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Contra G_N-Continuity in Neutrosophic Generalized Topological Spaces

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Abstract

One of the aims of this article is to promote some Contra Continuous Functions (CCF) by means of Neutrosophic Sets in Generalized Topological Spaces (G_N-TSs). Then, we deliberate certain properties of CCF in G_N-TSs. Further, we talk over about the associations among several types of CCF along with illustrations. Also, we dealt the concept of almost continuous and its contra characteristics in G_N-TSs. Finally, we discuss some separation axioms related to G_N-TSs.

Key words: Contra G_N-continuous function(G_N-CCF), contra G_N-α-continuous function(G_N-αCCF), contra G_N-semicontinuousfunction(G_N-σCCF),contraG_N-pre-continuousfunction(G_N-πCCF) and contra G_N-beta continuous function(G_N-βCCF).

Introduction

In 1965, fuzzy set idea was instigated byZadeh [19] that share out uncertainty in actual lifestyles conditions. A special note to the field of topology was originated by Chang [3] in 1968. Attanassov [1] in 1983, considered both membership and non-membership of the elements in intuitionistic fuzzy sets. Coker [4] located a place for Intuitionistic fuzzy topological space by extending the concepts of fuzziness. Likewise, Jeyaraman M and Yuvarani A [9] talk over about Contra Alpha Generalized Semi Continuous Mappings in Intuitionistic Fuzzy Topology. The significant conjoint studies of contra continuity on generalized topological spaces have been done by many researchers [8] & [10].

Smarandache [5], [6], [7] & [18] engrossed his interpretations en route for the degree of indeterminacy that directed into Neutrosophic Sets (NS). In a little while, Salama and Albowi [13] acquainted Neutrosophic Topological Spaces (NTS). In addition to that, the continuous (Cts) functions in NTS were offered by Salama, Smarandache and Valeri Kromov [14]. In [2], Contra-Continuity via topological ideals was introduced and analysed by Bhuvaneshwari and et. al., in Ideal Topological Spaces.

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Further, Vijaya and Santhi [16] investigated about the Characterization of Almost (α, μ)-Continuous functions and its properties in Generalized Topological Spaces. In addition to that, Contra Nα-I-Continuity over Nano Ideals in Nano Topological Spaces and Contra irresolute functions in Generalized Neutrosophic Topological spaces was look over by Vijaya and et. al.,[15],[17]. By way of retaining most of these works as an inspiration, Raksha Ben, Hari Siva Annam [11] & [12] contrived G_N-Topological Space and reflected its properties, in 2020.

Here we deliberate certain properties of CCF in G_N-TSs. Additionally, we presented several types of CCF along with illustrations. Also, we dealt the concept of almost continuous and its contra characteristics in G_N-TSs. To end with, we inspected some separation axioms related to G_N-TSs.

Prerequisites

Definition 2.1. [13]

Let Γ be a non-empty fixed set. A NS, $F = \{\langle \gamma, \mathcal{M}_F(\gamma), \mathcal{I}_F(\gamma), \mathcal{N}_F(\gamma) \rangle : \gamma \in \Gamma\}$ where $\mathcal{M}_F(\gamma)$, $\mathcal{I}_F(\gamma)$ and $\mathcal{N}_F(\gamma)$ represents the degree of membership, indeterminacy and non-membership functions respectively of every element $\gamma \in \Gamma$.

Remark 2.2. [13]

A NS, F can be recognized as a structured triple $F = \{\langle \gamma, \mathcal{M}_F(\gamma), \mathcal{I}_F(\gamma), \mathcal{N}_F(\gamma) \rangle : \gamma \in \Gamma\}$ in $]^{-}0, 1^{+}[$ on Γ.

Remark 2.3.[13]

The NS, 0_N and 1_N in Γ is defined as

(P_1) $0_N = \{\langle \gamma, 0, 0, 1 \rangle : \gamma \in \Gamma\}$; ($P_2$) $0_N = \{\langle \gamma, 0, 1, 1 \rangle : \gamma \in \Gamma\}$

(P_3) $0_N = \{\langle \gamma, 0, 1, 0 \rangle : \gamma \in \Gamma\}$; ($P_4$) $0_N = \{\langle \gamma, 0, 0, 0 \rangle : \gamma \in \Gamma\}$

(P_5) $1_N = \{\langle \gamma, 1, 0, 0 \rangle : \gamma \in \Gamma\}$; ($P_6$) $1_N = \{\langle \gamma, 1, 0, 1 \rangle : \gamma \in \Gamma\}$

(P_7) $1_N = \{\langle \gamma, 1, 1, 0 \rangle : \gamma \in \Gamma\}$; ($P_8$) $1_N = \{\langle \gamma, 1, 1, 1 \rangle : \gamma \in \Gamma\}$

Definition 2.4. [13]

If $F = \{\langle \mathcal{M}_F(\gamma), \mathcal{I}_F(\gamma), \mathcal{N}_F(\gamma) \rangle\}$, then the complement of F on Γ is

(P_9) $F^c = \{\langle \gamma, 1 - \mathcal{M}_F(\gamma), 1 - \mathcal{I}_F(\gamma)$ and $1 - \mathcal{N}_F(\gamma) \rangle : \gamma \in \Gamma\}$

(P_{10}) $F^c = \{\langle \gamma, \mathcal{N}_F(\gamma), \mathcal{I}_F(\gamma)$ and $\mathcal{M}_F(\gamma) \rangle : \gamma \in \Gamma\}$

(P_{11}) $F^c = \{ \langle \gamma, \mathcal{N}_F(\gamma), 1 - \mathcal{I}_F(\gamma)$ and $\mathcal{M}_F(\gamma) \rangle : \gamma \in \Gamma\}$

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Definition 2.5. [13]

Let Γ be a non-empty set and let F = $\{\langle \gamma, \mathcal{M}_F(\gamma), \mathcal{I}_F(\gamma), \mathcal{N}_F(\gamma) \rangle : \gamma \in \Gamma\}$ and T = $\{\langle \gamma, \mathcal{M}_T(\gamma), \mathcal{I}_T(\gamma), \mathcal{N}_T(\gamma) \rangle : \gamma \in \Gamma\}$. Then

(P_{12}) $F \subseteq T \Rightarrow \mathcal{M}_F(\gamma) \leq \mathcal{M}_T(\gamma), \mathcal{I}_F(\gamma) \leq \mathcal{I}_T(\gamma), \mathcal{N}_F(\gamma) \geq \mathcal{N}_T(\gamma), \forall\ \gamma \in \Gamma$

(P_{13}) $F \subseteq T \Rightarrow \mathcal{M}_F(\gamma) \leq \mathcal{M}_T(\gamma), \mathcal{I}_F(\gamma) \geq \mathcal{I}_T(\gamma), \mathcal{N}_F(\gamma) \geq \mathcal{N}_T(\gamma), \forall\ \gamma \in \Gamma$

Definition 2.6. [13]

Let Γ be a non-empty set and F = $\{\langle \gamma, \mathcal{M}_F(\gamma), \mathcal{I}_F(\gamma), \mathcal{N}_F(\gamma) \rangle : \gamma \in \Gamma\}$ and T = $\{\langle \gamma, \mathcal{M}_T(\gamma), \mathcal{I}_T(\gamma), \mathcal{N}_T(\gamma) \rangle : \gamma \in \Gamma\}$ are NSs. Then,

(P_{14}) $F \cap T = \langle \gamma, \mathcal{M}_F(\gamma) \wedge \mathcal{M}_T(\gamma), \mathcal{I}_F(\gamma) \vee \mathcal{I}_T(\gamma), \mathcal{N}_F(\gamma) \vee \mathcal{N}_T(\gamma) \rangle$

(P_{15}) $F \cap T = \langle \gamma, \mathcal{M}_F(\gamma) \wedge \mathcal{M}_T(\gamma), \mathcal{I}_F(\gamma) \wedge \mathcal{I}_T(\gamma), \mathcal{N}_F(\gamma) \vee \mathcal{N}_T(\gamma) \rangle$

(P_{16}) $F \cup T = \langle \gamma, \mathcal{M}_F(\gamma) \vee \mathcal{M}_T(\gamma), \mathcal{I}_F(\gamma) \wedge \mathcal{I}_T(\gamma), \mathcal{N}_F(\gamma) \wedge \mathcal{N}_T(\gamma) \rangle$

(P_{17}) $F \cup T = \langle \gamma, \mathcal{M}_F(\gamma) \vee \mathcal{M}_T(\gamma), \mathcal{I}_F(\gamma) \vee \mathcal{I}_T(\gamma), \mathcal{N}_F(\gamma) \wedge \mathcal{N}_T(\gamma) \rangle$

Definition 2.7. [12]

Let $\Gamma \neq \phi$. A family of Neutrosophic subsets of Γ is G_N-topology if it satisfies

(Δ_1) $0_N \in G_N$

(Δ_2) $F_1 \cup F_2 \in G_N$ for any $F_1, F_2 \in G_N$.

