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# Convergence of Fuzzy Neutrosophic Soft Circulant Matrices

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**Abstract.** The iterates of Fuzzy Neutrosophic Soft Circulant Matrices (FNSCMs) under the max-min product is examined right now. It is indicated that on the off chance that the first row of a FNSCM is in decreasing order, at that point the iterates of the circulant converge and on the off chance that the first row is in increasing order, at that point the iterates oscillate.

#### 1. Introduction

Uncertainty structures have a significant part in our day by day life. The customary methods may not be sufficient and simple, so Zadeh [34] gave the presentation of fuzzy set theory and this came out to be a contemporary for the investigation of some uncertainty types at whatever point old systems didn't work. Fuzzy theory and the speculations in regards to it added to some surprising outcomes, all things considered, that include uncertainty of specific kind. Ranjit Biswas[29] has broken down that whether the fuzzy theory is a suitable tool for the enormous size issue.

In 1995, Smarandache [30] established a theory called neutrosophic theory and neutrosophic set has ability to manage uncertainty, imprecise, inadequate and conflicting data which exist in the genuine world. The theory is an incredible asset which sums up the idea of the crisp set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, thus on.

In 1999, a Russian scientist Molodtsov [24] started the idea of soft set theory as a general numerical tool for dealing with uncertainty and ambiguity. After Molodtsov's work a few specialists were concentrated on soft set theory with applications. Magi et.al, [25] started the idea of fuzzy soft set with certain properties in regards to the fuzzy soft union, intersection and complement of a fuzzy soft set. In addition Maji et.al, [23] stretched out soft sets to intuitionistic fuzzy soft sets and neutrosophic soft sets.

The behaviour of composition iterates of fuzzy matrices (for example square matrices  $A = [a_{ij}], 0 \le a_{ij} \le 1$  for all i, j) under the max-min product has been concentrated in a few papers. In one of the early papers on this theme [32], Thomason demonstrated that the iterates of fuzzy matrix either converge or oscillate with a finite period. He got a few conditions, adequate for the convergence of the max-min iterates  $A^n$  of a fuzzy matrix A, and demonstrated that the limiting matrix is the permanently adjoint of A. (Thomason utilizes the term adjoint for the permanental adjoint.) From the few authors have discovered different conditions on fuzzy matrix that are either adequate for convergence or oscillation [10, 17, 18]. In the last case results

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about the time of the emphasizes have additionally showed up [18].

Bora et.al, [28] presented the intuitionistic fuzzy soft matrices and applied in the use of a Medical determination. Sumathi and Arokiarani [31] presented new procedure on fuzzy neutrosophic soft matrices (FNSMs).

First time Kavitha et.al, [12, 13, 14, 20, 21] presented the idea of unique solvability of maxmin operation through FNSM condition Ax = b and clarified strong regularity of FNSMs over fuzzy neutrosophic soft algebra based computing and registering the greatest X-eigenvector of FNSM. They additionally presented on the robustness of FNSM and Monotone interval fuzzy neutrosophic soft eigenproblem. Uma et.al, [33] presented two sorts of FNSMs.

In this article we study the max-min Iterates of Fuzzy Neutrosophic Soft Circulant Matrices ((I) FNSCMs). We indicated that the entries of the iterates and-when it exists-the entries of the limit of the iterates of certain circulant matrices can be described precisely.

#### 2. Preliminaries

This area essentially depicts Neutrosophic Set (NS), Fuzzy Neutrosophic Soft Set (FNSS), Fuzzy Neutrosophic Soft Matrix (FNSM) and FNSMs of type-I. For the fundamental definitions and outline see ([30],[1],[24],[2],[33]).

# 3. Background of the problem

This area presents a portion of the phrasing, definitions and results that are to be utilized in the paper. Let

$$C_n = \begin{bmatrix} \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \cdots & \cdots & \langle 1,1,0 \rangle \\ \langle 1,1,0 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \cdots & \cdots & \langle 0,0,1 \rangle \\ \langle 0,0,1 \rangle & \langle 1,1,0 \rangle & \langle 0,0,1 \rangle & \cdots & \cdots & \langle 0,0,1 \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \cdots & \cdots & \langle 0,0,1 \rangle \\ \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \cdots & \langle 1,1,0 \rangle & \langle 0,0,1 \rangle \end{bmatrix}.$$

Some effectively obvious realities about  $C_n$  are expressed straight away. (Multiplication can be normal multiplication or the max-min product). The FNSM  $C_n$  has period n, for example  $C_n^n = I$  and n is the smallest such whole number for which this is valid. Further,  $C_nC_n^t = I = C^tC_n$ . Hence  $C^{-1} = C_n^t$ . An  $n \times n$  CFNSM has the form

