Conjecture on the consecutive concatenation of the terms of an arithmetic progression

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Abstract. In this paper I make the following conjecture: for any arithmetic progression a + b*k, where at least one of a and b is different than 1, that also satisfies the conditions imposed by the Dirichlet's Theorem (a and b are positive coprime integers) is true that the sequence obtained by the consecutive concatenation of the terms a + b*k has an infinity of prime terms. Example: for [a, b] = [7, 11], the sequence obtained by consecutive concatenation of 7, 18, 29, 40, 51, 62, 73 (...) has the prime terms 718294051, 7182940516273 (...). If this conjecture were true, the fact that the *Smarandache consecutive numbers sequence* 1, 12, 123, 1234, 12345 (...) could have not any prime term (thus far there is no prime number known in this sequence, though there have been checked the first about 40 thousand terms) would be even more amazing.

Conjecture:

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Note:

If this conjecture were true, the fact that the *Smarandache consecutive numbers sequence* 1, 12, 123, 1234, 12345 (...) could have not any prime term (thus far there is no prime number known in this sequence, though there have been checked the first about 40 thousand terms) would be even more amazing.

The sequence of primes for [a, b] = [2, 1]:

: 2, 23, 23456789, 23456789101112131415161718192021222324252627 (...)

The sequence of primes for [a, b] = [3, 1]:

: 3, 345678910111213141516171819 (...)

The sequence of primes for [a, b] = [4, 1]:

: 4567, 45678910111213 (...)

The sequence of primes for [a, b] = [5, 1]:

: 5, 567891011121314151617 (...)

The sequence of primes for [a, b] = [6, 1]:

: 67, 678910111213 (...)

The sequence of primes for [a, b] = [25, 1]:

: 25262728293031323334353637383940414243444546474849 (...)

Note that 25 and 49 are both squares of primes (twins); question: are there any other pairs of squares of primes [p^2, q^2] having the property that concatenating the numbers p^2, p^2 + 1, ..., q^2 is obtained a prime?

The sequence of primes for [a, b] = [1, 2]:

: 13, 135791113151719, 135791113151719212325272931 (...)

This sequence is known as *Smarandache concatenated odd sequence* and Florentin Smarandache conjectured that there exist an infinity of prime terms of this sequence. The terms of this sequence are primes for the following values of n: 2, 10, 16, 34, 49, 2570 (the term corresponding to n = 2570 is a number with 9725 digits); there is no other prime term known though where checked the first about 26 thousand terms of this sequence.

The sequence of primes for [a, b] = [1, 3]:

: 14710131619, 14710131619222528313437 (...)

In my previous paper "Conjecture on the consecutive concatenation of the numbers nk+1 where k multiple of 3" I already conjectured that this sequence has an infinity of prime terms.

The sequence of primes for [a, b] = [1, 4]:

:   1591317212529333731454953576165697377781 (...)

The sequence of primes for [a, b] = [1, 20]:

:   121416181101121141161181 (...)

The sequence of primes for [a, b] = [3, 2]:

:   357911131517192123252729 (...)

The sequence of primes for [a, b] = [5, 2]:

:   57911131517, 57911131517192123252729313335373941 (...)

The sequence of primes for [a, b] = [17, 16]:

:   1733496581, 173349658197113, 173349658197113129145161177, 173349658197113129145161177193209 (...)

The sequence of primes for [a, b] = [19, 30]:

:   1949, 194979109139169199229259289319 (...)

The sequence of primes for [a, b] = [23, 30]:

:   23, 235383113143173203233263293323, 235383113143173203233263293323353383 (...)

The sequence of primes for [a, b] = [61, 30]:

:   61, 6191121151181, 6191121151181211241271301331 (...)

The sequence of primes for [a, b] = [1729, 30]:

:   17291759, 1729175917891819184918791909 (...)

The sequence of primes for [a, b] = [1729, 60]:

:   1729178918491909 (...)

The sequence of primes for [a, b] = [1729, 90]:

:   17291819, 1729181919091999 (...)

The sequence of primes for [a, b] = [341, 90]:

: 341431521611, 341431521611701 (...)

The sequence of primes for [a, b] = [341, 340]:

: 341681 (...)

The sequence of primes for [a, b] = [561, 560]:

: 5611121 (...)

The sequence of primes for [a, b] = [1729, 1728]:

: 17293457 (...)

Note that from a Poulet number P (see above 341, 561, 1729) is obtained often a prime Q when is concatenated with 2*P – 1 (it is also the case for P = 1387 (Q = 13872773), P = 3277 (Q = 32776553) etc.