

Decision-Making Method based on the Interval Valued Neutrosophic Graph

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Abstract—In this article, we extend the concept of neutrosophic graph-based multicriteria decision making method (NGMCDM) to the case of interval valued neutrosophic graph theory. The new concept is called interval valued neutrosophic graph-based multicriteria decision making method (IVNGMCDM for short). Finally, an illustrative example is given and a comparison analysis is conducted between the proposed approach and other existing methods, to verify the feasibility and effectiveness of the developed approach.

Keywords—interval valued neutrosophic set; interval valued neutrosophic graph; influence coefficient; decision making problem

I. INTRODUCTION

The Neutrosophic Set (NS), proposed by Smarandache [1, 2] as a generalization of fuzzy sets theory [3], intuitionistic fuzzy set [4, 5], interval-valued fuzzy set [6] and interval-valued intuitionistic fuzzy set [7], is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity-membership function (f) independently, which are within the real standard or nonstandard unit interval $]0, 1^+[$. In order to conveniently apply NS in real life applications, Wang et al. [8] introduced the concept of single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [9] also introduced the concept of interval valued neutrosophic set (IVNS), which is more precise and more flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which three membership functions are independent, and their value belong to the unit interval $[0, 1]$.

The theory of single valued neutrosophic set and interval valued neutrosophic set have been applied in a wide diversity of fields [10, 11, 12, 13, 14, 15, 16]. Multi-criteria decision making attempts to handle problems with imprecise goals, referring to a number of individual criteria by a set of alternatives at choice. Many scholars have begun to study the practical application of neutrosophic sets and interval valued neutrosophic sets in multi-attribute decision-making problems. [17]

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. The graph is a widely used tool for solving combinatorial problems in different areas, such as geometry, algebra, number theory, topology, optimization and computer science. When the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs and their extensions [18, 19, 20, 21, 22] fail. For this purpose, Smarandache [23, 24, 25] defined four main categories of neutrosophic graphs. Two of them, called I-edge neutrosophic graph and I-vertex neutrosophic graph, are based on literal indeterminacy (I); these concepts are deeply studied and gained popularity among the researchers due to applications via real world problems [26, 27, 28, 29]. The two other categories of graphs, called (t, i, f) -Edge neutrosophic graph and (t, i, f) -vertex neutrosophic graph, are based on (t, i, f) components, but they not at all developed. Later on, Broumi et al. [30, 31] introduced a third neutrosophic graph model, called single valued neutrosophic graph (SVNG), and investigated some of its properties. This model allows the attachment of truth-membership (t), indeterminacy-membership (i) and falsity-membership(f) degrees both to vertices and edges. The single valued neutrosophic graph is a generalization of fuzzy graph and intuitionistic fuzzy graph. Also, the same authors [32] introduced neighborhood degree

of a vertex and closed neighborhood degree of a vertex in single valued neutrosophic graph, as a generalization of neighborhood degree of a vertex and closed neighborhood degree of a vertex in fuzzy graph and intuitionistic fuzzy graph. Moreover, Broumi et al. [33] introduced the concept of interval valued neutrosophic graph as a generalization fuzzy graph, intuitionistic fuzzy graph, interval valued fuzzy graph, interval valued intuitionistic fuzzy graph and single valued neutrosophic graph, discussing some properties with proofs and examples. In addition, Broumi et al. [34] proposed some operations - such as Cartesian product, composition, union and join - on interval valued neutrosophic graphs, and investigated some properties. Withal, Broumi et al. [35] discussed a subclass of interval valued neutrosophic graph, called strong interval valued neutrosophic graph, and introduced as well some operations - such as Cartesian product, composition and join of two strong interval valued neutrosophic graph - with proofs. Recently, Broumi et al. [36, 37] propounded the concept of bipolar single valued neutrosophic graph as a generalization of fuzzy graph, intuitionistic fuzzy graph, N-graph [38], bipolar fuzzy graph [39] and single valued neutrosophic graph, and studied some properties.