Remark 2.8. [12]

Members of G_N-topology are G_N-Open Sets (G_N-Os) and their complements are G_N-Closed Sets (G_N-Cs).

Definition 2.9. [12]

Let (Γ, G_N) be a G_N-TS and F = $\{\langle \Gamma, \mathcal{M}_F(\gamma), \mathcal{I}_F(\gamma), \mathcal{N}_F(\gamma) : \gamma \in \Gamma \rangle\}$ be a NS in Γ.

(Δ_1) G_N-Closure(F) = $\cap\{T : F \subseteq T$, T is G_N-Cs$\}$

(Δ_1) G_N-Interior(F) = $\cup\{W : W \subseteq F$, W is G_N-Os$\}$

Definition 2.10. [11]

A NS, F in a G_N-TS is said to be

(Δ_1) G_N-σ-Open Set (G_N-σOs) if $F \subseteq G_N$-*clo*(G_N-*intr*(F))

(Δ_2) G_N-π-Open Set (G_N- πOs) if $F \subseteq G_N$- *intr*(G_N- *clo*(F)),

(Δ_3) G_N-α-Open Set (G_N-αOs) if $F \subseteq G_N$- *intr*(G_N-*clo*(G_N-*intr*(F))),

(Δ_4) G_N-β-Open Set (G_N-βOs) if $F \subseteq G_N$-*clo*(G_N-*intr*(G_N-*clo*(F))),

(Δ_5) G_N-regular-Open Set (G_N-rOs) if $F = G_N$-*intr*(G_N-*clo*(F)),

(Δ_6) G_N-b-Open Set (G_N-bOs) if $F \subseteq G_N$-*clo*(G_N-*intr*(F)) $\cup$ G_N-*intr*(G_N-*clo*(F)).

Lemma 2.11. [11]

Every G_N- αOs is G_N-σOs and G_N-πOs.

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Definition 2.12. [11]

Let the function $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ is defined to be G_N-Cts (resp.G_N-σCts, G_N-πCts, G_N-αCts) if the inverse image of G_N-Cs in (Γ_2, ρ_2) is a G_N-Cs (resp. G_N-σCs, G_N-πCs, G_N-αCs, G_N-βCs) in (Γ_1, ρ_1).

Contra G_N-Continuous Functions

Definition 3.1

Let (Γ_1, ρ_1) and (Γ_2, ρ_2) be G_N-TSs. Then $\psi: \Gamma_1 \rightarrow \Gamma_2$ is said to be

(Δ_1) Contra G_N-Continuous Function (G_N-CCF) if for each G_N-Os M in Γ_2, $\psi^{-1}(M)$ is a G_N-Cs in Γ_1,

(Δ_2) Contra G_N-α-Continuous Function (G_N-αCCF) if for each G_N-Os M in Γ_2, $\psi^{-1}(M)$ is a G_N-αCs in Γ_1,

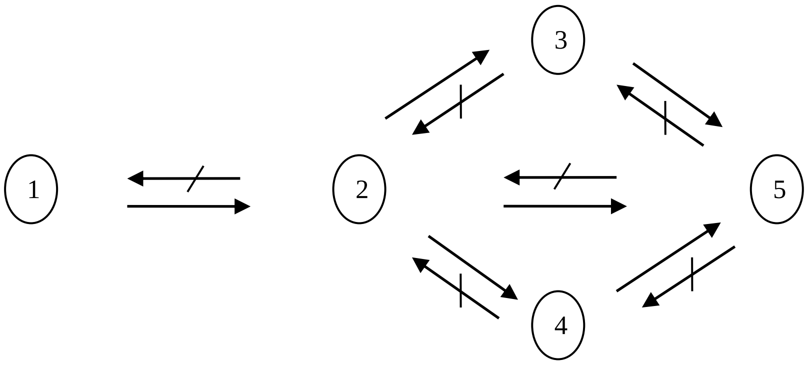
(Δ_3) Contra G_N-σ-Continuous Function (G_N-σCCF) if for each G_N-Os M in Γ_2, $\psi^{-1}(M)$ is a G_N-σCs in Γ_1,

(Δ_4) Contra G_N-π-Continuous Function (G_N-πCCF) if for each G_N-Os M in Γ_2, $\psi^{-1}(M)$ is a G_N-πCs in Γ_1,

(Δ_5) Contra G_N-β-Continuous Function (G_N-βCCF) if for each G_N-Os M in Γ_2, $\psi^{-1}(M)$ is a G_N-βCs in Γ_1.

Remark 3.2

Let $\psi: (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ be a function, where (Γ_1, ρ_1) and (Γ_2, ρ_2) be G_N-TSs. Then we obtain



Where $A \nrightarrow B$ means that A does not necessarily imply B and, moreover,

1 = G_N-CCF 2 = G_N-αCCF

3 = G_N-πCCF 4 = G_N-σCCF

5 = G_N-βCCF

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Example 3.3

The contrary implications may not be factual in general as shown below.

(i) G_N-αCCF $\not\rightarrow$ G_N-CCF

Let $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ be defined as $\psi(s) = v$ and $\psi(t) = u$, where $\Gamma_1 = \{s, t\}$ and $\Gamma_2 = \{u, v\}$, $\rho_1 = \{0_N, \mathcal{A}, \mathcal{B}\}$, $\rho_2 = \{0_N, \mathcal{C}, \mathcal{D}, \mathcal{H}\}$.

$$\mathcal{A} = \langle \left(\frac{2}{10},\frac{8}{10},\frac{9}{10}\right), \left(\frac{1}{10},\frac{7}{10},\frac{8}{10}\right) \rangle,\ \mathcal{B} = \langle \left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right), \left(\frac{4}{10},\frac{6}{10},\frac{7}{10}\right) \rangle,$$

$$\mathcal{C} = \langle \left(\frac{8}{10},\frac{3}{10},\frac{1}{10}\right), \left(\frac{9}{10},\frac{2}{10},\frac{2}{10}\right) \rangle,\ \mathcal{D} = \langle \left(\frac{7}{10},\frac{4}{10},\frac{4}{10}\right), \left(\frac{6}{10},\frac{5}{10},\frac{3}{10}\right) \rangle,$$

$$\mathcal{G} = \langle \left(\frac{3}{10},\frac{7}{10},\frac{8}{10}\right), \left(\frac{2}{10},\frac{6}{10},\frac{7}{10}\right) \rangle,\ \mathcal{H} = \langle \left(\frac{7}{10},\frac{4}{10},\frac{2}{10}\right), \left(\frac{8}{10},\frac{3}{10},\frac{3}{10}\right) \rangle.$$

Now $\{1_N, \mathcal{A}^c, \mathcal{B}^c, \mathcal{G}^c\}$ is G_N-αCs of (Γ_1, ρ_1). Here $\psi^{-1}(H) = \mathcal{G}^c$ which is G_N-αCs but not G_N-Cs. Hence, ψ is G_N-αCCF but not G_N-CCF.

