)})$ is a subgraph of $G^* = (X, Y)$.

Theorem 11: Let (V, ρ, T) be a KM -fuzzy metric space and $G^* = (V, E)$ be a simple graph.

- (i) If $T_E \leq \rho$, $I_E \geq \rho$ and $F_E \geq \rho$ then $G = (X, Y, \rho, T)$ is a KM -single valued neutrosophic metric graph on G^* .
- (ii) If $G = (X, Y, \rho, T_{min}, S_{max})$ is a KM -single valued neutrosophic metric graph on G^* and $T_E > \rho$, $I_E < \rho$ and $F_E < \rho$, then $G = (X, Y)$ is not a single valued neutrosophic graph on G^* .
- (iii) If $G = (X, Y, \rho, T_{min}, S_{max})$ is a strong KM -single valued neutrosophic metric graph on G^* , then $G = (X, Y)$ is a KM -single valued neutrosophic graph on G^* if and only if $\rho(x, y, t) \geq T_E(xy)$, $\rho(x, y, t) \leq I_E(xy)$ and $\rho(x, y, t) \leq F_E(xy)$.

Proof 11: Let $x, y \in V$. Then for some $t \in \mathbb{R}^{\geq 0}$:

(i) Since $T(T_E(xy), T(T_V(x), T_V(y))) \leq T_E(xy)$, $S(I_E(xy), S(I_V(x), I_V(y))) \geq I_E(xy)$ and $S(F_E(xy), S(F_V(x), F_V(y))) \geq F_E(xy)$ then $T_E \leq \rho$, $I_E \geq \rho$ and $F_E \geq \rho$ imply that $T(T_E(xy), T(T_V(x), T_V(y))) \leq \rho(x, y, t)$, $S(I_E(xy), S(I_V(x), I_V(y))) \geq \rho(x, y, t)$ and $S(F_E(xy), S(F_V(x), F_V(y))) \geq \rho(x, y, t)$. So $G = (X, Y, \rho, T)$ is a KM -single valued neutrosophic graph metric graph on G^* .

(ii) Let $G = (X, Y)$ be a single valued neutrosophic graph on G^* . For all $xy \in E$, since $G = (X, Y, \rho, T_{min}, S_{max})$ is a KM -single valued neutrosophic metric graph on G^* , using $T_E(xy) \leq T_{min}(T_V(x), T_V(y))$, $I_E(xy) \geq S_{max}(I_V(x), I_V(y))$ and $F_E(xy) \geq S_{max}(F_V(x), F_V(y))$, we get that $T_E(xy) = T_{min}(T_E(xy), T_{min}(T_V(x), T_V(y))) \leq \rho(x, y, t)$, $I_E(xy) = S_{max}(I_E(xy), S_{max}(I_V(x), I_V(y))) \geq \rho(x, y, t)$ and $F_E(xy) = S_{max}(F_E(xy), S_{max}(F_V(x), F_V(y))) \geq \rho(x, y, t)$ which it is a contradiction.

(iii) $G = (X, Y)$ is a single valued neutrosophic graph on G^* if and only if for all $xy \in E$, $T_E(xy) \leq T_{min}(T_V(x), T_V(y))$, $I_E(xy) \geq S_{max}(I_V(x), I_V(y))$ and $F_E(xy) \geq S_{max}(F_V(x), F_V(y))$. Then $G = (X, Y)$ is a KM -single valued neutrosophic graph on G^* if and only if $T_{min}(T_E(xy)(xy), T_{min}(T_V(x), T_V(y))) =$

$T_E(xy), S_{max}(I_E(xy)(xy), S_{max}(I_V(x), I_V(y))) = I_E(xy)$ and $S_{max}(F_E(xy)(xy), S_{max}(F_V(x), F_V(y))) = F_E(xy)$ if and only if $\rho(x, y, t) \geq T_E(xy), \rho(x, y, t) \leq I_E(xy)$ and $\rho(x, y, t) \leq F_E(xy)$.

Corollary 4: Let $G = (X, Y, \rho, T, S)$ be a KM -fuzzy metric connected graph on $G^* = (V, E)$. Then for starting time $G = (X, Y)$ is not a single valued neutrosophic graph on G^* .

Theorem 12: Let (V, ρ, T) be a KM -fuzzy metric space, $G^* = (V, E)$ be a simple graph and $xy \in E$. Then for $T_V, I_V, F_V : V \rightarrow [0, 1]$ and $T_E, I_E, F_E : E \rightarrow [0, 1]$,

- (i) If $T_V(x) + T_V(y) \leq 1, I_V(x) + I_V(y) = 1$ and $F_V(x) + F_V(y) = 1$ then $G = (X, Y, \rho, T_{lu}, S_{lu})$ is a KM -single valued neutrosophic metric graph on G^* .
- (ii) If $T_E(xy) + 1 \leq T_V(xy) + T_V(x) + T_V(y) \leq 2, I_V(x) + I_V(y) = F_V(x) + F_V(y) = 1$, then $G = (X, Y, \rho, T_{lu}, S_{lu})$ is a KM -single valued neutrosophic metric graph on G^* .

Proof 12: Let $x, y \in V$. Then for some $t \in \mathbb{R}^{\geq 0}$:

(i) W have

$$\begin{aligned} & T_{lu}(T_E(xy), T_{lu}(T_V(x), T_V(y))) \\ &= \max(0, T_E(xy) + T_{lu}(T_V(x), T_V(y)) - 1) \\ &= \max(0, T_E(xy) + \max(0, T_V(x) + T_V(y) - 1) - 1). \end{aligned}$$

If $T_V(x) + T_V(y) \leq 1$, then $T_{lu}(T_E(xy), T_{lu}(T_V(x), T_V(y))) = \max(0, T_E(xy) - 1) = 0$, since for all $x, y \in V$ we have $T_E(xy) \leq 1$. It concludes that for any time $t \in \mathbb{R}^{\geq 0}$ get that $T_{lu}(T_E(xy), T_{lu}(T_V(x), T_V(y))) \leq \rho(x, y, t)$. In addition, $I_V(x) + I_V(y) = 1$, implies that

$$\begin{aligned} & S_{lu}(I_E(xy), S_{lu}(I_V(x), I_V(y))) \\ &= \min(1, I_E(xy) + S_{lu}(I_V(x), I_V(y))) \\ &= \min(1, I_E(xy) + \min(1, I_V(x) + I_V(y))) \\ &= \min(1, I_E(xy) + 1) = 1 \geq \rho(x, y, t). \end{aligned}$$

In a similar way, one can prove that $S_{lu}(F_E(xy), S_{lu}(F_V(x), F_V(y))) \geq \rho(x, y, t)$ and so $G = (X, Y, \rho, T_{lu}, S_{lu})$ is a KM -single valued neutrosophic metric graph on G^* .

(ii) Because $T_E(xy) + 1 \leq T_V(xy) + T_V(x) + T_V(y) \leq 2$, we get that $T_V(x) + T_V(y) \geq 1$ and by item (i), have $T_{lu}(T_E(xy), T_{lu}(T_V(x), T_V(y))) = T_{lu}(0, T_E(xy) + T_V(x) + T_V(y) - 2) = 0$. Moreover, $I_V(x) + I_V(y) = F_V(x) + F_V(y) = 1$, implies that

$$\begin{aligned} & S_{lu}(F_E(xy), S_{lu}(F_V(x), F_V(y))) \\ &= \min(1, F_E(xy) + S_{lu}(F_V(x), F_V(y))) \\ &= \min(1, F_E(xy) + \min(1, F_V(x) + F_V(y))) \\ &= \min(1, F_E(xy) + 1) = 1 \geq \rho(x, y, t). \end{aligned}$$

KM -fuzzy metric graph on G^* . It follows that $G = (X, Y, \rho, T_{lu}, S_{lu})$ is a KM -single valued neutrosophic metric graph on G^* .

A. OPERATIONS ON KM -FUZZY METRIC GRAPHS

In this section, for any given two KM -single valued neutrosophic metric graphs, define some product operations and show that the product of KM -single valued neutrosophic metric graphs is a KM -fuzzy metric graph. From now on, we consider $G_1 = (X_1 = (T_V^{(1)}, I_V^{(1)}, F_V^{(1)}), Y_1 = (T_E^{(1)}, I_E^{(1)}, F_E^{(1)}), \rho_1, T, S)$, $G_2 = (X_2 = (T_V^{(2)}, I_V^{(2)}, F_V^{(2)}), Y_2 = (T_E^{(2)}, I_E^{(2)}, F_E^{(2)}), \rho_2, T, S)$ as KM -single valued neutrosophic metric graphs on simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively.