In this paper, we extend the concept of neutrosophic graph-based multicriteria decision making (NGMADM) method, introduced by Shain [40] to solve MCDM problems with interval valued neutrosophic information.

The paper is organized as follows: in the 2nd section, we give all the basic definitions to be employed in later sections, related to single valued neutrosophic graph and interval valued neutrosophic graph; in the 3rd section, we present the neutrosophic graph-based multicriteria decision making (NGMCDM) method; in the 4th section, we present an application of interval valued neutrosophic graphs in decision making; in the 5th section, an illustrative example is given, and then a comparison analysis is conducted between the proposed approach and other existing methods, in order to verify its feasibility and effectiveness. Finally, the conclusions are drawn in the 7th section.

II. PRELIMINARIES

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets, single valued neutrosophic graphs and interval valued neutrosophic graphs, relevant to the present paper. The readers are referred to [1, 8, 9, 14, 30, 31].

Definition 2.1 [1]. Let X be a space of points (objects) with generic elements in X denoted by x ; then, the neutrosophic set A (NS A) is an object having the form $A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle, x \in X \}$, where the functions $T, I, F: X \rightarrow]0, 1[$ define respectively a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A , with the condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+ \quad (1)$$

The functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0, 1[$.

Since it is difficult to apply NSs to practical problems, Wang et al. [8] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [8]. Let X be a space of points (objects) with generic elements in X denoted by x . A single valued neutrosophic set A (SVNS A) is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in $X, T_A(x), I_A(x), F_A(x) \in [0, 1]$.

A SVNS A can be written as

$$A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle, x \in X \}. \quad (2)$$

Definition 2.3 [9]. Let X be a universe of discourse and $\text{Int}[0, 1]$ be the set of all closed subsets of $[0, 1]$. Then, an interval neutrosophic set is defined as:

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle: x \in X \}, \quad (3)$$

Where $T_A: X \rightarrow \text{Int}[0, 1]$, $I_A: X \rightarrow \text{Int}[0, 1]$ and $F_A: X \rightarrow \text{Int}[0, 1]$ with $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$, for all $x \in X$.

The intervals $T_A(x), I_A(x)$ and $F_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A , respectively.

For convenience, if $T_A(x) = [T_A^L(x), T_A^U(x)]$, $I_A(x) = [I_A^L(x), I_A^U(x)]$ and $F_A(x) = [F_A^L(x), F_A^U(x)]$, then:

$$A = \{ \langle x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] \rangle: x \in X \}, \quad (4)$$

with the condition, $0 \leq \sup T_A^U(x) + \sup I_A^U(x) + \sup F_A^U(x) \leq 3$, for all $x \in X$.

Definition 2.4 [14]. Let $\alpha = \{ [t^l, t^u], [i^l, i^u], [f^l, f^u] \}$ be an interval neutrosophic number; a score function S of the interval valued neutrosophic number can be defined by

$$S(\alpha) = \frac{2+t^l+t^u-2i^l-2i^u-t^l-f^u}{4} \quad (5)$$

where $S(\alpha) \in [-1, 1]$.

Definition 2.5 [30]. Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be single valued neutrosophic sets on a set X .

If $A = (T_A, I_A, F_A)$ is a single valued neutrosophic relation on a set X , then $A = (T_A, I_A, F_A)$ is called a single valued neutrosophic relation on $B = (T_B, I_B, F_B)$, if:

$$\begin{aligned} T_B(x, y) &\leq \min(T_A(x), T_A(y)) \\ I_B(x, y) &\geq \max(I_A(x), I_A(y)) \\ \text{and} \\ F_B(x, y) &\geq \max(F_A(x), F_A(y)), \text{ for all } x, y \in X. \end{aligned} \quad (6)$$

A single valued neutrosophic relation A on X is called symmetric, if:

$$\begin{aligned} T_A(x, y) &= T_A(y, x), I_A(x, y) = I_A(y, x), F_A(x, y) = F_A(y, x), \\ T_B(x, y) &= T_B(y, x), I_B(x, y) = I_B(y, x) \text{ and } F_B(x, y) = F_B(y, x), \\ \text{for all } x, y &\in X. \end{aligned}$$

