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Decision-Making Problems under the Environment of m-Polar Diophantine Neutrosophic N-Soft Set

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ABSTRACT

Fuzzy models are present everywhere from natural to artificial structures, embodying the dynamic processes in physical, biological, and social systems. As real-life problems are often uncertain on account of inconsistent and indeterminate information, it seems very demanding for an expert to solve those problems using a fuzzy model. In this regard, we develop a hybrid new model m-polar Diophantine neutrosophic N-soft set which is based on neutrosophic set and soft set. Additionally, we define several different sorts of compliments on the proposed set. A proposed set is a generalized form of fuzzy, soft, Pythagorean fuzzy, Pythagorean fuzzy soft, and Pythagorean fuzzy N-soft sets. In this manner, m-polar Diophantine neutrosophic N-soft set is more proficient, a versatile model to oversee vulnerabilities as it likewise survives the downsides of existing models which are to be summed up. Furthermore, we give the application of the proposed set in multi-attribute decision-making problems by defining a new choice-value function.

KEYWORDS

Neutrosophic set; soft set; N-soft set; m-polar diophantine neutrosophic N-soft set; decision making

1 Introduction

The idea of a set and set theory are incredible assets in arithmetic. Shockingly, a non-condition basic set theory for example that the component can either have a place in a set or not, is frequently not appropriate in genuine a daily existence where numerous unclear terms as “enormous benefit,” “high pressing factor,” “moderate temperature,” “dependable instruments,” “safe conditions,” and so forth are broadly utilized. Tragically, such loose depictions cannot be sufficiently taken care of by ordinary mathematical tools.



In fuzzy theory, a recently characterized model by and large beats the downsides of recently characterized models. Because of uncertainty and weaknesses issues in numerous days by day life issues, routine math is not continuously accessible. To manage such issues, different methods such as the theory of possibility, rough set assumption, and fuzzy set theory has been considered as elective models and to keep away from weaknesses too. Inopportunately, the greater part of the options such as science have their own disadvantages and downsides. For example, a large portion of the words like expert, amazing, best, significant is most certainly not quantifiable and uncertain. The rules for words like superb, best, famous, and so forth, hesitate from individual to person.

To deal with such sort of equivocal and unsure data, Zadeh [1] investigated the idea of fuzzy set which is mapping from a universal set X to $[0, 1]$. Atanassov [2] proposed the fortuitous of intuitionistic fuzzy sets as an expansion of fuzzy sets by presenting the idea of membership and non-membership grades. Molodtsov [3] began the thought of soft set as a mathematical model to oversee vulnerabilities. The chance of soft set has another objective for the researchers due to them utilizes in a wide range of exuberant issues.

Ali et al. [4] presented some new operations on soft set theory. They introduced the ideas of expanded and restricted union and intersections in detail. In [5–7], Yager introduced and investigated several relations on Pythagorean fuzzy set. Peng et al. [8] discussed certain results on Pythagorean fuzzy sets and also defined the Pythagorean fuzzy number in. Peng et al. [9] set up some Pythagorean fuzzy data measures and their applications. Peng et al. [10] proposed some new approaches to manage single-regarded neutrosophic MADM reliant on MABAC, TOPSIS, and new closeness measure with score function. In [11–21], many decision-making problems and algebraic structures are discussed over different fuzzy environments.

Smarandache's introduced neutrosophic set and then proposed many operation on it [22–24]. Neutrosophic set (NS) based on three parameters namely, membership, indeterminacy, and non-membership. Wang et al. [25] propped the concept of single valued neutrosophic sets. Deli et al. [26] introduced the idea of bipolar neutrosophic set and their applications in multi-criteria decision making problems in. Fatimah et al. [22] introduced the notion of an N-soft set which is an extension of a soft set. Many problems related to decision-making are discussed by using different kind of environments in [27–30].

There are many problems regarding decision-making that need to improveby investigating a new set or model. In this regard, we develop a proposed setthat provides a more batter approximation than existing sets. The proposed work is arranged as follows. In Section 2, some preliminary concepts are given to understand the proposed work. In Section 3, we define the notion of m-polar Diophantine neutrosophic N-soft set and then define some operations on it. In Section 4, we discuss different types of compliments on the proposed set. We give the comparison table and application in multi-attribute decision-making problems in Section 5.

2 Preliminaries

In this section, we give some preliminary concepts related to previous existing sets.

Definition 2.1. [1] A fuzzy set is a mapping μ from universal set X to $[0, 1]$ such that, $\mu: X \rightarrow [0, 1]$. The fuzzy set can be written in the form of

$$F_X = \{(x, \mu_F(x)) : x \in X\}.$$

Definition 2.2. [2] An intuitionistic fuzzy set on universal set X is defined as

$$\mathcal{I}_X = \{(x, \mu_I(x), \nu_I(x)) : x \in I\},$$

where $\mu_I: X \rightarrow [0, 1]$ and $\nu_I: X \rightarrow [0, 1]$ are the membership and non-membership functions, respectively.

Definition 2.3. [31] The m-polar fuzzy set on a universal set M is a mapping $\mu: M \rightarrow [0, 1]^m$ and $m(X)$ is the collection of all m-polar fuzzy set on M .

Definition 2.4. [3] The soft set is defined by the set valued mapping $\varphi: \mathcal{I} \rightarrow 2^T$, where $\mathcal{I}$ denotes the set of parameters and 2^T is the power set of T . The soft set can be written as,

$$\varphi_{\mathcal{I}} = (\varphi, \mathcal{I}) = \left\{ (t, \varphi(t)) : t \in \mathcal{I}, \varphi(t) \in 2^T \right\}.$$

Definition 2.5. [32,33] A fuzzy soft set is defined as

$$\Gamma_S = \{(t, \gamma_S(t)) : t \in T, \gamma_S \in \mathcal{F}(L)\},$$

where $\gamma_S: T \rightarrow \mathcal{F}(L)$ and $\mathcal{F}(L)$ is the collection of all fuzzy sets on L and T is the set of parameters with $S \subseteq T$.

Definition 2.6. [34] Let X be the crisp set. Intuitionistic fuzzy soft set (IFSS) is interpreted by multi-valued mapping $\psi: B \rightarrow IF^X$, where IF^X represents the collection of all IF-subsets defined over crisp set X (where $B \subseteq X$). Thus the IFSS can be expressed as

$$\mathcal{U}_B = \{(e, \psi_B(e)) : e \in X, \psi_B \in IF^X\}.$$

Definition 2.7. [5] Let X be the crisp set. A *Pythagorean fuzzy set* (PFS) can be expressed as

$$P = \left\{ \langle \rho, \mu_P(\rho), \nu_P(\rho) \rangle : 0 \leq \mu_P^2(\rho) + \nu_P^2(\rho) \leq 1, \rho \in X \right\},$$

where $\mu_P: X \rightarrow [0, 1]$ and $\nu_P: X \rightarrow [0, 1]$ with the condition that $0 \leq \mu_P^2(\rho) + \nu_P^2(\rho) \leq 1$, is known as the degrees of membership and non-membership of $\rho \in X$ to the set P .

Definition 2.8. [35] The score function and accuracy function of Pythagorean fuzzy number $\alpha = (\mu_\alpha, \nu_\alpha)$ over X is defined as, $S(\gamma) = \mu_\gamma^2 - \nu_\gamma^2$ and $Q(\gamma) = \mu_\gamma^2 + \nu_\gamma^2$, with $-1 \leq \gamma \leq 1$ and $0 \leq Q(\gamma) \leq 1$.

Definition 2.9. [36] The ranking function of Pythagorean fuzzy number $\gamma = (\mu_\gamma, \nu_\gamma)$ over X is defined as

$$R(\gamma) = \frac{1}{2} + r_\gamma \left(\frac{1}{2} - \frac{2\theta_\gamma}{\pi} \right),$$

where $r_\gamma = \sqrt{\mu_\gamma^2 + \nu_\gamma^2}$ is called commitment strength and θ_γ is the angle between r_γ and μ_γ . The direction of commitment d_γ is $d_\gamma = 1 - \frac{2\theta_\gamma}{\pi}$, where $\mu_\gamma = r_\gamma \cos \theta_\gamma$, $\nu_\gamma = r_\gamma \sin \theta_\gamma$.

Definition 2.10. [37] Let X be a non-empty universal set, S be the set of attributes, and $Y \subseteq S$. Let $D = \{0, 1, 2, \dots, N-1\}$ be set of grading. The triple (F_p, Y, N) is said to be a Pythagorean fuzzy N-soft set on X , if F_p is a mapping $F_p: Y \rightarrow 2^{X \times D} \times PFN$, in which $F: Y \rightarrow 2^{X \times D}$, and $P: Y \rightarrow PFN$, where PFN is Pythagorean fuzzy number. That is $\mu: Y \rightarrow [0, 1]$ and $\nu: Y \rightarrow [0, 1]$ such that

$$0 \leq \mu_y^2(x) + \nu_y^2(x) \leq 1, \quad \forall y \in Y, \quad \forall x \in X.$$

Hence,

$$(F_p, Y, N) = ((x, d_y), (\mu_y(x), \nu_y(x))), \quad d_y \in D.$$

Definition 2.11. [38] A neutrosophic fuzzy set (NS), S over the universal set X is defined as $S = \{(\psi, \mu_S(\psi), \lambda_S(\psi), \nu_S(\psi))\}$,

where mappings μ_S, λ_S, ν_S stand for degree of truth, degree of indeterminacy, degree of falsity. $\mu_S, \lambda_S, \nu_S \in [0, 1]$, with $0 \leq \mu_S + \lambda_S + \nu_S \leq 3$.