$$\begin{bmatrix} \langle a_1^T, a_1^I, a_1^F \rangle & \langle a_2^T, a_2^I, a_2^F \rangle & \langle a_3^T, a_3^I, a_3^F \rangle & \cdots & \cdots & \langle a_{n-1}^T, a_{n-1}^I, a_{n-1}^F \rangle & \langle a_n^T, a_n^I, a_n^F \rangle \\ \langle a_n^T, a_n^I, a_n^F \rangle & \langle a_1^T, a_1^I, a_1^F \rangle & \langle a_2^T, a_2^I, a_2^F \rangle & \cdots & \cdots & \langle a_{n-2}^T, a_{n-2}^I, a_{n-2}^F \rangle & \langle a_{n-1}^T, a_{n-1}^I, a_{n-1}^F \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle a_3^T, a_3^I, a_3^F \rangle & \langle a_4^T, a_4^I, a_4^F \rangle & \langle a_5^T, a_5^I, a_5^F \rangle & \cdots & \cdots & \langle a_1^T, a_1^I, a_1^F \rangle & \langle a_2^T, a_2^I, a_2^F \rangle \\ \langle a_2^T, a_2^I, a_2^F \rangle & \langle a_3^T, a_3^I, a_3^F \rangle & \langle a_4^T, a_4^I, a_4^F \rangle & \cdots & \cdots & \langle a_n^T, a_n^I, a_n^F \rangle & \langle a_1^T, a_1^I, a_1^F \rangle \end{bmatrix}.$$

In this manner a FNSCM is dictated by its first column. We will call a circulant with sections in [0,1] a CFNSM. In conventional matrices, circulant matrices are described by the accompanying paradigm.

A similar portrayal holds for FNSMs under the max- min product.

# 3.1. Theorem

A  $n \times n$  FNSM A is a circulant if and only if  $A \odot C_n = C_n \odot A$ .

Proof: Let P be any permutation matrix and A a FNSM, at that point it is obvious that  $P \odot A = PA$  and  $A \odot P = AP$  where AP and PA are the max-min products. Henceforth  $A \odot C_n = C_n \odot A$  if and only if  $AC_n = C_n A$ .

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# 3.2. Theorem

Max-min products of FNSCMs will be FNSCMs. Specifically, the max-min iterates of a FNSCMs will be FNSCMs. Additionally, the transpose of a FNSCM is a FNSCM.

Proof: Let A and B be FNSCMs. At that point each of A and B commutes with  $C_n$ . Hence  $A \odot B$  commutes with  $C_n$ . Transposing the two sides of  $A \odot C_n = C_n \odot A$  yields  $C_n^t \odot A^t = A^t \odot C_n$ . Since  $C_n^t \odot C_n = I = C_n \odot C_n^t$ , we have  $A^t = C_n \odot A^t \odot C_n^t$  whence  $A^t \odot C_n = C_n \odot A^t$ .

In the spin-off, except if expressed something else, product will mean the max-min product. Accordingly we will discard the unique max-min product image of starting now and into the foreseeable future and compose AB for  $A \odot B$ . Also  $A^m$  will indicate the mth power of a FNSM A under the product. Another idea that we will utilize is the of convolution. In the event that  $\langle a_n^T, a_n^I, a_n^F \rangle$  and  $\langle b_n^T, b_n^I, b_n^F \rangle$  are both limited or unending successions, at that point their max-min convolution  $\langle a_n^T, a_n^I, a_n^F \rangle * \langle b_n^T, b_n^I, b_n^F \rangle$  is the arrangement whose nth term is given by  $\bigvee_{k=1}^n (\langle a_n^T, a_n^I, a_n^F \rangle \wedge \langle b_{n-(k-1)}^T, b_{n-(k-1)}^I, b_{n-(k-1)}^F \rangle)$ . We state that a grouping  $\langle a_n^T, a_n^I, a_n^F \rangle$  is decreasing (increasing) if  $\langle a_{n+1}^T, a_{n+1}^I, a_{n+1}^F \rangle \leq \langle a_n^T, a_n^I, a_n^F \rangle (\langle a_{n+1}^T, a_{n+1}^I, a_{n+1}^F \rangle \geq \langle a_n^T, a_n^I, a_n^F \rangle)$  for all n. If severe disparity holds for each n, then the arrangement is named  $\langle a_n^T, a_n^I, a_n^F \rangle$  is monotone (strictly

monotone) grouping is one that is either decreasing (strictly decreasing) or increasing (strictly increasing).

#### 4. Iterates of FNSCMs

Here we will talk about the behaviour of the iterates of any FNSCM when its first row is either in decreasing order or increasing order. Below we see a few perceptions in regards to the above concept.

**Observation I.** Let A be any  $n \times n$  FNSCM with first row  $[\langle a_1^T, a_1^I, a_1^F \rangle, \langle a_2^T, a_2^I, a_2^F \rangle, ..., \langle a_n^T, a_n^I, a_n^F \rangle]$ . Then the  $k^{th}$  segment of A is  $[\langle a_k^T, a_k^I, a_k^F \rangle, \langle a_{k-1}^T, a_{k-1}^I, a_{k-1}^F \rangle, ..., \langle a_1^T, a_1^I, a_1^F \rangle, \langle a_n^T, a_n^I, a_n^F \rangle]$ .

