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DSmT-Based Group DEMATEL Method with Reaching Consensus

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Abstract

The decision-making trial and evaluation laboratory (DEMATEL) method employs expert assessments expressed by crisp values to construct a group initial direct-relation (IDR) matrix. However, it tends to be a low-precision expression, especially in complex practical problems. Although significant efforts have been made to improve the DEMATEL method, these improvements tend to neglect individual characteristics and group consensus, resulting in unconvincing decision results. This study provides a Dezert–Smarandache theory-based group DEMATEL method with reaching consensus. In order to reasonably determine the group IDR matrix, basic belief assignment function is employed to extract expert assessments and the proportional conflict redistribution rule no.5 of DSmT is employed to make fusion to derive the temporary group IDR matrix. Moreover, the consensus measures at both expert level and pair-factors level are calculated to determine whether the acceptable consensus level has been reached or not. If the required consensus level is not reached, a feedback mechanism will be activated to help experts reach a consensus. A consensus group IDR matrix for the group DEMATEL can be obtained with the help of feedback mechanism, based on which an algorithm is summarized for the proposed method to identify major factors in a complex system. Finally, numerical comparison and discussion are introduced to verify the effectiveness and applicability of the proposed method and algorithm.

Keywords DEMATEL · Group decision making · Dezert–Smarandache theory (DSmT) · Consensus reaching · Evidence distance · Expert weight

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1 Introduction

Between 1972 and 1976, the Science and Human Affairs Program of Battelle Memorial Institute of Geneva developed the decision-making trial and evaluation laboratory (DEMATEL) method. This method aimed to describe the basic concept of contextual relations and identify cause–effect chain factors for a complex decision problem in an understandable manner by addressing the influence relations among factors given by experts (Fontela 1974; Gabus 1973). It was considered to be a credible decision-making method.

The DEMATEL method has been extensively used to solve complex decision problems because of its simplicity and effectiveness, including problems pertaining to hospital service quality (Shieh et al. 2010), decision making (Michnik 2013), sustainable supply chain management (Liang et al. 2016; Ren et al. 2013), etc. In special, in order to determine the weights of factors by considering their relations, the DEMATEL method is also extended into decision making fields, such as analytic hierarchy process (AHP) (Sara et al. 2015; Balsara et al. 2019), analytic network process (ANP) (Gölcük and Baykasoğlu 2016; Lan and Zhong 2016; Ghaemi Rad et al. 2018; Quezada et al. 2018; Chen et al. 2019; Dinçer et al. 2019), and technique for order preference by similarity to ideal solution (TOPSIS) (Baykasoğlu and Gölcük 2017; Baykasoğlu et al. 2013). In the DEMATEL method, the initial decision information is always subjectively given by experts in the form of crisp values and calculated to obtain an individual or group initial direct-relation (IDR) matrix by simple operations (e.g., weighted sum). However, such descriptions and operations are considered to hardly reflect the vagueness of the real world (Chen and Hwang 1992). Therefore, scholars have carried out some research to improve the DEMATEL combined with fuzzy theory (Wu and Lee 2007). Several fuzzy DEMATEL methods have been introduced. For examples, Abdullah et al. (2019) introduced interval-valued intuitionistic fuzzy numbers to improve the judgement of DEMATEL in a group decision-making (GDM) environment. Addae et al. (2019) used a two-step fuzzy DEMATEL method to solve a practical problem. Asan et al. (2018) proposed a new interval-valued hesitant fuzzy approach to DEMATEL to explicitly deal with hesitation in expert assessments and offered a better representation of uncertainty, etc. (Bhatia and Srivastava 2018; Acuña-Carvajal et al. 2019; Cui et al. 2019; Du and Zhou 2019).

Obviously, all these extensions have made great contributions to the DEMATEL method, and its ability of dealing with complex problems can be strengthened to some extent. As currently defined, instead of one expert, a panel gives the assessments on the influence relations of factors, and multiple experts arrive at an acceptable result (Cheng and Lin 2002). This process makes it necessary to extend the traditional DEMATEL method to a group DEMATEL method, which belongs to GDM problems. Although some literature considers the DEMATEL from the perspective of GDM, we believe that these group DEAMTEL methods have a lot of room for further improvement in both expert assessment extraction and group IDR matrix construction.

Firstly, the expert assessment extraction of group DEMATEL method should be improved to obtain accurate decision information in accordance with experts' cognitive competence. As introduced earlier, DEMATEL is a GDM method totally based on expert assessments to conduct later computation and analyzation. Considering the complexity of reality and individual characteristics of experts, there is no doubt that not every expert is proficient in all areas. In other words, experts may give incomplete or uncertain assessments to the influence relations among factors for a specific complex practical problem. However, all of the existing group DEMATEL methods default that each expert can give a definite assessment for every pair of factors by means of crisp values or fuzzy numbers, and neglect the problem that the assessments given by experts may be incomplete in practice. If experts who cannot give definite assessments with crisp values or fuzzy numbers are required to give the assessments in those forms, the final decision of DEMATEL resulted from the ineffective assessments may be erroneous. Therefore, understanding how to depict and fuse incomplete information from experts is of great importance to improve the properties of the group DEMATEL method. Fortunately, the basic belief assignment (BBA) function, as a key concept in the Dezert–Smarandache theory (DSmT) of evidence (Smarandache and Dezert 2004), can directly express the uncertainty by assigning probability to the subsets of a set composed of multiple objects rather than to each individual object (Du et al. 2018c). The BBA functions generated from different evidence sources (experts) could be well fused by the proportional conflict redistribution rule no. 5 (PCR5). All these features exactly meet the requirements of the group DEMATEL method. Accordingly, DSmT is used to extract and fuse expert assessments to derive the group IDR matrix in this paper. Moreover, the differences in experts' knowledge backgrounds and cognitive abilities on a particular problem are reflected by expert weights (Du et al. 2018b). Expert weights are reflected as discounting parameters to reflect one's relative importance in a group during fusion process. In this paper, expert weights are calculated based on similarity functions of expert assessments.

Secondly, the group IDR matrix construction of group DEMATEL method should be improved to obtain the acceptable decision results in accordance with experts' satisfaction. In the GDM, "group" refers to not only the number of experts merely, but also the experts who have common interests in reaching a consensus for the ultimate satisfactory results despite individual differences. This principle helps to reduce biased evaluations and inherent partiality in GDM processes (Büyüközkan and Çifçi 2012). Unfortunately, the group IDR matrix in the existing group DEMATEL methods is frequently constructed by making arithmetic average values for individual IDR matrices, while whether experts agree with the group results is scarcely concerned. The group IDR matrix plays a fundamental role in the entire DEMATEL processes and it has a significant influence on the effectiveness of final results. If strong inconsistency and conflict exist among experts, the group IDR matrix may not be able to precisely describe the real influence relations of factors. Therefore, it is particularly important to construct a group IDR matrix according to the consensus rules, that is, to construct a group consensus IDR matrix by reaching general or widespread agreement among the experts involved in the GDM processes (Pérez et al. 2018). Fortunately, Herrera-Viedma proposed a rational consensus model in GDM composed of a selection process and a consensus-reaching process (CRP), which had become a hot issue in the recent GDM area. The CRP has been successfully introduced to make GDM with different situations, such as hesitant fuzzy preference relations, Delphi processes, multi-attribute large-scale GDM, sentiment analysis, and virtual reality industry (F. Herrera et al. 1997; Liang et al. 2016; Michnik 2013; Ren et al. 2013; Shieh et al. 2010). Traditionally, unanimous agreement of all experts in CRP is required. However, the desired result can hardly be achieved because of the diversity of opinions, knowledge, and experiences of experts. Therefore, the concept of "soft consensus" has been provided, in which, "soft" means better reflecting all possible levels of agreement by setting an acceptable consensus level (CL) threshold value (such as 0.8 rather than 1) and guiding the CRP until a high-level agreement is achieved among the individuals. Soft consensus can be reached through an iterative dynamic process with several collection and adjustment rounds (Kacprzyk and Zadrożny 2010; Cabrerizo et al. 2017). Hence, in this paper, we consider a soft CRP in the group IDR matrix construction, and transform the original static group DEMATEL problems into dynamic ones.

The motivation of this paper is to improve group DEMATEL method according to the following three aspects. Firstly, the BBA function is used to extract expert assessments to accurately express uncertainty and incompleteness, thus reducing the loss of accuracy. Secondly, the initial assessments are discounted with expert weights by using Shafer's discounting method. Moreover, the PCR5 of DSmT is used to fuse the discounted assessments to overcome the defects in the intuitional paradox of Dempster's combination rule. Thirdly, we apply a soft CRP to help reach an acceptable CL in the construction of group IDR matrix to ensure the consistency and satisfaction among experts. This paper is organized as follows. Section 2 briefly introduces the basic knowledge of DEMATEL and DSmT. In Sect. 3, the DSmT-based group DEMATEL method with reaching consensus is proposed, and the corresponding algorithm is summarized. In Sect. 4, the numerical comparison and discussion are provided to demonstrate the performances of the proposed method and algorithm. In Sect. 5, the conclusion is drawn and the future research directions are briefly discussed.

2 Preliminaries

In order to facilitate the later formulation, some basic concepts of DEMATEL and the Dezert–Smarandache theory (DSmT) are given in this section.

2.1 DEMATEL Method

The DEMATEL method is an effective way to analyze the influence relations among factors of a system. Through an analysis of the total influence relation of factors by the DEMATEL method, we can obtain a better understanding of structural relations and an ideal way to solve complicated system problems. Consider that a group of experts are invited to assess the influence relations for a set of factors $F = \{f_1, \dots, f_L\}$

with a set of grade levels $\{0, 1, 2\}$, where the expert set is $E = \{e_1, \dots, e_K\}$, and the degree of influence to which he or she believes factor f_i has an effect on factor f_j (denoted by $f_i \to f_j$) is assessed by expert e_k and denoted as g_{ij}^k, $\forall i, j, k$. The meaning of each element in the set of grade levels is that 0 denotes "no influence", 1 denotes a "low influence" and 2 denotes a "very high influence". The central concepts in the DEMATEL are defined as follows.