3 m-Polar Diophantine Neutrosophic N-Soft Set

Definition 3.1 Let $\mathfrak{L} = \{0, 1, 2, \dots, N - 1\}$ be the set of grades where $N \in \{2, 3, 4, \dots\}$. If X is a non-empty set and E is the family of attributes. Let A be a non-empty subset of E . A *m-polar Diophantine neutrosophic N-soft* (MPDNNS) set on X is denoted as $(\mathcal{U}, A, m, N)$ or $\mathcal{U}_A^{(m,N)}$, where $\mathcal{U}: A \rightarrow \mathbf{P}(PF^X \times \mathfrak{L})$ is a mapping (where PF^X is the aggregate of all Diophantine neutrosophic subsets over X). That is

$$(\mathcal{U}, A, m, N) = \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_e(\rho) \rangle}{(\mu_1(\rho), \mu_2(\rho), \dots, \mu_m(\rho); \lambda_1(\rho), \lambda_2(\rho), \dots, \lambda_m(\rho); \nu_1(\rho), \nu_2(\rho), \dots, \nu_m(\rho))} \right\} \right) \right. \\ \left. | e \in A, \rho \in X, \mathfrak{l}_e(\rho) \in \mathfrak{L} \right\},$$

where $\mu_e: X \rightarrow [0, 1]^m$, $\nu_e: X \rightarrow [0, 1]^m$, and $\lambda_e: X \rightarrow [0, 1]^m$ are mappings along with the property,

$$0 \leq \sum_{i=1}^m \mu_i^m(\rho) + \sum_{i=1}^m \nu_i^m(\rho) + \sum_{i=1}^m \lambda_i^m(\rho) \leq 3m.$$

In particular, $\mu_i(\rho)$ represents the truth-membership, $\nu_i(\rho)$ denotes degree of falsity-membership, $\lambda_i(\rho)$ is the degree of indeterminacy and $\mathfrak{l}_e(\rho)$ denotes the grading value of the element $\rho \in X$ corresponding to the attribute $e \in A$ to the set $(\mathcal{U}, A, m, N)$. If we write $a_{ij} = \mu_{ei}(\rho_j)$, $b_{ij} = \nu_{ei}(\rho_j)$, $d_{ij} = \lambda_{ei}(\rho_j)$, and $c_{ij} = \mathfrak{l}_{e_j}(\rho_j)$ where i runs from 1 to m and j runs from 1 to n then the MPDNNS set $\mathcal{U}_A^{(m,N)}$ may be represented in tabular form as

$\mathcal{U}_A^{(m,N)}$	e_1	e_2	$\dots$	e_m
ρ_1	$\langle c_{11}, (a_{11}, a_{21}, \dots, a_{m1};$ $d_{11}, d_{21}, \dots, d_{m1};$ $b_{11}, b_{21}, \dots, b_{m1}) \rangle$	$\langle c_{12}, (a_{11}, a_{21}, \dots, a_{m1};$ $d_{11}, d_{21}, \dots, d_{m1};$ $b_{11}, b_{21}, \dots, b_{m1}) \rangle$	$\dots$	$\langle c_{1m}, (a_{11}, a_{21}, \dots, a_{m1};$ $d_{11}, d_{21}, \dots, d_{m1};$ $b_{11}, b_{21}, \dots, b_{m1}) \rangle$
ρ_2	$\langle c_{21}, (a_{12}, a_{22}, \dots, a_{m2};$ $d_{12}, d_{22}, \dots, d_{m2};$ $b_{12}, b_{22}, \dots, b_{m2}) \rangle$	$\langle c_{22}, (a_{12}, a_{22}, \dots, a_{m2};$ $d_{12}, d_{22}, \dots, d_{m2};$ $b_{12}, b_{22}, \dots, b_{m2}) \rangle$	$\dots$	$\langle c_{2m}, (a_{12}, a_{22}, \dots, a_{m2};$ $d_{12}, d_{22}, \dots, d_{m2};$ $b_{12}, b_{22}, \dots, b_{m2}) \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
ρ_n	$\langle c_{n1}, (a_{1n}, a_{2n}, \dots, a_{mn};$ $d_{1n}, d_{2n}, \dots, d_{mn};$ $b_{1n}, b_{2n}, \dots, b_{mn}) \rangle$	$\langle c_{n2}, (a_{1n}, a_{2n}, \dots, a_{mn};$ $d_{1n}, d_{2n}, \dots, d_{mn};$ $b_{1n}, b_{2n}, \dots, b_{mn}) \rangle$	$\dots$	$\langle c_{nm}, (a_{1n}, a_{2n}, \dots, a_{mn};$ $d_{1n}, d_{2n}, \dots, d_{mn};$ $b_{1n}, b_{2n}, \dots, b_{mn}) \rangle$

and in matrix form as

$$\begin{aligned}
 & (\mathcal{U}, A, m, N) \\
 &= [[c_{ij}, (a_{ij}, d_{ij}, b_{ij})]]_{n \times m} \\
 &= \begin{pmatrix} \langle c_{11}, (a_{11}, a_{21}, \dots, a_{m1}; \\ d_{11}, d_{21}, \dots, d_{m1}; \\ b_{11}, b_{21}, \dots, b_{m1}) \rangle & \langle c_{12}, (a_{11}, a_{21}, \dots, a_{m1}; \\ d_{11}, d_{21}, \dots, d_{m1}; \\ b_{11}, b_{21}, \dots, b_{m1}) \rangle & \dots & \langle c_{1m}, (a_{11}, a_{21}, \dots, a_{m1}; \\ d_{11}, d_{21}, \dots, d_{m1}; \\ b_{11}, b_{21}, \dots, b_{m1}) \rangle \\ \langle c_{21}, (a_{12}, a_{22}, \dots, a_{m2}; \\ d_{12}, d_{22}, \dots, d_{m2}; \\ b_{12}, b_{22}, \dots, b_{m2}) \rangle & \langle c_{22}, (a_{12}, a_{22}, \dots, a_{m2}; \\ d_{12}, d_{22}, \dots, d_{m2}; \\ b_{12}, b_{22}, \dots, b_{m2}) \rangle & \dots & \langle c_{2m}, (a_{12}, a_{22}, \dots, a_{m2}; \\ d_{12}, d_{22}, \dots, d_{m2}; \\ b_{12}, b_{22}, \dots, b_{m2}) \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle c_{n1}, (a_{1n}, a_{2n}, \dots, a_{mn}; \\ d_{1n}, d_{2n}, \dots, d_{mn}; \\ b_{1n}, b_{2n}, \dots, b_{mn}) \rangle & \langle c_{n2}, (a_{1n}, a_{2n}, \dots, a_{mn}; \\ d_{1n}, d_{2n}, \dots, d_{mn}; \\ b_{1n}, b_{2n}, \dots, b_{mn}) \rangle & \dots & \langle c_{nm}, (a_{1n}, a_{2n}, \dots, a_{mn}; \\ d_{1n}, d_{2n}, \dots, d_{mn}; \\ b_{1n}, b_{2n}, \dots, b_{mn}) \rangle \end{pmatrix}
 \end{aligned}$$

This matrix is called *m-Polar Diophantine neutrosophic N-Soft matrix* or shortly **MPDNNS** matrix.

Note: We use the expression $x^n + y^n = z^n$ in Definition 3.1, which is similar to Diophantine equation. Thats why we call m-polar Diophantine neutrosophic N-soft set instead of m-polar neutrosophic N-soft set.

Definition 3.2. An MPDNNS set $\mathcal{U}_A^{(m,N)}$ over X is known as *null MPDNNS set*, symbolized as $\mathcal{U}_{\phi}^{(m,0)}$ and defined as

$$\begin{aligned}
 \mathcal{U}_{\phi}^{(m,0)} = & \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_{\phi}(\rho) \rangle}{(\mu_{\phi_1}(\rho), \mu_{\phi_2}(\rho), \dots, \mu_{\phi_m}(\rho); \lambda_{\phi_1}(\rho), \lambda_{\phi_2}(\rho), \dots, \lambda_{\phi_m}(\rho); \nu_{\phi_1}(\rho), \nu_{\phi_2}(\rho), \dots, \nu_{\phi_m}(\rho))} \right\} \right) \right. \\
 & \left. | e \in A, \rho \in X, \mathfrak{l}_{\phi}(\rho) \in \mathfrak{L} \right\},
 \end{aligned}$$

where, $\mu_{\phi_i}(\rho) = 0$, $\lambda_{\phi_i}(\rho) = 1$, $\nu_{\phi_i}(\rho) = 1$, $1 \leq i \leq m$, and $\mathfrak{l}_{\phi}(\rho) = 0$.

Definition 3.3. An MPDNNS set $\mathcal{U}_A^{(m,N)}$ over X is known as *absolute MPDNNS set*, symbolized as $\mathcal{U}_E^{(m,N-1)}$ and defined as

$$\begin{aligned}
 \mathcal{U}_E^{(m,N-1)} = & \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_E(\rho) \rangle}{(\mu_{E_1}(\rho), \mu_{E_2}(\rho), \dots, \mu_{E_m}(\rho); \lambda_{E_1}(\rho), \lambda_{E_2}(\rho), \dots, \lambda_{E_m}(\rho); \nu_{E_1}(\rho), \nu_{E_2}(\rho), \dots, \nu_{E_m}(\rho))} \right\} \right) \right. \\
 & \left. | e \in A, \rho \in X, \mathfrak{l}_E(\rho) \in \mathfrak{L} \right\},
 \end{aligned}$$

where $\mu_{E_i}(\rho) = 1$, $\lambda_{E_i}(\rho) = 0$, $\nu_{E_i}(\rho) = 0$, $1 \leq i \leq m$, and $\mathfrak{l}_{\phi}(\rho) = N - 1$.