(ii) G_N-πCCF $\not\rightarrow$ G_N-αCCF and G_N-βCCF $\not\rightarrow$ G_N-σCCF

Let $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ be defined as $\psi(p) = w$, $\psi(q) = u$ and $\psi(r) = v$, where $\Gamma_1 = \{p, q, r\}$ and $\Gamma_2 = \{u, v, w\}$, $\rho_1 = \{0_N, \mathcal{A}, \mathcal{B}\}$, $\rho_2 = \{0_N, \mathcal{C}, \mathcal{D}, \mathcal{H}\}$.

$$\mathcal{A} = \langle \left(\frac{2}{10},\frac{7}{10},\frac{7}{10}\right), \left(\frac{3}{10},\frac{7}{10},\frac{8}{10}\right), \left(\frac{1}{10},\frac{8}{10},\frac{8}{10}\right) \rangle,\ \mathcal{B} = \langle \left(\frac{3}{10},\frac{7}{10},\frac{6}{10}\right), \left(\frac{4}{10},\frac{6}{10},\frac{7}{10}\right), \left(\frac{2}{10},\frac{7}{10},\frac{8}{10}\right) \rangle,$$

$$\mathcal{C} = \langle \left(\frac{9}{10},\frac{1}{10},\frac{1}{10}\right), \left(\frac{8}{10},\frac{2}{10},\frac{2}{10}\right), \left(\frac{8}{10},\frac{3}{10},\frac{2}{10}\right) \rangle,\ \mathcal{D} = \langle \left(\frac{8}{10},\frac{3}{10},\frac{2}{10}\right), \left(\frac{6}{10},\frac{3}{10},\frac{3}{10}\right), \left(\frac{7}{10},\frac{4}{10},\frac{4}{10}\right) \rangle,$$

$$\mathcal{G} = \langle \left(\frac{2}{10},\frac{8}{10},\frac{8}{10}\right), \left(\frac{2}{10},\frac{7}{10},\frac{8}{10}\right), \left(\frac{1}{10},\frac{9}{10},\frac{9}{10}\right) \rangle,\ \mathcal{H} = \langle \left(\frac{8}{10},\frac{2}{10},\frac{1}{10}\right), \left(\frac{7}{10},\frac{3}{10},\frac{2}{10}\right), \left(\frac{8}{10},\frac{3}{10},\frac{3}{10}\right) \rangle.$$

Now $\{1_N, \mathcal{A}^c, \mathcal{B}^c, \mathcal{G}^c\}$ and $\{1_N, \mathcal{A}^c, \mathcal{B}^c\}$ are G_N-πCs and G_N-αCs of (Γ_1, ρ_1) respectively. Here $\psi^{-1}(H) = \mathcal{G}^c$ which is G_N-πCs but not G_N-αCs. Hence, ψ is G_N-πCCF but not G_N-αCCF. Also $\{1_N, \mathcal{A}^c, \mathcal{B}^c, \mathcal{G}^c\}$ and $\{1_N, \mathcal{A}^c, \mathcal{B}^c\}$ are G_N-βCs and G_N-σCs of (Γ_1, ρ_1) respectively. Here $\psi^{-1}(H) = \mathcal{G}^c$ which is G_N-βCs but not G_N-σCs. Hence, ψ is G_N-βCCF but not G_N-σCCF.

(iii) G_N-σCCF $\not\rightarrow$ G_N-αCCF and G_N-βCCF $\not\rightarrow$ G_N-πCCF

Let $\psi : (\Gamma_1,\rho_1) \rightarrow (\Gamma_2,\rho_2)$ be defined as $\psi(p) = s$ and $\psi(q) = r$, where $\Gamma_1 = \{p, q\}$ and $\Gamma_2 = \{r, s\}$, $\rho_1 = \{0_N, \mathcal{A}, \mathcal{B}\}$, $\rho_2 = \{0_N, \mathcal{C}, \mathcal{D}, \mathcal{H}\}$.

$$\mathcal{A} = \langle \left(\frac{3}{10},\frac{7}{10},\frac{8}{10}\right), \left(\frac{2}{10},\frac{6}{10},\frac{8}{10}\right) \rangle,\ \mathcal{B} = \langle \left(\frac{4}{10},\frac{6}{10},\frac{7}{10}\right), \left(\frac{5}{10},\frac{5}{10},\frac{6}{10}\right) \rangle,$$

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$$\mathcal{C} = \langle\, (\tfrac{8}{10},\tfrac{4}{10},\tfrac{2}{10}), (\tfrac{8}{10},\tfrac{3}{10},\tfrac{3}{10})\,\rangle,\ \ \mathcal{D} = \langle\, (\tfrac{6}{10},\tfrac{5}{10},\tfrac{5}{10}), (\tfrac{7}{10},\tfrac{4}{10},\tfrac{4}{10})\,\rangle,$$

$$\mathcal{G} = \langle\, (\tfrac{5}{10},\tfrac{5}{10},\tfrac{6}{10}), (\tfrac{5}{10},\tfrac{5}{10},\tfrac{6}{10})\,\rangle,\ \ \mathcal{H} = \langle\, (\tfrac{6}{10},\tfrac{5}{10},\tfrac{5}{10}), (\tfrac{6}{10},\tfrac{5}{10},\tfrac{5}{10})\,\rangle.$$

Now $\{1_N, \mathcal{A}^c, \mathcal{B}^c, \mathcal{G}^c\}$ and $\{1_N, \mathcal{A}^c, \mathcal{B}^c\}$ areG_N-σCs and G_N-αCs of (Γ_1, ρ_1) respectively. Here $\psi^{-1}(H) = \mathcal{G}^c$ which isG_N-σCs but not G_N-αCs. Hence, ψ is G_N-σCCF but not G_N-αCCF. Also $\{1_N, \mathcal{A}^c, \mathcal{B}^c, \mathcal{G}^c\}$ and $\{1_N, \mathcal{A}^c, \mathcal{B}^c\}$ are G_N-βCs and G_N-πCs of (Γ_1, ρ_1) respectively. Here $\psi^{-1}(H) = \mathcal{G}^c$ which isG_N-βCs but not G_N-πCs. Hence, ψ is G_N-βCCF but not G_N-πCCF.

Remark 3.4

Let $\psi : (\Gamma_1, \rho_1)\rightarrow(\Gamma_2, \rho_2)$ be a function, where (Γ_1, ρ_1) and (Γ_2,ρ_2) be G_N-TSs. Then

$$G_N\text{-CF} \ \underset{\not\longrightarrow}{\overset{\not\longleftarrow}{}}\ G_N\text{-CCF}$$

Example 3.5

(i) G_N-CF $\not\rightarrow$ G_N-CCF

Let $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ be defined as $\psi(p) = s$ and $\psi(q) = r$, where $\Gamma_1 = \{p, q\}$ and $\Gamma_2 = \{r, s\}$, $\rho_1 = \{0_N, \mathcal{A}, \mathcal{B}\}$, $\rho_2 = \{0_N, \mathcal{C}, \mathcal{D}\}$.

$$\mathcal{A} = \langle\, (\tfrac{2}{10},\tfrac{8}{10},\tfrac{9}{10}), (\tfrac{1}{10},\tfrac{7}{10},\tfrac{8}{10})\,\rangle,\ \ \mathcal{B} = \langle\, (\tfrac{3}{10},\tfrac{5}{10},\tfrac{6}{10}), (\tfrac{4}{10},\tfrac{6}{10},\tfrac{7}{10})\,\rangle,$$

$$\mathcal{C} = \langle\, (\tfrac{4}{10},\tfrac{6}{10},\tfrac{7}{10}), (\tfrac{3}{10},\tfrac{5}{10},\tfrac{6}{10})\,\rangle,\ \ \mathcal{D} = \langle\, (\tfrac{1}{10},\tfrac{7}{10},\tfrac{8}{10}), (\tfrac{2}{10},\tfrac{8}{10},\tfrac{9}{10})\,\rangle,$$

Here, ψ is G_N-CF but not G_N-CCF.