Definition 1 (Singhal et al. 2018) Suppose a pairwise comparison of influence degree from the ith to the jth factor given by the kth expert e_k is denoted as g_{ij}^k with 0–2 grade levels, and the grade levels given by each expert form a $L \times L$ non-negative answer matrix $G^k = [g_{ij}^k]_{L\times L}$, $k = 1, \dots, K$. The group IDR matrix, which represents the initial direct relation between each pair of factors derived from experts, is obtained by calculating the average values of all experts' answer matrices as $G = [g_{ij}]_{L\times L}$, where $g_{ij} = \sum_{ij}^{k} g_{ij}^k / K, i, j = 1, \dots, L$.

In Definition 1, the 0, 1, 2 grade levels mean "no influence," "medium influence," and "high influence," respectively. Note that, the diagonal elements of each answer matrix G^k are all set to zero, which means that the factors do not influence themselves.

Definition 2 (Singhal et al. 2018) Suppose the maximal row-wise and column-wise sum of matrix G is $g' = \max\left(\max_{1\le i\le L} \sum_{j=1}^{L} g_{ij}, \max_{1\le j\le L} \sum_{i=1}^{L} g_{ij}\right)$; then the normalized IDR matrix $D = [d_{ij}]_{L\times L}$ can be computed according to Eq. (1).

$$D = G \big/ g' \tag{1}$$

Definition 3 (Singhal et al. 2018) Suppose the direct and indirect relations among several factors are represented by the total relation matrix, and it is defined as in Eq. (2).

$$A = \lim_{N\to\infty} (D + D^2 + \cdots + D^N) = D(I - D)^{-1} \tag{2}$$

Some kinds of extensions are further discussed to strengthen the original DEMATEL. One kind of extensions is used to overcome the drawback that raising the normalized IDR matrix to the power of infinity may not converge to zero, and hence, the total relation matrix may not converge (see Eq. (2)). A very small positive number μ (e.g., $\mu = 10^{-5}$) is introduced in the maximal row-wise and column-wise sum of matrix G as $g'' = \max\left(\max_{1\le i\le L} \sum_{j=1}^{L} g_{ij}, \mu + \max_{1\le j\le L} \sum_{i=1}^{L} g_{ij}\right)$. Other steps remained unchanged as in the original DEMATEL. The revised DEMATEL guarantees that the normalized IDR matrix to infinite power will converge to zero and that the total relation matrix can be obtained smoothly (Lee et al. 2013).

Definition 4 (Singhal et al. 2018) Suppose r and c represent the sum of rows and the sum of columns of the total relation matrix A. According to $A = [a_{ij}]_{L\times L}$, r and c can be defined as follows:

$$r = [r_i]_{L\times 1} = \left(\sum_{j=1}^{L} a_{ij}\right)_{L\times 1}$$
$$c = [c_i]'_{1\times L} = \left(\sum_{j=1}^{L} a_{ji}\right)'_{1\times L} \tag{3}$$

where r_i shows the total influence, both direct and indirect, given by the factor f_i to other factors; c_i shows the total influence, both direct and indirect, received by the factor f_i from other factors; $r_i + c_i$ is defined as the prominence, showing the degree of the important role that the factor f_i plays in the complex system; and $r_i - c_i$ shows the net influence that the factor f_i contributes to the complex system. Note that, if $r_i - c_i$ is positive, the factor f_i is a net causer; if $r_i - c_i$ is negative, the factor f_i is a net receiver.

2.2 Dezert–Smarandache Theory

DSmT, jointly proposed by Smarandache and Dezert (2006), can be used to obtain more accurate fusion results of BBA functions especially in high conflicting information cases. It has a series of proportional conflict redistribution rules to make fusion for evidences (Du et al. 2019), among which, PCR5 is the most widely used one with the advantages in dealing with conflict belief functions. For example, it provides the appropriate redistribution of conflict beliefs and can produce a reasonable fusion result even in highly conflicting cases. These attractive features motivate the use of DSmT in GDM problems, such as map reconstruction of robot (Singh et al. 2008), decision making support (Huang et al. 2014; Liu et al. 2011), target type tracking (Faux and Luthon 2012), image processing (Liu et al. 2012a), data classification (Lian et al. 2015; Liu et al. 2013, 2015a, b, 2016), clustering (Denœux et al. 2015; Liu et al. 2012b, 2015b), and so on.

In DSmT framework, the frame $\Theta = \{\theta_1, \ldots, \theta_Y\}$ is a finite set of Y exhaustive propositions that are not necessarily mutually exclusive. The hyper-power set D^Θ is defined as the set of all composite propositions built from elements of Θ with $\cup$ and $\cap$ operators, such that (Guo et al. 2016):

(i) $\emptyset, \theta_1, \ldots, \theta_Y \in D^\Theta$;

(ii) If $\theta_y, \theta_{y'} \in D^\Theta$, then $\theta_y \cup \theta_{y'} \in D^\Theta$ and $\theta_y \cap \theta_{y'} \in D^\Theta$;

(iii) No other elements belong to D^Θ, except those obtained by using rules (i) or (ii).

Definition 5 (Smarandache and Dezert 2006) Suppose $\Theta = \{\theta_1, \ldots, \theta_Y\}$ is a set of exhaustive propositions; then the basic belief assignment is defined over the hyper-power set D^Θ. If the mapping function $m : D^\Theta \to [0, 1]$ could fulfill the following:

$$m(\emptyset) = 0, \sum_{\theta \subseteq D^{\Theta}} m(\theta) = 1 \tag{4}$$

then $m(\cdot)$ is called the BBA function. If $m(\theta) > 0$, θ is called a focal element.

In DSmT framework, PCR5 for making fusion for two pieces of evidence is introduced as follows.

Definition 6 (Smarandache and Dezert 2006) Suppose the BBA functions of two pieces of evidence are m_1 and m_2 on D^{Θ}; then, PCR5 to fuse m_1 and m_2 can be defined as follows:

$$m_{PCR5}(\theta) = \begin{cases} \sum\limits_{\substack{\theta' \cap \theta'' = \theta_1 \\ \theta', \theta'' \subseteq D^{\Theta}}} m_1(\theta')m_2(\theta'') + \sum\limits_{\substack{\theta''' \in D^{\Theta} \\ \theta \cap \theta''' = \emptyset}} \left[\frac{m_1(\theta)^2 \cdot m_2(\theta''')}{m_1(\theta) + m_2(\theta''')} + \frac{m_2(\theta)^2 \cdot m_1(\theta''')}{m_2(\theta) + m_1(\theta''')} \right], & \theta \in D^{\Theta} \ and \ \theta \neq \emptyset \\ 0, \quad \theta = \emptyset \end{cases} \tag{5}$$

3 The Proposed Method

In the proposed method, we construct the group IDR matrix in the DEMATEL by following two steps. Firstly, we develop an expert assessments extraction and fusion mechanism with DSmT to obtain a temporary group IDR matrix (see Sect. 3.1). Secondly, we activate the soft CRP based on the current individual and group IDR matrices to reach a soft consensus among experts (see Sect. 3.2).

3.1 Expert Assessment Extraction and Fusion

In traditional DEMATEL, experts are advised to give their assessments of the influence relations among factors by means of crisp values to construct the group IDR matrix $G = [g_{ij}]_{L \times L}$ (see Definition 1), which is a rough extraction method with low precision. When restricted by expert experiences and knowledge, the assessments given by experts may be ignorant and partially credible. Taking the influence degree for $f_i \rightarrow f_j$ and expert e_k as an example, the expert may well know this problem and he or she can give a definite result by using one of the given grade levels $\{0, 1, 2\}$. On the contrary, the expert may not know this problem at all, in which case he or she cannot give any assessment information about the influence degree. In most instances, the expert knows this problem to a certain extent and he or she may point out that the influence degree belongs to several grade levels but may not be sure which is the best one. Obviously, experts cannot give their assessments by means of crisp values in the latter two situations—that is, the original extraction method is not practical and does not consider experts' personality. Thus, we employed the BBA function to extract expert assessments in this paper as shown in Fig. 1.

As shown in Fig. 1, the expert assessment extraction and fusion mechanism includes three major steps. (1) Extracting expert assessments with BBA function; (2) Discounting these assessments with Shafer's discounting method; (3) Fusing the discounted

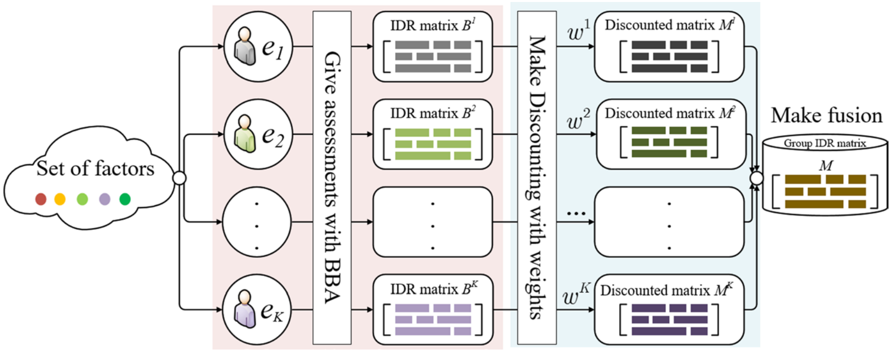

Fig. 1 Expert assessments extraction and fusion mechanism

assessments with PCR5. Next, we will give a detailed definition and explanation of the procedures involved in the above processes.

Let the frame of discernment be $\Theta = \{\theta_1, \theta_2, \theta_3\} = \{0, 1, 2\}$, which can be seen as the discernment frame of DSmT. Expert e_k is asked to assess the influence degree for $f_i \to f_j$ and his/her assessment is allowed to be expressed by the BBA function as given in Definition 5 and shown in Eq. (6).

$$b_{ij}^k = \left\{ (\theta, b_{ij}^k(\theta)) | \sum_{\theta \subseteq \Theta} b_{ij}^k(\theta) = 1;\ b_{ij}^k(\theta) \geq 0,\ \theta \subseteq \Theta,\ \forall i, j, k \right\} \tag{6}$$

All of the BBA functions make up the individual IDR matrix for expert e_k, denoted as $B^k = [b_{ij}^k]_{L \times L}$, with $i, j = 1, \ldots, L$ and $k = 1, \ldots, K$. Note that the basic beliefs in b_{ij}^k can be assigned not only to singleton grade levels but also to any subsets of Θ, thereby it is allowed such an assessment (also called a piece of evidence) to be profiled by a BBA defined on the hyper-power set D^Θ. It is capable of reflecting ignorance in expert assessments, and the basic beliefs in b_{ij}^k can be given to Θ (global ignorance) or to $\theta \subset \Theta$ (local ignorance) according to the unknown and partial assessments (Du and Xu 2017). Expert assessments could be expressed precisely through BBA functions as discussed, laying a great foundation for later DEMATEL procedures.