Example 3.1. Let $X = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7\}$, $E = \{e_1, e_2, e_3\}$ and $\mathcal{L} = \{0, 1, 2, \dots, 5\}$. Suppose that $A = \{e_1, e_3\}$. Then

$$\begin{aligned} \mathcal{U}_A^{(3,6)} = & \{(e_1, \{\langle \rho_1, 3, (.3, .4, .5; .5, .4, .3; .1, .6, .4) \rangle, \langle \rho_4, 4, (.4, .3, .1; .4, .4, .2; .3, .4, .5) \rangle, \\ & \langle \rho_7, 5, (.6, .2, .55; .3, .1, .2; .4, .3, .5) \rangle\}), (e_3, \{\langle \rho_2, 2, (.2, .1, .7; .6, .6, .5; .8, .4, .6) \rangle, \\ & \langle \rho_3, 1, (.5, .6, .7; .7, .5, .4; .4, .3, .1) \rangle, \langle \rho_5, 2, (.4, .6, .2; .7, .3, .5; .7, .5, .3) \rangle\})\} \end{aligned}$$

is a 3PDN6S set over X . The tabular representation of $\mathcal{U}_A^{(3,6)}$ is

$\mathcal{U}_A^{(3,6)}$	e_1	e_2	e_3
ρ_1	$\langle 3, (.3, .4, .5; .5, .4, .3; .1, .6, .4) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$
ρ_2	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 2, (.2, .1, .7; .6, .6, .5; .8, .4, .6) \rangle$
ρ_3	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 1, (.5, .6, .7; .7, .5, .4; .4, .3, .1) \rangle$
ρ_4	$\langle 4, (.4, .3, .1; .4, .4, .2; .3, .4, .5) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$
ρ_5	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 2, (.4, .6, .2; .7, .3, .5; .7, .5, .3) \rangle$
ρ_6	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$
ρ_7	$\langle 5, (.6, .2, .55; .3, .1, .2; .4, .3, .5) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle$

The corresponding 3PDN6S matrix is $(\mathcal{U}, A, 3, 6) = [a_{ij}, d_{ij}, b_{ij}]_{7 \times 3}$

$$= \begin{pmatrix} \langle 3, (.3, .4, .5; .5, .4, .3; .1, .6, .4) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 2, (.2, .1, .7; .6, .6, .5; .8, .4, .6) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 1, (.5, .6, .7; .7, .5, .4; .4, .3, .1) \rangle \\ \langle 4, (.4, .3, .1; .4, .4, .2; .3, .4, .5) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 2, (.4, .6, .2; .7, .3, .5; .7, .5, .3) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 5, (.6, .2, .55; .3, .1, .2; .4, .3, .5) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix}$$

Definition 3.4. Let $\mathcal{U}_A^{(m, N_1)}$ and $\mathcal{U}_B^{(m, N_2)}$ be MPDNNS sets over X . Then $\mathcal{U}_A^{(m, N_1)}$ is *MPDNNS* subset of $\mathcal{U}_B^{(m, N_2)}$, i.e., $\mathcal{U}_A^{(m, N_1)} \subseteq \mathcal{U}_B^{(m, N_2)}$, if

- $A \subseteq B$,
- $N_1 \leq N_2$,
- $\mu_{i,A}(\rho) \leq \mu_{i,B}(\rho)$, $1 \leq i \leq m$,
- $\lambda_{i,A}(\rho) \geq \lambda_{i,B}(\rho)$, $1 \leq i \leq m$,
- $\nu_{i,A}(\rho) \geq \nu_{i,B}(\rho)$, $1 \leq i \leq m$, and
- $\mathfrak{l}_A(\rho) \leq \mathfrak{l}_B(\rho)$.

It is worth mentioning that $\mathcal{U}_A^{(m, N_1)} \subseteq \mathcal{U}_B^{(m, N_2)}$ it is not necessary each element of $\mathcal{U}_A^{(m, N_1)}$ is also in $\mathcal{U}_B^{(m, N_2)}$.

Definition 3.5. Let $\mathcal{U}_A^{(m,N_1)}$ and $\mathcal{U}_B^{(m,N_2)}$ be MPDNNS sets over X . Then $\mathcal{U}_A^{(m,N_1)}$ is *MPDNNS* equal of $\mathcal{U}_B^{(m,N_2)}$ i.e., $\mathcal{U}_A^{(m,N_1)} \cong \mathcal{U}_B^{(m,N_2)}$, if

- $A = B$,
- $N_1 = N_2$,
- $\mu_{i,A}(\rho) = \mu_{i,B}(\rho)$, $1 \leq i \leq m$,
- $\lambda_{i,A}(\rho) = \lambda_{i,B}(\rho)$, $1 \leq i \leq m$,
- $\nu_{i,A}(\rho) = \nu_{i,B}(\rho)$, $1 \leq i \leq m$, and
- $\mathfrak{l}_A(\rho) = \mathfrak{l}_B(\rho)$.

Proposition 3.1. If $\mathcal{U}_A^{(m,N)}$ is any MPDNNS set over X , then

- (i) $\mathcal{U}_\phi^{(m,0)} \cong \mathcal{U}_A^{(m,N)}$,
- (ii) $\mathcal{U}_A^{(m,N)} \cong \mathcal{U}_E^{(m,N-1)}$.

Proof.

(i) The truth-membership, degree of indeterminacy and degree of falsity always fall in $[0, 1]$ according to Definition 3.1 of MPDNNS set. So, $0 \leq \mu_{i,A}(\rho)$, $\nu_{i,A}(\rho) \leq 1$, $\lambda_{i,A}(\rho) \leq 1$, and grading value $0 \leq \mathfrak{l}_A(\rho)$, $\forall \rho \in X$ and $1 \leq i \leq m$.

Thus, it follows from Definitions 3.4 and 3.2 of null MPDNNS set $\mathcal{U}_\phi^{(m,0)}$. Hence, $\mathcal{U}_\phi^{(m,0)} \cong \mathcal{U}_A^{(m,N)}$.

(ii) Clearly, $\mu_{i,A}(\rho) \leq 1$, $\nu_{i,A}(\rho) \geq 0$ and $\lambda_{i,A}(\rho) \geq 0$ and grading value $\mathfrak{l}_A(\rho) \leq N-1$ for all $\rho \in X$ and $1 \leq i \leq m$. Thus, it follows from Definitions 3.4 and 3.3 of absolute MPDNNS set. Hence, $\mathcal{U}_A^{(m,N)} \cong \mathcal{U}_E^{(m,N-1)}$.

Proposition 3.2. If $\mathcal{U}_A^{(m,N)}$, $\mathcal{U}_B^{(m,N)}$ and $\mathcal{U}_C^{(m,N)}$ are MPDNNS sets over X , then

- (i) $\mathcal{U}_A^{(m,N)} \cong \mathcal{U}_A^{(m,N)}$.
- (ii) $\mathcal{U}_A^{(m,N)} \cong \mathcal{U}_B^{(m,N)}$ and $\mathcal{U}_B^{(m,N)} \cong \mathcal{U}_C^{(m,N)} \Rightarrow \mathcal{U}_A^{(m,N)} \cong \mathcal{U}_C^{(m,N)}$.

Definition 3.6. Let $\mathcal{U}_A^{(m,N_1)}$ and $\mathcal{U}_B^{(m,N_2)}$ be MPDNNS sets over X . Then their *extended union* is defined as $\mathcal{U}_C^{(m,N)^*} = \mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_E \mathcal{U}_B^{(m,N_2)}$,

$$\mathcal{U}_C^{(m,N)^*} = \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_C(\rho) \rangle}{(\mu_{1,C}(\rho), \mu_{2,C}(\rho), \dots, \mu_{m,C}(\rho); \lambda_{1,C}(\rho), \lambda_{2,C}(\rho) \dots, \lambda_{m,C}(\rho); \nu_{1,C}(\rho), \nu_{2,C}(\rho) \dots, \nu_{m,C}(\rho))} \right\} \right) \mid e \in C, \rho \in X, \mathfrak{l}_C(\rho) \in \mathfrak{L} \right\}$$

where,

- $C = A \cup B$,
- $(m, N)^* = (m, \max\{N_1, N_2\})$,

- $\mu_C(\rho) = \max\{\mu_{i,A}(\rho), \mu_{i,B}(\rho)\}, 1 \leq i \leq m,$
- $\lambda_C(\rho) = \min\{\lambda_{i,A}(\rho), \lambda_{i,B}(\rho)\} 1 \leq i \leq m,$
- $\nu_C(\rho) = \min\{\nu_{i,A}(\rho), \nu_{i,B}(\rho)\} 1 \leq i \leq m,$ and
- $\mathfrak{l}_C(\rho) = \max\{\mathfrak{l}_A(\rho), \mathfrak{l}_B(\rho)\} \forall e \in C.$