(ii) G_N-CCF $\not\rightarrow$ G_N-CF

Let $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ be defined as $\psi(p) = v$, $\psi(q) = w$ and $\psi(r) = u$, where $\Gamma_1 = \{p, q, r\}$ and $\Gamma_2 = \{u, v, w\}$, $\rho_1 = \{0_N, \mathcal{A}, \mathcal{B}\}$, $\rho_2 = \{0_N, \mathcal{C}, \mathcal{D}\}$.

$$\mathcal{A} = \langle\, (\tfrac{2}{10},\tfrac{6}{10},\tfrac{8}{10}), (\tfrac{1}{10},\tfrac{7}{10},\tfrac{9}{10}), (\tfrac{2}{10},\tfrac{8}{10},\tfrac{9}{10})\,\rangle,\ \ \mathcal{B} = \langle\, (\tfrac{3}{10},\tfrac{5}{10},\tfrac{7}{10}), (\tfrac{2}{10},\tfrac{5}{10},\tfrac{8}{10}), (\tfrac{4}{10},\tfrac{6}{10},\tfrac{7}{10})\,\rangle,$$

$$\mathcal{C} = \langle\, (\tfrac{9}{10},\tfrac{3}{10},\tfrac{1}{10}), (\tfrac{9}{10},\tfrac{2}{10},\tfrac{2}{10}), (\tfrac{8}{10},\tfrac{4}{10},\tfrac{2}{10})\,\rangle,\ \ \mathcal{D} = \langle\, (\tfrac{8}{10},\tfrac{5}{10},\tfrac{2}{10}), (\tfrac{7}{10},\tfrac{4}{10},\tfrac{4}{10}), (\tfrac{7}{10},\tfrac{5}{10},\tfrac{3}{10})\,\rangle.$$

Here, ψ is G_N-CCF but not G_N-CF

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Theorem 3.6

Let $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ be a function. Then the following are equivalent:

(1) ψ is G_N-CCF.

(2) $\psi^{-1}(K) \in G_N\text{-Os}(\Gamma_1)$ for any $K \in G_N\text{-Cs}(\Gamma_2)$.

(3) For each $k \in \Gamma_1$ and each $L \in G_N\text{-Os}(\Gamma_2)$ with $\psi(k) \notin L$, there exists $M \in G_N\text{-Cs}(\Gamma_2)$ such that $k \notin M$ and $\psi^{-1}(L) \subset M$.

(4) ψ is G_N-CCF at any $k \in \Gamma_1$.

(5) $\psi^{-1}(K) \subset G_N\text{-}intr(\psi^{-1}(K))$ for any $K \in G_N\text{-Cs}(\Gamma_2)$

(6) $G_N\text{-}clo(\psi^{-1}(L)) \subset \psi^{-1}(L)$ for any $L \in G_N\text{-Os}(\Gamma_2)$.

(7) $G_N\text{-}clo(\psi^{-1}(G_N\text{-}intr(Q)) \subset \psi^{-1}(G_N\text{-}intr(Q))$ for any $Q \subset \Gamma_2$.

(8) $\psi^{-1}(G_N\text{-}clo(Q)) \subset G_N\text{-}intr(\psi^{-1}(G_N\text{-}clo(Q)))$ for any $Q \subset \Gamma_2$.

Proof :

(1) $\Rightarrow$ (2) Let $K \in G_N\text{-Cs}(\Gamma_2)$. Then $\Gamma_2 - K \in G_N\text{-Os}(\Gamma_2)$. By (1), $\psi^{-1}(\Gamma_2 - K) = \Gamma_1 - \psi^{-1}(K) \in G_N\text{-Cs}(\Gamma_1)$. Thus $\psi^{-1}(K) \in G_N\text{-Os}(\Gamma_1)$.

(1) $\Rightarrow$ (3) Let $k \in \Gamma_1$ and $L \in G_N\text{-Os}(\Gamma_2)$ with $\psi(k) \notin L$. Then $k \notin \psi^{-1}(L)$. By (1), $\psi^{-1}(L) \in G_N\text{-Cs}(\Gamma_1)$. Put $M = \psi^{-1}(L)$. Then $\psi^{-1}(L) \subset M$ and $k \notin M$.

(3) $\Rightarrow$ (1) Let $L \in G_N\text{-Os}(\Gamma_2)$. For each $k \in \psi^{-1}(\Gamma_2 - L)$, $\psi(k) \notin L$. By (3), there exists $M_k \in G_N\text{-Cs}(\Gamma_1)$ such that $k \notin M_k$ and $\psi^{-1}(L) \subset M_k$. Then $k \in \Gamma_1 - M_k \subset \Gamma_1 - \psi^{-1}(L) = \psi^{-1}(\Gamma_2 - L)$. We have, $\bigcup_{k \in \psi^{-1}(\Gamma_2 - L)} \{k\} \subset \bigcup_{k \in \psi^{-1}(\Gamma_2 - L)} \{\Gamma_1 - M_k\} \subset \psi^{-1}(\Gamma_2 - L)$.

Thus $\psi^{-1}(\Gamma_2 - L) = \bigcup_{k \in \psi^{-1}(\Gamma_2 - L)} \{\Gamma_1 - M_k\} \in G_N\text{-Os}(\Gamma_1)$. This implies $\psi^{-1}(L) \in G_N\text{-Cs}(\Gamma_1)$. Hence ψ is G_N-CCF.

(2) $\Rightarrow$ (4) Let $k \in \Gamma_1$ and $L \in G_N\text{-Cs}(\Gamma_2, \psi(k))$. By (2), $\psi^{-1}(L) \in G_N\text{-Os}(\Gamma_1)$. Put $M = \psi^{-1}(L)$. We have $M \in G_N\text{-Os}(\Gamma_1, k)$ and $\psi(M) \subset L$.

(4) $\Rightarrow$ (5) Let $K \in G_N\text{-Cs}(\Gamma_2)$. For each $k \in \psi^{-1}(K)$, $\psi(k) \in K$. By (4), there exists $M \in G_N\text{-Os}(\Gamma_1, k)$ such that $\psi(M) \subset K$. Since $k \in M \subset \psi^{-1}(K)$, we have $k \in G_N\text{-}intr(\psi^{-1}(K))$. This implies $\psi^{-1}(K) \subset G_N\text{-}intr(\psi^{-1}(K))$.

(5) $\Rightarrow$ (6) Let $L \in G_N\text{-Os}(\Gamma_2)$. Then $\Gamma_2 - L \in G_N\text{-Cs}(\Gamma_2)$. By (5), $\psi^{-1}(\Gamma_2 - L) \subset G_N\text{-}intr(\psi^{-1}(\Gamma_2 - L) = G_N\text{-}intr(\Gamma_1 - \psi^{-1}(L)) = \Gamma_1 - G_N\text{-}clo(\psi^{-1}(L))$. Thus $G_N\text{-}clo(\psi^{-1}(L) \subset \psi^{-1}(L)$.

(6) $\Rightarrow$ (7). Let $Q \subset \Gamma_2$. Since $G_N\text{-}intr(Q) \in G_N\text{-Os}(\Gamma_2)$, By (6), we have $G_N\text{-}clo(\psi^{-1}(G_N\text{-}intr(Q))) \subset \psi^{-1}(G_N\text{-}intr(Q))$.

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(7) ⇒ (8). Let $Q \subset \Gamma_2$. By (7), G_N-$clo(\psi^{-1}(G_N$-$intr(\Gamma_2 - Q))) \subset \psi^{-1}(G_N$-$intr(\Gamma_2 - Q))$. Then, G_N-$clo(\psi^{-1}(G_N$-$intr(\Gamma_2 - Q))) = G_N$-$clo(\psi^{-1}(\Gamma_2 - G_N$-$clo(Q))) = G_N$-$clo(\Gamma_1 - \psi^{-1}(G_N$-$clo(Q))) = \Gamma_1 - G_N$-$intr(\psi^{-1}(G_N$-$clo(Q)))$, and $\psi^{-1}(G_N$-$intr(\Gamma_2 - Q))) = \Gamma_1 - \psi^{-1}(G_N$-$clo(Q))$. Thus $\psi^{-1}(G_N$-$clo(Q)) \subset G_N$-$intr(\psi^{-1}(G_N$-$clo(Q)))$.