Definition 2.6 [30]. A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair $G=(A, B)$ where:

1) The functions $T_A:V \rightarrow [0, 1]$, $I_A:V \rightarrow [0, 1]$ and $F_A:V \rightarrow [0, 1]$ denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in V$, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \text{ for all } v_i \in V (i=1, 2, \dots, n). \quad (7)$$

2) The functions $T_B: E \subseteq V \times V \rightarrow [0, 1]$, $I_B: E \subseteq V \times V \rightarrow [0, 1]$ and $F_B: E \subseteq V \times V \rightarrow [0, 1]$ are defined by

$$T_B(v_i, v_j) \leq \min [T_A(v_i), T_A(v_j)], I_B(v_i, v_j) \geq \max [I_A(v_i), I_A(v_j)], \text{ and } F_B(v_i, v_j) \geq \max [F_A(v_i), F_A(v_j)] \quad (8)$$

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where:

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3, \text{ for all } (v_i, v_j) \in E (i, j = 1, 2, \dots, n) \quad (9)$$

they called A the single valued neutrosophic vertex set of V , B the single valued neutrosophic edge set of E , respectively; note that B is a symmetric single valued neutrosophic relation on A .

Example 2.7 [30] Figure 1 is an example for SVN G , $G=(A, B)$ defined on a graph $G^*=(V, E)$ such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$, A is single valued neutrosophic set of V

$A = \{ \langle v_1, (0.5, 0.1, 0.4) \rangle, \langle v_2, (0.6, 0.3, 0.2) \rangle, \langle v_3, (0.2, 0.3, 0.4) \rangle, \langle v_4, (0.4, 0.2, 0.5) \rangle \}$, and B single valued neutrosophic set of $E \subseteq V \times V$

$B = \{ \langle v_1v_2, (0.5, 0.4, 0.5) \rangle, \langle v_2v_3, (0.2, 0.3, 0.4) \rangle, \langle v_3v_4, (0.2, 0.4, 0.5) \rangle, \langle v_4v_1, (0.4, 0.3, 0.6) \rangle \}$

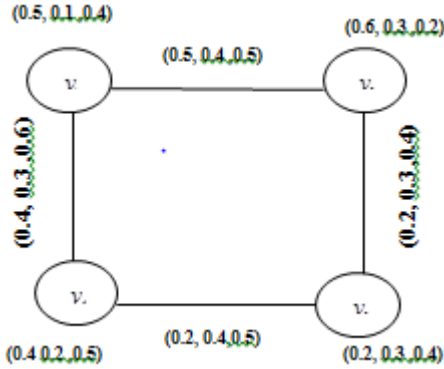


Fig. 1. Single valued neutrosophic graph

Definition 2.8 [30]. A single valued neutrosophic graph $G=(A, B)$ of $G^*=(V, E)$ is called strong single valued neutrosophic graph, if:

$$T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)], I_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)], F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)], \text{ for all } (v_i, v_j) \in E. \quad (10)$$

Definition 2.9 [30]. A single valued neutrosophic graph $G=(A, B)$ is called complete, if:

$$T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)], I_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)] \text{ and } F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)], \text{ for all } v_i, v_j \in V. \quad (11)$$

Definition 2.10 [31]. By an interval-valued neutrosophic graph of a graph $G^*=(V, E)$ we mean a pair $G=(A, B)$, where $A = \langle [T_{AL}, T_{AU}], [I_{AL}, I_{AU}], [F_{AL}, F_{AU}] \rangle$ is an interval-valued neutrosophic set on V , and $B = \langle [T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}] \rangle$ is an interval-valued neutrosophic relation on E satisfying the following condition:

1) $V = \{v_1, v_2, \dots, v_n\}$ such that $T_{AL}:V \rightarrow [0, 1]$, $T_{AU}:V \rightarrow [0, 1]$, $I_{AL}:V \rightarrow [0, 1]$, $I_{AU}:V \rightarrow [0, 1]$, and $F_{AL}:V \rightarrow [0, 1]$, $F_{AU}:V \rightarrow [0, 1]$ denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in V$, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \text{ for all } v_i \in V (i=1, 2, \dots, n). \quad (12)$$