Example 1 Assume expert e_1 points out the influence degree for $f_i \to f_j$ has 30% probabilities belonging to θ_1 and 70% probabilities belonging to θ_2. Thus, his or her assessment is described as $b_{ij}^1(\theta)$ =$\{(\theta_1, 0.30), (\theta_2, 0.70)\}$. Expert e_2 points out the influence degree has 20% probabilities belonging to θ_1 and has 80% probabilities belonging to $(\theta_2 \cup \theta_3)$ but is not sure which is the best one. Therefore, his or her assessment is described as $b_{ij}^2(\theta) = \{(\theta_1, 0.20), ((\theta_2 \cup \theta_3), 0.80)\}$. Expert e_3 points out the influence degree for $f_i \to f_j$ has 100% probabilities belonging to $(\theta_2 \cap \theta_3)$. Thus, his or her assessment is described as $b_{ij}^3(\theta) = \{((\theta_2 \cap \theta_3), 1.00)\}$. Expert e_4

cannot give any information about the influence degree. Therefore, his or her assessment is described as $b_{ij}^{4}(\theta) = \{(\Theta, 1.00)\}$.

Normally, expert weight is subjective and relative to reflect the importance of one's assessments in a group, and it is usually denoted by w in $[0, 1]$, with 0 and 1 respectively standing for not important at all and the most important (Du and Wang 2017; Du et al. 2018a). It is obvious to find that the expert weight can be determined by AHP (Saaty 2003), ANP (Saaty 2007) or Delphi (Stebler et al. 2015), and it can also be determined subjectively according to the requirements of actual issues (Li et al. 2015). As discussed, expert weight is used to account for one's relative importance among all experts—that is, the closer one's assessments are to others' assessments, the more important the expert is likely to be (Huang et al. 2014). Hence, in our opinion, expert weight could be calculated indirectly based on the similarity between one expert and other experts—that is, the higher the similarity of assessments between the expert and others, the larger the weight of the expert. Expert weight is directly proportional to the similarity between expert assessments and others' assessments. Thus, we use evidential distance to depict the similarity of expert assessments, based on which we could calculate expert weight in simple ways. The Euclidean evidential distance and Euclidean evidential similarity, which have little computation complexity and fast convergence speed, are defined as follows.

Definition 7 (Li et al. 2011) Suppose m_1 and m_2 are two BBA functions for the same frame of discernment Θ, θ_n is the n th element of D^{Θ}, and $\left|D^{\Theta}\right|$ is the cardinality of D^{Θ}. The distance between m_1 and m_2 is defined as follows.

$$Dist_E(m_1, m_2) = \frac{1}{\sqrt{2}} \sqrt{\sum\nolimits_{n=1}^{\left|D^{\Theta}\right|} [m_1(\theta_n) - m_2(\theta_n)]^2} \tag{7}$$

Definition 8 (Li et al. 2011) Suppose m_1 and m_2 are two BBA functions for the same frame of discernment Θ, θ_n is the n th element of D^{Θ}, and $\left|D^{\Theta}\right|$ is the cardinality of D^{Θ}. The Euclidean similarity function $Sim_E(m_1, m_2)$ is defined based on the Euclidean evidential distance as follows.

$$Sim_E(m_1, m_2) = 1 - \frac{1}{\sqrt{2}} \sqrt{\sum\nolimits_{n=1}^{\left|D^{\Theta}\right|} [m_1(\theta_n) - m_2(\theta_n)]^2} \tag{8}$$

Example 2 Assume two pieces of evidence, $b_1 = \{(\theta_2, 1)\}$ and $b_2 = \{(\theta_1, 0.4), (\theta_2, 0.6)\}$, and that the distance between them could be computed by Eq. (7): $Dist_E(b_1, b_2) = \sqrt{(0 - 0.4)^2 + (1 - 0.6)^2} / \sqrt{2} = 0.40$. The similarity between them is computed by Eq. (8): $Sim_E(b_1, b_2) = 1 - 0.40 = 0.60$.

On the basis of the previous definitions, the similarity between experts can be directly calculated. Thus, we define the following expert weight calculation method.

Definition 9 (Yang and Wang 2010) Suppose the individual IDR matrix consisting of all influence relations among factors given by expert e_k is $B^k = [b_{ij}^k]_{L\times L}$, with $i,j = 1,\dots,L$ and $k = 1,\dots,K$. The similarity between experts e_k and $e_{k'}$ on each pair of factors $f_i \to f_j$ is $Sim_E(b_{ij}^k, b_{ij}^{k'})$, and the similarity between any two experts is computed as $s^{kk'} = \sum_{i,j=1,i\neq j}^{L} Sim_E(b_{ij}^k, b_{ij}^{k'})/L^2$, where L^2 denotes the quantity of factor pairs, making up the similarity matrix as follows.

$$S = [s^{kk'}]_{K\times K} = \begin{bmatrix} 1 & \cdots & s^{1k} & \cdots & s^{1K} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ s^{k1} & \cdots & 1 & \cdots & s^{kK} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ s^{K1} & \cdots & s^{Kk} & \cdots & 1 \end{bmatrix} \tag{9}$$

The support of e_k can be obtained by adding all of the elements in the similarity matrix S that are related to expert e_k except for self-similarity, i.e., $Sup(e_k) = \sum_{k'=1,k'\neq k}^{K} s^{kk'}$, $k = 1,\dots,K$, where $Sup(e_k)$ represents the support degree of expert e_k received from other experts. By normalizing them, we could obtain the expert credibility $Crd(e_k)$ which is generally regarded as expert weight w^k as follows.

$$w^k = Crd(e_k) = Sup(e_k)/\sum_{k=1}^{K} Sup(e_k),\ k = 1,\dots,K \tag{10}$$

Example 3 Assume experts e_1, e_2, e_3 give their assessments for the influence relations among factor set $F = \{f_1, f_2, f_3\}$, respectively, as follows:

$$B^1 = \begin{bmatrix} \{(\theta_1, 1.0)\} & \{(\theta_2, 1.0)\} & \{(\theta_3, 1.0)\} \\ \{(\theta_2, 0.7), (\theta_3, 0.3)\} & \{(\theta_1, 1.0)\} & \{(\theta_1, 0.4), (\theta_3, 0.6)\} \\ \{(\theta_1, 1.0)\} & \{(\theta_2, 1.0)\} & \{\theta_1, 1.0)\} \end{bmatrix}$$

$$B^2 = \begin{bmatrix} \{(\theta_1, 1.0)\} & \{(\theta_2, 1.0)\} & \{(\theta_3, 1.0)\} \\ \{(\theta_1, 0.8), (\theta_2, 0.2)\} & \{(\theta_1, 1.0)\} & \{(\theta_1, 1.0)\} \\ \{(\theta_1, 1.0)\} & \{(\theta_1, 0.5), (\theta_2, 0.5)\} & \{(\theta_1, 1.0)\} \end{bmatrix}$$

$$B^3 = \begin{bmatrix} \{(\theta_1, 1.0)\} & \{(\theta_2, 1.0)\} & \{(\theta_1, 0.2), (\theta_2 \cup \theta_3, 0.8)\} \\ \{(\theta_2 \cup \theta_3, 1.0)\} & \{(\theta_1, 1.0)\} & \{(\theta_1, 1.0)\} \\ \{(\theta_1, 1.0)\} & \{(\theta_1, 1.0)\} & \{(\theta_1, 1.0)\} \end{bmatrix}$$

The similarity matrix can be calculated by Definition 9 as follows:

$$S=\begin{bmatrix} 1.00 & 0.47 & 0.29 \\ 0.47 & 1.00 & 0.41 \\ 0.29 & 0.41 & 1.00 \end{bmatrix}$$

We calculate the support degree of experts as $Sup(e_1) = 0.47 + 0.29 = 0.76$, $Sup(e_2) = 0.47 + 0.41 = 0.88$, $Sup(e_3) = 0.29 + 0.41 = 0.70$. Then we compute the expert weight as $w^1 = Crd(e_1) = 0.76/(0.76 + 0.88 + 0.70) \approx \{(\theta_1, P_{ij}^1), (\theta_2, P_{ij}^2), (\theta_3, P_{ij}^3)\}$, w^2 =$0.87/2.34 \approx 0.37$, $w^3 = 0.70/2.34 \approx 0.30$.

As mentioned above, expert weight should be considered through Shafer's discounting method, which multiplies the masses of focal elements by the expert weight and transfers all of the remaining discounted mass to the full ignorance Θ. Mathematically, Shafer's discounting method can be given as in Definition 10.

Definition 10 (Shafer 1996) Suppose the BBA function is b as in Eq. (4), and w is a parameter to discount the evidence, $0 \le w \le 1$. Then, Shafer's discounting method is defined as follows:

$$m(\theta) = \begin{cases} 0, & \theta = \emptyset \\ w \cdot b(\theta), \theta \subset \Theta, & \theta \ne \emptyset \\ w \cdot b(\theta) + 1 - w, & \theta = \Theta \end{cases} \tag{11}$$

If the sum of weights is equal to 1, then the discounting parameter is usually derived by standardizing the weights as $\bar{w}^k = w^k / \max(w^k | k = 1, \dots, K)$. The discounted expert assessments could be described as follows.

$$m_{ij}^k = \left\{ (\theta, m_{ij}^k(\theta)) | \sum_{\theta \subseteq \Theta} m_{ij}^k(\theta) = 1; m_{ij}^k(\theta) \ge 0, \theta \subseteq \Theta, \forall i, j, k \right\} \tag{12}$$

The discounted individual IDR matrix is denoted as $M^k = [m_{ij}^k]_{L \times L}$, with $i.j = 1, \dots, L$ and $k = 1, \dots, K$.