(8) ⇒ (1) Let $Q \in G_N$-$Os(\Gamma_2)$. Then $\Gamma_2 - Q \in G_N$-$Cs(\Gamma_2))$. By (8), $\Gamma_1 - \psi^{-1}(Q) = \psi^{-1}(\Gamma_2 - Q) = \psi^{-1}(G_N$-$clo(\Gamma_2 - Q)) \subset G_N$-$intr(\psi^{-1}(G_N$-$clo(\Gamma_2 - Q)) = G_N$-$intr(\psi^{-1}(\Gamma_2 - Q))$. Now, G_N-$intr(\psi^{-1}(\Gamma_2 - Q)) = G_N$-$intr(\Gamma_1 - \psi^{-1}(Q))) = \Gamma_1 - G_N$-$clo(\psi^{-1}(Q)$. Then $\psi^{-1}(Q) \supset G_N$-$clo(\psi^{-1}(Q)$. Thus $\psi^{-1}(Q) \in G_N$-$Cs(\Gamma_1)$. This shows that ψ is G_N-CCF.

Theorem 3.7

A mapping $\psi: (\Gamma_1, \rho_1) \to (\Gamma_2, \rho_2)$ is G_N-αCCF if and only if it is both G_N-πCCF and G_N-σCCF.

Proof

Necessity. It is clear from Remark 3.2

Sufficiency. Let K be a G_N-$Os(\Gamma_2)$. Then by hypothesis, $\psi^{-1}(K)$ is both G_N-$\pi Cs(\Gamma_1)$ and G_N-$\sigma Cs(\Gamma_1)$. Therefore G_N-$clo(G_N$-$intr(\psi^{-1}(K))) \subset \psi^{-1}(K)$ and G_N-$intr(G_N$-$clo(\psi^{-1}(K)))$ $\subset \psi^{-1}(K)$. We have G_N-$intr(G_N$-$intr(G_N$-$clo(\psi^{-1}(K)))) \subset G_N$-$intr(\psi^{-1}(K))$. That is G_N-$intr(G_N$-$clo(\psi^{-1}(K))) \subset G_N$-$intr(\psi^{-1}(K))$. Now G_N-$clo(G_N$-$intr(G_N$-$clo(\psi^{-1}(K))) \subset G_N$-$clo(G_N$-$intr(\psi^{-1}(K))) \subset \psi^{-1}(K)$. Hence $\psi^{-1}(K)$ is a G_N-$\alpha Cs(\Gamma_1)$. Thus ψ is G_N-αCCF.

Theorem 3.8

Let $\psi : (\Gamma_1, \rho_1) \to (\Gamma_2, \rho_2)$ be a function from two G_N-TSs. Then the following conditions are equivalent:

(1) ψ isG_N-πCCF,

(2) $\psi^{-1}(M)$ is G_N-$\pi Os(\Gamma_1)$for every G_N-CsM in Γ_2,

(3) $\psi^{-1}(M) \subset G_N$-$intr(G_N$-$clo(\psi^{-1}(G_N$-$clo(M))))$ for every subset M in Γ_2,

(4) G_N-$clo(G_N$-$intr(\psi^{-1}(G_N$-$intr(M)))) \subset \psi^{-1}(M)$ for every subset M in Γ_2,

(5) $M \subset G_N$-$intr(G_N$-$clo(\psi^{-1}(G_N$-$clo(\psi(M)))))$ for every subset M in Γ_1.

Proof

(1) ⇔ (2) is obvious.

(2) ⇒ (3) Let $M \subset \Gamma_2$. Then G_N-$clo(M)$ is a G_N-Cs in Γ_2. (2) implies that $\psi^{-1}(G_N$-$clo(M))$ is a G_N-πOsin Γ_1. Therefore $\psi^{-1}(G_N$-$clo(M)) \subset G_N$-$intr(G_N$-$clo(\psi^{-1}(G_N$-$clo(M))))$. Hence $\psi^{-1}(M) \subset \psi^{-1}(G_N$-$clo(M)) \subset G_N$-$intr(G_N$-$clo(\psi^{-1}(G_N$-$clo(M))))$.

(3) ⇔ (4) can be proved by taking complement.

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(3) $\Rightarrow$ (5). Let $M\subset\Gamma_1$. Then $\psi(M)\subset\Gamma_2$. (iii) implies that $\psi^{-1}(\psi(M))\subset G_N$-*intr*($G_N$-*clo*($\psi^{-1}$($G_N$-*clo*($\psi(M)$)))). Therefore $M\subset\psi^{-1}(\psi(M))\subset G_N$-*intr*($G_N$-*clo*($\psi^{-1}$($G_N$-*clo*($\psi(M)$)))).

(5) $\Rightarrow$ 2). Let Mbe G_N-Csin Γ_2. Then $\psi^{-1}(M)\subset\Gamma_1$. By hypothesis $\psi^{-1}(M)\subset G_N$-*intr*(G_N-*clo*(ψ^{-1}(G_N-*clo*($\psi(\psi^{-1}(M))$)))) $\subset$ G_N-*intr*(G_N-*clo*(ψ^{-1}(G_N-*clo*(M)))) = G_N-*intr*(G_N-*clo*($\psi^{-1}(M)$)). Hence $\psi^{-1}(M)$ is G_N-πOs in Γ_1.

Theorem 3.9

Let $\psi:(\Gamma_1,\rho_1)\rightarrow(\Gamma_2,\rho_2)$ be a function from two G_N-TSs. Then the following conditions are equivalent:

(1) ψ is G_N-σCCF,

(2) $\psi^{-1}(M)$ is G_N-σOs(Γ_1) for every G_N-CsM in Γ_2,

(3) $\psi^{-1}(M)\subset G_N$-*clo*(G_N-*intr*(ψ^{-1}(G_N-*clo*(M)))) for every subset M in Γ_2,

(4) G_N-*clo*(G_N-*intr*(ψ^{-1}(G_N-*intr*(M)))) $\subset\psi^{-1}(M)$ for every subset M in Γ_2,

(5) $M\subset G_N$-*clo*(G_N-*intr*(ψ^{-1}(G_N-*clo*($\psi(M)$)))) for every subset M in Γ_1.

Proof

Proof is similar to Theorem 3.8.

Remark 3.10

We can obtain the above equivalent conditions for G_N-αCCF by Theorem 3.7.

Theorem 3.11

Let $\psi:(\Gamma_1,\rho_1)\rightarrow(\Gamma_2,\rho_2)$ be a function from two G_N-TSs. Suppose that one of the following conditions hold:

(1) ψ^{-1}(G_N-*clo*(K)) $\subset G_N$-*intr*(G_N-α*clo*($\psi^{-1}(K)$)) for each subset K in Γ_2,

(2) G_N-*clo*(G_N-α*intr*($\psi^{-1}(K)$)) $\subset\psi^{-1}$(G_N-*intr*(K)) for each subset K in Γ_2,

(3) ψ(G_N-*clo*(G_N-α*intr*(M))) $\subset G_N$-*intr*($\psi(M)$) for each subset M in Γ_1,

(4) ψ(G_N-*clo*(M)) $\subset G_N$-*intr*($\psi(M)$) for each G_N-αOs M in Γ_1.

Then ψ is G_N-αCCF.

Proof

(1) $\Rightarrow$ (2) is obvious by taking complement in (1).

(2) $\Rightarrow$ (3). Let $M\subset\Gamma_1$, then $\psi(M)\subset\Gamma_2$. Now (ii) implies G_N-*clo*(G_N-α*intr*($\psi^{-1}(\psi(M))$)) $\subset\psi^{-1}$(G_N-*intr*($\psi(M)$)). That is G_N-*clo*(G_N-α*intr*(M)) $\subset$ G_N-*clo*(G_N-α*intr*($\psi^{-1}(\psi(M))$)) $\subset$ ψ^{-1}(G_N-*intr*($\psi(M)$)). Hence ψ(G_N-*clo*(G_N-α*intr*(M))) $\subset$ $\psi(\psi^{-1}$(G_N-*intr*($\psi(M)$))) $\subset$ G_N-*intr*($\psi(M)$).