2) The functions $T_{BL}:V \times V \rightarrow [0, 1]$, $T_{BU}:V \times V \rightarrow [0, 1]$, $I_{BL}:V \times V \rightarrow [0, 1]$, $I_{BU}:V \times V \rightarrow [0, 1]$ and $F_{BL}:V \times V \rightarrow [0, 1]$, $F_{BU}:V \times V \rightarrow [0, 1]$ are such that:

$$T_{BL}(v_i, v_j) \leq \min [T_{AL}(v_i), T_{AL}(v_j)], T_{BU}(v_i, v_j) \leq \min [T_{AU}(v_i), T_{AU}(v_j)], I_{BL}(v_i, v_j) \geq \max [I_{BL}(v_i), I_{BL}(v_j)], I_{BU}(v_i, v_j) \geq \max [I_{BU}(v_i), I_{BU}(v_j)] \text{ and}$$

$$F_{BL}(v_i, v_j) \geq \max [F_{BL}(v_i), F_{BL}(v_j)], F_{BU}(v_i, v_j) \geq \max [F_{BU}(v_i), F_{BU}(v_j)] \quad (13)$$

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where:

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3, \text{ for all } (v_i, v_j) \in E (i, j = 1, 2, \dots, n). \quad (14)$$

They called A the interval valued neutrosophic vertex set of V , and B the interval valued neutrosophic edge set of E , respectively; note that B is a symmetric interval valued neutrosophic relation on A .

Example 2.11 [33] Figure 2 is an example for IVNG, $G=(A, B)$ defined on a graph $G^*=(V, E)$ such that $V = \{v_1, v_2, v_3\}$, $E = \{v_1v_2, v_2v_3, v_3v_1\}$, A is an interval valued neutrosophic set of V

$A = \{ \langle v_1, ([0.3, 0.5], [0.2, 0.3], [0.3, 0.4]) \rangle, \langle v_2, ([0.2, 0.3], [0.2, 0.3], [0.1, 0.4]) \rangle, \langle v_3, ([0.1, 0.3], [0.2, 0.4], [0.3, 0.5]) \rangle \}$, and B an interval valued neutrosophic set of $E \subseteq V \times V$

$B = \{ \langle v_1v_2, ([0.1, 0.2], [0.3, 0.4], [0.4, 0.5]) \rangle, \langle v_2v_3, ([0.1, 0.3], [0.4, 0.5], [0.4, 0.5]) \rangle, \langle v_3v_1, ([0.1, 0.2], [0.3, 0.5], [0.4, 0.6]) \rangle \}$

Remark 2.12: -The underlying set V is vertex set of usual graph that we use it in neutrosophic graph as vertex.

- $G^*=(V, E)$ denoted a usual graph where a neutrosophic graphs obtained from it that truth –membership, indeterminacy –membership and non-membership values are 0 to 1.

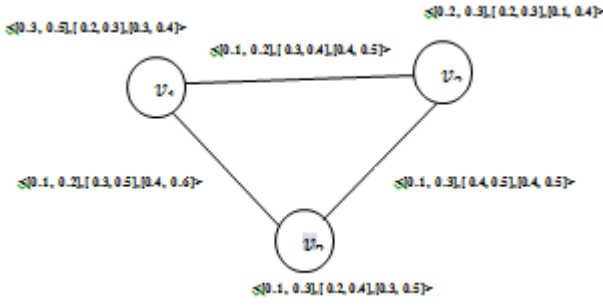


Fig. 2. Interval valued neutrosophic graph

III. NEUTROSOPHIC GRAPH-BASED MULTICRITERIA DECISION MAKING (NGMCDM) METHOD

Shain [40] proposed a procedure for the decision-maker to select the best choice with neutrosophic information. The method implies the following steps:

Step1. Compute the influence coefficient between the criteria α_i and α_j ($i, j = 1, 2, \dots, n$) in decision process by

$$\xi_{ij} = \frac{t_{ij} + (1 - i_{ij})(1 - f_{ij})}{3}, \quad (15)$$

Where $\varphi_{ij} = (t_{ij}, i_{ij}, f_{ij})$ is the neutrosophic relation edge between the vertexes α_i and α_j ($i, j = 1, 2, \dots, n$). We have $\xi_{ii} = 1$ and $\xi_{ij} = \xi_{ji}$ for $i = j$.