Example 4 Assume expert weight is $W = \{w^1 = 0.40, w^2 = 0.20, w^3 = 0.20, w^4 = 0.20\}$. The assessments given by four experts are the same as in Example 1. The discounting parameters for the four experts can be standardized as $\bar{w}^1 = w^1/\max(w^1, \dots, w^4)$= $0.40/0.40$=1.00, and similarly we get $\overline{w}^2 = \overline{w}^3 = \overline{w}^4 = 0.20/0.40 = 0.50$. Taking them into Eq. (11), the discounted assessments are as follows: $m_{ij}^1(\theta) = \{(\theta_1, 0.30), (\theta_2, 0.70)\}$,$m_{ij}^2(\theta) = \{(\theta_1, 0.10), ((\theta_2 \cup \theta_3), 0.40), (\Theta, 0.50)\}$, $m_{ij}^3(\theta) = \{(\theta_2 \cap \theta_3, 0.50), (\Theta, 0.50)\}$, and $m_{ij}^4(\theta) = \{(\Theta, 1.00)\}$.

The DSmT framework with PCR5 rule can be used to make fusion for individual discounted assessments as in Eq. (12) and the fusion result can be described as follows:

$$m_{ij} = \left\{ (\theta, m_{ij}(\theta)) | \sum_{\theta \subseteq \Theta} m_{ij}(\theta) = 1; \quad m_{ij}(\theta) \geq 0, \ \theta \subseteq \Theta, \ \forall i, j \right\} \tag{13}$$

The determined group IDR matrix is denoted as $M = [m_{ij}]_{L \times L}$.

Example 5 Assume two pieces of evidence are: $m_{ij}^1(\theta) = \{(\theta_1, 0.20), (\theta_2, 0.30), (\Theta, 0.50)\}$,$m_{ij}^2(\theta) = \{(\theta_2, 0.70), (\Theta, 0.30)\}$. Taking them into Eq. (5), the fusion results are obtained as $m_{ij}(\theta) = \{(\theta_1, 0.09), (\theta_2, 0.76), (\Theta, 0.15)\}$.

3.2 The Soft CRP for Group DEMATEL

Soft CRP can be designed with the aim of supporting experts until a group consensus is reached by following several discussion and adjustment rounds. As discussed in Sect. 1, situations in which all of the experts agree with each other unanimously are rare or not desirable in the decision making process. Since the "unanimous consensus" of conflict tolerance, which has the ability to satisfy all pairs of BBA functions, rarely exists, the choice of a "soft consensus" is largely subjective and application oriented (Liu 2006). That is, setting an acceptable CL threshold value (suppose that the value is 0.8 there) to guide the whole process to reach group consensus. Basically, this process relies on making assumptions about experts' willingness to change their opinion or preferences (Herrera-Viedma et al. 2007; Pérez et al. 2011). Initially, consensus measures are computed based on individual and group IDR matrices to determine whether an acceptable CL has been reached or not. If so ($CL \geq 0.8$), the soft CRP is finished and a consensus group IDR matrix can be obtained. Otherwise ($CL < 0.8$), the feedback mechanism will be activated, and the experts who are not contributing to the consensus are identified and the advice about how to alter their assessments is generated (Xu and Wu 2011).

This consensus measure can indicate the current consensus situation throughout the soft CRP. According to the characteristics of group DEMATEL, the consensus measure is categorized into two levels, i.e., pair-factors level and expert level. The pair-factors level is the most basic level and reflects the original conflict degree. At this level, experts make some adjustments to increase consistency among the group. The expert level is the highest level, which reflects the conflict degree between a specific expert and the group as a whole for the collected results on all pairs of factors (Wu et al. 2017). We employ these two levels of consensus measures to identify inconsistencies between experts and group.

The calculation methods for consensus measures can be derived based on the similarities between these assessments. At the pair-factors level, the consensus measure values can be calculated based on the Euclidean similarity function as in Definition 11. At the expert level, the consensus measure values can be calculated by adding the consensus measure values on all of pair-factors for an expert as in Definition 12. Obviously, the order for calculating the two levels of consensus measure is pair-factors level at first and then expert level. When determining the inconsistency assessments that need to be modified, the order is the highest level at first and then the most basic level. Experts only adjust the conflicting assessments based on the recommendations at the pair-factors level.

Definition 11 In the t th round, suppose the assessments of the pair of factors $f_i \to f_j$ given/calculated by expert e_k and group are $b_{ij}^{k,t}$ and m_{ij}^{t}. Then, the consensus measure values can be calculated by the Euclidean similarity function as follows.

$$c_{ij}^{k,t} = 1 - \frac{1}{\sqrt{2}}\sqrt{\sum\nolimits_{n=1}^{|D^{\Theta}|}[b_{ij}^{k,t}(\theta_n) - m_{ij}^{t}(\theta_n)]^2} \tag{14}$$

Definition 12 In the t th round, suppose the consensus measure value of expert e_k for $f_i \to f_j$ at pair-factors level is $c_{ij}^{k,t}$, $\forall i, j, k$, then the consensus measure value of expert e_k at expert level in this round can be calculated as follows.

$$c^{k,t} = \sum_{i,j=1,i\neq j}^{L} c_{ij}^{k,t} \Big/ L^2 \tag{15}$$

where L^2 denotes the quantity of factor pairs.

The closer the value $c^{k,t}$ is to 0, the greater the conflict degree; the closer the value is to 1, the smaller the conflict. Obviously, $c^{k,t} = 0$ indicates complete conflict between expert e_k and group, and $c^{k,t} = 1$ indicates no conflict between them. If each expert's consensus measure $c^{k,t}$ is larger than the acceptable CL threshold value, indicating that existing conflict degree can be accepted by all experts and group consensus is reached, then the soft CRP is finished and the current collected group IDR matrix $M^t = [m_{ij}^t]_{L\times L}$ is taken as a consensus result. Otherwise, if one's consensus measure $c^{k,t}$ is lower than the acceptable CL threshold value, indicating that strong conflict degree among experts, then the feedback mechanism is carried out to help the inconsistent experts adjust their assessments to enhance group consensus, and a new round $(t + 1)$ of group IDR matrix construction should be initiated.

Note that, expert weights may be not fixed during several of the rounds for the reason that expert assessments may be changed in the context of not reaching the acceptable CL. Thus, in each new round, we need to recalculate expert weights based on the latest assessments. In order to prevent the collective assessments from failing to converge after several discussion rounds, we incorporate a maximum number of rounds (t_{MAX}) in the soft CRP to develop. It can be ensured that the feedback mechanism will not be carried out when $t = t_{MAX}$ and the current collected group IDR matrix will be taken as the final result even if the acceptable CL has not been reached yet.

Suppose the acceptable CL threshold values at expert level and pair-factors level are ε_e and ε_f. Then the processes of feedback mechanism can be divided into the following two steps.

Step 1 Experts whose consensus measure values at expert level are lower than the threshold value ε_e in the t th round are identified as follows.

$$\tilde{E}_t = \{k | c^{k,t} < \varepsilon_e\} \tag{16}$$

Step 2 For the identified experts in step 1, their assessments for such pairs of factors that the consensus measure values are lower than the threshold value ε_f are identified as follows.

$$\tilde{F}_t = \{k, i \to j | c_{ij}^{k,t} < \varepsilon_f \wedge k \in \tilde{E}\} \tag{17}$$

When $\tilde{E}$ and $\tilde{F}$ have been identified, personal advices for experts will be generated to reduce the conflict caused by the moderator. Since pair-factors level is the most basic level and contains the original assessments of experts, only the experts whose consensus measure values $c^{k,t}$ are lower than the threshold value ε_e may obtain the adjusted advices. After the identified experts all finish their adjustments, a new round of group IDR matrix construction will be carried out to determine the consensus situation. It is obvious to find that the above procedure is very similar to the Delphi method.

3.3 DSmT-Based Group DEMATEL Method and Algorithm

The construction of a group IDR matrix includes two main processes. The first is collection process, which focuses on expert assessment extraction and fusion; and the second is soft consensus reaching process, which aims at reaching an acceptable CL among experts. After the above two processes, the final group IDR matrix M can be constructed. Obviously, M is different from the original IDR matrix G as given in Definition 1 for two reasons. (1) M is a consensus group IDR matrix that satisfies all experts, while G does not consider consensus and satisfaction of experts. (2) M is made up of BBA functions that is capable of reflecting local or global ignorance in experts' mind, while G is made up of crisp values. Note that, the IDR matrix included in the DEMATEL is required to be exact influence degrees rather than BBA functions. Therefore, we apply the generalized pignistic probability as in Definition 13, which is an extension of the original pignistic probability (Smets 2005), to reassign the local and global ignorance in the BBA functions to singleton grade levels. Then, we calculate the expected value to derive the exact influence degrees.

Definition 13 (Smarandache and Dezert 2009) Suppose the frame of discernment in the DSmT framework is $\Theta = \{\theta_1, \dots, \theta_Y\}$, then the generalized pignistic probability for $\forall A \in D^{\Theta}$ can be calculated by Eq. (18).

$$P(A) = \sum_{X \in D^{\Theta}} \frac{|X \cap A|}{|X|} m(X) \tag{18}$$

where $|X|$ denotes the cardinal of proposition X.