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(3) ⇒ (4). Let M ⊂ $\Gamma_1$be G_N-αOs. Then ψ(G_N-*clo*(G_N-α*intr*(M))) ⊂ G_N-*intr*(ψ(M)). That is ψ(G_N-*clo*(M)) = ψ(G_N-*clo*(G_N-α*intr*(M))) ⊂ G_N-*intr*(ψ(M)), since G_N-α*intr*(M) = M. Hence ψ(G_N-*clo*(M)) ⊂ G_N-*intr*(ψ(M)).

Suppose (4) holds: Let M ⊂ $\Gamma_2$be G_N-Os. Then ψ^{-1}(M) ⊂ Γ_1 and G_N-α*intr*(ψ^{-1}(M)) is G_N-αOs in Γ_1. (4) implies that ψ(G_N-*clo*(G_N-α*intr*(ψ^{-1}(M)))) ⊂ G_N-*intr*(ψ(G_N-α*intr*(ψ^{-1}(M)))) ⊂ G_N-*intr*(ψ(ψ^{-1}(M))) ⊂ G_N-*intr*(M) = M. Now G_N-*clo*(G_N-α*intr*(ψ^{-1}(M))) ⊂ ψ^{-1}(ψ(G_N-*clo*(G_N-α*intr*(ψ^{-1}(M))))) ⊂ψ^{-1}(M). We have G_N-*clo*(G_N-*intr*(ψ^{-1}(M))) ⊂ G_N-*clo*(G_N-α*intr*(ψ^{-1}(M))) ⊂ψ^{-1}(M). Therefore ψ^{-1}(M) is a G_N-αCs and hence M is a G_N-αCs. Thus ψ is G_N-αCCF.

Theorem 3.12

Let ψ : (Γ_1, ρ_1)→(Γ_2, ρ_2) be a function from two G_N-TSs. Then ψ is G_N-σCCF if ψ^{-1}(M) ⊂ G_N-*clo*(G_N-*intr*(ψ^{-1}(G_N-*clo*(M)))) for every subset M in Γ_2.

Proof

Let M be G_N-Os in Γ_2. Then M^c is G_N-Cs in Γ_2. By hypothesis ψ^{-1}(M^c) ⊂ G_N-*clo*(G_N-*intr*(ψ^{-1}(G_N-*clo*(M^c)))) = G_N-*clo*(G_N-*intr*(ψ^{-1}(M^c))). This implies (ψ^{-1}(M))^c ⊂ G_N-*clo*(G_N-*intr*(ψ^{-1}(M^c))) = (G_N-*intr*(G_N-*clo*(ψ^{-1}(M))))^c. Thus G_N-*intr*(G_N-*clo*(ψ^{-1}(M))) ⊂ ψ^{-1}(M). Hence ψ^{-1}(M) is a G_N-σCs in Γ_1. Thus ψ is G_N-σCCF.

Theorem 3.13

Let ψ : (Γ_1, ρ_1)→(Γ_2,ρ_2) be a function from two G_N-TSs. Suppose one of the following conditions hold:

(1) ψ(G_N-α*clo*(K)) ⊂ G_N-*intr*(ψ(K)) for each subset K in Γ_1.

(2) G_N-α*clo*(ψ^{-1}(M)) ⊂ ψ^{-1}(G_N-*intr*(M)) for each subset M in Γ_2.

(3) ψ^{-1}(G_N-*clo*(M)) ⊂ G_N-α*intr*(ψ^{-1}(M)) for each subset M in Γ_2.

Then ψ is G_N-αCCF.

Proof

(1) ⇒ (2) Let M ⊂ Γ_2. Then ψ^{-1}(M) ⊂ Γ_1. (1) implies that ψ(G_N-α*clo*(ψ^{-1}(M))) ⊂ G_N-*intr*(ψ(ψ^{-1}(M))) ⊂ G_N-*intr*(M). Now ψ^{-1}(ψ(G_N-α*clo*(ψ^{-1}(M)))) ⊂ ψ^{-1}(G_N-*intr*(M)). Therefore G_N-α*clo*(ψ^{-1}(M)) ⊂ ψ^{-1}(ψ(G_N-α*clo*(ψ^{-1}(M)))) ⊂ ψ^{-1}(G_N-*intr*(M)). Hence G_N-α*clo*(ψ^{-1}(M)) ⊂ ψ^{-1}(G_N-*intr*(M)).

(2) ⇒ (3) is obvious by taking complement in (2).

Suppose (3) holds: Let M ⊂ $\Gamma_2$be G_N-Cs. Then, by hypothesis, ψ^{-1}(G_N-*clo*(M)) ⊂ G_N-α*intr*(ψ^{-1}(M)). That is ψ^{-1}(M) = ψ^{-1}(G_N-*clo*(M)) ⊂ G_N-α*intr*(ψ^{-1}(M)) ⊂ψ^{-1}(M). Therefore ψ^{-1}(M) is G_N-αOs in Γ_1. Hence ψ is G_N-αCCF.

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Theorem 3.14

Let $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ be a bijective function from two G_N-TSs. Then ψ is G_N-πCCF if G_N-$clo(\psi(M)) \subset \psi(G_N$-$\pi intr(M))$ for each subset M of Γ_1.

Proof

Let $K \subset \Gamma_2$ be G_N-Cs. Then $\psi^{-1}(K) \subset \Gamma_1$. By hypothesis G_N-$clo(\psi(\psi^{-1}(K))) \subset \psi(G_N$-$\pi intr(\psi^{-1}(K)))$. Now $K = G_N$-$clo(K) = G_N$-$clo(\psi(\psi^{-1}(K))) \subset \psi(G_N$-$\pi intr(\psi^{-1}(K)))$. Therefore $\psi^{-1}(K) \subset \psi^{-1}(\psi(G_N$-$\pi intr(\psi^{-1}(K)))) = G_N$-$\pi intr(\psi^{-1}(K)) \subset \psi^{-1}(K)$. Hence $\psi^{-1}(K)$ is G_N-πOs and hence ψ is a G_N-πCCF.

Theorem 3.15

Let $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ be a G_N-αCCF from two G_N-TSs. Then the following properties hold:

(1) G_N-$\alpha clo(\psi^{-1}(M)) \subset \psi^{-1}(G_N$-$intr(G_N$-$\alpha clo(M)))$ for each G_N-Os M in Γ_2.

(2) $\psi^{-1}(G_N$-$clo(G_N$-$\alpha intr(M))) \subset G_N$-$\alpha intr(\psi^{-1}(M))$ for each G_N-Cs M in Γ_2.

Proof

(1) Let $M \subset \Gamma_2$ be G_N-Os. By hypothesis, $\psi^{-1}(M)$ is G_N-αCs in Γ_1. Then G_N-$\alpha clo(\psi^{-1}(M)) = \psi^{-1}(M) = \psi^{-1}(G_N$-$intr(M)) \subset \psi^{-1}(G_N$-$intr(G_N$-$\alpha clo(M)))$. Hence G_N-$\alpha clo(\psi^{-1}(M)) \subset \psi^{-1}(G_N$-$intr(G_N$-$\alpha clo(M)))$.

(2) can be proved easily by taking complement of (1).

Theorem 3.16

Let $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ be a function from two G_N-TSs. Then the following conditions are equivalent:

(1) ψ is G_N-αCCF,

(2) for each $k \in \Gamma_1$ and each G_N-Cs M containing $\psi(k)$, there exists a G_N-αOs K in Γ_1 and $k \in K$ such that $K \subset \psi^{-1}(M)$,

(3) for each $k \in \Gamma_1$ and each G_N-Cs M containing $\psi(k)$, there exists a G_N-αOs K in Γ_1 and $k \in K$ such that $\psi(K) \subset M$.

Proof

(1) $\Rightarrow$ (2) Let $M \subset \Gamma_2$ be G_N-Cs and $\psi(k) \in M$. By hypothesis, $\psi^{-1}(M)$ is G_N-αOs in Γ_1. Therefore G_N-$\alpha intr(\psi^{-1}(M)) = \psi^{-1}(M)$. Put $K = G_N$-$\alpha intr(\psi^{-1}(M))$. Then K is a G_N-αOs in Γ_1 and $K \subset \psi^{-1}(M)$.