Definition 7: Let G_1, G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. Define the categorical product (tensor product) of fuzzy subsets $X_1 \times X_2 = (T_V^{(1)} \times T_V^{(2)}, I_V^{(1)} \times I_V^{(2)}, F_V^{(1)} \times F_V^{(2)})$, $Y_1 \times Y_2 = (T_E^{(1)} \times T_E^{(2)}, I_E^{(1)} \times I_E^{(2)}, F_E^{(1)} \times F_E^{(2)})$, where $T_V^{(1)} \times T_V^{(2)}, I_V^{(1)} \times I_V^{(2)}, F_V^{(1)} \times F_V^{(2)} : V(G_1^* \times G_2^*) \rightarrow [0, 1]$ by

$$\begin{aligned} (T_V^{(1)} \times T_V^{(2)})(x_1, x_2) &= T_{\min}(T_V^{(1)}(x_1), T_V^{(2)}(x_2)), \\ (I_V^{(1)} \times I_V^{(2)})(x_1, x_2) &= S_{\max}(I_V^{(1)}(x_1), I_V^{(2)}(x_2)), \\ (F_V^{(1)} \times F_V^{(2)})(x_1, x_2) &= S_{\max}(F_V^{(1)}(x_1), F_V^{(2)}(x_2)), \end{aligned}$$

and $T_E^{(1)} \times T_E^{(2)}, I_E^{(1)} \times I_E^{(2)}, F_E^{(1)} \times F_E^{(2)} : E(G_1^* \times G_2^*) \rightarrow [0, 1]$ by

$$\begin{aligned} (T_E^{(1)} \times T_E^{(2)})((x_1, x_2)(y_1, y_2)) &= T_{\min}(T_E^{(1)}(x_1y_1), \\ & T_E^{(2)}(x_2y_2)), (I_E^{(1)} \times I_E^{(2)})((x_1, x_2)(y_1, y_2)) = S_{\max}(\\ & I_E^{(1)}(x_1y_1), I_E^{(2)}(x_2y_2)), (F_E^{(1)} \times F_E^{(2)})((x_1, x_2)(y_1, y_2)) \\ &= S_{\max}(F_E^{(1)}(x_1y_1), F_E^{(2)}(x_2y_2)). \end{aligned}$$

Theorem 13: Let G_1 and G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. Then $G_1 \times G_2 = (X_1 \times X_2, Y_1 \times Y_2, T_{\min}(\rho), T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* \times G_2^*$.

Proof 13: Firstly, by Theorem 6, $(V_1 \times V_2, T_{\min}(\rho), T)$ is a KM -fuzzy metric space. Let $(x_1, x_2)(y_1, y_2) \in E(G_1^* \times G_2^*)$. Since G_1 is a KM -single valued neutrosophic metric graph on G_1^* and G_2 is a KM -single valued neutrosophic metric graph on G_2^* , for some $t_1, t_2 \in \mathbb{R}^{\geq 0}$, we get that

$$\begin{aligned} & T((T_E^{(1)} \times T_E^{(2)})((x_1, x_2)(y_1, y_2)), T((T_V^{(1)} \times T_V^{(2)})(x_1, x_2), (T_V^{(1)} \times T_V^{(2)})(y_1, y_2))) = T(T_{\min}(T_E^{(1)}(x_1y_1), \\ & T_E^{(2)}(x_2y_2)), T((T_{\min}(T_V^{(1)}(x_1), T_V^{(1)}(x_2)), (T_{\min}(T_V^{(1)}(y_1), T_V^{(1)}(y_2)))))) \leq T(T_E^{(1)}(x_1y_1), T(T_V^{(1)}(x_1), \\ & T_V^{(1)}(y_1))) \leq \rho_1(x_1, y_1, t_1) \text{ and} \\ & T((T_E^{(1)} \times T_E^{(2)})((x_1, x_2)(y_1, y_2)), T((T_V^{(1)} \times T_V^{(2)})(x_1, x_2), (T_V^{(1)} \times T_V^{(2)})(y_1, y_2))) = T(T_{\min}(T_E^{(1)}(x_1y_1), \\ & T_E^{(2)}(x_2y_2)), T((T_{\min}(T_V^{(1)}(x_1), T_V^{(1)}(x_2)), (T_{\min}(T_V^{(1)}(y_1), T_V^{(1)}(y_2)))))) \leq T(T_E^{(2)}(x_2y_2), T(T_V^{(2)}(x_2), \\ & T_V^{(2)}(y_2))) \leq \rho_2(x_2, y_2, t_2). \end{aligned}$$

Consider $t = \max\{t_1, t_2\}$, so by Theorem 1, we obtain

$$\begin{aligned} & T((T_E^{(1)} \times T_E^{(2)}((x_1, x_2)(y_1, y_2)), T((T_V^{(1)} \times T_V^{(2)} \\ & (x_1, x_2), (T_V^{(1)} \times T_V^{(2)})(y_1, y_2)) \leq T_{\min}(\rho_1(x_1, y_1, t_1), \\ & \rho_2(x_2, y_2, t_2)) \leq T_{\min}(\rho)((x_1, x_2), (y_1, y_2), t). \end{aligned}$$

In addition,

$$\begin{aligned} & S((I_E^{(1)} \times I_E^{(2)}((x_1, x_2)(y_1, y_2)), S((I_V^{(1)} \times I_V^{(2)} \\ & (x_1, x_2), (I_V^{(1)} \times I_V^{(2)})(y_1, y_2)) \\ & = S(S_{\max}(I_E^{(1)}(x_1y_1), I_E^{(2)}(x_2y_2)), S((S_{\max}(I_V^{(1)}(x_1), \\ & I_V^{(1)}(x_2)), (S_{\max}(I_V^{(1)}(y_1), I_V^{(2)}(y_2)))) \\ & \geq S(I_E^{(1)}(x_1y_1), S(I_V^{(1)}(x_1), I_V^{(1)}(y_1))) \geq \rho_1(x_1, y_1, t_1) \\ & \text{and } S((I_E^{(1)} \times I_E^{(2)}((x_1, x_2)(y_1, y_2)), S((I_V^{(1)} \times I_V^{(2)} \\ & (x_1, x_2), (I_V^{(1)} \times I_V^{(2)})(y_1, y_2)) = S(S_{\max}(I_E^{(1)}(x_1y_1), \\ & I_E^{(2)}(x_2y_2)), S((S_{\max}(I_V^{(1)}(x_1), I_V^{(2)}(x_2)), (S_{\max}(I_V^{(1)} \\ & (y_1), I_V^{(2)}(y_2)))) \geq S(I_E^{(2)}(x_2y_2), S(I_V^{(2)}(x_2), I_V^{(2)}(y_2) \\ &)) \geq \rho_2(x_2, y_2, t_2). \end{aligned}$$

Consider $t = \min\{t_1, t_2\}$, so by Theorem 1, we obtain

$$\begin{aligned} & S((I_E^{(1)} \times I_E^{(2)}((x_1, x_2)(y_1, y_2)), S((I_V^{(1)} \times I_V^{(2)} \\ & (x_1, x_2), (I_V^{(1)} \times I_V^{(2)})(y_1, y_2)) \\ & \geq S_{\max}(\rho_1(x_1, y_1, t_1), \rho_2(x_2, y_2, t_2)) \geq S_{\max}(\rho)((x_1, \\ & x_2), (y_1, y_2), t). \end{aligned}$$

In a similar way, can see that $S((F_E^{(1)} \times F_E^{(2)}((x_1, x_2)(y_1, y_2)), S((F_V^{(1)} \times F_V^{(2)}(x_1, x_2), (F_V^{(1)} \times F_V^{(2)})(y_1, y_2)) \geq S_{\max}(\rho)((x_1, x_2), (y_1, y_2), t)$. Thus $G_1 \times G_2 = (X_1 \times X_2, Y_1 \times Y_2, T_{\min}(\rho), T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* \times G_2^*$.

Definition 8: Let G_1, G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. Define the Cartesian product (or product) fuzzy subsets $X_1 \otimes X_2 = (T_V^{(1)} \otimes T_V^{(2)}, I_V^{(1)} \otimes I_V^{(2)}, F_V^{(1)} \otimes F_V^{(2)})$, $Y_1 \otimes Y_2 = (T_E^{(1)} \otimes T_E^{(2)}, I_E^{(1)} \otimes I_E^{(2)}, F_E^{(1)} \otimes F_E^{(2)})$, where $T_V^{(1)} \otimes T_V^{(2)}, I_V^{(1)} \otimes I_V^{(2)}, F_V^{(1)} \otimes F_V^{(2)} : V(G_1^* \times G_2^*) \rightarrow [0, 1]$ by $(T_V^{(1)} \otimes T_V^{(2)})(x_1, x_2) = T_{\min}(T_V^{(1)}(x_1), T_V^{(2)}(x_2))$, $(I_V^{(1)} \otimes I_V^{(2)})(x_1, x_2) = S_{\max}(I_V^{(1)}(x_1), I_V^{(2)}(x_2))$, $(F_V^{(1)} \otimes F_V^{(2)})(x_1, x_2) = S_{\max}(F_V^{(1)}(x_1), F_V^{(2)}(x_2))$, and $T_E^{(1)} \otimes T_E^{(2)}, I_E^{(1)} \otimes I_E^{(2)}, F_E^{(1)} \otimes F_E^{(2)} : E(G_1^* \times G_2^*) \rightarrow [0, 1]$ by $(T_E^{(1)} \otimes T_E^{(2)})((x, x_2)(x, y_2)) = T_{\min}(T_E^{(1)}(x), T_E^{(2)}(x_2y_2))$, $(T_E^{(1)} \otimes T_E^{(2)})((x_1, y)(y_1, y)) = T_{\min}(T_E^{(1)}(y), T_E^{(2)}(x_1y_1))$, $(I_E^{(1)} \otimes I_E^{(2)})((x, x_2)(x, y_2)) = S_{\max}(I_E^{(1)}(x), I_E^{(2)}(x_2y_2))$, $(I_E^{(1)} \otimes I_E^{(2)})((x_1, y)(y_1, y)) = S_{\max}(I_E^{(1)}(y), I_E^{(2)}(x_1y_1))$, $(F_E^{(1)} \otimes F_E^{(2)})((x, x_2)(x, y_2)) = S_{\max}(F_E^{(1)}(x), F_E^{(2)}(x_2y_2))$, $(F_E^{(1)} \otimes F_E^{(2)})((x_1, y)(y_1, y)) = S_{\max}(F_E^{(1)}(y), F_E^{(2)}(x_1y_1))$.