 $\langle a_{n-1}^T, a_{n-1}^I, a_{n-1}^F \rangle, ..., \langle a_{k+1}^T, a_{k+1}^I, a_{k+1}^F \rangle]^t.$  Assume now that A, B are  $n \times n$  FNSCMs whose first row are, separately,  $[\langle a_1^T, a_1^I, a_1^F \rangle, \langle a_2^T, a_2^I, a_2^F \rangle, ..., \langle a_n^T, a_n^I, a_n^F \rangle].$  and  $[\langle b_1^T, b_1^I, b_1^F \rangle, \langle b_2^T, b_2^I, b_2^F \rangle, ..., \langle b_n^T, b_n^I, b_n^F \rangle].$  As reference of the second content o enced before, AB is a FNSCM, whence it is totally controlled by its first row. Let  $[\langle c_1^T, c_1^I, c_1^F \rangle, \langle c_2^T, c_2^I, c_2^F \rangle, ..., \langle c_n^T, c_n^I, c_n^F \rangle]$  signify the principal column of AB. Let us consider how the qualities  $\langle c_k^T, c_k^I, c_k^F \rangle$  are processed:

$$\begin{split} \langle c_k^T, c_k^I, c_k^F \rangle &= (\langle a_1^T, a_1^I, a_1^F \rangle \wedge \langle b_k^T, b_k^I, b_k^F \rangle) \vee (\langle a_2^T, a_2^I, a_2^F \rangle \wedge \langle b_{k-1}^T, b_{k-1}^I, b_{k-1}^F \rangle) \vee \ldots \vee \\ (\langle a_k^T, a_k^I, a_k^F \rangle \wedge \langle b_1^T, b_1^I, b_1^F \rangle) \vee (\langle a_{k+1}^T, a_{k+1}^I, a_{k+1}^F \rangle \wedge \langle b_n^T, b_n^I, b_n^F \rangle) \vee (\langle a_{k+1}^T, a_{k+1}^I, a_{k+1}^F \rangle \wedge \langle b_{n-1}^T, b_{n-1}^I, b_{n-1}^F \rangle) \vee \ldots \vee (\langle a_n^T, a_n^I, a_n^F \rangle \wedge \langle b_{k+1}^T, b_{k+1}^I, b_{k+1}^F \rangle). \end{split}$$

Accept now that both of the FNSVs  $[\langle a_1^T, a_1^I, a_1^F \rangle, \langle a_2^T, a_2^I, a_2^F \rangle, ..., \langle a_n^T, a_n^I, a_n^F \rangle]$  and  $[\langle b_1^T, b_1^I, b_1^F \rangle, \langle b_2^T, b_2^I, b_2^F \rangle, ..., \langle b_n^T, b_n^I, b_n^F \rangle]$ , are decreasing order (i.e.  $\langle a_i^T, a_i^I, a_i^F \rangle \geq \langle a_{i+1}^T, a_{i+1}^I, a_{i+1}^F \rangle$ and  $\langle b_i^T, b_i^I, b_i^F \rangle \geq \langle b_{i+1}^T, b_{i+1}^I, b_{i+1}^F \rangle$  for i = 1, 2, ..., n-1). At that point it is anything but difficult to see that, for any k = 2, 3, ..., n and for i = 1, ..., k

$$\langle a_i^T, a_i^I, a_i^F \rangle \wedge \langle b_{k-i+1}^T, b_{k-i+1}^I, b_{k-i+1}^F \rangle \geq (\langle a_{k+1}^T, a_{k+1}^I, a_{k+1}^F \rangle \wedge \langle b_n^T, b_n^I, b_n^F \rangle) \vee (\langle a_{k+2}^T, a_{k+2}^I, a_{k+2}^F \rangle \wedge \langle b_{n-1}^T, b_{n-1}^I, b_{n-1}^F \rangle) \vee \ldots \vee (\langle a_n^T, a_n^I, a_n^F \rangle \wedge \langle b_{k+1}^T, b_{k+1}^I, b_{k+1}^F \rangle).$$

Along these lines  $\langle c_k^T, c_k^I, c_k^F \rangle = (\langle a_1^T, a_1^I, a_1^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_{k-1}^T, b_{k-1}^I, b_{k-1}^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_{k-1}^T, b_{k-1}^I, b_{k-1}^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, b_k^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, a_2^F \rangle \land \langle b_k^T, b_k^I, a_2^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, a_2^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, b_k^I, a_2^F \rangle) \lor (\langle a_2^T, a_2^I, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle \land \langle b_k^T, a_2^F \rangle) \lor (\langle a_2^T, a_2^F \rangle) \lor (\langle a_2^T$ ...  $\vee (\langle a_k^T, a_k^I, a_k^F \rangle \wedge \langle b_1^T, b_1^I, b_1^F \rangle)$  for k = 1, 2, ..., n. The accompanying perception is unimportant from perception I.

**Observation II.** If the first row of A and B are in decreasing order, at that point the first column of AB is the convolution of the first row of A with the first row of B.

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We next infer an outcome, Lemma 4.1, about the multifold convolution of a decreasing sequence with itself. This outcome will assume a significant job in depicting the emphasizes of a FNSCMs.

# 4.1. Lemma

Let  $S = (\langle a_n^T, a_n^I, a_n^F \rangle)$  be a infinite sequence in [0,1] and indicate by  $S^m$ , the m-fold max-min convolution of S with itself. On the off chance that the sequence  $(\langle a_n^T, a_n^I, a_n^F \rangle)$  is decreasing, at that point

$$S^{m} = \{\langle a_{1}^{T}, a_{1}^{I}, a_{1}^{F} \rangle, \underbrace{\langle a_{2}^{T}, a_{2}^{I}, a_{2}^{F} \rangle, ..., \langle a_{2}^{T}, a_{2}^{I}, a_{2}^{F} \rangle}_{m}, \underbrace{\langle a_{3}^{T}, a_{3}^{I}, a_{3}^{F} \rangle, ..., \langle a_{3}^{T}, a_{3}^{I}, a_{3}^{F} \rangle}_{m}, ...\}$$

for example  $S^m(1) = \langle a_1^T, a_1^I, a_1^F \rangle$  and for  $q=0,1,2,...,S^m(n)=a_{q+2}$  when n fulfills  $qm+1 < n \leq (q+1)m+1$ .