(2) $\Rightarrow$ (3) Let $M \subset \Gamma_2$ be G_N-Cs and $\psi(k) \in M$. By hypothesis, there exists a G_N-αOs K in Γ_1 and $k \in K$ such that $K \subset \psi^{-1}(M)$. Therefore $\psi(K) \subset \psi(\psi^{-1}(M)) \subset M$. Thus $\psi(K) \subset M$.

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(3) ⇒ (1) Let M be G_N-Cs in Γ_2. Let $k \in \Gamma_1$and $\psi(k) \in$ M. By hypothesis, there exists a G_N-αOs K in $\Gamma_1$and $k \in$ K such that $\psi(K) \subset M$. This implies $k \in K \subset \psi^{-1}(\psi(K)) \subset \psi^{-1}(M)$. That is $k \in \psi^{-1}(M)$. Since K is G_N-αOs, K= G_N-α*intr*(K) ⊂ G_N-α*intr*(ψ^{-1}(M)). Hence $k \in G_N$-α*intr*(ψ^{-1}(M)). Therefore $\psi^{-1}(M) = \bigcup_{k \in \psi^{-1}(M)} \{k\} \subset G_N\text{-}\alpha intr(\psi^{-1}(M)) \subset \psi^{-1}(M)$. Thus G_N-α*intr* (ψ^{-1}(M)) = ψ^{-1}(M) and ψ^{-1}(M) is G_N-αOs. Hence ψ is G_N-αCCF.

Theorem 3.17

(1) A function $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ is a G_N-σCCF from two G_N-TSs iff ψ^{-1}(G_N-σ*clo*(M)) ⊂ G_N-σ*intr*(ψ^{-1}(G_N-*clo*(M))) for each subset M in Γ_2.

(2) A function $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ is a G_N-πCCF from two G_N-TSs iff ψ^{-1}(G_N-π*clo*(M)) ⊂ G_N-π*intr*(ψ^{-1}(G_N-*clo*(M))) for each subset M in Γ_2.

Proof

(1) **Necessity.** Let M ⊂ Γ_2. Then G_N-*clo*(M) is G_N-Cs in Γ_2. By hypothesis, ψ^{-1}(G_N-*clo*(M)) is G_N-σOs in Γ_1. Therefore ψ^{-1}(G_N-σ*clo*(M)) ⊂ ψ^{-1}(G_N-*clo*(M)) = G_N-σ*intr*(ψ^{-1}(G_N-*clo*(M))). Hence ψ^{-1}(G_N-σ*clo*(M)) ⊂ G_N-σ*intr*(ψ^{-1}(G_N-*clo*(M))).

Sufficiency. Let M ⊂ $\Gamma_2$be G_N-Cs. Then G_N-*clo*(M) = M. By hypothesis ψ^{-1}(G_N-σ*clo*(M)) ⊂ G_N-σ*intr*(ψ^{-1}(G_N-*clo*(M))) = G_N-σ*intr*(ψ^{-1}(M)). Now ψ^{-1}(M) ⊂ ψ^{-1}(G_N-σ*clo*(M)) ⊂ G_N-σ*intr*(ψ^{-1}(M)) ⊂ ψ^{-1}(M). This implies ψ^{-1}(M) = G_N-σ*intr*(ψ^{-1}(M)). Hence ψ^{-1}(M) is G_N-σOs in $\Gamma_1$and hence ψ is G_N-σCCF.

(2) Proof is alike to (1).

Theorem 3.18

A function $\psi : (\Gamma_1, \rho_1) \rightarrow (\Gamma_2, \rho_2)$ is a G_N-αCCF from two G_N-TSs iff ψ^{-1}(G_N-α*clo*(M)) ⊂ G_N-α*intr*(ψ^{-1}(G_N-*clo*(M))) for each subset M in Γ_2.

Proof:

We can obtain the proof for G_N-αCCF by Theorem 3.7 and 3.17.

Related Separation Axioms in G_N-TSs

Definition 4.1

Let (Γ_1, ρ_1) and (Γ_2, ρ_2) be G_N-TSs. Then $\psi : \Gamma_1 \rightarrow \Gamma_2$ is said to be

(Δ_1) G_N-Open if for each G_N-Os M in Γ_1, ψ(M) is a G_N-Os in Γ_2,

(Δ_2) G_N-Closed if for each G_N-Cs M in Γ_1, ψ(M) is a G_N-Cs in Γ_2,

(Δ_3) Almost G_N-α-Continuous Function (G_N-AαCF) if for each G_N-rOs M in Γ_2, ψ^{-1}(M) is a G_N-αOs in Γ_1.

(Δ_4) Almost Contra G_N-Continuous Function (G_N-ACCF) if for each G_N-rOs M in Γ_2, ψ^{-1}(M) is a G_N-Cs in Γ_1,

(Δ_5) Almost Contra G_N-α-Continuous Function (G_N-AαCCF) if for each G_N-

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rOs M in Γ_2, $\psi^{-1}(M)$ is a G_N-αCs in Γ_1,

(Δ_6) Almost Contra G_N-σ-Continuous Function (G_N-AσCCF) if for each G_N-rOs M in Γ_2, $\psi^{-1}(M)$ is a G_N-σCs in Γ_1,

(Δ_7) Almost Contra G_N-π-Continuous Function (G_N-AπCCF) if for each G_N-rOs M in Γ_2, $\psi^{-1}(M)$ is a G_N-πCs in Γ_1.

Definition 4.2

A G_N-TS(Γ_1, ρ_1) is said to be

(Δ_1) G_N-connected if it cannot be expressed as the union of two nonempty, disjoint G_N-Os,

(Δ_2) G_N-α-connected if it cannot be expressed as the union of two nonempty, disjoint G_N-αOs.

Theorem 4.3

If $\psi : (\Gamma_1, \rho_1)\rightarrow(\Gamma_2, \rho_2)$ is G_N-AαCCF and surjective function and Γ_1 is G_N-α-connected space, then Γ_2 is G_N-connected space.

Proof

Suppose that Γ_2 is not G_N-connected space. Then there exists non-empty disjoint G_N-Os K and L such that $\Gamma_2 = K \cup L$. Therefore, K and L are G_N-rOs in Γ_2. Since ψ is G_N-AαCCF, then $\psi^{-1}(K)$ and $\psi^{-1}(L)$ are G_N-αCs in Γ_1. Moreover, $\psi^{-1}(K)$ and $\psi^{-1}(L)$ are nonempty disjoint and $\Gamma_1 = \psi^{-1}(K) \cup \psi^{-1}(L)$. This shows that Γ_1 is not G_N-α-connected. This is a contradiction. By contradiction, Γ_2 is G_N-connected.

Definition 4.4

A G_N-TS(Γ_1, ρ_1) is said to be G_N-α-normal if every pair of nonempty disjoint G_N-Cs can be separated by disjoint G_N-αOs.

Definition 4.5

A G_N-TS(Γ_1, ρ_1) is said to be strongly G_N-normal if for every pair of non empty disjoint G_N-Cs K and L in Γ_1 there exist disjoint G_N-Os P and Q such that $K \subseteq P$, $L \subseteq Q$ and G_N-clo(P) $\cap$ G_N-clo(Q) = ϕ.

Theorem 4.6

If (Γ_2, ρ_2) is strongly G_N-normal and $\psi : (\Gamma_1, \rho_1)\rightarrow(\Gamma_2, \rho_2)$ is G_N-AαCCF and G_N-Closed injective function, then (Γ_1, ρ_1) is G_N-α-normal.