The (t, i, f) is a neutrosophic number. Because truth degree prove a positive impact while indeterminacy degree and falsity degree prove a negative impact in the relationship. If this relationship has maximum i.e., $(t, i, f) = (1, 0, 0)$ then we should have the biggest impact, $\xi_{ij} = 1$. If two criteria are independent, this relationships should be $(0, 1, 1)$ i.e., $\xi_{ij} = 0$

Step 2. Obtain the overall criterion value of the alternative p_k ($k = 1, 2, \dots, m$) by

$$\tilde{p}_k = \sum_{j=1}^n \omega_j \left(\sum_{i=1}^n e_{ki} \xi_{ij} \right), \quad (16)$$

Where $e_{ki} = (t_{ki}, i_{ki}, f_{ki})$ is clearly a neutrosophic number.

Step 3: Compute the score value of the alternative p_k ($k = 1, 2, \dots, m$) which is defined by:

$$s(\tilde{p}_k) = \frac{1 + \tilde{t} - 2\tilde{i} - \tilde{f}}{2}. \quad (17)$$

Step 4. Rank all the alternatives p_k ($k = 1, 2, \dots, m$) and select the best one(s) in concordance with $s(\tilde{p}_k)$.

Step5. End.

IV. DECISION-MAKING METHOD BASED ON THE INTERVAL VALUED NEUTROSOPHIC GRAPH

The interval valued neutrosophic set proposed by Wang et al. [9] is independently characterized by the truth-membership, the indeterminacy-membership and the falsity-membership, which is a powerful tool to deal with incomplete, indeterminate and inconsistent information. Recently, the interval valued neutrosophic set became an interesting topic research and attracted wide attention. The interval valued neutrosophic graph can well describe the uncertainty in real-

life world. Therefore, we will extend the NGMCDM method introduced by Shain [40] to solve MCDM problems with interval valued neutrosophic information. The new model to solve the decision-making problems is called interval valued neutrosophic graph-based multicriteria decision making (IVNGMCDM) method.

We firstly describe the decision making problem.

Suppose that $P = \{p_1, p_2, \dots, p_m\}$ is a collection of alternatives, $B = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is a collection of criteria, which weight vector is $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ satisfying $\omega_j \in [0, 1]$, $\sum_{j=1}^n \omega_j = 1$. If the decision maker provide a neutrosophic value for the alternative p_k ($k = 1, 2, \dots, m$) under the criteria α_j ($j = 1, 2, \dots, n$), these values can be characterized as an IVNN $e_{kj} = \{[t_{kj}^l, t_{kj}^u], [i_{kj}^l, i_{kj}^u], [f_{kj}^l, f_{kj}^u]\}$ ($j = 1, 2, \dots, n; k = 1, 2, \dots, m$). Assume that $E = [e_{kj}]_{m \times n}$ is the decision matrix, where e_{kj} is expressed by an interval valued neutrosophic element. If there exists an interval valued neutrosophic relation between two criteria $\alpha_i = \{[t_i^l, t_i^u], [i_i^l, i_i^u], [f_i^l, f_i^u]\}$ and $\alpha_j = \{[t_j^l, t_j^u], [i_j^l, i_j^u], [f_j^l, f_j^u]\}$, we denote the interval valued neutrosophic relation as $\varphi_{ij} = \{[t_{ij}^l, t_{ij}^u], [i_{ij}^l, i_{ij}^u], [f_{ij}^l, f_{ij}^u]\}$, with the properties:

$$\begin{aligned} t_{ij}^l &\leq \min(t_i^l, t_j^l), \quad t_{ij}^u \leq \min(t_i^u, t_j^u), \\ i_{ij}^l &\geq \max(i_i^l, i_j^l), \quad i_{ij}^u \geq \max(i_i^u, i_j^u), \\ f_{ij}^l &\geq \max(f_i^l, f_j^l), \quad f_{ij}^u \geq \max(f_i^u, f_j^u), \end{aligned}$$

for all $(i, j = 1, 2, \dots, m)$; otherwise, $\varphi_{ij} = \{[0, 0], [1, 1], [1, 1]\}$.