Definition 3.7. Let $\mathcal{U}_A^{(m,N_1)}$ and $\mathcal{U}_B^{(m,N_2)}$ be MPDNNS sets over X . Then their *restricted union* is defined as $\mathcal{U}_D^{(m,N)^\diamond} = \mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_R \mathcal{U}_B^{(m,N_2)},$

$$\mathcal{U}_D^{(m,N)^\diamond} = \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_D(\rho) \rangle}{(\mu_{1,D}(\rho), \mu_{2,D}(\rho), \dots, \mu_{m,D}(\rho); \lambda_{1,D}(\rho), \lambda_{2,D}(\rho) \dots, \lambda_{m,D}(\rho); \nu_{1,D}(\rho), \nu_{2,D}(\rho) \dots, \nu_{m,D}(\rho))} \right\} \right) \right. \\ \left. | e \in D, \rho \in X, \mathfrak{l}_D(\rho) \in \mathfrak{L} \right\}$$

where,

- $D = A \cap B,$
- $(m, N)^\diamond = (m, \max\{N_1, N_2\}),$
- $\mu_D(\rho) = \max\{\mu_{i,A}(\rho), \mu_{i,B}(\rho)\}, 1 \leq i \leq m,$
- $\lambda_D(\rho) = \min\{\lambda_{i,A}(\rho), \lambda_{i,B}(\rho)\} 1 \leq i \leq m,$
- $\nu_D(\rho) = \min\{\nu_{i,A}(\rho), \nu_{i,B}(\rho)\} 1 \leq i \leq m,$ and
- $\mathfrak{l}_D(\rho) = \max\{\mathfrak{l}_A(\rho), \mathfrak{l}_B(\rho)\} \forall e \in D.$

Example 3.2. Let $X = \{\rho_i: i = 1, 2, \dots, 8\}, E = \{e_i: i = 1, 2, 3\}$ and $\mathfrak{L} = \{0, 1, 2, \dots, 12\}.$ Assume that $A = \{e_1, e_3\} \subseteq E$ and $B = \{e_2, e_3\} \subseteq E.$ Then

$$\mathcal{U}_A^{(3,13)} = \{(e_1, \{\langle \rho_2, 11, (.1, .9, .7; .5, .3, 2; .4, .6, .8) \rangle, \langle \rho_5, 9, (.2, .1, .2; .3, .4, .5; .6, .7, .8) \rangle, \\ \langle \rho_7, 8, (.3, .4, .5; .6, .7, .6; .5, .4, .3) \rangle\}), (e_3, \{\langle \rho_3, 3, (.4, .3, .4; .5, .6, .8; .7, .6, .5) \rangle, \\ \langle \rho_4, 6, (.5, .4, .5; .6, .7, .7; .6, .4, .2) \rangle, \langle \rho_6, 5, (.6, .5, .6; .7, .8, .9; .8, .7, .6) \rangle\})\}$$

is a 3PDN13S set over X . The corresponding $\mathcal{U}_A^{(3,13)}$ matrix is

$$\mathcal{U}_A^{(3,13)} = [[c_i, (a_{ij}; d_{ij}; b_{ij})]]_{8 \times 3}$$

$$= \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 11, (.1, .9, .7; .5, .3, 0.2; .4, .6, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 3, (.4, .3, .4; .5, .6, .8; .7, .6, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 6, (.5, .4, .5; .6, .7, .7; .6, .4, .2) \rangle \\ \langle 9, (.2, .1, .2; .3, .4, .5; .6, .7, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (.6, .5, .6; .7, .8, .9; .8, .7, .6) \rangle \\ \langle 8, (.3, .4, .5; .6, .7, .6; .5, .4, .3) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (1)$$

$$\begin{aligned} \mathcal{U}_B^{(3,10)} = \{ & (e_2, \{ \langle \rho_1, 9, (.7, .6, .4; .2, .1, .4; .3, .5, .7) \rangle, \langle \rho_4, 6, (.8, .6, .5; .6, .4, .7; .5, .8, .9) \rangle, \\ & \langle \rho_8, 8, (.1, .3, .4; .8, .7, .4; .7, .3, .1) \rangle \}), (e_3, \{ \langle \rho_2, 1, (.2, .1, .3; .6, .7, .9; .7, .3, .6) \rangle, \\ & \langle \rho_5, 5, (.1, .2, .4; .8, .7, .1; .8, .3, .7) \rangle, \langle \rho_7, 3, (.2, .3, .6; .9, .8, .7; .8, .7, .5) \rangle \}) \} \end{aligned}$$

is a 3PDF10S set over X . The corresponding $\mathcal{U}_B^{(3,10)}$ matrix is

$$\begin{aligned} \mathcal{U}_B^{(3,10)} &= [[c_i, (a_{ij}; b_{ij})]]_{8 \times 3} \\ \mathcal{U}_B^{(3,10)} &= \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 9, (.7, .6, .4; .2, .1, .4; .3, .5, .7) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 1, (.2, .1, .3; .6, .7, .9; .7, .3, .6) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 6, (.8, .6, .5; .6, .4, .7; .5, .8, .9) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (.1, .2, .4; .8, .7, .1; .8, .3, .7) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 3, (.2, .3, .6; .9, .8, .7; .8, .7, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 8, (.1, .3, .4; .8, .7, .4; .7, .3, .1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (2) \end{aligned}$$

Extended union of $\mathcal{U}_A^{(3,13)}$ and $\mathcal{U}_B^{(3,10)}$ is given below in Eq. (3):

$$\mathcal{U}_C^{(3,13)} = \mathcal{U}_A^{(3,13)} \widetilde{\cup} \mathcal{U}_B^{(3,10)} \text{ where } C = \{e_1, e_2, e_3\}$$

$$= \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 9, (.7, .6, .4; .2, .1, .4; .3, .5, .7) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 11, (.1, .9, .7; .5, .3, .2; .4, .6, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 1, (.2, .1, .3; .6, .7, .9; .7, .3, .6) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 3, (.4, .3, .4; .5, .6, .8; .7, .6, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 6, (.8, .6, .5; .6, .4, .7; .5, .8, .9) \rangle & \langle 6, (.5, .4, .5; .6, .7, .7; .6, .4, .2) \rangle \\ \langle 9, (.2, .1, .2; .3, .4, .5; .6, .7, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (.1, .2, .4; .8, .7, .1; .8, .3, .7) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (.6, .5, .6; .7, .8, .9; .8, .7, .6) \rangle \\ \langle 8, (.3, .4, .5; .6, .7, .6; .5, .4, .3) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 3, (.2, .3, .6; .9, .8, .7; .8, .7, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 8, (.1, .3, .4; .8, .7, .4; .7, .3, .1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (3)$$

Restricted union of $\mathcal{U}_A^{(3,13)}$ and $\mathcal{U}_B^{(3,10)}$ is given below in Eq. (4)

$$\mathcal{U}_D^{(3,13)} = \mathcal{U}_A^{(3,13)} \widetilde{\cap}_R \mathcal{U}_B^{(3,10)} \text{ where } D = \{e_3\}$$

$$= \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 1, (.2, .1, .3; .6, .7, .9; .7, .3, .6) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 3, (.4, .3, .4; .5, .6, .8; .7, .6, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 6, (.5, .4, .5; .6, .7, .7; .6, .4, .2) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (.1, .2, .4; .8, .7, .1; .8, .3, .7) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (.6, .5, .6; .7, .8, .9; .8, .7, .6) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 3, (.2, .3, .6; .9, .8, .7; .8, .7, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (4)$$

Definition 3.8. Let $\mathcal{U}_A^{(m,N_1)}$ and $\mathcal{U}_B^{(m,N_2)}$ be MPDFNS set over X . Then their *extended intersection* is defined as

$$\mathcal{U}_C^{(m,N)*} = \mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_E \mathcal{U}_B^{(m,N_2)},$$

$$\mathcal{U}_C^{(m,N)*} = \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_C(\rho) \rangle}{(\mu_{1,C}(\rho), \mu_{2,C}(\rho), \dots, \mu_{m,C}(\rho); \nu_{1,C}(\rho), \nu_{2,C}(\rho) \dots, \nu_{m,C}(\rho))} \right\} \right) : \right.$$

$$\left. e \in C, \rho \in X, \mathfrak{l}_C(\rho) \in \mathfrak{L} \right\}$$

where,

- $C = A \cup B$,
- $(m, N)^* = (m, \min\{N_1, N_2\})$,
- $\mu_C(\rho) = \min\{\mu_{i,A}(\rho), \mu_{i,B}(\rho)\}$, $1 \leq i \leq m$,
- $\nu_C(\rho) = \max\{\nu_{i,A}(\rho), \nu_{i,B}(\rho)\}$ $1 \leq i \leq m$, and
- $\mathfrak{l}_C(\rho) = \min\{\mathfrak{l}_A(\rho), \mathfrak{l}_B(\rho)\} \forall e \in C$.

Definition 3.9. Let $\mathcal{U}_A^{(m,N_1)}$ and $\mathcal{U}_B^{(m,N_2)}$ be MPDFNS set over X . Then their *restricted intersetion* is defined as

$$\mathcal{U}_D^{(m,N)^\diamond} = \mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_R \mathcal{U}_B^{(m,N_2)},$$

$$\mathcal{U}_D^{(m,N)^\diamond} = \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_D(\rho) \rangle}{(\mu_{1,D}(\rho), \mu_{2,D}(\rho), \dots, \mu_{m,D}(\rho); \nu_{1,D}(\rho), \nu_{2,D}(\rho) \dots, \nu_{m,D}(\rho))} \right\} \right) : \right. \\ \left. e \in D, \rho \in X, \mathfrak{l}_D(\rho) \in \mathfrak{L} \right\}$$

where,

- $D = A \cap B$,
- $(m, N)^\diamond = (m, \min\{N_1, N_2\})$,
- $\mu_D(\rho) = \min\{\mu_{i,A}(\rho), \mu_{i,B}(\rho)\}$, $1 \leq i \leq m$,
- $\nu_D(\rho) = \max\{\nu_{i,A}(\rho), \nu_{i,B}(\rho)\}$ $1 \leq i \leq m$, and
- $\mathfrak{l}_D(\rho) = \min\{\mathfrak{l}_A(\rho), \mathfrak{l}_B(\rho)\} \forall e \in D$.