Proof

Let K and L be disjoint nonempty G_N-Cs of Γ_1. Since ψ is injective and G_N-Closed, ψ(K) and ψ(L) are disjoint G_N-Cs. Since (Γ_2, ρ_2) is strongly G_N-normal, there exists G_N-Os P and Q such that $\psi(K) \subseteq P$, $\psi(L) \subseteq Q$ and G_N-clo(P) $\cap$ G_N-clo(Q) = ϕ. Then, since G_N-clo(P)

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andG_N-*clo*(Q) are G_N-rCs and ψ is G_N-AαCCF, ψ^{-1}(G_N-*clo*(P)) andψ^{-1}(G_N-*clo*(Q)) are G_N-αOs. Since K $\subseteq \psi^{-1}$(G_N-*clo*(P)), L $\subseteq \psi^{-1}$(G_N-*clo*(Q)), and ψ^{-1}(G_N-*clo*(P)) and ψ^{-1}(G_N-*clo*(Q)) are disjoint, (Γ_1, ρ_1) is G_N-α-normal.

Definition 4.7

A G_N-TS (Γ, G_N) is said to be a G_N-P_Σ if for any G_N-Os M of Γ and each k $\in \Gamma$, there exists a G_N-rCs N containing k such that k $\in$ N $\subseteq$ M.

Theorem 4.8

If ψ : (Γ_1, ρ_1)$\rightarrow$ (Γ_2, ρ_2) is G_N-AαCCF and Γ_2 is G_N-P_Σ, then ψ is G_N-αCF.

Proof

Let M be a G_N-Os in Γ_2. Since $\Gamma_2$is G_N-P_Σ, there exists a family Ω whose members are G_N-rCs of Γ_2 such that M = $\cup$ {N: N $\in \Omega$}. Since ψ is G_N-AαCCF, ψ^{-1}(N) is G_N-αOs in Γ_1 for each N $\in \Omega$ and hence ψ^{-1}(M) is G_N-αOs in Γ_1. Therefore ψ is G_N-αCF.

Definition 4.9

A G_N-TS (Γ, G_N) is said to be weakly G_N-P_Σ if for any G_N-rOs M of Γ and each k $\in \Gamma$, there exists a G_N-rCs N containing k such that k $\in$ N $\subseteq$ M.

Theorem 4.10

If ψ : (Γ_1, ρ_1)$\rightarrow$ (Γ_2, ρ_2) is G_N-AαCCF and Γ_2 is weakly G_N-P_Σ, then ψ is G_N-AαCF.

Proof

Let M be a G_N-rOs in Γ_2. Since Γ_2 is weakly G_N-P_Σ, there exists a family Ω whose members are G_N-rCs of Γ_2 such that M = $\cup$ {N: N $\in \Omega$}. Since ψ is G_N-AαCCF, ψ^{-1}(N) is G_N-αOs in Γ_1 for each N $\in \Omega$ and hence ψ^{-1}(M) is G_N-αOs in Γ_1. Therefore ψ is G_N-AαCF.

Theorem 4.11

Let (Γ_1, ρ_1), (Γ_2, ρ_2) and (Γ_3, ρ_3) be G_N-TSs and let ψ_1 : (Γ_1, ρ_1)$\rightarrow$(Γ_2, ρ_2) and ψ_2 : (Γ_2, ρ_2)$\rightarrow$(Γ_3, ρ_3) be functions. If ψ_1 is G_N-α-irresolute and ψ_2 is G_N-AαCCF, then $\psi_2 \circ \psi_1$: (Γ_1, ρ_1)$\rightarrow$(Γ_3, ρ_3) is G_N-AαCCF.

Proof

Let M $\subseteq \Gamma_3$ be any G_N-rCs and let ($\psi_2 \circ \psi_1$)(k) $\in$M. Then ψ_2(ψ_1(k)) $\in$M. Since ψ_2 is G_N-AαCCF, it follows that there exists a G_N-αOs N containing ψ_1(k) such that ψ_2(N)$\subseteq$ M. Since ψ_1 is λ-α-irresolute function, it follows that there exists a G_N-αOs P

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containing k such that $\psi_1(P) \subseteq N$. From here we obtain that $(\psi_2 \circ \psi_1)(P) = \psi_2(\psi_1(P)) \subseteq \psi_2(N) \subseteq M$. Thus we show that $\psi_2 \circ \psi_1$ is G_N-AαCCF.

Conclusion

In this paper we studied about Contra Continuous Functions (CCF) by means of Neutrosophic Sets in Generalized Topological Spaces (G_N-TSs). Then, we deliberate certain properties of CCF in G_N-TSs. Further, we talk over about the associations among several types of CCF along with illustrations. Also, we dealt the concept of almost continuous and its contra characteristics in G_N-TSs. Finally, we discuss some separation axioms related to G_N-TSs. This paper can be further developed into several continuous functions and its contra continuity such as G_N-b-continuous, G_N-b^*-continuous function, contra G_N-b-continuous function in Neutrosophic Generalized Topological Spaces.

On account that the whole lot inside the world is complete of indeterminacy so, the neutrosophic becomes seem and observed their location into research. There exists quite a few utility in all area inclusive of in records era, information system and decision assist device as an instance, relational database structures, semantic web offerings, economic records set detection, new economy's growth, decline evaluation and many others. These notions can also help the researcher in making algorithm to clear up problems.

References

[1] Atanassov.K.T: Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87–96.

[2] Bhuvaneshwari J, Keskin A and Rajesh N: Contra-Continuity via topological ideals, Journal of Advanced Research in Pure Mathematics, 3(1)(2011), 40-51, Online ISSN: 1943-2380.

[3] Chang.C.L: Fuzzy topological spaces, Journal of Mathematical Analysis and Application, 24(1968), 183–190.

[4] Dogan Coker: An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets andSystems, 88(1997), 81–89.

[5] Floretin Smarandache: Neutrosophy and Neutrosophic Logic, First InternationalConference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA, 2002.

[6] Floretin Smarandache: Neutrosophic Set: A Generalization of Intuitionistic Fuzzy set, Journal of Defense Resources Management, 1(2010), 107–116.

[7] Floretin Smarandache: A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic set, Neutrosophic Probability, American Research Press, Rehoboth, NM, 1999.

[8] Jayanthi D, Contra continuity on generalized topological spaces, Acta Math Hungar, (2012), ISSN No. 0236-5294, doi: 10.1007/s10474-012-0211-x.

[9] Jeyaraman M and Yuvarani A: Intuitionistic Fuzzy Contra Alpha Generalized Semi Continuous Mappings, The Journal of Fuzzy Mathematics, 24 (1)(2016), 1-12.

[10] Li Z and Zhu W: Contra continuity on generalized topological spaces, Acta Math Hungar, (2012), ISSN No. 0236-5294,DOI: 10.1007/s10474-012-0215-6.

[11] Raksha Ben .N, Hari Siva Annam. G: Some new open sets in μ_N topological space, Malaya Journal of Matematik, 9(1)(2021), 89-94.

[12] Raksha Ben .N, Hari Siva Annam. G: Generalized Topological Spaces via Neutrosophic Sets, J. Math. Comput. Sci., 11(2021), 716-734.

[13] Salama A.A and Alblowi S.A: Neutrosophic set and Neutrosophic topological space, ISOR J. Mathematics, 3(4)(2012), 31–35.

[14] Salama A.A, Florentin Smarandache and Valeri Kroumov: Neutrosophic Closed set and Neutrosophic Continuous Function, Neutrosophic Sets and Systems, 4(2014), 4–8.

[15] Santhi P, Yuvarani A and Vijaya S, Irresolute and its Contra Functions in Generalized Neutrosophic Topological Spaces, Neutrosophic Sets and Systems, 51(2022), 123-133,doi: 10.5281/zenodo.7135261.

[16] Vijaya S and Santhi P: Characterization of Almost (α, μ)-Continuous Functions and itsproperties, International Science and Technology Journal, 7(3)(2018), 1-8, Online ISSN: 1632-2882.

[17] Vijaya S, Santhi P and Yuvarani A: Contra Nα-I-Continuity over Nano Ideals, Ratio Mathematica, 41(2021), 283-290.

[18] Wadel Faris Al-Omeri and Florentin Smarandache: New Neutrosophic Sets via Neutrosophic Topological Spaces, New Trends in Neutrosophic Theory and Applications, (2)(2016), 1-10.

[19] Zadeh. L.A: Fuzzy set, Inform and Control, 8(1965), 338– 353.