On the basis of the developed graph structure, we can propose a procedure for the decision-maker to select the best choice with interval valued neutrosophic information.

The method is described by the following steps:

Step 1. Compute the influence coefficient between the criteria α_i and α_j ($i, j = 1, 2, \dots, n$) in decision process by

$$\xi_{ij} = \frac{(t_{ij}^l + t_{ij}^u) + (2 - (i_{ij}^l + i_{ij}^u))(2 - (f_{ij}^l + f_{ij}^u))}{6}, \quad (18)$$

Where $\varphi_{ij} = \{[t_{ij}^l, t_{ij}^u], [i_{ij}^l, i_{ij}^u], [f_{ij}^l, f_{ij}^u]\}$ is the interval valued neutrosophic edge between the vertexes α_i and α_j ($i, j = 1, 2, \dots, n$). We have $\xi_{ii} = 1$ and $\xi_{ij} = \xi_{ji}$ for $i = j$.

Step 2. Obtain the overall criterion value of the alternative p_k ($k = 1, 2, \dots, m$) by

$$\tilde{p}_k = \sum_{j=1}^n \omega_j \left(\sum_{i=1}^n e_{ki} \xi_{ij} \right), \quad (19)$$

Where $e_{ki} = \{[t_{ki}^l, t_{ki}^u], [i_{ki}^l, i_{ki}^u], [f_{ki}^l, f_{ki}^u]\}$ is clearly an interval valued neutrosophic number.

Step 3. Compute the score value of the alternative p_k ($k = 1, 2, \dots, m$), which is defined by:

$$s(\tilde{p}_k) = \frac{2 + \tilde{t}^l + \tilde{t}^u - 2\tilde{i}^l - 2\tilde{i}^u - \tilde{f}^l - \tilde{f}^u}{4} \quad (20)$$

Step 4. Rank all the alternatives p_k ($k = 1, 2, \dots, m$) and select the best one(s) in concordance with $s(\tilde{p}_k)$

Step 5. End.

V. AN ILLUSTRATIVE EXAMPLE

In this section, an example for an IVNGMCDM problem with interval-valued neutrosophic information is used to prove the application and effectiveness of the proposed decision-making method.

Let us consider the decision-making problem adapted from Zhao et al. [41].

Example 5.1. An investment company wants to invest a sum of money in the best option. There is a panel with four possible alternatives in which to invest the money: (1) p_1 is a car company, (2) p_2 is a food company, (3) p_3 is a computer company, and (4) p_4 is an armament company. The investment company must take a decision according to three criteria: (1) α_1 is the risk analysis; (2) α_2 is the growth analysis, and (3) α_3 is the environmental impact analysis. Then, the weight vector of the criteria is given by $\omega = (0.2, 0.25, 0.55)^T$. The four possible alternatives are to be evaluated under these three criteria and presented in the form of interval valued neutrosophic information by decision maker, consistent to criteria α_j ($j = 1, 2, 3$) and the information evaluation on the alternative p_k ($k = 1, 2, 3, 4$) under the factors α_j ($j = 1, 2, 3$); it results the interval valued neutrosophic decision matrix D :

$$D = \begin{bmatrix} \langle [0.4, 0.5], [0.2, 0.3], [0.3, 0.4] \rangle & \langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle & \langle [0.4, 0.5], [0.2, 0.3], [0.7, 0.9] \rangle \\ \langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle & \langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle & \langle [0.8, 0.9], [0.3, 0.5], [0.3, 0.6] \rangle \\ \langle [0.3, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle & \langle [0.5, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle & \langle [0.7, 0.9], [0.2, 0.4], [0.4, 0.5] \rangle \\ \langle [0.7, 0.8], [0.0, 0.1], [0.1, 0.2] \rangle & \langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle & \langle [0.8, 0.9], [0.3, 0.4], [0.6, 0.7] \rangle \end{bmatrix}$$