Proof: The confirmation follows by numerical enlistment on m. The outcome obviously evident when m=1. Suppose that it holds for some number  $m\geq 1$ . Now,  $S^{m+1}=S^m*S$ , so the main term of  $S^{m+1}$  is  $\langle a_1^T, a_1^I, a_1^F \rangle$ . Let us explore how  $S^{m+1}(n)=(S^m*S)(n)$  is processed when n>1. The accompanying records clarify how the particulars of the individual sequences are consolidated to yield the  $n^{th}$  term of the convolution. The main summary contains the first n terms of  $S^m$  while the subsequent rundown comprises of the first n terms of S written backward order.

$$S^{m}: \{\langle a_{1}^{T}, a_{1}^{I}, a_{1}^{F} \rangle, \underbrace{\langle a_{2}^{T}, a_{2}^{I}, a_{2}^{F} \rangle, ..., \langle a_{2}^{T}, a_{2}^{I}, a_{2}^{F} \rangle}_{m}, \underbrace{\langle a_{3}^{T}, a_{3}^{I}, a_{3}^{F} \rangle, ..., \langle a_{3}^{T}, a_{3}^{I}, a_{3}^{F} \rangle}_{m}, ..., \underbrace{\langle a_{4}^{T}, a_{4}^{I}, a_{4}^{F} \rangle, ..., \langle a_{t+1}^{T}, a_{t+1}^{I}, a_{t+1}^{F} \rangle, ..., \langle a_{t+1}^{T}, a_{t+1}^{I}, a_{t+1}^{F} \rangle}_{m}, \underbrace{\langle a_{1}^{T}, a_{1}^{I}, a_{1}^{F} \rangle, ..., \langle a_{t+1}^{T}, a_{t+1}^{I}, a_{t+1}^{F} \rangle, ..., \langle a_{t+1}^{T}, a_{t+1}^{I}, a_{t+1}^{F} \rangle}_{m}, \underbrace{\langle a_{t+2}^{T}, a_{t+2}^{I}, a_{t+2}^{F} \rangle, ..., \langle a_{t+2}^{T}, a_{t+2}^{I}, a_{t+2}^{F} \rangle}_{m} \}, \underbrace{\langle a_{n-1}^{T}, a_{n}^{I}, a_{n}^{F} \rangle, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle}_{m}, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle, ..., \langle a_{n-2m}^{T}, a_{n-2m}^{I}, a_{n-2m}^{F} \rangle}_{m}, \underbrace{\langle a_{n-2m-1}^{T}, a_{n-2m-1}^{I}, a_{n-2m-1}^{F} \rangle, ..., \langle a_{n-3m}^{T}, a_{n-3m}^{I}, a_{n-3m}^{F} \rangle, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle}_{m}, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{I}, a_{n-1}^{F} \rangle, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle}_{m}, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{I}, a_{n-1}^{F} \rangle, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle}_{m}, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle}_{m}, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{I}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle}_{m}, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{I}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle}_{m}, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F} \rangle}_{m}, \underbrace{\langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{I}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{F}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{F}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{F}, ..., \langle a_{n-1}^{T}, a_{n-1}^{F}, ..., \langle a$$

Here 
$$t = \lfloor (n-1)/m \rceil$$
 and  $u = (n-1) - tm$ . The nth term of  $S^{m+1}$  is 
$$(\langle a_1^T, a_1^I, a_1^F \rangle \wedge \langle a_n^T, a_n^I, a_n^F \rangle) \vee (\langle a_2^T, a_2^I, a_2^F \rangle \wedge (\underbrace{\langle a_{n-1}^T, a_{n-1}^I, a_{n-1}^F \rangle \vee ... \vee \langle a_{n-m}^T, a_{n-m}^I, a_{n-m}^F \rangle}_{m})) \vee \underbrace{(\langle a_3^T, a_3^I, a_3^F \rangle \wedge (\underbrace{\langle a_{n-2}^T, a_{n-2}^I, a_{n-2}^F \rangle \vee ... \vee \langle a_{n-2m}^T, a_{n-2m}^I, a_{n-2m}^F \rangle}_{m})) \vee ... \vee (\langle a_{t+1}^T, a_{t+1}^I, a_{t+1}^F \rangle \wedge \underbrace{\langle a_{n-2}^T, a_{n-2}^I, a_{n-2}^F \rangle \vee ... \vee \langle a_{n-2m}^T, a_{n-2m}^I, a_{n-2m}^F \rangle}_{m})) \vee ... \vee (\langle a_{t+1}^T, a_{t+1}^I, a_{t+1}^F \rangle \wedge \underbrace{\langle a_{n-2m}^T, a_{n-2m}^I, a_{n-2m}^I, a_{n-2m}^I, a_{n-2m}^I \rangle}_{m})) \vee ... \vee (\langle a_{t+1}^T, a_{t+1}^I, a_{t+1}^I, a_{t+1}^I, a_{t+1}^I \rangle \wedge \underbrace{\langle a_{n-2m}^T, a_{n-2m}^I, a_{n-2m}^I, a_{n-2m}^I, a_{n-2m}^I, a_{n-2m}^I \rangle}_{m})) \vee ... \vee (\langle a_{n-2m}^T, a_{n-2m}^I, a_{n$$