Example 3.3. Consider $\mathcal{U}_A^{(3,13)}$ and $\mathcal{U}_B^{(3,10)}$ as given in Example 3.2 by Eqs. (1) and (2), respectively. Extended intersection of $\mathcal{U}_A^{13}$ and $\mathcal{U}_B^{10}$ is given below in Eq. (5):

$$\mathcal{U}_C^{(3,10)} = \mathcal{U}_A^{(3,13)} \widetilde{\cap}_E \mathcal{U}_B^{(3,10)} \text{ where } C = \{e_1, e_2, e_3\}$$

$$\mathcal{U}_C^{(3,10)} = \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (5)$$

Restricted intersection of $\mathcal{U}_A^{(3,13)}$ and $\mathcal{U}_B^{(3,10)}$ is given below Eq. (6):

$$\mathcal{U}_D^{(3,10)} = \mathcal{U}_A^{(3,13)} \widetilde{\cap}_R \mathcal{U}_B^{(3,10)} \text{ where } D = \{e_3\}$$

$$\mathcal{U}_D^{(3,10)} = \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (6)$$

Proposition 3.3. If $\mathcal{U}_A^{(m,N_1)}$, $\mathcal{U}_B^{(m,N_2)}$, $\mathcal{U}_C^{(m,N_3)}$ are MPDNFNS set over X , then

- (i) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_E \mathcal{U}_A^{(m,N_1)} = \mathcal{U}_A^{(m,N_1)}$.
- (ii) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_E \mathcal{U}_A^{(m,N_1)} = \mathcal{U}_A^{(m,N_1)}$.
- (iii) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_E \mathcal{U}_B^{(m,N_2)} = \mathcal{U}_B^{(m,N_2)} \widetilde{\cap}_E \mathcal{U}_A^{(m,N_1)}$.
- (iv) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_E \mathcal{U}_B^{(m,N_2)} = \mathcal{U}_B^{(m,N_2)} \widetilde{\cup}_E \mathcal{U}_A^{(m,N_1)}$.
- (v) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_E \left(\mathcal{U}_B^{(m,N_2)} \widetilde{\cap}_E \mathcal{U}_C^{(m,N_3)} \right) = \left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_E \mathcal{U}_B^{(m,N_2)} \right) \widetilde{\cap}_E \mathcal{U}_C^{(m,N_3)}$.
- (vi) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_E \left(\mathcal{U}_B^{(m,N_2)} \widetilde{\cup}_E \mathcal{U}_C^{(m,N_3)} \right) = \left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_E \mathcal{U}_B^{(m,N_2)} \right) \widetilde{\cup}_E \mathcal{U}_C^{(m,N_3)}$.
- (vii) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_R \mathcal{U}_A^{(m,N_1)} = \mathcal{U}_A^{(m,N_1)}$.
- (viii) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_R \mathcal{U}_A^{(m,N_1)} = \mathcal{U}_A^{(m,N_1)}$.
- (ix) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_R \mathcal{U}_B^{(m,N_2)} = \mathcal{U}_B^{(m,N_2)} \widetilde{\cap}_R \mathcal{U}_A^{(m,N_1)}$.
- (x) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_R \mathcal{U}_B^{(m,N_2)} = \mathcal{U}_B^{(m,N_2)} \widetilde{\cup}_R \mathcal{U}_A^{(m,N_1)}$.
- (xi) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_R \left(\mathcal{U}_B^{(m,N_2)} \widetilde{\cap}_R \mathcal{U}_C^{(m,N_3)} \right) = \left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_R \mathcal{U}_B^{(m,N_2)} \right) \widetilde{\cap}_R \mathcal{U}_C^{(m,N_3)}$.
- (xii) $\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_R \left(\mathcal{U}_B^{(m,N_2)} \widetilde{\cup}_R \mathcal{U}_C^{(m,N_3)} \right) = \left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_R \mathcal{U}_B^{(m,N_2)} \right) \widetilde{\cup}_R \mathcal{U}_C^{(m,N_3)}$.

Proof. The proof is obvious from Definitions 3.4, 3.5, 3.6, 3.7, 3.8 and 3.9.

Proposition 3.4. If $\mathcal{U}_A^{(m,N_1)}$ and $\mathcal{U}_B^{(m,N_2)}$ are MPDNFNS sets over X , then

- (i) $\left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_E \mathcal{U}_B^{(m,N_2)} \right) \cong \mathcal{U}_A^{(m,N_1)} \cong \left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_E \mathcal{U}_B^{(m,N_2)} \right)$.
- (ii) $\left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_E \mathcal{U}_B^{(m,N_2)} \right) \cong \mathcal{U}_B^{(m,N_2)} \cong \left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_E \mathcal{U}_B^{(m,N_2)} \right)$.
- (iii) $\left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_R \mathcal{U}_B^{(m,N_2)} \right) \cong \mathcal{U}_A^{(m,N_1)} \cong \left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_R \mathcal{U}_B^{(m,N_2)} \right)$.
- (iv) $\left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cap}_R \mathcal{U}_B^{(m,N_2)} \right) \cong \mathcal{U}_B^{(m,N_2)} \cong \left(\mathcal{U}_A^{(m,N_1)} \widetilde{\cup}_R \mathcal{U}_B^{(m,N_2)} \right)$.

Proof. Proof of this proposition is obvious from Definitions 3.4, 3.6, 3.8 and 3.9.

4 Complements of m-Polar Neutrosophic Diophantine N-Soft Set

Definition 4.1. Let $\mathcal{U}_A^{(m,N)}$ be MPDNNS sets over X . Then their *weak complement* is represented as $\left(\mathcal{U}_A^{(m,N)}\right)^{wc}$ and defined by

$$\left(\mathcal{U}_A^{(m,N)}\right)^{wc} = \left\{ \left(e, \left\{ \frac{\langle \rho, l_A^{wc}(\rho) \rangle}{\left(\mu_{1,A}^{wc}(\rho), \mu_{2,A}^{wc}(\rho), \dots, \mu_{m,A}^{wc}(\rho); \lambda_{1,A}^{wc}(\rho), \lambda_{2,A}^{wc}(\rho), \dots, \lambda_{m,A}^{wc}(\rho); v_{1,A}^{wc}(\rho), v_{2,A}^{wc}(\rho), \dots, v_{m,A}^{wc}(\rho) \right)} \right\} \right) \mid e \in A, \rho \in X, l_A^{wc}(\rho) \in \mathcal{L} \right\}$$

where,

- $\mu_{i,A}^{wc}(\rho) = v_{i,A}(\rho), 1 \leq i \leq m,$
- $\lambda_{i,A}^{wc}(\rho) = 1 - \lambda_{i,A}(\rho), 1 \leq i \leq m,$
- $v_{i,A}^{wc}(\rho) = \mu_{i,A}(\rho), 1 \leq i \leq m,$ and
- $l_A^{wc}(\rho) \cap l_A(\rho) = \emptyset, \forall e \in A.$

Example 4.1. Consider $\mathcal{U}_A^{(3,13)}$ and $\mathcal{U}_B^{(3,10)}$ as given in Example 3.2 by Eqs. (1) and (2), respectively. Weak complement of $\mathcal{U}_A^{(3,13)}$ and $\mathcal{U}_B^{(3,10)}$ are given below in Eqs. (7) and (8):

$$\mathcal{U}_A^{(3,13)wc} = \left(\begin{array}{ccc} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 10, (.4, .6, .8; .5, .7, 0.8; .1, .9, .7) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (.7, .6, .5; .5, .4, .2; .4, .3, .4) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 8, (.6, .4, .2; .4, .3, .3; .5, .4, .5) \rangle \\ \langle 8, (.6, .7, .8; .7, .6, .5; .2, .1, .2) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 7, (.8, .7, .6; .3, .2, 1; .6, .5, .6) \rangle \\ \langle 6, (.5, .4, .3; .4, .3, .4; .3, .4, .5) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{array} \right) \quad (7)$$

$$\begin{aligned}
& \mathcal{U}_B^{(3,13)wc} \\
&= \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 8, (.3, .5, .7; .8, .9, .6; .7, .6, .4) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 7, (.7, .3, .6; .4, .3, .1; .2, .1, .3) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 4, (.5, .8, .9; .4, .6, .3; .8, .6, .5) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 6, (.8, .3, .7; .2, .3, .9; .1, .2, .4) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 2, (.8, .7, .5; .1, .2, .3; .2, .3, .6) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 6, (.7, .3, .1; .8, .7, .4; .1, .3, .4) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (8)
\end{aligned}$$

Remark 4.1 Again taking week complement of $\left(\mathcal{U}_A^{(3,13)}\right)^{wc}$ and $\left(\mathcal{U}_B^{(3,10)}\right)^{wc}$ which given above in Eqs. (7), (8), respectively. There compliments are given below in Eqs. (9) and (10), respectively.

$$\begin{aligned}
& \left(\left(\mathcal{U}_A^{(3,13)wc}\right)^{wc}\right) \\
&= \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 6, (.1, .9, .7; .5, .3, .0; .4, .6, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 4, (.4, .3, .4; .5, .6, .8; .7, .6, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (.5, .4, .5; .6, .7, .7; .6, .4, .2) \rangle \\ \langle 7, (.2, .1, .2; .3, .4, .5; .6, .7, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 7, (.6, .5, .6; .7, .8, .9; .8, .7, .6) \rangle \\ \langle 7, (.3, .4, .5; .6, .7, .6; .5, .4, .3) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (9)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\mathcal{U}_A^{(3,10)wc}\right)^{wc}\right) \\
&= \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 4, (.7, .6, .4; .2, .1, .4; .3, .5, .7) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 3, (.2, .1, .3; .6, .7, .9; .7, .3, .6) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 7, (.8, .6, .5; .6, .4, .7; .5, .8, .9) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 6, (.1, .2, .4; .8, .7, .1; .8, .3, .7) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (.2, .3, .6; .9, .8, .7; .8, .7, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 9, (.1, .3, .4; .8, .7, .4; .7, .3, .1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (10)
\end{aligned}$$

Hence from Eqs. (1), (2), (9) and (10). We conclude that $((\mathcal{U}_A^{(3,13)})^{wc})^{wc} \neq \mathcal{U}_A^{(3,13)}$ and $((\mathcal{U}_B^{(3,10)})^{wc})^{wc} \neq \mathcal{U}_B^{(3,10)}$.