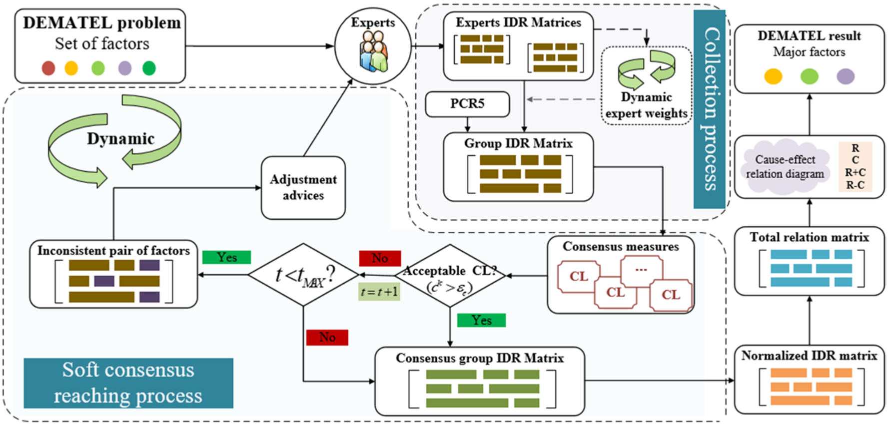

Fig. 2 The process of proposed method

Taking m_{ij} in the final group IDR matrix M into Eq. (18), the transformation results can be obtained and recorded as $\{(\theta_1, P_{ij}^1), \dots, (\theta_Y, P_{ij}^Y)\}$, where P_{ij}^y denotes the probability of influence degree θ_y,$y = 1, \dots, Y$. The subscript Y is equal to 3 for the reason that the frame of discernment is defined as $\Theta = \{\theta_1, \theta_2, \theta_3\} = \{0, 1, 2\}$ in this paper. Which influence grade level or influence degree is attached to the relationship of $f_i \to f_j$? We follow two kinds of principles to solve this problem: (1) Highest probability principle. The grade level with the highest probability $\tilde{g}_{ij} = \theta_*$ is chosen as the final influence grade level, where $\{\theta_* | P_{ij}^* = \max(P_{ij}^1, \dots, P_{ij}^Y)\}$. (2) Expected value principle. The expected value $\tilde{g}_{ij} = \sum_{y=1}^{Y} \theta_y \times P_{ij}^y, \forall i, j$ is calculated as the final influence degree. According to the real situation, we obtain the final influence degree for $f_i \to f_j$ by using one of the two principles, and then the final group IDR matrix $\tilde{G} = [\tilde{g}_{ij}]_{L\times L}$ can be constructed.

The process of DSmT-based group DEMATEL is illustrated as in Fig. 2. As shown in Fig. 2, the complete steps of DSmT-based group DEMATEL can be summarized as follows.

Step 1 *Define group DEMATEL problem and soft CRP parameters* Suppose the set of factors is $F = \{f_1, \dots, f_L\}$, the set of experts is $E = \{e_1, \dots, e_K\}$, the set of grade levels is $\Theta = \{\theta_1, \theta_2, \theta_3\}$, and the threshold value to filter out major factors is η. Then, the acceptable CL threshold values at expert level and pair-factors level are ε_e and ε_f, the round counter is t (t is set to one at first), and the maximum number of rounds is t_{MAX}. A moderator is invited to participate in the group decision making process and responsible for managing the whole process such as consensus measure value calculation, inconsistent expert identification, the maximum number of round judgement.

Step 2 *Expert assessments extraction and fusion* Experts are advised to indicate the influence degree to which he or she believes factor f_i has an effect on factor f_j (denoted by $f_i \rightarrow f_j$) under the framework of DSmT. The BBA functions assessed for each pair of factors $f_i \rightarrow f_j$ by expert e_k are described as b_{ij}^k (as in Eq. (6)), and make up an individual IDR matrix $B^K = [b_{ij}^k]_{L \times L}$. On the basis of expert assessments, expert weights $W = \{w^1, \dots, w^K\}$ are calculated by Definition 8. Initial assessments b_{ij}^k are discounted by Shafer's discounting method with expert weight as in Definition 10 to obtain the discounted assessments m_{ij}^k. Then PCR5 is applied to fuse those BBA functions to obtain group assessments m_{ij}, making up group IDR matrix $M = [m_{ij}]_{L \times L}$.

Step 3 *Soft consensus reaching process* The moderator calculates the consensus measure values $c_{ij}^{k,t}$ and $c^{k,t}$ by means of expert and group IDR matrices to identify whether an acceptable CL is reached in the current round. If so, the soft CRP is finished and a consensus group IDR matrix M has been obtained. Generalized pignistic probability is introduced to deal with global ignorance and local ignorance as in Eq. (18) and the influence degree for $f_i \rightarrow f_j$ $(\forall i, j)$ is determined by one of the two principles as mentioned above to construct $\tilde{G} = [\tilde{g}_{ij}]_{L \times L}$, and proceeding to step 4. Otherwise, let $t = t + 1$ and carry out the feedback mechanism. The moderator identifies the experts who differed strongly from the group by Eq. (16) and provides advices for them by Eq. (17). Then, return to step 2 and follow the same two processes. Note that the moderator also should detect whether the maximum number of rounds (means by $t = t_{MAX}$) has been reached before carrying out the feedback mechanism. If so, the mechanism will be ceased, taking the current group IDR matrix M as the final result and applying the generalized pignistic probability to obtain $\tilde{G} = [\tilde{g}_{ij}]_{L \times L}$. Then proceed to step 4.

Step 4 *Calculate the normalized IDR matrix* To guarantee that the normalized IDR matrix to infinite power will converge to zero and that the total relation matrix can be smoothly obtained, $\tilde{g}''_{ij}$=max(max$_{1 \le i \le L} \sum_{i=1}^{L} \tilde{g}_{ij}, \mu +$ max$_{1 \le j \le L} \sum_{j=1}^{L} \tilde{g}_{ij})$ is introduced as shown in Eq. (1) to calculate the normalized IDR matrix $D = [d_{ij}]_{L \times L}$, where $d_{ij} = \tilde{g}_{ij} / \tilde{g}''_{ij}, \forall i, j$.

Step 5 *Compute total relation matrix* The total relation matrix is calculated by $A = D(I - D)^{-1} = [a_{ij}]_{L \times L}$. The influencing degree of f_i is computed by $r_i = \sum_{j=1}^{L} a_{ij}$, the influenced degree of f_i is computed by $c_i = \sum_{j=1}^{L} a_{ji}$, the prominence degree of f_i is computed by $r_i + c_i$, and the net influence degree of f_i is computed by $r_i - c_i$.

Step 6 *Obtain the major factors with the threshold value* To simplify the complexity of a system to a manageable level, negligible factors should be filtered out. Only those factors whose prominence degrees are greater than the threshold value $(r_i + c_i \ge \eta)$ should be chosen and considered in the complex system. Generally, the cause-effect relation diagram can be made for the major factors based on their prominence degrees and net influence degrees.

Through the analysis of total relation of the factors by DEMATEL, a better understanding of the structural relation and an ideal way to solve complicate system problems can be obtained. The algorithm can be summarized to help moderator manage the whole processes.

Algorithm 1 The algorithm for DSmT-based group DEMATEL method with reaching consensus

Inputs: Set of factors $F=\{f_1,\ldots,f_L\}$, set of grade levels $\Theta=\{\theta_1,\theta_2,\theta_3\}$, set of experts $E=\{e_1,\ldots,e_K\}$, DEMATEL threshold value η, threshold values at expert level and pair-factors level ε_e and ε_f, round counter t, accepted maximum number of rounds t_{MAX}.
Outputs: Set of major factors $\hat{F}=\{f_1,\ldots,f_L\}$.
Begin
t=1, $\tilde{F}_t=\{k,i\rightarrow j\,|\,\forall k,\forall i,\forall j\}$, $b_{ij}^{k,0}=\{(\theta_1,1)\}$ for $\forall i,j,k$
Step 1: Collection process
For i=1 to L
  For j=1 to L
    If $i\neq j$
      For k=1 to K
        If $\{k,i\rightarrow j\}\in\tilde{F}_t$ Then
          Give/modify the influence degree $b_{ij}^{k,t}$ by expert e_k
        Else
          $b_{ij}^{k,t}=b_{ij}^{k,t-1}$
        EndIf
      EndFor
      Calculate expert weight $W_t=\{w^{k,t}\,|\,k=1,\ldots,K\}$ by Eqs.(8)-(10)
      Calculate $m_{ij}^{k,t}$ in Eq.(12) by discounting $b_{ij}^{k,t}$ with $w^{k,t}$
      Make fusion to get $m_{ij}^{t}=m_{ij}^{1,t}\oplus\ldots\oplus m_{ij}^{K,t}$ with Eq.(5)
    Else
      Let $m_{ij}^{t}=\{(\theta_1,1)\}$
    EndIf
  EndFor
EndFor
Get temporary group IDR matrix in the t th round $M^t=[m_{ij}^t]_{L\times L}$

Step 2: Soft consensus reaching process
For i=1 to L
  For j=1 to L
    Calculate consensus measure value at pair-factors level by $c_{ij}^{k,t}=1-\sqrt{\sum_{n=1}^{|D^\Theta|}[b_{ij}^{k,t}(\theta_n)-m_{ij}^{t}(\theta_n)]^2}\Big/\sqrt{2}$
  EndFor
EndFor
Calculate consensus measure value at expert level by $c^{k,t}=\sum_{i\neq j}c_{ij}^{k,t}\Big/L^2$
Derive inconsistent expert set by Eq.(16) and derive $\tilde{E}_t=\{k\,|\,c^{k,t}<\varepsilon_e\}$
Derive inconsistent pair of factors set by Eq.(17) and derive $\tilde{F}_t=\{k,i\rightarrow j\,|\,c_{ij}^{k,t}<\varepsilon_f\wedge k\in\tilde{E}\}$
If $\tilde{F}_t\neq\varnothing$ and $t\leq t_{MAX}$ Then
  For each $\{k,i\rightarrow j\}\in\tilde{F}_t$
    Give the advices that expert e_k is suggested to make corrections for $b_{ij}^{k,t}$
  EndFor
  Let t=t+1
  Return to Step 1
Else
  Get final BBA-formed group IDR matrix $M^t=[m_{ij}^t]_{L\times L}$
  Turn to Step 3
EndIf