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$$\underbrace{(\langle a_{n-(t-1)m-1}^T, a_{n-(t-1)m-1}^I, a_{n-(t-1)m-1}^F \rangle \vee \ldots \vee \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle))}_{m}) \vee (\langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \underbrace{(\langle a_{n-tm-1}^T, a_{n-tm-1}^I, a_{n-tm-1}^F \rangle \vee \ldots \vee \langle a_{1}^T, a_{1}^I, a_{1}^F \rangle)}_{n})).$$

Since  $\langle a_n^T, a_n^I, a_n^F \rangle$  is a decreasing sequence, the maximum of the terms within the marked blocks in the preceding expression are, respectively,

$$\langle a_{n-m}^T, a_{n-m}^I, a_{n-m}^F \rangle, \langle a_{n-2m}^T, a_{n-2m}^I, a_{n-2m}^F \rangle, \langle a_{n-3m}^T, a_{n-3m}^I, a_{n-3m}^F \rangle, ..., \\ \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle, \langle a_{1}^T, a_{1}^I, a_{1}^F \rangle.$$

Thus

$$(S^m * S)(n) = (\langle a_1^T, a_1^I, a_1^F \rangle \wedge \langle a_n^T, a_n^I, a_n^F \rangle) \vee (\langle a_2^T, a_2^I, a_2^F \rangle \wedge \langle a_{n-m}^T, a_{n-m}^I, a_{n-m}^F \rangle) \vee (\langle a_3^T, a_3^I, a_3^F \rangle \wedge \langle a_{n-2m}^T, a_{n-2m}^I, a_{n-2m}^F \rangle) \vee ... \vee (\langle a_{t+1}^T, a_{t+1}^I, a_{t+1}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee (\langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee (\langle a_3^T, a_3^I, a_3^F \rangle \wedge \langle a_{n-2m}^T, a_{n-2m}^I, a_{n-2m}^F \rangle) \vee ... \vee (\langle a_{t+1}^T, a_{t+1}^I, a_{t+1}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^F \rangle \rangle \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^I, a_{n-tm}^I \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^I, a_{n-tm}^I \rangle) \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^I \rangle \rangle \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle \wedge \langle a_{n-tm}^T, a_{n-tm}^I, a_{n-tm}^I \rangle \rangle \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^I,$$

Consider the general term  $\langle a_k^T, a_k^I, a_k^F \rangle \wedge \langle a_{n-(k-1)m}^T, a_{n-(k-1)m}^I, a_{n-(k-1)m}^F \rangle$ . Because  $\langle a_n^T, a_n^I, a_n^F \rangle$  is decreasing,  $\langle a_k^T, a_k^I, a_k^F \rangle \wedge \langle a_{n-(k-1)m}^T, a_{n-(k-1)m}^I, a_{n-(k-1)m}^F \rangle = \langle a_{n-(k-1)m}^T, a_{n-(k-1)m}^I, a_{n-(k-1)m}^I, a_{n-(k-1)m}^I \rangle$  for k < n - (k-1)m and  $\langle a_k^T, a_k^I, a_k^F \rangle \wedge \langle a_{n-(k-1)m}^T, a_{n-(k-1)m}^I, a_{n-(k-1)m}^I, a_{n-(k-1)m}^F \rangle = \langle a_k^T, a_k^I, a_k^F \rangle$  if  $k \ge n - (k-1)m$ . Let q be the unique integer such that  $q(m+1)+1 < n \le (q+1)(m+1)+1$ . Then  $q+1 < (n+m)/(1+m) \le q+2$ . Since k < n-(k-1)m or  $k \ge n-(k-1)m$  according as k < (n+m)/(1+m) or  $k \ge (n+m)/(1+m)$  we have that

$$\begin{split} \langle a_k^T, a_k^I, a_k^F \rangle \wedge \langle a_{n-(k-1)m}^T, a_{n-(k-1)m}^I, a_{n-(k-1)m}^F \rangle &= \langle a_{n-(k-1)m}^T, a_{n-(k-1)m}^I, a_{n-(k-1)m}^F \rangle \text{ if } \\ & k \leq q+1, \\ \langle a_k^T, a_k^I, a_k^F \rangle \wedge \langle a_{n-(k-1)}^T, a_{n-(k-1)}^I, a_{n-(k-1)}^F \rangle &= \langle a_k^T, a_k^I, a_k^F \rangle \text{ if } k \geq q+2. \end{split}$$

Hence

$$(S^m * S)(n) = \\ \langle a_n^T, a_n^I, a_n^F \rangle \vee \langle a_{n-m}^T, a_{n-m}^I, a_{n-m}^F \rangle \vee \langle a_{n-2m}^T, a_{n-2m}^I, a_{n-2m}^F \rangle \vee \ldots \vee \langle a_{n-qm}^T, a_{n-qm}^I, a_{n-qm}^F \rangle \vee \\ \langle a_{q+2}^T, a_{q+2}^I, a_{q+2}^F \rangle \vee \langle a_{q+3}^T, a_{q+3}^I, a_{q+3}^F \rangle \vee \ldots \vee \langle a_{t+1}^T, a_{t+1}^I, a_{t+1}^F \rangle \vee \langle a_{t+2}^T, a_{t+2}^I, a_{t+2}^F \rangle.$$

The largest term in the disjunction on the right is  $\langle a_{q+2}^T, a_{q+2}^I, a_{q+2}^F \rangle$ . Thus, as stated in the assertion,

$$(S^m * S)(n) = \langle a_{q+2}^T, a_{q+2}^I, a_{q+2}^F \rangle$$
, for  $q(1+m) < n-1 \le (q+1)(1+m)$ .