Example 2: Consider the KM -fuzzy metric spaces $(V_1 = \{1, 2\}, \rho_1, T_{\min})$, $(V_2 = \{3, 4, 5\}, \rho_2, T_{\min})$, where $\rho_1(1, 1, t > 0) = 1, \rho_1(2, 2, t > 0) = 1, \rho_1(1, 2, t > 0) = \frac{1}{1+t}$, $\rho_1(x, y, 0) = 0, x, y \in V_1$ and for all $x, y \in \{3, 4, 5\}$,

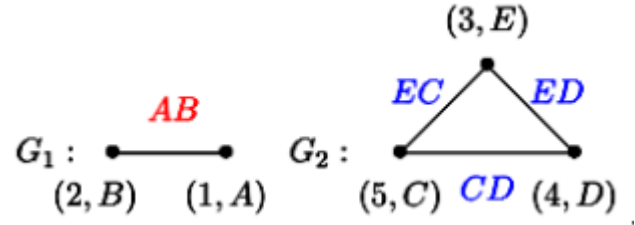


FIGURE 2. KM -single valued neutrosophic metric graphs G_1 and G_2 for $t = 1$.

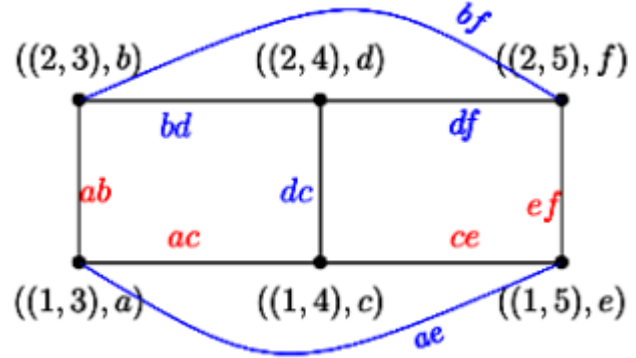


FIGURE 3. KM -single valued neutrosophic metric graph $G_1 \otimes G_1$ for $t = 1$.

$$\rho_2(x, y, t) = \begin{cases} \frac{\min\{x, y\} + t}{\max\{x, y\} + t} & \text{if } t > 0 \\ 0 & \text{if } t = 0 \end{cases}. \text{ We take the}$$

negation $\eta(m) = 1 - m$ ($m \in [0, 1]$) and obtain the KM -single valued neutrosophic metric graphs $G_1 = (V_1, (X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho_1, T_{\min}, S_{\max}))$ in unit time $t_1 = 1$ and $G_2 = (V_2, (X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho_2, T_{\min}, S_{\max}))$ in unit time $t_2 = 1$ on G_1^* and G_2^* in Figure 2, where $A = (0.6, 0.4, 0.2)$, $B = (0.5, 0.1, 0.3)$, $C = (0.3, 0.5, 0.7)$, $D = (0.5, 0.6, 0.2)$, $E = (0.1, 0.2, 0.5)$, $AB = (0.5, 0.97, 0.95)$, $CE = (0.1, 0.96, 0.91)$, $ED = (0.5, 0.98, 0.99)$ and $DC = (0.3, 0.93, 0.96)$. Now, we obtain the KM -fuzzy metric graph $G_1 \times G_2$ in Figure 3, where $a = (0.1, 0.4, 0.5)$, $b = (0.1, 0.4, 0.5)$, $c = (0.5, 0.6, 0.2)$, $d = (0.5, 0.6, 0.3)$, $e = (0.3, 0.5, 0.7)$, $f = (0.3, 0.5, 0.7)$, $ab = (0.1, 0.97, 0.95)$, $dc = (0.5, 0.97, 0.95)$, $ef = (0.3, 0.97, 0.95)$, $bd = (0.5, 0.98, 0.99)$, $df = (0.3, 0.93, 0.96)$, $ac = (0.5, 0.98, 0.99)$, $ce = (0.3, 0.93, 0.96)$, $bf = (0.1, 0.96, 0.961)$ and $ae = (0.1, 0.96, 0.91)$.

Theorem 14: Let G_1 and G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively.

- (i) If $(G_1^* \otimes G_2^*, T(\rho), T)$ is a KM -fuzzy metric space, then $T(\rho) = \rho_1$ or $T(\rho) = \rho_2$, where $T \in \mathcal{C}_T$.
- (ii) $G_1 \otimes G_2 = (X_1 \otimes X_2, Y_1 \otimes Y_2, T_{\min}(\rho), T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* \otimes G_2^*$.

Proof 14: (i) Let $(x_1, x_2), (y_1, y_2) \in E(G_1^* \otimes G_2^*)$. Then $x_1 = y_1$ and $x_2y_2 \in E(G_2^*)$ or $x_2 = y_2$ and $x_1y_1 \in E(G_1^*)$. If $x_1 = y_1$ and

$x_2y_2 \in E(G_2^*)$, then $T(\rho)((x_1, x_2), (x_1, y_2), t) = T(\rho_1(x_1, x_1, t), \rho_2(x_2, y_2, t)) = T(1, \rho_2(x_2, y_2, t)) = \rho_2(x_2, y_2, t)$. If $x_2 = y_2$ and $x_1y_1 \in E(G_1^*)$, then $T(\rho)((x_1, x_2), (y_1, x_2), t) = T(\rho_1(x_1, y_1, t), \rho_2(x_2, x_2, t)) = T(\rho_1(x_1, y_1, t), 1) = \rho_1(x_1, y_1, t)$.

(ii) Firstly, by Theorem 6, $(V_1 \times V_2, T_{\min}(\rho), T)$ is a KM -fuzzy metric space. Let $(x_1, x_2), (y_1, y_2) \in E(G_1^* \otimes G_2^*)$. Since G_1 is a KM -single valued neutrosophic metric graph on G_1^* and G_2 is a KM -single valued neutrosophic metric graph on G_2^* , for some $t_1, t_2 \in \mathbb{R}^{\geq 0}$, give $t = \max\{t_1, t_2\}$, so by item (i) and Theorem 1, we get that

$$\begin{aligned} & T((T_E^{(1)} \otimes T_E^{(2)})((x, x_2)(x, y_2)), T((T_V^{(1)} \otimes T_V^{(2)})(x, x_2), (T_V^{(1)} \otimes T_V^{(2)})(x, y_2))) \\ &= T(T_{\min}(T_V^{(1)}(x), T_E^{(2)}(x_2y_2)), T(T_{\min}(T_V^{(1)}(x), T_V^{(2)}(x_2)), T_{\min}(T_V^{(1)}(x), T_V^{(2)}(y_2)))) \\ &\leq T(T_E^{(2)}(x_2y_2), T(T_V^{(2)}(x), T_V^{(2)}(y_2))) \leq \rho_2(x_2, y_2, t_2) \\ &\leq T_{\min}(\rho)((x, x_2)(x, y_2), t) \text{ and} \\ & T((T_E^{(1)} \otimes T_E^{(2)})((x_1, y)(y_1, y)), T((T_V^{(1)} \otimes T_V^{(2)})(x_1, y), (T_V^{(1)} \otimes T_V^{(2)})(y_1, y))) \\ &= T(T_{\min}(T_V^{(2)}(y), T_E^{(1)}(x_1y_1)), T(T_{\min}(T_V^{(1)}(x_1), T_V^{(2)}(y)), T_{\min}(T_V^{(1)}(y_1), T_V^{(2)}(y)))) \\ &\leq T(T_E^{(1)}(x_1y_1), T(T_V^{(1)}(x_1), T_V^{(1)}(y_1))) \leq \rho_1(x_1, y_1, t_1) \\ &\leq T_{\min}(\rho)((x_1, y)(y_1, y), t) \end{aligned}$$