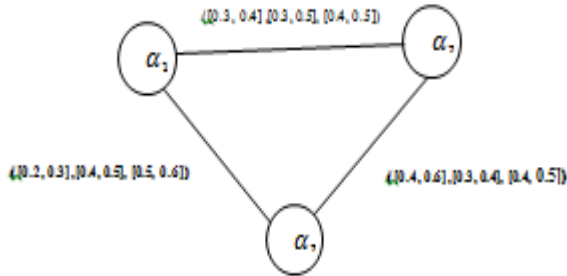


Fig. 3. The graph relationship among the criteria

Moreover, we assume that the relationships among the factors α_j ($j = 1, 2, 3$) can be described by a complete graph $G = (A, E)$, where $A = \{\alpha_1, \alpha_2, \alpha_3\}$ and $E = \{\alpha_1\alpha_2, \alpha_1\alpha_3, \alpha_2\alpha_3\}$ (see Fig. 3). Employing Eq. (18), we can obtain all influence coefficients to quantify the relationships among the criteria.

Suppose that the neutrosophic edges denoting the connection among the criteria are described as follows:

$$\begin{aligned} e_{12} &= \langle [t_{12}^l, t_{12}^u], [i_{12}^l, i_{12}^u], [f_{12}^l, f_{12}^u] \rangle = \langle [0.3, 0.4], [0.3, 0.5], [0.4, 0.5] \rangle, \\ e_{13} &= \langle [t_{13}^l, t_{13}^u], [i_{13}^l, i_{13}^u], [f_{13}^l, f_{13}^u] \rangle = \langle [0.2, 0.3], [0.4, 0.5], [0.5, 0.6] \rangle, \\ e_{23} &= \langle [t_{23}^l, t_{23}^u], [i_{23}^l, i_{23}^u], [f_{23}^l, f_{23}^u] \rangle = \langle [0.4, 0.6], [0.3, 0.4], [0.4, 0.5] \rangle. \end{aligned}$$

Note that $G = (A, E)$ describes an interval valued neutrosophic graph according to the relationship among criteria for each alternative.

To get the best alternative(s), the following steps are involved:

Step 1. We apply all computations only in the alternative p_1 . Others can be similarly proved.

The influence coefficients between criteria was computed as follows:

$$\begin{aligned} \xi_{12}^1 &= \frac{(t_{12}^l + t_{12}^u) + (2 - (i_{12}^l + i_{12}^u))(2 - (f_{12}^l + f_{12}^u))}{6} \\ &= \frac{(0.3 + 0.4) + (2 - (0.3 + 0.5))(2 - (0.4 + 0.5))}{6} = 0.337, \\ \xi_{13}^1 &= \frac{(t_{13}^l + t_{13}^u) + (2 - (i_{13}^l + i_{13}^u))(2 - (f_{13}^l + f_{13}^u))}{6} \\ &= \frac{(0.2 + 0.3) + (2 - (0.4 + 0.5))(2 - (0.5 + 0.6))}{6} = 0.248, \\ \xi_{23}^1 &= \frac{(t_{23}^l + t_{23}^u) + (2 - (i_{23}^l + i_{23}^u))(2 - (f_{23}^l + f_{23}^u))}{6} \\ &= \frac{(0.4 + 0.6) + (2 - (0.3 + 0.4))(2 - (0.4 + 0.5))}{6} = 0.405. \end{aligned}$$