The next result describes the iterates of any FNSCM whose first row is in decreasing order.

# 4.2. Theorem

Let A be an  $n \times n$  FNSCM with its first row  $[\langle a_1^T, a_1^I, a_1^F \rangle, \langle a_2^T, a_2^I, a_2^F \rangle, ..., \langle a_n^T, a_n^I, a_n^F \rangle]$  decreasing order. Then for any positive integer m, the first row of  $A^m$  is

$$\underbrace{\langle a_{1}^{T}, a_{1}^{I}, a_{1}^{F} \rangle, \underbrace{\langle a_{2}^{T}, a_{2}^{I}, a_{2}^{F} \rangle, ..., \langle a_{2}^{T}, a_{2}^{I}, a_{2}^{F} \rangle}_{m}, \underbrace{\langle a_{3}^{T}, a_{3}^{I}, a_{3}^{F} \rangle, ..., \langle a_{3}^{T}, a_{3}^{I}, a_{3}^{F} \rangle}_{m}, ..., \underbrace{\langle a_{t+1}^{T}, a_{t+1}^{I}, a_{t+1}^{F} \rangle, ..., \langle a_{t+1}^{T}, a_{t+1}^{I}, a_{t+1}^{F} \rangle}_{m}, \underbrace{\langle a_{t+2}^{T}, a_{t+2}^{I}, a_{t+2}^{F} \rangle, ..., \langle a_{t+2}^{T}, a_{t+2}^{I}, a_{t+2}^{F} \rangle}_{n}$$

where  $t = \lfloor (n-1)/m \rfloor$  and u = (n-1) - tm. Moreover, the iterates  $A^m$  converges to the FNSCM whose first row is  $[\langle a_1^T, a_1^I, a_1^F \rangle, \langle a_2^T, a_2^I, a_2^F \rangle, ..., \langle a_2^T, a_2^I, a_2^F \rangle]$ . The number of iterations until convergence is  $\lceil (n-1)/(r-1) \rceil$ , where integer  $r = \max\{k : \langle a_k^T, a_k^I, a_k^F \rangle = \langle a_2^T, a_2^I, a_2^F \rangle\}$ . Proof: The proof follows immediately from Lemma 4.1

We illustrate the preceding results by two examples.

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### 4.3. Example

Suppose that the first row of a  $7 \times 7$  FNSCM A is  $[\langle 0.7, 0.6, 0.3 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.3, 0.2, 0.7 \rangle, \langle 0.3, 0.2, 0.7 \rangle, \langle 0.2, 0.1, 0.8 \rangle, \langle 0.1, 0.1, 0.9 \rangle].$ 

Then the first rows of  $A^2$  and  $A^3$  are, respectively,

 $[\langle 0.7, 0.6, 0.3 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.3, 0.2, 0.7 \rangle, \langle 0.3, 0.2, 0.7 \rangle]$  and

 $[\langle 0.7, 0.6, 0.3 \rangle, \langle 0.4, 0.3, 0.6 \rangle]$ . The sequence of iterates converges after 3 iterations to the circulant whose first row is  $[\langle 0.7, 0.6, 0.3 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3,$ 

 $\langle 0.4, 0.3, 0.6 \rangle$ ,  $\langle 0.4, 0.3, 0.6 \rangle$ ,  $\langle 0.4, 0.3, 0.6 \rangle$ ,  $\langle 0.4, 0.3, 0.6 \rangle$ ]. In the notation of Theorem 4.2, n=7 and r=3 and as stated in Theorem 4.2 convergence occurs after  $\lceil (7-1)/(3-1) \rceil = 3$  iterations.

# 4.4. Example

If the first row of a FNSCM is  $[\langle 0.8, 0.7, 0.2 \rangle, \langle 0.8, 0.7, 0.2 \rangle, \langle 0.8, 0.7, 0.2 \rangle, \langle 0.6, 0.5, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.3, 0.2, 0.7 \rangle, \langle 0.2, 0.1, 0.8 \rangle, \langle 0.1, 0.1, 0.9 \rangle]$ , then the first row of the iterates are  $[\langle 0.8, 0.7, 0.2 \rangle, \langle 0.8, 0.7,$ 

 $\begin{array}{l} [\langle 0.8, 0.7, 0.2 \rangle, \langle 0.$ 

A characteristic inquiry that emerges is whether there is a straightforward portrayal of the emphasizes of a FNSCM whose first column is in increasing order. This is for sure conceivable. Truth be told, Theorem 4.2 can be used to depict the conduct of the iterates of a FNSCM with increasing first columns. The accompanying perception shows how this comes about.