Since null 3PDN10S set is

$$\mathcal{U}_{\emptyset}^{(3,0)} = \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (11)$$

Since absolute 3PDN10S set is

$$\mathcal{U}_E^{(3,10)} = \begin{pmatrix} \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 9, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \end{pmatrix} \quad (12)$$

Weak complement of $\mathcal{U}_{\emptyset}^{(3,0)}$ and $\mathcal{U}_E^{(3,10)}$ are given below in Eqs. (13) and (14), respectively.

$$(\mathcal{U}_{\Phi}^{(3,0)})^{wc} = \begin{pmatrix} \langle 7, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 6, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 4, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 1, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 8, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 7, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 6, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 7, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 3, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 5, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 2, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 4, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 7, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 8, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 6, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 8, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 7, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 2, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \\ \langle 1, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 6, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle & \langle 5, (1, 1, 1; 0, 0, 0; 0, 0, 0) \rangle \end{pmatrix} \quad (13)$$

$$(\mathcal{U}_E^{(3,10)})^{wc} = \begin{pmatrix} \langle 9, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 1, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 3, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 9, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 8, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 9, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 6, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 2, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 7, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 4, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 7, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 4, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 6, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 5, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 1, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 8, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 9, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (14)$$

From Eqs. (11)–(14), we have $(\mathcal{U}_\emptyset^{(3,0)})^{wc} \neq \mathcal{U}_E^{(3,10)}$ and $(\mathcal{U}_E^{(3,10)})^{wc} \neq \mathcal{U}_\emptyset^{(3,0)}$. Hence, we have following result.

Remark 4.2 Let $\mathcal{U}_A^{(m,N)}$ be MPDNNS sets, $\mathcal{U}_\emptyset^{(m,0)}$ be null MPDNNS and $\mathcal{U}_E^{(m,N-1)}$ absolute MPDNNS over X , then following results that hold in crisp set theory but not hold in MPDNNS set theory

- (i) $(\mathcal{U}_\emptyset^{(m,0)})^{wc} \neq \mathcal{U}_E^{(m,N-1)}$.
- (ii) $(\mathcal{U}_E^{(m,N-1)})^{wc} \neq \mathcal{U}_\emptyset^{(m,0)}$.
- (iii) $((\mathcal{U}_A^{(m,N)})^{wc})^{wc} \neq \mathcal{U}_A^{(m,N)}$.

Definition 4.2. Let $\mathcal{U}_A^{(m,N)}$ be MPDNNS set over X . Then their *top weak complement* is represented as $(\mathcal{U}_A^{(m,N)})^{twc}$ and defined by

$$(\mathcal{U}_A^{(m,N)})^{twc} = \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_A^{twc}(\rho) \rangle}{(\mu_{1,A}^{twc}(\rho), \mu_{2,A}^{twc}(\rho), \dots, \mu_{m,A}^{twc}(\rho); \lambda_{1,A}^{twc}(\rho), \lambda_{2,A}^{twc}(\rho), \dots, \lambda_{m,A}^{twc}(\rho); v_{1,A}^{twc}(\rho), v_{2,A}^{twc}(\rho) \dots, v_{m,A}^{twc}(\rho))} \right\} \right) \mid e \in A, \rho \in X, \mathfrak{l}_A^{twc}(\rho) \in \mathfrak{L} \right\}$$

where,

- $\mu_{i,A}^{twc}(\rho) = v_{i,A}(\rho), 1 \leq i \leq m,$
- $\lambda_{i,A}^{twc}(\rho) = 1 - \lambda_{i,A}(\rho), 1 \leq i \leq m,$
- $v_{i,A}^{twc}(\rho) = \mu_{i,A}(\rho), 1 \leq i \leq m,$ and
- $\mathfrak{l}_A^{twc}(\rho) = \begin{cases} N-1, & \text{if } \mathfrak{l}_A(\rho) < N-1, \\ 0, & \text{if } \mathfrak{l}_A(\rho) = N-1, \forall e \in A. \end{cases}$

The top weak complement of $\mathcal{U}_A^{(3,13)}$ and $\mathcal{U}_B^{(3,10)}$ as given in Example 3.2 by Eqs. (1) and (2) are given below in Eqs. (15) and (16), respectively.

$$\begin{aligned}
 & (\mathcal{U}_A^{(3,13)})^{twc} \\
 &= \left(\begin{array}{ccc}
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\
 \langle 12, (.4, .6, .8; .5, .7, 0.8; .1, .9, .7) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.7, .6, .5; .5, .4, .2; .4, .3, .4) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.6, .4, .2; .4, .3, .3; .5, .4, .5) \rangle \\
 \langle 12, (.6, .7, .8; .7, .6, .5; .2, .1, .2) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.8, .7, .6; .3, .2, .1; .6, .5, .6) \rangle \\
 \langle 12, (.5, .4, .3; .4, .3, .4; .3, .4, .5) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle
 \end{array} \right) \quad (15)
 \end{aligned}$$

$$\begin{aligned}
 & (\mathcal{U}_B^{(3,10)})^{twc} \\
 &= \left(\begin{array}{ccc}
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.3, .5, .7; .8, .9, .6; .7, .6, .4) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.7, .3, .6; .4, .3, .1; .2, .1, .3) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.5, .8, .9; .4, .6, .3; .8, .6, .5) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.8, .3, .7; .2, .3, .9; .1, .2, .4) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.8, .7, .5; .1, .2, .3; .2, .3, .6) \rangle \\
 \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.7, .3, .1; .8, .7, .4; .1, .3, .4) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle
 \end{array} \right) \quad (16)
 \end{aligned}$$

Proposition 4.1 Let $\mathcal{U}_A^{(m,N)}$ be MPDNNS set, $\mathcal{U}_{\emptyset}^{(m,0)}$ be null MPDNNS and $\mathcal{U}_E^{(m,N-1)}$ absolute MPDNNS over X , then

- (i) $(\mathcal{U}_{\emptyset}^{(m,0)})^{twc} = \mathcal{U}_E^{(m,N-1)}$.
- (ii) $(\mathcal{U}_E^{(m,N-1)})^{twc} = \mathcal{U}_{\emptyset}^{(m,0)}$.

Remark 4.3. Again taking top weak complement of $(\mathcal{U}_A^{(3,13)})^{twc}$ given in Eq. (15) we have Eq. (17)

$$\begin{aligned}
 & ((\mathcal{U}_A^{(3,13)})^{twc})^{twc} \\
 &= \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (.1, .9, .7; .5, .3, 0.2; .4, .6, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.4, .3, .4; .5, .6, .8; .7, .6, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.5, .4, .5; .6, .7, .7; .6, .4, .2) \rangle \\ \langle 0, (.2, .1, .2; .3, .4, .5; .6, .7, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.6, .5, .6; .7, .8, .9; .8, .7, .6) \rangle \\ \langle 0, (.3, .4, .5; .6, .7, .6; .5, .4, .3) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix}
 \end{aligned} \tag{17}$$

Hence from Eqs. (1) and (17) we conclude that $\left((\mathcal{U}_A^{(3,13)})^{twc}\right)^{twc} \neq \mathcal{U}_A^{(3,13)}$. Hence we have the following result:

Remark 4.4. Let $\mathcal{U}_A^{(m,N)}$ be MPDNNs set, over X , then $((\mathcal{U}_A^{(m,N)})^{twc})^{twc} \neq \mathcal{U}_A^{(m,N)}$.

Definition 4.3. Let $\mathcal{U}_A^{(m,N)}$ be MPDNNs set over X . Then their *bottom weak complement* is represented as $(\mathcal{U}_A^{(m,N)})^{bwc}$ and defined by

$$\begin{aligned}
 & (\mathcal{U}_A^{(m,N)})^{bwc} \\
 &= \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_A^{bwc}(\rho) \rangle}{(\mu_{1,A}^{bwc}(\rho), \mu_{2,A}^{bwc}(\rho), \dots, \mu_{m,A}^{bwc}(\rho); \lambda_{1,A}^{bwc}(\rho), \lambda_{2,A}^{bwc}(\rho), \dots, \lambda_{m,A}^{bwc}(\rho); v_{1,A}^{bwc}(\rho), v_{2,A}^{bwc}(\rho), \dots, v_{m,A}^{bwc}(\rho))} \right\} \right) \right\} \\
 & \quad | e \in A, \rho \in X, \mathfrak{l}_A^{bwc}(\rho) \in \mathcal{L} \}
 \end{aligned}$$

where,

- $\mu_{i,A}^{bwc}(\rho) = v_{i,A}(\rho), 1 \leq i \leq m,$
- $\lambda_{i,A}^{bwc}(\rho) = 1 - \lambda_{i,A}(\rho), 1 \leq i \leq m,$
- $v_{i,A}^{bwc}(\rho) = \mu_{i,A}(\rho), 1 \leq i \leq m,$ and
- $\mathfrak{l}_A^{bwc}(\rho) = \begin{cases} N-1, & \text{if } \mathfrak{l}_A(\rho) = 0, \\ 0, & \text{if } \mathfrak{l}_A(\rho) > 0, \forall e \in A. \end{cases}$