Now, give $t = \min\{t_1, t_2\}$, so by item (i) and Theorem 1, we get that

$$\begin{aligned} & S((I_E^{(1)} \otimes I_E^{(2)})((x, x_2)(x, y_2)), S((I_V^{(1)} \otimes I_V^{(2)})(x, x_2), (I_V^{(1)} \otimes I_V^{(2)})(x, y_2))) \\ &= S(S_{\max}(I_V^{(1)}(x), I_E^{(2)}(x_2y_2)), S(S_{\max}(I_V^{(1)}(x), I_V^{(2)}(x_2)), S_{\max}(I_V^{(1)}(x), I_V^{(2)}(y_2)))) \\ &\geq S(I_E^{(2)}(x_2y_2), S(I_V^{(2)}(x), I_V^{(2)}(y_2))) \geq \rho_2(x_2, y_2, t_2) \\ &\geq S_{\max}(\rho)((x, x_2)(x, y_2), t) \text{ and} \\ & S((I_E^{(1)} \otimes I_E^{(2)})((x_1, y)(y_1, y)), S((T_V^{(1)} \otimes I_V^{(2)})(x_1, y), (T_V^{(1)} \otimes I_V^{(2)})(y_1, y))) \\ &= S(S_{\max}(I_V^{(2)}(y), I_E^{(1)}(x_1y_1)), T(S_{\max}(I_V^{(1)}(x_1), I_V^{(2)}(y)), S_{\max}(I_V^{(1)}(y_1), I_V^{(2)}(y)))) \\ &\geq S(I_E^{(1)}(x_1y_1), S(I_V^{(1)}(x_1), I_V^{(1)}(y_1))) \geq \rho_1(x_1, y_1, t_1) \\ &\geq S_{\max}(\rho)((x_1, y)(y_1, y), t). \end{aligned}$$

In a similar way, can see that $S((F_E^{(1)} \otimes F_E^{(2)})((x_1, x_2)(y_1, y_2)), S((F_V^{(1)} \otimes F_V^{(2)})(x_1, x_2), (F_V^{(1)} \otimes F_V^{(2)})(y_1, y_2))) \geq S_{\max}(\rho)((x_1, x_2), (y_1, y_2), t)$. Thus $G_1 \otimes G_2 = (X_1 \otimes X_2, Y_1 \otimes Y_2, T_{\min}(\rho), T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* \otimes G_2^*$.

Definition 9: Let G_1, G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively.

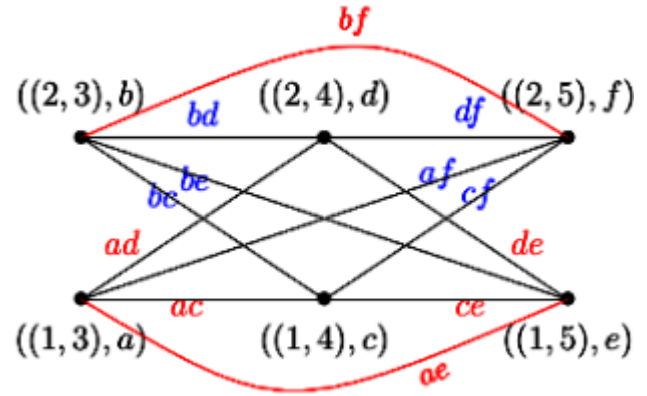


FIGURE 4. KM -single valued neutrosophic metric graph $G_1 \bullet G_2$ for $t = 1$.

Define the semi-strong product of fuzzy subsets $X_1 \bullet X_2 = (T_V^{(1)} \bullet T_V^{(2)}, I_V^{(1)} \bullet I_V^{(2)}, F_V^{(1)} \bullet F_V^{(2)}), Y_1 \bullet Y_2 = (T_E^{(1)} \bullet T_E^{(2)}, I_E^{(1)} \bullet I_E^{(2)}, F_E^{(1)} \bullet F_E^{(2)})$, where $T_V^{(1)} \bullet T_V^{(2)}, I_V^{(1)} \bullet I_V^{(2)}, F_V^{(1)} \bullet F_V^{(2)} : V(G_1^* \times G_2^*) \rightarrow [0, 1]$ by $(T_V^{(1)} \bullet T_V^{(2)})(x_1, x_2) = T_{\min}(T_V^{(1)}(x_1), T_V^{(2)}(x_2)), (I_V^{(1)} \bullet I_V^{(2)})(x_1, x_2) = S_{\max}(I_V^{(1)}(x_1), I_V^{(2)}(x_2)), (F_V^{(1)} \bullet F_V^{(2)})(x_1, x_2) = S_{\max}(F_V^{(1)}(x_1), F_V^{(2)}(x_2))$, and $T_E^{(1)} \bullet T_E^{(2)}, I_E^{(1)} \bullet I_E^{(2)}, F_E^{(1)} \bullet F_E^{(2)} : E(G_1^* \times G_2^*) \rightarrow [0, 1]$ by $(T_E^{(1)} \bullet T_E^{(2)})((x, x_2)(x, y_2)) = T_{\min}(T_E^{(1)}(x), T_E^{(2)}(x_2y_2)), (T_E^{(1)} \bullet T_E^{(2)})((x_1, x_2)(y_1, y_2)) = T_{\min}(T_E^{(1)}(x_1y_1), T_E^{(2)}(x_2y_2)), (I_E^{(1)} \bullet I_E^{(2)})((x, x_2)(x, y_2)) = S_{\max}(I_E^{(1)}(x), I_E^{(2)}(x_2y_2)), (I_E^{(1)} \bullet I_E^{(2)})(x_1, x_2)(y_1, y_2) = S_{\max}(I_E^{(1)}(x_1y_1), I_E^{(2)}(x_2y_2)), (F_E^{(1)} \bullet F_E^{(2)})((x, x_2)(x, y_2)) = S_{\max}(F_E^{(1)}(x), F_E^{(2)}(x_2y_2)), (F_E^{(1)} \bullet F_E^{(2)})(x_1, x_2)(y_1, y_2) = S_{\max}(F_E^{(1)}(x_1y_1), F_E^{(2)}(x_2y_2))$.

Example 3: Consider the KM -single valued neutrosophic metric graphs G_1 and G_2 in Example 2. So we obtain the KM -fuzzy metric graph $G_1 \bullet G_2$ in Figure 4, where $a = (0.1, 0.4, 0.5), b = (0.1, 0.4, 0.5), c = (0.5, 0.6, 0.2), d = (0.5, 0.6, 0.3), e = (0.3, 0.5, 0.7), f = (0.3, 0.5, 0.7), be = (0.1, 0.97, 0.95), bc = (0.5, 0.98, 0.99), af = (0.1, 0.97, 0.95), bd = (0.5, 0.98, 0.99), df = (0.3, 0.93, 0.96), ac = (0.5, 0.98, 0.99), ce = (0.3, 0.93, 0.96), bf = (0.1, 0.96, 0.961), cf = (0.3, 0.97, 0.96), de = (0.3, 0.97, 0.96), ad = (0.5, 0.98, 0.99) and $ae = (0.1, 0.96, 0.91)$.$

Theorem 15: Let G_1 and G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. Then $G_1 \bullet G_2 = (X_1 \bullet X_2, Y_1 \bullet Y_2, T_{\min}(\rho), T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* \bullet G_2^*$.

Proof 15: It is similar to Theorems 13 and 14.

Definition 10: Let G_1, G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. Define the strong product of fuzzy subsets $X_1 \odot X_2 = (T_V^{(1)} \odot T_V^{(2)}, I_V^{(1)} \odot I_V^{(2)}, F_V^{(1)} \odot F_V^{(2)}), Y_1 \odot Y_2 = (T_E^{(1)} \odot T_E^{(2)}, I_E^{(1)} \odot I_E^{(2)}, F_E^{(1)} \odot F_E^{(2)})$, where

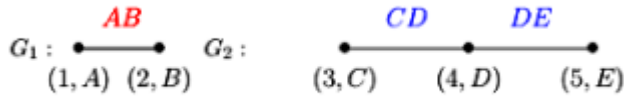


FIGURE 5. KM -single valued neutrosophic metric graphs G_1, G_2 for $t_1 = 2, t_2 = 1$.