Step 3: Group DEMATEL process
For i=1 to L
  For j=1 to L
    Take m_{ij}^t into Eq.(18) and get its generalized pignistic probability $\{(\theta_1,P_{ij}^1),(\theta_2,P_{ij}^2),(\theta_3,P_{ij}^3)\}$
    Use the highest probability or the expected value principle to obtain the final influence degree $\tilde{g}_{ij}$
  EndFor
EndFor
Construct final group IDR matrix $\tilde{G}=[\tilde{g}_{ij}]_{L\times L}$
Calculate normalized IDR matrix by $D=\tilde{G}/\max(\max_{1\leq i\leq L}\sum_{i=1}^{L}\tilde{g}_{ij},\mu+\max_{1\leq j\leq L}\sum_{j=1}^{L}\tilde{g}_{ij})$
Calculate total-relation matrix by $A=(a_{ij})_{L\times L}=D(I-D)^{-1}$
Initialize set of major factors by $\hat{F}=\varnothing$
For i=1 to L
  Calculate the important degree of f_i by $r_i+c_i=\sum_{j=1}^{L}a_{ij}+\sum_{j=1}^{L}a_{ji}$
  If $r_i+c_i\geq\eta$
    Add f_i into the set of major factors by $\hat{F}=\hat{F}\cup f_i$
  EndIf
EndFor
End

Table 1 Assessments of expert e_1 in the first round

	$f_1 \to f_2$	$f_1 \to f_3$	$f_1 \to f_4$	$f_1 \to f_5$	$f_2 \to f_3$	$f_2 \to f_4$	$f_3 \to f_4$	$f_4 \to f_5$	$f_5 \to f_2$	$f_5 \to f_3$
θ_1	0.00	0.50	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.00
θ_2	0.00	0.50	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.00
$\theta_1 \cup \theta_2$	0.30	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.80	0.80
θ_3	0.70	0.00	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00
$\theta_1 \cup \theta_3$	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.00
$\theta_2 \cup \theta_3$	0.00	0.00	0.00	0.20	0.50	0.20	0.00	0.00	0.00	0.00
$\theta_1 \cup \theta_2 \cup \theta_3$	0.00	0.00	1.00	0.80	0.00	0.60	0.00	1.00	0.20	0.20

4 Numerical Comparison and Discussion

In this section, we apply the proposed method and the fuzzy DEMATEL method to conduct a numerical simulation case, in which the key factors will be selected by two methods respectively. Afterwards, the results are compared and discussed.

4.1 The Proposed Method

Step 1 *Define DEMATEL problem and soft CRP parameters*

Similar to the examples in the literatures about DSmT (Smarandache et al. 2011), here the group IDR matrix construction of the given example works on the classical power set 2^{Θ}, not on the hyper-power set D^{Θ}. In the numerical simulation case, suppose the set of factors is $F = \{f_1, f_2, f_3, f_4, f_5\}$, the set of experts is $E = \{e_1, e_2, e_3, e_4, e_5\}$, the set of grade levels is $\Theta = \{\theta_1, \theta_2, \theta_3\} = \{0, 1, 2\}$, the threshold value is η=0.35, and the acceptable CL threshold values at expert level is ε_e=0.55, and at pair-factors level is ε_f=0.50, the round counter is t, and the maximum number of rounds is $t_{MAX} = 5$.

Step 2 *Expert assessments extraction and fusion*

Experts are advised to indicate the influence degree to which he or she believes factor f_i has an effect on factor f_j (denoted by $f_i \to f_j$) under the framework of DSmT. As an example, the BBA functions on each pair of factors $f_i \to f_j$ from expert e_1 are shown in Table 1. Due to limited space, other experts' assessments are not given in this paper.

On the basis of the experts' assessments, we calculate the similarity matrix by the Euclidean similarity function as given in Definition 8 and shown in Eq. (19).

$$S = \begin{bmatrix} 1.00 & 0.20 & 0.16 & 0.13 & 0.18 \\ 0.20 & 1.00 & 0.15 & 0.17 & 0.19 \\ 0.16 & 0.15 & 1.00 & 0.14 & 0.18 \\ 0.13 & 0.17 & 0.14 & 1.00 & 0.18 \\ 0.18 & 0.19 & 0.18 & 0.18 & 1.00 \end{bmatrix} \tag{19}$$

Table 2 Fusion results of expert assessments in the first round

	$f_1 \to f_2$	$f_1 \to f_3$	$f_1 \to f_4$	$f_1 \to f_5$	$f_2 \to f_3$	$f_2 \to f_4$	$f_3 \to f_4$	$f_4 \to f_5$	$f_5 \to f_2$	$f_5 \to f_3$
θ_1	0.00	0.23	0.00	0.13	0.00	0.05	0.46	0.00	0.07	0.05
θ_2	0.00	0.22	0.39	0.26	0.35	0.20	0.17	0.15	0.25	0.05
$\theta_1 \cup \theta_2$	0.08	0.16	0.00	0.00	0.03	0.05	0.08	0.00	0.30	0.32
θ_3	0.66	0.23	0.19	0.28	0.20	0.58	0.12	0.56	0.22	0.43
$\theta_1 \cup \theta_3$	0.02	0.06	0.00	0.00	0.00	0.05	0.16	0.06	0.01	0.06
$\theta_2 \cup \theta_3$	0.02	0.00	0.17	0.07	0.20	0.04	0.00	0.05	0.07	0.04
$\theta_1 \cup \theta_2 \cup \theta_3$	0.22	0.13	0.26	0.28	0.23	0.06	0.03	0.18	0.09	0.06

Accordingly, $Sup(e_1)^1 = 0.20 + 0.16 + 0.13 + 0.18 = 0.68$, $Sup(e_2)^1 = 0.72$, $Sup(e_3)^1 = 0.64$, $Sup(e_4)^1 = 0.62$, and $Sup(e_5)^1 = 0.73$; the expert weight is computed as $w^{1,1} = Sup(e_1)^1 / \sum_{k=1}^{5} Sup(e_k)^1 = 0.68/3.40 = 0.20$, $w^{2,1} = 0.21$, $w^{3,1} = 0.19$, $w^{4,1} = 0.18$, and $w^{5,1} = 0.22$; the discounting parameters are derived by $\bar{w}^{1,1} = w^{1,1}/\max\{w^{k,1}|k = 1, \ldots, 5\} = 0.20/0.22 = 0.93$, $\bar{w}^{2,1} = 0.98$, $\bar{w}^{3,1} = 0.87$,$\bar{w}^{4,1} = 0.82$, and $\bar{w}^{5,1} = 1.00$. Taking $b_{ij}^{k,1}$ and $\bar{w}^{k,1}$ into Shafer's discounting method (as in Eq. (11)), we derive the discounted assessments $m_{ij}^{k,1}$, $i,j = 1, \ldots, 5$, and $k = 1, 2, 3, 4, 5$. With the help of the MATLAB code for PCR5, we fuse the BBA functions and derive a holism result for the influence degrees between any two factors, as shown in Table 2.

Step 3 *Soft consensus reaching process*

The moderator calculates the consensus measure values $c_{ij}^{k,1}$ and $c^{k,1}$ by means of $b_{ij}^{k,1}$ and m_{ij}^1 to identify whether an acceptable CL has been reached. Taking expert and fusion assessment BBA functions into Eq. (13) and Eq. (14), we calculate the consensus measure values at two levels, as shown in Tables 3 and 4.

Table 3 The consensus measure values at pair-factors level in the first round

	$f_1 \to f_2$	$f_1 \to f_3$	$f_1 \to f_4$	$f_1 \to f_5$	$f_2 \to f_3$	$f_2 \to f_4$	$f_3 \to f_4$	$f_4 \to f_5$	$f_5 \to f_2$	$f_5 \to f_3$
e_1	0.78	0.65	0.38	0.53	0.58	0.40	0.80	0.29	0.56	0.53
e_2	0.74	0.61	0.50	0.53	0.35	0.42	0.78	0.79	0.56	0.79
e_3	0.78	0.68	<u>0.32</u>	0.57	0.57	<u>0.20</u>	0.56	<u>0.45</u>	0.52	0.55
e_4	<u>0.27</u>	0.57	0.53	0.68	<u>0.37</u>	<u>0.35</u>	0.80	<u>0.38</u>	0.66	0.70
e_5	0.78	0.67	0.44	0.57	0.49	0.67	0.57	0.77	0.66	0.60

The underlined data indicates the pair of factors that do not reach the set consensus level

Table 4 The consensus measure values at expert level in the first round

	e_1	e_2	e_3	e_4	e_5
$c^{k,1}$	0.55	0.61	<u>0.52</u>	<u>0.53</u>	0.62

The underlined data indicates the experts that do not reach the set consensus level

Table 5 Adjusted assessments of expert e_3 in the second round

	$f_1 \to f_2$	$f_1 \to f_3$	$f_1 \to f_4$	$f_1 \to f_5$	$f_2 \to f_3$	$f_2 \to f_4$	$f_3 \to f_4$	$f_4 \to f_5$	$f_5 \to f_2$	$f_5 \to f_3$
θ_1	0.00	0.00	0.00	0.50	0.00	0.04	0.00	0.00	0.00	0.00
θ_2	0.00	0.00	0.22	0.50	0.80	0.16	0.00	0.28	0.00	0.00
$\theta_1 \cup \theta_2$	0.30	0.30	0.00	0.00	0.20	0.04	0.20	0.00	0.00	0.70
θ_3	0.70	0.30	0.37	0.00	0.00	0.46	0.40	0.28	0.00	0.00
$\theta_1 \cup \theta_3$	0.00	0.00	0.00	0.00	0.00	0.04	0.40	0.07	0.20	0.30
$\theta_2 \cup \theta_3$	0.00	0.00	0.07	0.00	0.00	0.03	0.00	0.07	0.50	0.00
$\theta_1 \cup \theta_2 \cup \theta_3$	0.00	0.40	0.34	0.00	0.00	0.24	0.00	0.30	0.30	0.00