**Observation III.** Suppose that A is a  $n \times n$  FNSCM whose first row is in decreasing order. At that point  $C_nA^t$  is a FNSCM whose first row is in increasing order. On the other hand, on the off chance that B is any FNSCM whose first row is in increasing order, at that point  $A = C_nB^t$  is a CFNSM whose first row is decreasing. Rather than giving a proper evidence of the statements in Observation III, we will outline them by a model.

#### 4.5. Example

$$B = \begin{bmatrix} \langle 0.1, 0.1, 0.9 \rangle & \langle 0.2, 0.1, 0.8 \rangle & \langle 0.3, 0.2, 0.7 \rangle & \langle 0.4, 0.3, 0.6 \rangle \\ \langle 0.4, 0.3, 0.6 \rangle & \langle 0.1, 0.1, 0.9 \rangle & \langle 0.2, 0.1, 0.8 \rangle & \langle 0.3, 0.2, 0.7 \rangle \\ \langle 0.3, 0.2, 0.7 \rangle & \langle 0.4, 0.3, 0.6 \rangle & \langle 0.1, 0.1, 0.9 \rangle & \langle 0.2, 0.1, 0.8 \rangle \\ \langle 0.2, 0.1, 0.8 \rangle & \langle 0.3, 0.2, 0.7 \rangle & \langle 0.4, 0.3, 0.6 \rangle & \langle 0.1, 0.1, 0.9 \rangle \end{bmatrix}.$$

Then

$$B^t = \begin{bmatrix} \langle 0.1, 0.1, 0.9 \rangle & \langle 0.4, 0.3, 0.6 \rangle & \langle 0.3, 0.2, 0.7 \rangle & \langle 0.2, 0.1, 0.8 \rangle \\ \langle 0.2, 0.1, 0.8 \rangle & \langle 0.1, 0.1, 0.9 \rangle & \langle 0.4, 0.3, 0.6 \rangle & \langle 0.3, 0.2, 0.7 \rangle \\ \langle 0.3, 0.2, 0.7 \rangle & \langle 0.2, 0.1, 0.8 \rangle & \langle 0.1, 0.1, 0.9 \rangle & \langle 0.4, 0.3, 0.6 \rangle \\ \langle 0.4, 0.3, 0.6 \rangle & \langle 0.3, 0.2, 0.7 \rangle & \langle 0.2, 0.1, 0.8 \rangle & \langle 0.1, 0.1, 0.9 \rangle \end{bmatrix},$$

and

$$A = \begin{bmatrix} \langle 0.4, 0.3, 0.6 \rangle & \langle 0.3, 0.2, 0.7 \rangle & \langle 0.2, 0.1, 0.8 \rangle & \langle 0.1, 0.1, 0.9 \rangle \\ \langle 0.1, 0.1, 0.9 \rangle & \langle 0.4, 0.3, 0.6 \rangle & \langle 0.3, 0.2, 0.7 \rangle & \langle 0.2, 0.1, 0.8 \rangle \\ \langle 0.2, 0.1, 0.8 \rangle & \langle 0.1, 0.1, 0.9 \rangle & \langle 0.4, 0.3, 0.6 \rangle & \langle 0.3, 0.2, 0.7 \rangle \\ \langle 0.3, 0.2, 0.7 \rangle & \langle 0.2, 0.1, 0.8 \rangle & \langle 0.1, 0.1, 0.9 \rangle & \langle 0.4, 0.3, 0.6 \rangle \end{bmatrix}$$

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$$= \begin{bmatrix} \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 1,1,0 \rangle \\ \langle 1,1,0 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle \\ \langle 0,0,1 \rangle & \langle 1,1,0 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle \\ \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 1,1,0 \rangle & \langle 0,0,1 \rangle \end{bmatrix}$$

$$\begin{bmatrix} \langle 0.1,0.1,0.9 \rangle & \langle 0.4,0.3,0.6 \rangle & \langle 0.3,0.2,0.7 \rangle & \langle 0.2,0.1,0.8 \rangle \\ \langle 0.2,0.1,0.8 \rangle & \langle 0.1,0.1,0.9 \rangle & \langle 0.4,0.3,0.6 \rangle & \langle 0.3,0.2,0.7 \rangle \\ \langle 0.3,0.2,0.7 \rangle & \langle 0.2,0.1,0.8 \rangle & \langle 0.1,0.1,0.9 \rangle & \langle 0.4,0.3,0.6 \rangle \\ \langle 0.4,0.3,0.6 \rangle & \langle 0.3,0.2,0.7 \rangle & \langle 0.2,0.1,0.8 \rangle & \langle 0.1,0.1,0.9 \rangle \end{bmatrix} = C_4 B^t.$$

We presently use perception III and Theorem 4.2 to portray the FNSMs  $B^m$  when B is FNSCM whose first column is increasing. Taking into account perception III, B can be communicated as  $C_nA^t$ , where A is a FNSCM with decreasing first column.