Example 4.2. Consider $\mathcal{U}_A^{(3,13)}$ and $\mathcal{U}_B^{(3,10)}$ as given in Example 3.2 by Eqs. (1) and (2), respectively. Bottom weak complement of $\mathcal{U}_A^{(3,13)}$ and $\mathcal{U}_B^{(3,10)}$ are given below in Eqs. (18) and (19):

$$(\mathcal{U}_A^{(3,13)})^{bwc} = \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (.4, .6, .8; .5, .7, 0.8; .1, .9, .7) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.7, .6, .5; .5, .4, .2; .4, .3, .4) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.6, .4, .2; .4, .3, .3; .5, .4, .5) \rangle \\ \langle 0, (.6, .7, .8; .7, .6, .5; .2, .1, .2) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.8, .7, .6; .3, .2, .1; .6, .5, .6) \rangle \\ \langle 0, (.5, .4, .3; .4, .3, .4; .3, .4, .5) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (18)$$

$$(\mathcal{U}_B^{(3,13)})^{bwc} = \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.3, .5, .7; .8, .9, .6; .7, .6, .4) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.7, .3, .6; .4, .3, .1; .2, .1, .3) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.5, .8, .9; .4, .6, .3; .8, .6, .5) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.8, .3, .7; .2, .3, .9; .1, .2, .4) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.8, .7, .5; .1, .2, .3; .2, .3, .6) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (.7, .3, .1; .8, .7, .4; .1, .3, .4) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (19)$$

Proposition 4.2. Let $\mathcal{U}_A^{(m,N)}$ be MPDNNS set, $\mathcal{U}_{\emptyset}^{(m,0)}$ be null MPDNNS and $\mathcal{U}_E^{(m,N-1)}$ absolute MPDNNS over X , then

$$(i) (\mathcal{U}_{\emptyset}^{(m,0)})^{bwc} = \mathcal{U}_E^{(m,N: amp: minus; 1)}.$$

$$(ii) (\mathcal{U}_E^{(m,N-1)})^{bwc} = \mathcal{U}_{\emptyset}^{(m,0)}.$$

Remark 4.5. Again taking bottom weak complement of $(\mathcal{U}_A^{(3,13)})^{bwc}$ as given above in Eq. (18) we have Eq. (20) given below:

$$\begin{aligned}
 & ((\mathcal{U}_A^{(3,13)})^{bwc})^{bwc} \\
 &= \begin{pmatrix} \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 12, (.1, .9, .7; .5, .3, 0.2; .4, .6, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.4, .3, .4; .5, .6, .8; .7, .6, .5) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.5, .4, .5; .6, .7, .7; .6, .4, .2) \rangle \\ \langle 12, (.2, .1, .2; .3, .4, .5; .6, .7, .8) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 12, (.6, .5, .6; .7, .8, .9; .8, .7, .6) \rangle \\ \langle 12, (.3, .4, .5; .6, .7, .6; .5, .4, .3) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1) \rangle \end{pmatrix} \quad (20)
 \end{aligned}$$

Hence from Eqs. (1) and (20) we conclude that $\left((\mathcal{U}_A^{(3,13)})^{bwc} \right)^{bwc} \neq \mathcal{U}_A^{(3,13)}$.

5 Relationships with Existing Models and Application in Multi-Attribute Group Decision-Making

The comparison with existing sets is shown in Tab. 1.

The following proposed algorithm for the choice values of MPDNNs:

An Algorithm for the Choice Values of MPDNNs

- 1: $S = \{\rho_1, \rho_2, \dots, \rho_m\}$ is the universal set.
- 2: $T = \{e_1, e_2, \dots, e_n\}$ set of attributes.
- 3: Input MPDFNSs $\psi_A^{(m,n)}$ with $N = \{0, 1, 2, \dots, N - 1\}$.
- 4: Calculate $M_i^m = \left(\sum_{j=1}^m d_{\alpha_{ij}}, \sum_{j=1}^m R_{\alpha_{ij}} \right)$. Here,

$$d_{\alpha_{ij}} = 1 - \frac{2\theta_a}{\pi}, \quad R_{\alpha_{ij}} = \frac{1}{2} + r_a \left(\frac{1}{2} - \frac{2\theta_a}{\pi} \right),$$

$$r_a = \left(\mu_1^2 + \mu_2^2 + \dots + \mu_m^2 + \lambda_1^2 + \lambda_2^2 + \dots + \lambda_m^2 + v_1^2 + v_2^2 + \dots + v_m^2 \right)^{1/2},$$

$$\theta_a = \tan^{-1} \left(\frac{\langle (\lambda_1, \lambda_2, \dots, \lambda_m), (v_1, v_2, \dots, v_m) \rangle}{\|(\mu_1, \mu_2, \dots, \mu_m)\|} \right),$$

where, $\langle (\lambda_1, \lambda_2, \dots, \lambda_m), (v_1, v_2, \dots, v_m) \rangle = \lambda_1 v_1 + \lambda_2 v_2 + \dots + \lambda_m v_m$.

- 5: Calculate $M_t^m = \max \{M_i^m\}$ with $i = 1, 2, \dots, n$.
- 6: $M_t^m = \max \{M_i^m\}$ can be chosen for any alternative.

Table 1: Comparison of proposed model with existing models

Sets	Truth membership	Falsity membership	Indeterminacy	Parametrization	Non-binary evaluation	Multi-polarity
Fuzzy set	✓	×	×	×	×	×
Intuitionistic fuzzy set	✓	✓	×	×	×	×
Neutrosopic set	✓	✓	✓	×	×	×
Soft set	×	×	×	✓	×	×
N-soft set	×	×	×	✓	✓	×
Fuzzy N-soft set	✓	×	×	✓	✓	×
Intuitionistic N-soft set	✓	✓	×	✓	✓	×
Neutrosopic N-soft set	✓	✓	✓	✓	✓	×
m-polar fuzzy set	✓	×	×	×	×	✓
m-polar diophantine neutrosopic N-soft set	✓	✓	✓	✓	✓	✓

Example 5.1. We assume that there are five candidates which appear in the interview and their interviews will be completed in three days. There are three judges which are observed to each candidate separately. The following data are elaborated in [Tab. 2](#), of each candidate c_i , $i = 1, 2, 3, 4, 5$ which are observed by from judges j_i , $i = 1, 2, 3$.

Table 2: Data of interview of each candidates observed by judges

$\mathcal{U}_C^{(3,11)}$	j_1	j_2	j_3
c_1	$\langle 4, (.3, .2, .5; .2, .7, .8; .1, .4, .6) \rangle$	$\langle 6, (.5, .4, .7; .6, .5, .4; .3, .6, .5) \rangle$	$\langle 7, (.3, .5, .5; .3, .6, .7; .6, .4, .3) \rangle$
c_2	$\langle 6, (.6, .4, .6; .3, .5, .7; .5, .4, .3) \rangle$	$\langle 5, (.2, .5, .6; .4, .6, .9; .1, .2, .3) \rangle$	$\langle 8, (.2, .6, .5; .5, .8, .6; .3, .1, .4) \rangle$
c_3	$\langle 7, (.4, .6, .5; .3, .5, .7; .1, .4, .3) \rangle$	$\langle 2, (.4, .5, .3; .5, .6, .2; .5, .4, .5) \rangle$	$\langle 3, (.5, .3, .5; .4, .3, .7; .4, .3, .6) \rangle$
c_4	$\langle 3, (.5, .3, .4; .4, .5, .6; .3, .2, .5) \rangle$	$\langle 8, (.3, .6, .4; .4, .9, .6; .5, .4, .3) \rangle$	$\langle 9, (.5, .4, .5; .3, .7, .6; .4, .3, .2) \rangle$
c_5	$\langle 1, (.6, .5, .4; .3, .5, .7; .3, .4, .4) \rangle$	$\langle 4, (.6, .5, .4; .4, .7, .1; .3, .5, .5) \rangle$	$\langle 7, (.4, .3, .5; .2, .5, .7; .4, .6, .4) \rangle$

In [Tab. 3](#), we find the choice values of all candidates by using proposed algorithm.

From [Tab. 4](#), the candidates c_5 , c_2 , c_2 are leading with respect to days first, second, and third, respectively.

The working of proposed algorithm is shown in [Fig. 1](#).