Table 6 Adjusted assessments of expert e_4 in the second round

	$f_1 \to f_2$	$f_1 \to f_3$	$f_1 \to f_4$	$f_1 \to f_5$	$f_2 \to f_3$	$f_2 \to f_4$	$f_3 \to f_4$	$f_4 \to f_5$	$f_5 \to f_2$	$f_5 \to f_3$
θ_1	0.00	0.00	0.00	0.00	0.00	0.03	0.50	0.00	0.00	0.00
θ_2	0.00	0.00	0.00	0.50	0.22	0.14	0.30	0.08	0.00	0.00
$\theta_1 \cup \theta_2$	0.06	0.30	0.00	0.00	0.02	0.02	0.20	0.00	0.40	0.00
θ_3	0.48	0.70	0.50	0.50	0.13	0.39	0.00	0.36	0.60	0.60
$\theta_1 \cup \theta_3$	0.02	0.00	0.00	0.00	0.00	0.05	0.00	0.14	0.00	0.20
$\theta_2 \cup \theta_3$	0.02	0.00	0.50	0.00	0.13	0.12	0.00	0.12	0.00	0.20
$\theta_1 \cup \theta_2 \cup \theta_3$	0.43	0.00	0.00	0.00	0.51	0.27	0.00	0.30	0.00	0.00

According to the selected threshold value $\varepsilon_e = 0.55$ and Eq. (16), we find that the acceptable CL is not reached in this round for $\tilde{F}_t \neq \emptyset$. Then, the moderator lets $t = 1 + 1 = 2$, which is lower than t_{MAX}, and carries out the feedback mechanism. The order from expert level to pair-factors level should be followed to determine the inconsistent experts and the pairs of factors that need to be modified (for those values less than $\varepsilon_f = 0.50$). Tables 3 and 4 show that expert e_3 greatly differs from the group results on the pairs of factors $f_1 \to f_4$, $f_2 \to f_4$, and $f_4 \to f_5$, and e_4 greatly differs from the group results on the pairs of factors $f_1 \to f_2$, $f_2 \to f_3$, $f_2 \to f_4$, and $f_4 \to f_5$ (see those underlined data in Tables 3 and 4). Experts e_3 and e_4 need to modify those assessments on the mentioned pairs of factors. Suppose the adjusted assessments for expert e_3 and e_4 are shown in Tables 5 and 6.

Because some of the expert assessments are changed. In the second round, expert weights are recalculated to be $w^{1,2} = 0.19$, $w^{2,2} = 0.20$, $w^{3,2} = 0.20$, $w^{4,2} = 0.21$, and $w^{5,2} = 0.21$, and the corresponding discounting parameters are standardized as $\bar{w}^{1,2} = 0.89$, $\bar{w}^{2,2} = 0.93$, $\bar{w}^{3,2} = 0.96$, $\bar{w}^{4,2} = 1.00$, and $\bar{w}^{5,2} = 1.00$. Taking $b_{ij}^{k,2}$ and $\bar{w}^{k,2}$ into Eq. (11), we obtain the discounted assessments $m_{ij}^{k,2}$, $(\forall i,j)$ and fuse these results by PCR5 to construct the group IDR matrix in the second round. The moderator calculates the new consensus measure values at two levels. Fusion results and consensus measures at expert level in the second round are shown in Tables 7 and 8.

Table 7 Fusion results of experts' assessments in the second round

	$f_1 \to f_2$	$f_1 \to f_3$	$f_1 \to f_4$	$f_1 \to f_5$	$f_2 \to f_3$	$f_2 \to f_4$	$f_3 \to f_4$	$f_4 \to f_5$	$f_5 \to f_2$	$f_5 \to f_3$
θ_1	0.00	0.21	0.00	0.12	0.00	0.02	0.44	0.00	0.06	0.06
θ_2	0.00	0.20	0.48	0.26	0.42	0.10	0.17	0.06	0.22	0.05
$\theta_1 \cup \theta_2$	0.16	0.17	0.00	0.00	0.05	0.12	0.08	0.00	0.26	0.29
θ_3	0.69	0.27	0.13	0.23	0.20	0.53	0.14	0.74	0.28	0.43
$\theta_1 \cup \theta_3$	0.04	0.06	0.00	0.00	0.00	0.05	0.17	0.04	0.02	0.07
$\theta_2 \cup \theta_3$	0.04	0.00	0.16	0.06	0.27	0.08	0.00	0.02	0.08	0.05
$\theta_1 \cup \theta_2 \cup \theta_3$	0.07	0.11	0.24	0.33	0.08	0.14	0.01	0.15	0.09	0.06

Table 8 The consensus measure values at the expert level in the second round

	e_1	e_2	e_3	e_4	e_5
$c^{k,2}$	0.55	0.63	0.66	0.68	0.62

As shown in Table 8, the acceptable CL has been reached in the second round, which means that the current group IDR matrix is a consensus result and can be applied to the group DEMATEL method. Then we derive pignistic probability transformation for each element in final BBA-formed group IDR matrix $M^2 = [m_{ij}^2]_{5\times5}$. Take the pair of factors $f_1 \to f_2$ as an example, $m_{12}(\theta_1) = m(\theta_1) + \frac{1}{2}(\theta_1 \cup \theta_2) + \frac{1}{2}(\theta_1 \cup \theta_3) + \frac{1}{3}m(\theta_1 \cup \theta_2 \cup \theta_3) = 0.13$, $m_{12}(\theta_2) = 0.13$, $m_{12}(\theta_3) = 0.76$, and $\tilde{g}_{12} = 0 \times 0.13 + 1 \times 0.13 + 2 \times 0.76 = 1.64$. After pignistic probability transformation, the final group IDR matrix is constructed as in Eq. (20).

$$\tilde{G} = \begin{bmatrix} 0.00 & 1.64 & 0.99 & 1.22 & 1.13 \\ 0.00 & 0.00 & 1.33 & 1.53 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.67 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.76 \\ 0.00 & 1.12 & 1.25 & 0.00 & 0.00 \end{bmatrix} \tag{20}$$

Step 4 *Calculate the normalized IDR matrix*

Let $\mu = 0.00001$ and we have $\tilde{g}_{ij}''=\max(\max_{1\le i\le L}\sum_{i=1}^{L}\tilde{g}_{ij}, \mu + \max_{1\le j\le L}\sum_{j=1}^{L}\tilde{g}_{ij}) \approx 4.98$, and the normalized IDR matrix can be calculated by Eq. (1). The result is shown as in Eq. (21).

$$D = \begin{bmatrix} 0.00 & 0.33 & 0.20 & 0.24 & 0.23 \\ 0.00 & 0.00 & 0.27 & 0.31 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.13 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.35 \\ 0.00 & 0.22 & 0.25 & 0.00 & 0.00 \end{bmatrix} \tag{21}$$

Table 9 The attribute parameters of factors

	r	c	$r+c$	$r-c$
f_1	0.41	0.00	0.41	0.41
f_2	0.19	0.19	0.38	0.00
f_3	0.02	0.24	0.26	−0.22
f_4	0.13	0.23	0.36	−0.10
f_5	0.13	0.22	0.35	−0.09

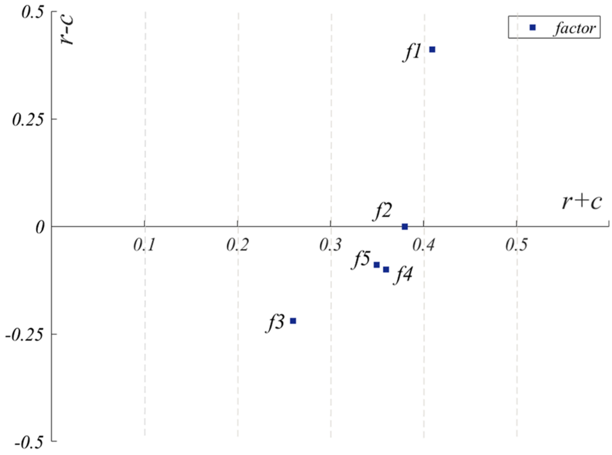

Fig. 3 The cause-effect relation diagram derived by the proposed method

Step 5 *Compute the total relation matrix*

The total relation matrix can be calculated by Eq. (2). The result is shown as

$$A = \begin{bmatrix} 0.00 & 0.14 & 0.08 & 0.10 & 0.09 \\ 0.00 & 0.00 & 0.08 & 0.11 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.02 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.13 \\ 0.00 & 0.05 & 0.08 & 0.00 & 0.00 \end{bmatrix} \tag{22}$$

According to $A = [a_{ij}]_{5\times5}$ and Eq. (3), we derive the following parameters: the total influence $r_i = \sum_{j=1}^{5} a_{ij}$ given by the factor f_i to other factors, the total influence $c_i = \sum_{j=1}^{5} a_{ji}$ received by the factor f_i from other factors, the degree of the important role $r_i + c_i$, and the net influence $r_i - c_i$. These parameters are listed in Table 9. Additionally, we construct the cause-effect relation diagram of factors with the horizontal axis $r + c$ and the vertical axis $r - c$, as shown in Fig. 3. According to Definition 4 and Fig. 3, f_1 is known as net causer, whereas f_3,f_4,f_5 are net receivers.

Step 6 *Obtain major factors with the threshold value*

Because of the threshold value $\eta = 0.35$, we select the factors whose degrees of the important role are greater than the threshold ($r_i + c_i \geq \eta$) as major factors. As a result, the major factors in the complex system are $\hat{F} = \{f_1, f_2, f_4, f_5\}$.

Table 10 The fuzzy linguistic scale

Linguistic terms	Triangular fuzzy numbers
High influence (H)	(0.75, 1.00, 1.00)
Low influence (L)	(0.25, 0.50, 0.75)
No influence (No)	(0.00, 0.00, 0.25)

We find that the Algorithm 1 can be programmed easily. Thus, the proposed method in this study is valid and applicable for solving group DEMATEL problems.

4.2 The Fuzzy DEMATEL Method

The fuzzy DEMATEL method proposed by Wu and Lee (2007) is a vital and significant improvement of DEMATEL. Since the fuzzy DEMATEL method is not only a major extension of DEMATEL but also extracts information with fuzzy linguistic terms, here we employ it to make a comparison with the proposed method. To ensure the results of two kinds of methods can be compared with each other, the set of factors $F = \{f_1, f_2, f_3, f_4, f_5\}$, the set of experts $E = \{e_1, e_2, e_3, e_4, e_5\}$ and the threshold value is η=0.35 are all the same as in Sect. 4.1. Besides, the inputs of fuzzy DEMATEL method should be generated from the initial assessments given by experts in the proposed method to ensure its comparability. According to the procedure of fuzzy DEMATEL method, its computation processes can be summarized as follows.