Let  $[\langle a_1^T, a_1^I, a_1^F \rangle, \langle a_2^T, a_2^I, a_2^F \rangle, ..., \langle a_n^T, a_n^I, a_n^F \rangle]$  signify the primary line of A. Then the first row of B is

 $[\langle a_n^T, a_n^I, a_n^F \rangle, \langle a_{n-1}^T, a_{n-1}^I, a_{n-1}^F \rangle, ..., \langle a_1^T, a_1^I, a_1^F \rangle]. \text{ What we will do is portray the first row of } B^m \text{ in quite a while of the entries } \langle a_1^T, a_1^I, a_1^F \rangle, \langle a_2^T, a_2^I, a_2^F \rangle, ..., \langle a_n^T, a_n^I, a_n^F \rangle]. \text{ Towards this end, } let <math>r = \max\{k : \langle a_k^T, a_k^I, a_k^F \rangle = \langle a_2^T, a_2^I, a_2^F \rangle\}, \text{ compose } r_0 = \lceil (n-1)/(r-1) \rceil \text{ and let } L \text{ mean the FNSCM whose first column is } [\langle a_1^T, a_1^I, a_1^F \rangle, \langle a_2^T, a_2^I, a_2^F \rangle, ..., \langle a_2^T, a_2^I, a_2^F \rangle].$ 

The FNSM  $A^t$  is likewise a FNSCM, thus it commutes with  $C_n$ . Therefore  $B^m = C_n^m (A^m)^t$  for m = 1, 2, 3, ... From Theorem 4.2, the FNSMs  $A, A^2, ..., A^{r_0}$  are for the most part extraordinary yet  $A^{r_0+s} = L$  for s = 0, 1, 2, ... Therefore,

$$B^{r_0+s} = C_n^{r_0+s}(L)^t = C_n^{r_0+s}(L), \text{ for } s \ge 0.$$

In any case,  $C_n^{r_0} = C_n^{r_0+n}$  with the goal that when  $m \ge r_0$  the succession  $(B^m)$  has period n and its terms are  $C_n^{r_0}L, C_n^{r_0+1}L, ..., C_n^{r_0+n-1}L$ .

The structure of these n FNSMs is anything but difficult to portray. They are all FNSCMs. The principal line of  $C_n^{r_0}L$  has  $\langle a_1^T, a_1^I, a_1^F \rangle$  as its  $n-(r_0-1)st$  section and the various passages are  $\langle a_2^T, a_2^I, a_2^F \rangle$ . The main column of  $C_n^{r_0+s}L$  is gotten by consistently moving the entries of the first row of  $C_n^{r_0}Ls$  positions to one side. When M < n, the principal column of  $B^m = C_n^m(A^m)^t$  is push n-(m-1) of  $(A^m)^t$ . This is on the grounds that premultiplying a FNSM X by  $C_n^m$  has the impact of lifting line n-(m-1) of X to the first row. Line n-(m-1) of  $A^m$ . Since Theorem 4.2 depicts  $A^m$ , it likewise yields, by means of the first contention, a portrayal of the FNSM  $B^m$ .

The above idea is represented beneath.

### 4.6. Example

Assume that the first column of B is

 $[\langle 0.1, 0.1, 0.9 \rangle, \langle 0.2, 0.1, 0.8 \rangle, \langle 0.3, 0.2, 0.7 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.5, 0.4, 0.4 \rangle].$ 

Then the main line of A is the FNSV

 $[\langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.3, 0.2, 0.7 \rangle, \langle 0.2, 0.1, 0.8 \rangle, \langle 0.1, 0.1, 0.9 \rangle].$  Shown underneath are the first rows of the  $A^m$  and  $B^m, m = 1, 2, ..., 8$ 

First row of  $A^m$ 

$$m = 1 \left[ \langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.3, 0.2, 0.7 \rangle, \langle 0.2, 0.1, 0.8 \rangle, \langle 0.1, 0.1, 0.9 \rangle \right]$$

$$m = 2 \left[ \langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.3, 0.2, 0.7 \rangle \right]$$

$$m = 3 \left[ \langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle \right]$$

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$$\begin{split} m &= 4 \; [\langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ m &= 5 \; [\langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ m &= 6 \; [\langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ m &= 7 \; [\langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ m &= 8 \; [\langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ \text{First row of } B^m \\ m &= 1 \; [\langle 0.1, 0.1, 0.9 \rangle, \langle 0.2, 0.1, 0.8 \rangle, \langle 0.3, 0.2, 0.7 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.5, 0.4, 0.4 \rangle] \\ m &= 2 \; [\langle 0.4, 0.3, 0.6 \rangle, \langle 0.5, 0.4, 0.4 \rangle, \langle 0.3, 0.2, 0.7 \rangle] \\ m &= 3 \; [\langle 0.4, 0.3, 0.6 \rangle, \langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ m &= 4 \; [\langle 0.4, 0.3, 0.6 \rangle, \langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ m &= 5 \; [\langle 0.4, 0.3, 0.6 \rangle, \langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ m &= 6 \; [\langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ m &= 7 \; [\langle 0.4, 0.3, 0.6 \rangle, \langle 0.4, 0.3, 0.6 \rangle, \langle 0.5, 0.4, 0.4 \rangle] \\ m &= 8 \; [\langle 0.4, 0.3, 0.6 \rangle, \langle 0.5, 0.4, 0.4 \rangle, \langle 0.4, 0.3, 0.6 \rangle] \\ \end{array}$$

(The columns of  $A^m$  is figured utilizing perception I. For  $m \geq 3$  the emphasizes of  $B^m$  oscillate with period 6.)

#### 5. Conclusion

In this article the max-min combination of FNSCMs is analyzed when the first row of the FN-SCM is monotone. It is verified through Examples 4.3, 4.4 and 4.6. As a future work we are trying to find the t value if the first row of a FNSM is not monotone.

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