$T_V^{(1)} \odot T_V^{(2)}, I_V^{(1)} \odot I_V^{(2)}, F_V^{(1)} \odot F_V^{(2)} : V(G_1^* \times G_2^*) \rightarrow [0, 1]$ by $(T_V^{(1)} \odot T_V^{(2)})(x_1, x_2) = T_{\min}(T_V^{(1)}(x_1), T_V^{(2)}(x_2)), (I_V^{(1)} \odot I_V^{(2)})(x_1, x_2) = S_{\max}(I_V^{(1)}(x_1), I_V^{(2)}(x_2)), (F_V^{(1)} \odot F_V^{(2)})(x_1, x_2) = S_{\max}(F_V^{(1)}(x_1), F_V^{(2)}(x_2))$, and $T_E^{(1)} \odot T_E^{(2)}, I_E^{(1)} \odot I_E^{(2)}, F_E^{(1)} \odot F_E^{(2)} : E(G_1^* \times G_2^*) \rightarrow [0, 1]$ by $(T_E^{(1)} \odot T_E^{(2)})((x, x_2)(x, y_2)) = T_{\min}(T_E^{(1)}(x), T_E^{(2)}(x_2y_2)), (T_E^{(1)} \odot T_E^{(2)})((x_1, y)(x_2, y)) = T_{\min}(T_E^{(1)}(y), T_E^{(2)}(x_1y_1)), (T_E^{(1)} \odot T_E^{(2)})((x_1, x_2)(y_1, y_2)) = T_{\min}(T_E^{(1)}(x_1y_1), T_E^{(2)}(x_2y_2)), (I_E^{(1)} \odot I_E^{(2)})((x, x_2)(x, y_2)) = S_{\max}(I_V^{(1)}(x), I_E^{(2)}(x_2y_2)), (I_E^{(1)} \odot I_E^{(2)})((x_1, y)(x_2, y)) = T_{\min}(I_V^{(2)}(y), I_E^{(1)}(x_1y_1)), (I_E^{(1)} \odot I_E^{(2)})(x_1, x_2)(y_1, y_2) = S_{\max}(I_E^{(1)}(x_1y_1), I_E^{(2)}(x_2y_2)), (F_E^{(1)} \odot F_E^{(2)})((x, x_2)(x, y_2)) = S_{\max}(F_V^{(1)}(x), F_E^{(2)}(x_2y_2)), (F_E^{(1)} \odot F_E^{(2)})((x_1, y)(x_2, y)) = T_{\min}(F_V^{(2)}(y), F_E^{(1)}(x_1y_1)), (F_E^{(1)} \odot F_E^{(2)})((x_1, x_2)(y_1, y_2)) = S_{\max}(F_E^{(1)}(x_1y_1), F_E^{(2)}(x_2y_2)).$

Example 4: Consider the KM -fuzzy metric spaces $(V_1 = \{1, 2\}, \rho_1, T_{\min})$, $(V_2 = \{3, 4, 5\}, \rho_2, T_{\min})$, where for all $x, y \in \{1, 2\}$, $\rho_1(x, y, 0) = 0, \rho_1(x, y, t > 0) = \frac{\min\{x, y\} + t}{\max\{x, y\} + t}$ and for all $x, y \in \{3, 4, 5\}$, $\rho_2(x, y, 0) = \frac{\min\{x, y\} + t}{\max\{x, y\} + t}$ and for all $x, y \in \{3, 4, 5\}$, $\rho_2(x, y, 0) = \frac{\min\{x, y\} + t}{\max\{x, y\} + t}$.

$0, \rho_2(x, y, t > 0) = \begin{cases} 1 & \text{if } x = y \\ \frac{5+t}{10+t} & \text{if } x \neq y \end{cases}$. We take the

negation $\eta(m) = 1 - m(m \in [0, 1])$ and obtain the KM -single valued neutrosophic metric graphs $G_1 = (V_1, (X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho_1, T_{\min}, S_{\max}))$ in unit time $t_1 = 2$ and $G_2 = (V_2, (X = (T_V, I_V, F_V), Y = (T_E, I_E, F_E), \rho_2, T_{\min}, S_{\max}))$ in unit time $t_2 = 1$ on G_1^* and G_2^* in Figure 5, where $A = (0.1, 0.5, 0.4), B = (0.2, 0.3, 0.3), C = (0.3, 0.4, 0.5), D = (0.4, 0.6, 0.5), E = (0.5, 0.2, 0.1), AB = (0.5, 0.97, 0.95), DE = (0.5, 0.98, 0.99)$ and $DC = (0.3, 0.93, 0.96)$. Now, we obtain the KM -single valued neutrosophic metric graph $G_1 \odot G_2$ in Figure 6, where $a = (0.1, 0.5, 0.5), b = (0.1, 0.6, 0.5), c = (0.1, 0.5, 0.4), d = (0.2, 0.4, 0.5), e = (0.2, 0.6, 0.5), f = (0.2, 0.3, 0.3), ab = (0.1, 0.93, 0.96), bc = (0.1, 0.98, 0.99), de = (0.2, 0.93, 0.96), ef = (0.2, 0.98, 0.99), ad = (0.3, 0.97, 0.95), be = (0.4, 0.97, 0.95), cf = (0.5, 0.97, 0.95), ae = (0.3, 0.97, 0.96), bf = (0.5, 0.98, 0.99), bd = (0.3, 0.97, 0.96), ce = (0.5, 0.98, 0.99).$

Theorem 16: Let G_1 and G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. Then $G_1 \odot G_2 = (X_1 \odot X_2, Y_1 \odot Y_2, T_{\min}(\rho), T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* \odot G_2^*$.

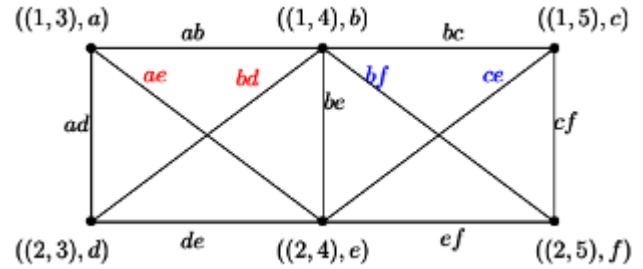


FIGURE 6. KM -single valued neutrosophic $G_1 \odot G_2$ for $t = 2$.

Proof 16: It is similar to Theorems 13 and 14.

Definition 11: Let G_1, G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. Define the union of fuzzy subsets $X_1 \cup X_2 = (T_V^{(1)} \cup T_V^{(2)}, I_V^{(1)} \cup I_V^{(2)}, F_V^{(1)} \cup F_V^{(2)}), Y_1 \cup Y_2 = (T_E^{(1)} \cup T_E^{(2)}, I_E^{(1)} \cup I_E^{(2)}, F_E^{(1)} \cup F_E^{(2)})$, where $T_V^{(1)} \cup T_V^{(2)}, I_V^{(1)} \cup I_V^{(2)}, F_V^{(1)} \cup F_V^{(2)} : (V_1 \cup V_2) \rightarrow [0, 1]$ by $(T_V^{(1)} \cup T_V^{(2)})(x_1, x_2) = \begin{cases} T_V^{(1)}(x) & \text{if } x \in V_1 \setminus V_2 \\ T_V^{(2)}(x) & \text{if } x \in V_2 \setminus V_1 \\ T_{\min}(T_V^{(1)}(x), T_V^{(2)}(x)) & \text{if } x \in V_2 \cap V_1 \end{cases}$, $(I_V^{(1)} \cup I_V^{(2)})(x_1, x_2) = \begin{cases} I_V^{(1)}(x) & \text{if } x \in V_1 \setminus V_2 \\ I_V^{(2)}(x) & \text{if } x \in V_2 \setminus V_1 \\ S_{\max}(I_V^{(1)}(x), I_V^{(2)}(x)) & \text{if } x \in V_2 \cap V_1 \end{cases}$, $(F_V^{(1)} \cup F_V^{(2)})(x_1, x_2) = \begin{cases} F_V^{(1)}(x) & \text{if } x \in V_1 \setminus V_2 \\ F_V^{(2)}(x) & \text{if } x \in V_2 \setminus V_1 \\ S_{\max}(F_V^{(1)}(x), F_V^{(2)}(x)) & \text{if } x \in V_2 \cap V_1 \end{cases}$ and $T_E^{(1)} \cup T_E^{(2)}, I_E^{(1)} \cup I_E^{(2)}, F_E^{(1)} \cup F_E^{(2)} : (E_1 \cup E_2) \rightarrow [0, 1]$, by $(T_E^{(1)} \cup T_E^{(2)})(xy) = \begin{cases} T_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ T_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ T_{\min}(T_E^{(1)}(xy), T_E^{(2)}(xy)) & \text{if } xy \in E_2 \cap E_1 \end{cases}$, $(I_E^{(1)} \cup I_E^{(2)})(xy) = \begin{cases} I_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ I_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ S_{\max}(I_E^{(1)}(xy), I_E^{(2)}(xy)) & \text{if } xy \in E_2 \cap E_1 \end{cases}$, $(F_E^{(1)} \cup F_E^{(2)})(xy) = \begin{cases} F_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ F_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ S_{\max}(F_E^{(1)}(xy), F_E^{(2)}(xy)) & \text{if } xy \in E_2 \cap E_1 \end{cases}$.