Table 3: Tabular representation of choice value of $\psi_C^{(3,11)}$

$\mathcal{U}_C^{(3,11)}$	j_1	j_2	j_3	M_i^1	M_i^2	M_i^3
c_1	$\langle 4, (.3, .2, .5; .2, .7, .8; .1, .4, .6) \rangle$	$\langle 6, (.5, .4, .7; .6, .5, .4; .3, .6, .5) \rangle$	$\langle 7, (.3, .5, .5; .3, .6, .7; .6, .4, .3) \rangle$	(17, 2.59680)	(17, 1.78017)	(17, 1.64489)
c_2	$\langle 6, (.6, .4, .6; .3, .5, .7; .5, .4, .3) \rangle$	$\langle 5, (.2, .5, .6; .4, .6, .9; .1, .2, .3) \rangle$	$\langle 8, (.2, .6, .5; .5, .8, .6; .3, .1, .4) \rangle$	(19, 2.30871)	(19, 2.36016)	(19, 2.24883)
c_3	$\langle 7, (.4, .6, .5; .3, .5, .7; .1, .4, .3) \rangle$	$\langle 2, (.4, .5, .3; .5, .6, .2; .5, .4, .5) \rangle$	$\langle 3, (.5, .3, .5; .4, .3, .7; .4, .3, .6) \rangle$	(12, 2.39995)	(12, 2.13054)	(12, 1.91588)
c_4	$\langle 3, (.5, .3, .4; .4, .5, .6; .3, .2, .5) \rangle$	$\langle 8, (.3, .6, .4; .4, .9, .6; .5, .4, .3) \rangle$	$\langle 9, (.5, .4, .5; .3, .7, .6; .4, .3, .2) \rangle$	(20, 2.32576)	(20, 2.05034)	(20, 1.89522)
c_5	$\langle 1, (.6, .5, .4; .3, .5, .7; .3, .4, .4) \rangle$	$\langle 4, (.6, .5, .4; .4, .7, .1; .3, .5, .5) \rangle$	$\langle 7, (.4, .3, .5; .2, .5, .7; .4, .6, .4) \rangle$	(12, 2.65388)	(12, 2.08484)	(12, 1.94623)

Table 4: Comparison table of MPDFNSs with pervious models

$\mathcal{U}_C^{(3,11)}$	$NSS(\sigma_i)$	$FNSS(Q_i)$	$IFNSS(S_i)$	$PFNSS(H_i)$	M_i^1	M_i^2	M_i^3
c_1	17	(17, 0.89)	(17, -0.03)	(17, 1.7462)	(17, 2.59680)	(17, 1.78017)	(17, 1.64489)
c_2	19	(19, 0.79)	(19, 0.09)	(19, 1.6368)	(19, 2.30871)	(19, 2.36016)	(19, 2.24883)
c_3	12	(12, 0.99)	(12, 0.15)	(12, 1.8940)	(12, 2.39995)	(12, 2.13054)	(12, 1.91588)
c_4	20	(20, 1.09)	(20, 0.09)	(20, 1.5705)	(20, 2.32576)	(20, 2.05034)	(20, 1.89522)
c_5	12	(12, 1.22)	(12, 0.54)	(12, 1.9097)	(12, 2.65388)	(12, 2.08484)	(12, 1.94623)

5.1 Relationships with Existing Sets

In this subsection, we establish the relationship with existing sets.

Definition 5.1. Let n be a threshold lies between 0 and N for the level, the PFNSS and PFSS over X associated with $\mathcal{U}_A^{(m,N)}$ and n , symbolized by $\mathcal{U}_A^{(n,m,N)}$, defined by, for all $a \in A$,

$$\mathcal{U}_a^{(n,m,N)} = \begin{cases} \langle x, (\mu(a), \nu(a)) \rangle, & \text{if } m = 1, \\ (x, d_a) (\mu(a), \nu(a)), & \text{if } (x, d_a) \in F(a), \text{ and } d_a \geq n, \\ (0, 0.5), & \text{if } d_a/N \geq 0.5, \\ (0, 1), & \text{if } d_a/N < 0.5. \end{cases}$$

Definition 5.2. Let k be a threshold with $k \in [-1, 1]$ for the score function, the N-soft over X associated with $\mathcal{U}_A^{(m,N)}$ and k , symbolized by $\mathcal{U}_A^{(k,m,N)}$, defined by, for all $a \in A$,

$$\mathcal{U}_a^{(k,m,N)} = \begin{cases} (x, d_a), & \text{if } (x, d_a) \in F(a), \text{ and } S_a(x) \geq k, m = 1, \\ 1, & \text{if } S_a(x) > 0, m = 1, \\ 0, & \text{if } S_a(x) \leq 0, m = 1. \end{cases}$$

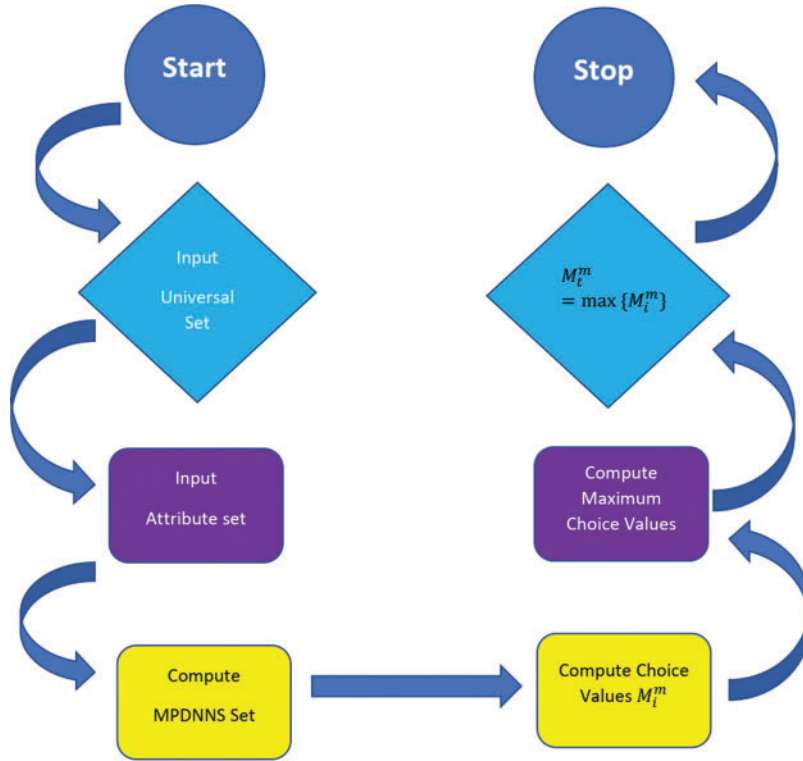


Figure 1: Flow chart of proposed algorithm

Definition 5.3. Let n be a threshold lies between 0 and N and threshold k with $k \in [-1, 1]$ for the score function, the soft over X associated with $\psi_A^{(m,N)}$ and (n, k) , symbolized by $\mathcal{U}_A^{((n,k),m,N)}$, defined by, for all $a \in A$,

$$\mathcal{U}_A^{((n,k),1,N)} = \left\{ x \in X \mid S_a^{\mathcal{U}_A^{((n,k),1,N)}}(x) > k, \right\}$$

For soft set associated with $\psi_A^{(3,6)}$ and threshold $(n, k) = (3, 0.3)$ is $\mathcal{U}_A^{((3,0.3),1,6)} = \{ \}$.

From Example 3.1, we have the following outcome which are elaborate in [Tab. 5](#).

Table 5: Pythagorean fuzzy N-soft set associated with $\mathcal{U}_A^{(3,6)}$ and $m = 1$

$\mathcal{U}_A^{(1,7)}$	e_1	e_2	e_3
ρ_1	$\langle 4, (0.3, 0.1) \rangle$	$\langle 0, (0, 1) \rangle$	$\langle 0, (0, 1) \rangle$
ρ_2	$\langle 0, (0, 1) \rangle$	$\langle 0, (0, 1) \rangle$	$\langle 3, (0.2, 0.8) \rangle$
ρ_3	$\langle 0, (0, 1,) \rangle$	$\langle 0, (0, 1) \rangle$	$\langle 2, (0.5, 0.4) \rangle$
ρ_4	$\langle 5, (0.5, 0.3) \rangle$	$\langle 0, (0, 1) \rangle$	$\langle 0, (0, 1) \rangle$
ρ_5	$\langle 0, (0, 1) \rangle$	$\langle 0, (0, 1) \rangle$	$\langle 2, (0.4, 0.7) \rangle$
ρ_6	$\langle 0, (0, 1) \rangle$	$\langle 0, (0, 1) \rangle$	$\langle 0, (0, 1) \rangle$
ρ_7	$\langle 6, (0.6, 0.4) \rangle$	$\langle 0, (0, 1) \rangle$	$\langle 0, (0, 1) \rangle$

In Tabs. 6 and 7, we deduce Pythagorean fuzzy soft set and N-soft set from MPDFNSs.

Table 6: Pythagorean fuzzy soft set associated with $\mathcal{U}_A^{(3,6)}$ and threshold $n = 3$

$\mathcal{U}_A^{(3,1,6)}$	e_1	e_2	e_3
ρ_1	(0.3, 0.1)	(0, 1)	(0, 1)
ρ_2	(0, 1)	(0, 1)	(0.2, 0.8)
ρ_3	(0, 1,)	(0, 1)	(0, 1)
ρ_4	(0.5, 0.3)	(0, 1)	(0, 1)
ρ_5	(0, 1)	(0, 1)	(0, 1)
ρ_6	(0, 1)	(0, 1)	(0, 1)
ρ_7	(0.6, 0.4)	(0, 1)	(0, 1)

Table 7: N-soft set associated with $\mathcal{U}_A^{(3,6)}$ and threshold $k = 0.3$

$\mathcal{U}_A^{(0.3,1,6)}$	e_1	e_2	e_3
ρ_1	1	0	0
ρ_2	0	0	0
ρ_3	0	0	1
ρ_4	1	0	1
ρ_5	0	0	0
ρ_6	0	0	0
ρ_7	1	0	0

6 Conclusion

In this paper, we investigate a new set namely the m-polar Diophantine neutrosophic N-soft set which is based on neutrosophic set and soft set. We are discussed different types of compliments on the proposed set and elaborate these compliments with examples. The purposed set is a generalized form of fuzzy, soft, Pythagorean fuzzy, Pythagorean fuzzy soft, and Pythagorean fuzzy N-soft sets. Moreover, as an application, we proposed an algorithm for multi-attribute decision-making problems by defining the new score function. In future work, one can discuss algebraic structures and topological properties on m-polar Diophantine neutrosophic N-soft set. Moreover, ones can develop the concept of m-polar Diophantine neutrosophic N-soft graph and then discuss their properties.

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