Step 1 *Define DEMATEL problem and fuzzy linguistic scale*

The DEMATEL problem is defined as the same as in Sect. 4.1 and the fuzzy linguistic scale is defined as in Table 10, where {No, L, H} are equal to $\{\theta_1, \theta_2, \theta_3\}$ as defined in this paper.

Step 2 *Exact and fuse expert assessments*

In order to ensure the comparability, the inputs of fuzzy DEMATEL method should be generated from the initial assessments given by experts in the proposed method. Because the assessments given by experts in Sect. 4.1 are in the form of BBA functions, how to make a transformation from the BBA functions to the inputs of fuzzy DEMATEL method is quite important. It is reasonable and logical to choose the grade level with the highest probability as expert assessments in the fuzzy DEMATEL method. Following this thought, the inputs of fuzzy

Table 11 The transformed assessments of expert e_1

	f_1	f_2	f_3	f_4	f_5
f_1	No	H	L	No	No
f_2	No	No	H	H	No
f_3	No	No	No	L	No
f_4	No	No	No	No	H
f_5	No	No	L	No	No

DEMATEL method are transformed from the experts' assessments in Sect. 4.1. For example, the transformed assessments of expert e_1 are shown in Table 11.

As introduced in fuzzy DEMATEL, the Converting Fuzzy data into Crisp Scores (CFCS) defuzzification method is applied to aggregate these assessments by five experts. Due to limited space, the detailed CFCS steps can be referred to Wu and Lee (2007) and are not repeated here. The IDR matrix $G' = [g'_{ij}]_{5\times5}$ is produced as in Eq. (23).

$$G' = \begin{bmatrix} 0.04 & 0.19 & 0.87 & 0.19 & 0.50 \\ 0.19 & 0.04 & 0.96 & 0.96 & 0.04 \\ 0.04 & 0.04 & 0.04 & 0.41 & 0.19 \\ 0.04 & 0.04 & 0.04 & 0.04 & 0.87 \\ 0.04 & 0.50 & 0.59 & 0.04 & 0.04 \end{bmatrix} \tag{23}$$

Step 3 *Calculate the normalized IDR matrix*

The normalizing method for IDR matrix in fuzzy DEMATEL is the same as in traditional DEMATEL. The normalized IDR matrix is calculated by taking Eq. (23) into Eq. (1) and it is shown as in Eq. (24).

$$D' = \begin{bmatrix} 0.02 & 0.08 & 0.35 & 0.08 & 0.20 \\ 0.08 & 0.02 & 0.38 & 0.38 & 0.02 \\ 0.02 & 0.02 & 0.02 & 0.16 & 0.08 \\ 0.02 & 0.02 & 0.02 & 0.02 & 0.35 \\ 0.02 & 0.20 & 0.24 & 0.02 & 0.02 \end{bmatrix} \tag{24}$$

Step 4 *Compute the total relation matrix*

The computing method for total-relation matrix in fuzzy DEMATEL is also the same as in traditional DEMATEL. The total-relation IDR matrix is calculated by taking Eq. (24) into Eq. (2) and it is shown as in Eq. (25).

$$A' = \begin{bmatrix} 0.02 & 0.01 & 0.18 & 0.02 & 0.07 \\ 0.01 & 0.02 & 0.21 & 0.20 & 0.00 \\ 0.00 & 0.00 & 0.02 & 0.03 & 0.01 \\ 0.00 & 0.00 & 0.00 & 0.02 & 0.14 \\ 0.00 & 0.05 & 0.09 & 0.00 & 0.02 \end{bmatrix} \tag{25}$$

Table 12 The factors' attributes parameters

	r'	c'	$r' + c'$	$r' - c'$
f_1	0.30	0.03	0.33	0.27
f_2	0.44	0.08	0.52	0.36
f_3	0.07	0.50	0.57	−0.43
f_4	0.17	0.28	0.44	−0.11
f_5	0.16	0.25	0.41	−0.09

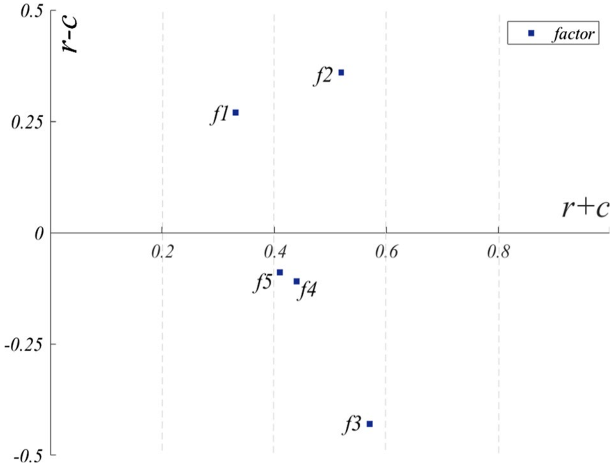

Fig. 4 The cause-effect relation diagram derived by fuzzy DEMATEL method

According to Eq. (3), we derive the following parameters as shown in Table 12 and the cause-effect relation diagram as shown in Fig. 4. According to Definition 4 and Fig. 4, it is obvious to find that f_1,f_2 are net causers, while f_3,f_4,f_5 are net receivers.

Step 5 *Set a threshold value and obtain the major factors*

Because of the threshold value $\eta = 0.35$, only the factors with $r_i' + c_i' \geq \eta$ should be chosen as the major factors. As a result, the major factors in the complex system are $\hat{F}' = \{f_2,f_3,f_4,f_5\}$.

4.3 Discussion

It is obvious to find that the major factors in the complex system determined by the proposed method are $\hat{F} = \{f_1,f_2,f_4,f_5\}$, while those determined by the fuzzy DEMATEL method are $\hat{F}' = \{f_2,f_3,f_4,f_5\}$. The results of two kinds of methods are different from each other. Which one is more reasonable? Now we make discussions from the following three aspects.

(1) Expert assessment extraction mechanism. The assessments of experts are used as the fundamental inputs of DEMATEL whether in the proposed method or in the fuzzy DEMATEL method. The proposed method allows experts to give assessments with BBA functions, while the fuzzy DEMATEL method employs the fuzzy linguistic scale to express their assessments. The local or global ignorance in experts' minds can be well reflected in the proposed method (e.g., $b_{12}^{1,1} = \{((\theta_1, \theta_2), 0.30), (\theta_3, 0.70)\}$), but it is unfortunate to find that such the ignorance cannot included in the fuzzy DEMATEL method (it seems that only the grade level/fuzzy linguistic scale with the highest probability may be allowed to express assessments, e.g., $b_{12}^{1,1} = \{L\} = \{\theta_3\}$). Consequently, it is believed that

the proposed method is more feasible than the fuzzy DEMATEL method in the aspect of expert assessment extraction mechanism.

(2) The importance roles of experts. As discussed in Sect. 3.1, the importance roles of experts are reflected by expert weights that are calculated based on expert assessments information, hence, the weight parameters could effectively express the relative importance of experts in the group and they have a significant impact on the group DEMATEL decision results. If the importance roles of experts are neglected in the process of decision making, the decision results may lose effectiveness. The proposed method calculates expert weights based on expert assessments with the aid of evidence distance and employs Shafer's discounting method to modify the subjective assessments. Unfortunately, the importance roles of experts are unconsidered in the whole processes of fuzzy DEMATEL method. Consequently, it is believed that the proposed method is more accurate than the fuzzy DEMATEL method in the aspect of reflecting the importance roles of experts.

(3) Group consensus reaching. Group consensus reaching is a key problem in the GDM field. Only when the assessments given by experts reach an acceptable CL, the GDM results are seen to be valuable and credible. In other worlds, the GDM results that lack of consensus may be ineffective and have few reference value for decision-making. From Sect. 4.2, we know that the fuzzy DEMATEL method is a static method without taking the group consensus reaching into consideration. In the proposed method, we apply the soft CRP into the construction of group IDR matrix to help the experts group reach a high CL, among which the feedback and modification mechanism are introduced. Consequently, it is believed that the proposed method is more reasonable than the fuzzy DEMATEL in the aspect of reaching group consensus.

5 Conclusions

In the present study, the DSmT is used to extract and fuse expert assessments, and a soft CRP is introduced to construct the group IDR matrix for the DEMATEL. The DSmT-based group DEMATEL method and the corresponding algorithm are proposed. Moreover, a numerical comparison is performed to discuss the applicability of the proposed method and algorithm. The main contributions of the present study can be summarized into three aspects.

Firstly, an expert assessment extraction and fusion mechanism is established on the basis of DSmT. Expert assessments on the influence relations among factors are extracted by BBA functions, which can help experts to express uncertainty and incompleteness assessments. The PCR5 of DSmT, which can overcome the defects of intuitional paradox in Dempster's combination rule, is employed to make fusion for the individual BBA functions discounted by Shafter's discounting method.

Secondly, expert weights are defined and introduced to reflect importance roles of experts in a group. Following the principle of pairwise comparisons, expert weights are calculated by the Euclidean similarity function to reflect experts' relative importance in the group. Expert weights are not fixed during the whole processes of group IDR matrix construction and they may vary dynamically with different expert assessments in each round. Shafer's discounting method is used to discount expert assessments in each round so as to reflect the importance roles of experts in the group dynamically. The above processes are beneficial to obtain an accurate group IDR matrix for the DEMATEL.

Thirdly, a soft CRP is established to construct the group IDR matrix with consensus and an algorithm is summarized for the group DSmT-based DEMATEL. The consensus measures at expert and pair-factors levels are defined to help establish the feedback/modification mechanism, based on which a soft CRP is established for the construction of consensus group IDR matrix, so that experts can reach a high CL. An algorithm for DSmT-based group DEMATEL method with reaching consensus is proposed to identify major factors in a complex system. The proposed algorithm can be programmed easily and is valid and applicable for solving group DEMATEL problems.

The proposed method forms an expert assessment extraction mechanism on the basis of the BBA function. However, the belief degrees or probabilities in BBA functions given by experts may hardly be assessed with exact values in more complex situations. Therefore, investigating how to deal with the group DEMATEL with the interval BBA function may be a good direction for future research.

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