$(T_V^{(1)} \cup T_V^{(2)})(x_1, x_2) = \begin{cases} T_V^{(1)}(x) & \text{if } x \in V_1 \setminus V_2 \\ T_V^{(2)}(x) & \text{if } x \in V_2 \setminus V_1 \\ T_{\min}(T_V^{(1)}(x), T_V^{(2)}(x)) & \text{if } x \in V_2 \cap V_1 \end{cases}$, $(I_V^{(1)} \cup I_V^{(2)})(x_1, x_2) = \begin{cases} I_V^{(1)}(x) & \text{if } x \in V_1 \setminus V_2 \\ I_V^{(2)}(x) & \text{if } x \in V_2 \setminus V_1 \\ S_{\max}(I_V^{(1)}(x), I_V^{(2)}(x)) & \text{if } x \in V_2 \cap V_1 \end{cases}$, $(F_V^{(1)} \cup F_V^{(2)})(x_1, x_2) = \begin{cases} F_V^{(1)}(x) & \text{if } x \in V_1 \setminus V_2 \\ F_V^{(2)}(x) & \text{if } x \in V_2 \setminus V_1 \\ S_{\max}(F_V^{(1)}(x), F_V^{(2)}(x)) & \text{if } x \in V_2 \cap V_1 \end{cases}$

and $T_E^{(1)} \cup T_E^{(2)}, I_E^{(1)} \cup I_E^{(2)}, F_E^{(1)} \cup F_E^{(2)} : (E_1 \cup E_2) \rightarrow [0, 1]$, by $(T_E^{(1)} \cup T_E^{(2)})(xy) = \begin{cases} T_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ T_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ T_{\min}(T_E^{(1)}(xy), T_E^{(2)}(xy)) & \text{if } xy \in E_2 \cap E_1 \end{cases}$, $(I_E^{(1)} \cup I_E^{(2)})(xy) = \begin{cases} I_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ I_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ S_{\max}(I_E^{(1)}(xy), I_E^{(2)}(xy)) & \text{if } xy \in E_2 \cap E_1 \end{cases}$, $(F_E^{(1)} \cup F_E^{(2)})(xy) = \begin{cases} F_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ F_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ S_{\max}(F_E^{(1)}(xy), F_E^{(2)}(xy)) & \text{if } xy \in E_2 \cap E_1 \end{cases}$.

Example 5: Consider the KM -single valued neutrosophic metric graphs G_1 and G_2 in Example 4. It is easy to see that KM -single valued neutrosophic metric graph $G_1 \cup G_2$ with $t = 2$ in Figure 4.

Theorem 17: Let $G_1 = (X_1, Y_1, \rho_1, T, S)$, $G_2 = (X_2, Y_2, \rho_2, T, S)$ be KM -single valued neutrosophic metric graphs on simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. If $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ are two simple graphs, where $V_1 \cap V_2 = \emptyset$, then $G_1 \cup G_2 = (X_1 \cup X_2, Y_1 \cup Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM -single valued

neutrosophic metric graph on $G_1^* \cup G_2^*$.

Proof 17: Firstly, by Theorem 7, $(V_1 \cup V_2, T_{\min}(\rho), T)$ is a KM -fuzzy metric space. Let $xy \in E(G_1^* \cup G_2^*)$. Since $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ are two simple graphs, $xy \in E_1 \setminus E_2$ implies that $(x, y \in V_1 \setminus V_2)$ and $xy \in E_2 \setminus E_1$ implies that $(x, y \in V_2 \setminus V_1)$. Since G_1 is a KM -single valued neutrosophic metric graph on G_1^* and G_2 is a KM -single valued neutrosophic metric graph on G_2^* , for some $t_1, t_2 \in \mathbb{R}^{\geq 0}$, take $t = \max\{t_1, t_2\}$ so by Theorems 1 and 7, if $xy \in E_1 \setminus E_2$, we have

$$\begin{aligned} & T((T_E^{(1)} \cup T_E^{(2)})(xy), T((T_V^{(1)} \cup T_V^{(2)})(x), (T_V^{(1)} \cup T_V^{(2)})(y))) \\ &= T(T_V^{(1)}(xy), T(T_V^{(1)}(x), T_V^{(1)}(y))) \\ &\leq \rho_1(x, y, t) \leq (\rho_1 \cup \rho_2)(x, y, t). \end{aligned}$$

In a similar way, if $xy \in E_2 \setminus E_1$, one can see that $T_{\min}((T_E^{(1)} \cup T_E^{(2)})(xy), T_{\min}((T_V^{(1)} \cup T_V^{(2)})(x), (T_V^{(1)} \cup T_V^{(2)})(y))) \leq \rho_2(x, y, t) = (\rho_1 \cup \rho_2)(x, y, t)$. Other cases is similar to.

Now consider $t = \min\{t_1, t_2\}$ so by Theorems 1 and 7, if $xy \in E_1 \setminus E_2$, we have

$$\begin{aligned} & S((I_E^{(1)} \cup I_E^{(2)})(xy), S((I_V^{(1)} \cup I_V^{(2)})(x), (I_V^{(1)} \cup I_V^{(2)})(y))) \\ &= S(I_V^{(1)}(xy), T(I_V^{(1)}(x), I_V^{(1)}(y))) \\ &\geq \rho_1(x, y, t) \geq (\rho_1 \cup \rho_2)(x, y, t). \end{aligned}$$

In a similar way, if $xy \in E_2 \setminus E_1$, one can see that $S((I_E^{(1)} \cup I_E^{(2)})(xy), S((I_V^{(1)} \cup I_V^{(2)})(x), (I_V^{(1)} \cup I_V^{(2)})(y))) \geq \rho_2(x, y, t) = (\rho_1 \cup \rho_2)(x, y, t)$. Other cases is similar to and in a similar way, we can prove that $S((F_E^{(1)} \cup F_E^{(2)})(xy), S((F_V^{(1)} \cup F_V^{(2)})(x), (F_V^{(1)} \cup F_V^{(2)})(y))) \geq (\rho_1 \cup \rho_2)(x, y, t)$. Thus $G_1 \cup G_2 = (X_1 \cup X_2, Y_1 \cup Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* \cup G_2^*$.

Definition 12: Let G_1, G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. Define the semi-ring sum of fuzzy subsets $X_1 \otimes X_2 = (T_V^{(1)} \otimes T_V^{(2)}, I_V^{(1)} + I_V^{(2)}, F_V^{(1)} \otimes F_V^{(2)})$, $Y_1 \otimes Y_2 = (T_E^{(1)} \otimes T_E^{(2)}, I_E^{(1)} + I_E^{(2)}, F_E^{(1)} \otimes F_E^{(2)})$, where $T_V^{(1)} \otimes T_V^{(2)}, I_V^{(1)} \otimes I_V^{(2)}, F_V^{(1)} \otimes F_V^{(2)} : (V_1 \otimes V_2) \rightarrow [0, 1]$ by $(T_V^{(1)} \otimes T_V^{(2)})(x_1, x_2) = (T_V^{(1)} \cup T_V^{(2)})(x_1, x_2)$, $(I_V^{(1)} \otimes I_V^{(2)})(x_1, x_2) = (I_V^{(1)} \cup I_V^{(2)})(x_1, x_2)$, $(F_V^{(1)} \otimes F_V^{(2)})(x_1, x_2) = (F_V^{(1)} \cup F_V^{(2)})(x_1, x_2)$ and $T_E^{(1)} \otimes T_E^{(2)}, I_E^{(1)} \otimes I_E^{(2)}, F_E^{(1)} \otimes F_E^{(2)} : (E_1 \otimes E_2) \rightarrow [0, 1]$, by $(T_E^{(1)} \otimes T_E^{(2)})(xy) = \begin{cases} T_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ T_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ 0 & \text{if } xy \in E_2 \cap E_1 \end{cases}$, $(I_E^{(1)} \otimes I_E^{(2)})(xy) = \begin{cases} I_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ I_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ 1 & \text{if } xy \in E_2 \cap E_1 \end{cases}$, $(F_E^{(1)} \otimes F_E^{(2)})(xy) = \begin{cases} F_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ F_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ 1 & \text{if } xy \in E_2 \cap E_1 \end{cases}$.

$$\begin{cases} F_E^{(1)}(xy) & \text{if } xy \in E_1 \setminus E_2 \\ F_E^{(2)}(xy) & \text{if } xy \in E_2 \setminus E_1 \\ 1 & \text{if } xy \in E_2 \cap E_1 \end{cases}$$

Theorem 18: Let G_1 and G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. If G_1^* and G_2^* are two simple graphs, where $V_1 \cap V_2 = \emptyset$, then $G_1 \otimes G_2 = (X_1 \otimes X_2, Y_1 \otimes Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* \cup G_2^*$.

Proof 18: It is similar to Theorem 17.