Step 2. By applying Eq.(19) we can obtain the overall criterion value of the alternative p_1 as follows:

$$\begin{aligned} \tilde{p}_1 &= w_1 \times (e_{11}\xi_{11} + e_{12}\xi_{21} + e_{13}\xi_{31}) + w_2 \times (e_{11}\xi_{12} + e_{12}\xi_{22} + e_{13}\xi_{32}) + w_3 \times (e_{11}\xi_{13} + e_{12}\xi_{23} + e_{13}\xi_{33}) \\ &= 0.2 \times (\langle [0.4, 0.5], [0.2, 0.3], [0.3, 0.4] \rangle + 0.337 \times \langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle + 0.248 \times \langle [0.4, 0.5], [0.2, 0.3], [0.7, 0.9] \rangle) \\ &\quad + 0.25 \times (0.337 \times \langle [0.4, 0.5], [0.2, 0.3], [0.3, 0.4] \rangle + \langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle + 0.405 \times \langle [0.4, 0.5], [0.2, 0.3], [0.7, 0.9] \rangle) \\ &\quad + 0.55 \times (0.248 \times (\langle [0.4, 0.5], [0.2, 0.3], [0.3, 0.4] \rangle + \langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle + 0.405 \times \langle [0.4, 0.5], [0.2, 0.3], [0.7, 0.9] \rangle) \\ &\quad + \langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle + \langle [0.4, 0.5], [0.2, 0.3], [0.7, 0.9] \rangle) \\ \tilde{p}_1 &= \langle [0.6718, 0.8937], [0.2819, 0.5038], [0.7373, 1.0311] \rangle. \end{aligned}$$

Similarly,

$$\begin{aligned} \tilde{p}_2 &= \langle [1.1514, 1.3193], [0.3117, 0.5515], [0.4077, 0.7194] \rangle, \\ \tilde{p}_3 &= \langle [0.8993, 1.2232], [0.3359, 0.5757], [0.5757, 0.7436] \rangle \\ \text{and} \\ \tilde{p}_4 &= \langle [1.1934, 1.3614], [0.2696, 0.4375], [0.5272, 0.7792] \rangle. \end{aligned}$$

Step 3. By applying Eq.(20) we can obtain $s(\tilde{p}_i)$ ($i=1, 2, 3, 4$) as follows:

$$s(\tilde{p}_1) = 0.0564, \quad s(\tilde{p}_2) = 0.4043, \quad s(\tilde{p}_3) = 0.2450 \quad \text{and} \quad s(\tilde{p}_4) = 0.4660.$$

Step 4. Since $s(\tilde{p}_4) > s(\tilde{p}_2) > s(\tilde{p}_3) > s(\tilde{p}_1)$, the ranking order of four alternatives is $p_4 > p_2 > p_3 > p_1$. Therefore, we can see that the alternative p_4 is the best choice among all the alternatives

VI. COMPARISON ANALYSIS

In order to verify the feasibility and effectiveness of the proposed decision-making approach, a comparison analysis with interval valued neutrosophic decision method, used by Zhao et al. [41], is given, based on the same illustrative example.

Clearly, the ranking order results are consistent with the result obtained in [41]; however, the best alternative is the same as A_4 . because the ranking principle is different, these two methods produced the same best and worst alternatives.

VII. CONCLUSION

The interval neutrosophic set, as a concept combining single valued neutrosophic set and interval fuzzy set, provides additional capability to deal with uncertainty, inconsistent, incomplete and imprecise information by including a truth-membership interval, an indeterminacy-membership interval and a falsity membership interval. Therefore, it plays a significant role in the uncertainty system. An Interval valued neutrosophic models provide more precision, flexibility and compatibility to the system in comparison to classic, fuzzy models and neutrosophic model. In this study, we consider the importance of relationships among criteria in decision process, we developed a new model, called interval valued neutrosophic graph-based multicriteria decision making (IVNGMADM) method, to solve complex problems within the interval valued neutrosophic information. That is, the relationships among criteria for each alternative are included by this method in the decision process. In this case, we can select the alternative(s) according to the overall criteria values resulting from the model. Finally, an illustrative example was given to prove the application of proposed method. The developed method is more suitable to handle indeterminate information and inconsistent information in complex decision making problems with interval valued neutrosophic information.

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