Definition 13: Let G_1, G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. Define the join(or suspension) of fuzzy subsets $X_1 + X_2 = (T_V^{(1)} + T_V^{(2)}, I_V^{(1)} + I_V^{(2)}, F_V^{(1)} + F_V^{(2)})$, $Y_1 + Y_2 = (T_E^{(1)} + T_E^{(2)}, I_E^{(1)} + I_E^{(2)}, F_E^{(1)} + F_E^{(2)})$, where $T_V^{(1)} + T_V^{(2)}, I_V^{(1)} + I_V^{(2)}, F_V^{(1)} + F_V^{(2)} : (V_1 \otimes V_2) \rightarrow [0, 1]$ by $(T_V^{(1)} + T_V^{(2)})(x_1, x_2) = (T_V^{(1)} \cup T_V^{(2)})(x_1, x_2)$, $(I_V^{(1)} + I_V^{(2)})(x_1, x_2) = (I_V^{(1)} \cup I_V^{(2)})(x_1, x_2)$, $(F_V^{(1)} + F_V^{(2)})(x_1, x_2) = (F_V^{(1)} \cup F_V^{(2)})(x_1, x_2)$ and $T_E^{(1)} + T_E^{(2)}, I_E^{(1)} + I_E^{(2)}, F_E^{(1)} + F_E^{(2)} : (E_1 \otimes E_2) \rightarrow [0, 1]$, by $(T_E^{(1)} + T_E^{(2)})(xy) = \begin{cases} T_E^{(1)}(xy) \cup T_E^{(2)}(xy) & \text{if } xy \in E_1 \cup E_2 \\ (\rho_1 \cup \rho_2)(x, y, t) & \text{if } xy \in E'(x \in V_1, y \in V_2) \end{cases}$, $(I_E^{(1)} + I_E^{(2)})(xy) = \begin{cases} I_E^{(1)}(xy) \cup I_E^{(2)}(xy) & \text{if } xy \in E_1 \cup E_2 \\ (\rho_1 \cup \rho_2)(x, y, t) & \text{if } xy \in E'(x \in V_1, y \in V_2) \end{cases}$, $(F_E^{(1)} + F_E^{(2)})(xy) = \begin{cases} F_E^{(1)}(xy) \cup F_E^{(2)}(xy) & \text{if } xy \in E_1 \cup E_2 \\ (\rho_1 \cup \rho_2)(x, y, t) & \text{if } xy \in E'(x \in V_1, y \in V_2) \end{cases}$, where E' is the set of all edges joining the vertices of V_1 and V_2 and $t \in \mathbb{R}^{\geq 0}$.

Theorem 19: Let G_1 and G_2 be KM -single valued neutrosophic metric graphs on simple graphs G_1^* and G_2^* , respectively. If G_1^* and G_2^* are two simple graphs, where $V_1 \cap V_2 = \emptyset$, then $G_1 + G_2 = (X_1 + X_2, Y_1 + Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* + G_2^*$.

Proof 19: Let $xy \in E(G_1^* + G_2^*)$. Then $xy \in E_1 \setminus E_2$, $xy \in E_2 \setminus E_1$ or $xy \in E'$. We only consider $xy \in E'$ and other cases are similar to Theorem 17. Since $xy \in E'$, we get that $(x \in V_1 \setminus V_2, y \in V_2 \setminus V_1)$ or $(y \in V_1 \setminus V_2, x \in V_2 \setminus V_1)$. If $x \in V_1 \setminus V_2, y \in V_2 \setminus V_1$ ($y \in V_1 \setminus V_2, x \in V_2 \setminus V_1$ is proved in a similar way), for some $t_1, t_2 \in \mathbb{R}^{\geq 0}$, take $t = \max\{t_1, t_2\}$ so by Theorem 1, we have $T((T_E^{(1)} + T_E^{(2)})(xy), T((T_V^{(1)} + T_V^{(2)})(x), (T_V^{(1)} + T_V^{(2)})(y))) \leq T((\rho_1 \cup \rho_2)(x, y, t), T(T_V^{(1)}(x), T_V^{(1)}(y))) \leq (\rho_1 \cup \rho_2)(x, y, t)$. Now, consider $t = \min\{t_1, t_2\}$ so by Theorem 1, we have $S((I_E^{(1)} + I_E^{(2)})(xy), S((I_V^{(1)} + I_V^{(2)})(x), (I_V^{(1)} + I_V^{(2)})(y))) \geq S((\rho_1 \cup \rho_2)(x, y, t), S(I_V^{(1)}(x), I_V^{(1)}(y))) \geq (\rho_1 \cup \rho_2)(x, y, t)$ and $S((F_E^{(1)} + F_E^{(2)})(xy), S((F_V^{(1)} + F_V^{(2)})(x), (F_V^{(1)} + F_V^{(2)})(y))) \geq S((\rho_1 \cup \rho_2)(x, y, t), S(F_V^{(1)}(x), F_V^{(1)}(y))) \geq (\rho_1 \cup \rho_2)(x, y, t)$. It follows that $G_1 + G_2 = (X_1 + X_2, Y_1 + Y_2, \rho_1 \cup \rho_2, T, S)$ is a KM -single valued neutrosophic metric graph on $G_1^* + G_2^*$.

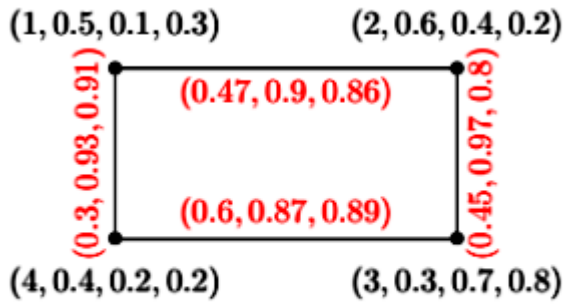


FIGURE 7. KM -single valued neutrosophic metric graph \bar{G} .

Definition 14: Let (V, ρ, T) be a KM -single valued neutrosophic metric space and $G^* = (V, E)$ be a simple graph. If $G = (X, Y, \rho, T, S)$ is a KM -fuzzy metric graph on G^* , then define the complement of fuzzy subsets $\bar{X} = (\bar{T}_V, \bar{I}_V, \bar{F}_V)$, $\bar{Y} = (\bar{T}_E, \bar{I}_E, \bar{F}_E)$, where $\bar{T}_V, \bar{I}_V, \bar{F}_V : V \rightarrow [0, 1]$ and $\bar{T}_E, \bar{I}_E, \bar{F}_E : E \rightarrow [0, 1]$ by $\bar{T}_V(x) = T_V(x)$, $\bar{I}_V(x) = I_V(x)$, $\bar{F}_V(x) = F_V(x)$ and $\bar{T}_E(xy) = \rho(x, y, t) - T(T_E(xy), T(T_V(x), T_V(y)))$, $\bar{I}_E(xy) = S(I_E(xy), S(I_V(x), I_V(y)))$, $\bar{F}_E(xy) = S(F_E(xy), S(F_V(x), F_V(y)))$, where $x, y \in V$. We will denote the complement of a KM -single valued neutrosophic metric graph $G = (X, Y, \rho, T, S)$, by $\bar{G} = (\bar{X}, \bar{Y}, \rho, T, S)$.

Theorem 20: Let (V, ρ, T) be a KM -fuzzy metric space and $G^* = (V, E)$ be a simple graph. If $G = (X, Y, \rho, T, S)$ is a KM -single valued neutrosophic metric graph on G^* , then $\bar{G} = (\bar{X}, \bar{Y}, \rho, T, S)$ is a KM -single valued neutrosophic metric graph.

Proof 20: Let $x, y \in V$. Since G is a KM -single valued neutrosophic metric graph on G^* , for some $t \in \mathbb{R}^{\geq 0}$,

$$\begin{aligned} & T(\bar{T}_E(xy), T(\bar{T}_V(x), \bar{T}_V(y))) \\ &= T(\rho(x, y, t) - T(T_E(xy), T(T_V(x), T_V(y))), T(T_V(x), T_V(y))) \\ &\leq \rho(x, y, t) - T(T_E(xy), T(T_V(x), T_V(y))) \\ &\leq \rho(x, y, t). \end{aligned}$$

In addition,

$$\begin{aligned} & S(\bar{I}_E(xy), S(\bar{I}_V(x), \bar{I}_V(y))) \\ &= S(S(I_E(xy), S(I_V(x), I_V(y))), S(I_V(x), I_V(y))) \\ &\geq S(I_E(xy), S(I_V(x), I_V(y))) \geq \rho(x, y, t). \end{aligned}$$

In a similar way, it is easy to see that $S(\bar{F}_E(xy), S(\bar{F}_V(x), \bar{F}_V(y))) \geq \rho(x, y, t)$. It follows that $\bar{G} = (\bar{X}, \bar{Y}, \rho, T, S)$ is a KM -single valued neutrosophic metric graph.

Example 6: Consider the KM -single valued neutrosophic metric graph G in Example 1. So obtain a KM -single valued neutrosophic metric graph \bar{G} on the cycle graph C_4 for $t = 1$, in Figure 7.

V. CONCLUSIONS

The current paper has introduced a novel concept fuzzy algebra as KM -single valued neutrosophic metric graph and a new generalization of graphs based on KM -fuzzy metric spaces. This work extended and obtained some properties in KM -fuzzy metric spaces. Also it showed that every non empty set converted to a KM -fuzzy metric space, the product and union of KM -fuzzy metric spaces is a KM -fuzzy metric space, the extended KM -fuzzy metric spaces are constructed using the some algebraic operations on KM -fuzzy metric spaces, the concept of complement of KM -single valued neutrosophic metric graph is defined and investigated some its properties. We hope that these results are helpful for further studies in theory of graphs. In our future studies, we hope to obtain more results regarding intuitionistic metric graphs, neutrosophic metric graphs, KM -single valued neutrosophic metric hypergraphs, bipolar KM -single valued neutrosophic metric graphs, automorphism KM -single valued neutrosophic metric graphs and their applications.

REFERENCES

- [1] S. Broumi, M. Talea, A. Bakkali and F. Samarandache, Single Valued Neutrosophic Graph, Journal of New theory, 2016, 10, 86–101.
- [2] A. Dey, A. Pal and T. Pal, Interval type 2 fuzzy set in fuzzy shortest path problem, 2016, Mathematics, 4(4), 62.
- [3] A. Dey, R. Pradhan, A. Pal and T. Pal, A Genetic Algorithm For Solving Fuzzy Shortest Path Problems With Interval Type-2 Fuzzy Arc Lengths, Malaysian Journal of Computer Science, 2018, 31(4), 255–270.
- [4] A. Dey, L. Son, Anita Pal, Fuzzy Minimum Spanning Tree with Interval Type 2 Fuzzy Arc Length: Formulation and A New Genetic Algorithm, Soft computing, 2020, 24, 3963–3974.
- [5] M. A. Erceg, Metric spaces in fuzzy set theory, J. Math. Anal. Appl., 1979, 69 (1), 205–230.
- [6] M. Grabiec, Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems, 1988, 27, 385–389.
- [7] A. George, P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets and Systems, 1994, 64, 395–399.
- [8] L. Huang, Y. Hu, Y. LiP. K. K. Kumar, D. Koley and A. Dey, A Study of Regular and Irregular Neutrosophic Graphs with Real Life Applications, Mathematics, 2019, 7(6) 551, 1–20.
- [9] M. Hamidi, S. Jahanpanah and A. Radfar, Extended graphs based on KM -fuzzy metric spaces, Iranian Journal of Fuzzy Systems, 2020, (Accepted).
- [10] M. Hamidi and A. Borumand Saied, Creating and Computing Graphs from Hypergraphs, Kragujevac Journal of Mathematics, 2019, 43(1), 139–164.
- [11] M. Hamidi and F. Smarandache, Single-Valued Neutrosophic Directed (Hyper)Graphs And Applications in Networks, Journal of Intelligent & Fuzzy Systems, 2019, 37(2), 2869–2885.
- [12] M. Hamidi and A. Borumand Saied, Achievable Single-Valued Neutrosophic Graphs in Wireless Sensor Networks, New Mathematics and Natural Computation, 2018, 14 (2), 157–185.
- [13] M. Hamidi, A. Borumand Saied, On Derivable Tree, Transactions on Combinatorics, 2019, 8(2), 21–43.
- [14] F. Harary, Graph Theory, Addison-Wesley Publishing Company, Inc, 1969.
- [15] K. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetika, 1975, 11, 336–344.
- [16] S. Kalathian, S. Ramalingam, N. Srinivasan, S. waran Raman, S. Broumi, Embedding of fuzzy graphs on topological surfaces, Neural Computing and Applications, 2019.
- [17] K. Mohanta, A. Dey, A. Pal, H. V. Long, L. H. Son, A Study of m-Polar Neutrosophic Graph with Applications, Journal of Intelligent & Fuzzy Systems, 2020, 38(4), 4809–4828.
- [18] L. Mancinska, D. E. Roberson, Quantum homomorphisms, J. Combin. Theory Ser. B, 2016, 118, 228–267.
- [19] I. Mardones-Perez, M.A.de Prada Vicente, Fuzzy pseudometric spaces vs fuzzy-fying structures, Fuzzy Sets and Systems, 2015, 267 117–132.

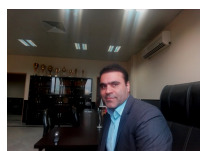
- [20] S. Mathew, J. N. Mordeson, H. L. Yang, Incidence cuts and connectivity in fuzzy incidence graphs, *Iranian Journal of Fuzzy Systems*, 2019, 16(2), 31–43.
- [21] J. N. Mordeson, S. Mathew, t -Norm Fuzzy Graphs, *New Mathematics and Natural Computation*, 2018, 14(1), 129–143.
- [22] M. Mullai, Said Broumi, Dominating Energy in Neutrosophic graphs, *International Journal of Neutrosophic Science*, 2020, 5(1), 38–58.
- [23] A. Rosenfeld, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu, K. Tanaka, M. Shimura (Eds.), *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*, Academic Press, New York, 1975, 77–95.
- [24] S. Rana, Muhammad Saeed, Midha Qayyum, Florentin Smarandache, Plithogenic Subjective Hyper-Super-Soft Matrices with New Definitions & Local, Global, Universal Subjective Ranking Model, *International Journal of Neutrosophic Science*, 2020, 6(2), 56–79.
- [25] P. K. Singh, Plithogenic set for multi-variable data analysis, *International Journal of Neutrosophic Science*, 2020, 1(2), 81–89.
- [26] F. Smarandache, *Neutrosophy, Neutrosophic Probability, Set, and Logic*, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 1998, 105.
- [27] F. Smarandache, Neutrosophic Set, a generalisation of the intuitionistic fuzzy sets, *Int. J. Pure Appl. Math.* 2005, 24, 287–297.
- [28] F. Smarandache, Extension of HyperGraph to n -SuperHyperGraph and to Plithogenic n -SuperHyperGraph, and Extension of HyperAlgebra to n -ary (Classical-/Neuro-/Anti-)HyperAlgebra, *Neutrosophic Sets and Systems*, 2020, 33, 290–296.
- [29] F. Smarandache, Neutro Algebra is a Generalization of Partial Algebra, *International Journal of Neutrosophic Science*, 2020, 2(1), 08–17.
- [30] F. Smarandache, Set is a Generalization of Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), q -Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and n -HyperSpherical Fuzzy Set, while Neutrosophication is a Generalization of Regret Theory, Grey System Theory, and Three-Ways Decision (revisited), *arXiv*, Cornell University, New York City, NY, USA, (Submitted 2019).
- [31] F. Smarandache, n -SuperHyperGraph and Plithogenic n -SuperHyperGraph, in *Nidus Idearum*, second edition, Pons asbl, Bruxelles, 2019, 7, 107–113.
- [32] F. Smarandache, The neutrosophic triplet (n -ary HyperAlgebra, n -ary NeutroHyperAlgebra, n -ary AntiHyperAlgebra), in *Nidus Idearum*, second edition, Pons asbl, Bruxelles, 2019, 7, 104–106.
- [33] F. Smarandache, Nivetha Martin, Plithogenic n -Super Hypergraph in Novel Multi-Attribute Decision Making, *International Journal of Neutrosophic Science*, 7(1), 2020, 08–30.
- [34] L. A. Zadeh, Fuzzy sets, *Information and Control*, 1965, 8(3), 338–353.
- [35] C. Zuo, A. Pal and A. Dey, New Concepts of Picture Fuzzy Graphs with Application, *mathematics*, 2019, 7(5)740, 1–20.



Florentin Smarandache is a professor of mathematics at the University of New Mexico, United States. He got his MSc in Mathematics and Computer Science from the University of Craiova, Romania, PhD in Mathematics from the State University of Kishinev, and Postdoctoral in Applied Mathematics from Okayama University of Sciences, Japan. He is the founder of neutrosophy (generalization of dialectics), neutrosophic set, logic, probability and statistics since 1995 and has

published hundreds of papers and books on neutrosophic physics, superluminal and instantaneous physics, unmatter, quantum paradoxes, absolute theory of relativity, redshift and blueshift due to the medium gradient and refraction index besides the Doppler effect, paradoxism, outerart, neutrosophy as a new branch of philosophy, Law of Included Multiple- Middle, multispace and multistructure, hypersoft set, degree of dependence and independence between neutrosophic components, refined neutrosophic set, neutrosophic over-under-off-set, plithogenic set, neutrosophic triplet and duplet structures, quadruple neutrosophic structures, extension of algebraic structures to NeutroAlgebras and AntiAlgebras, DSMT and so on to many peer-reviewed international journals and many books and he presented papers and plenary lectures to many international conferences around the world.

...



MOHAMMAD HAMIDI was born in Vidouja Village, Kashan, Isfahan, Iran in 1979. He received the B.S. degrees in mathematics from the University of Kashan, Kashan, in 2002, the M.S. degrees in mathematics(algebra) from the Isfahan University of Technology, Isfahan in 2005 and the Ph.D. degree in in mathematics(algebras and hyperalgebras and fuzzy logic) from Tehran Payame Noor University, Tehran, in 2014.

From 2002 to 2005, he was a Research Assistant with the Mathematics University. Since 2014, he has been an Assistant Professor with the Mathematics Department, Payame Noor University of Tehran. Since 2019, he has been an Associate Professor with the Mathematics Department, Payame Noor University of Tehran. He is the author of two books, more than 70 articles, and more than 3 research project. His research interests include applications of fuzzy graph and single valued neutrosophic graphs in hypernetworks and complex hypernetworks such as Wireless sensor networks.