The Concept of Complex Neutrosophic Hesitant Fuzzy Graph is Used to Solve a Problem Related to Cellular Network

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***Abstract*—In applied sciences, obtaining the shortest distance between the networks gives solutions to various significant problems. Therefore, the objective of the present article is to combine the two valuable and influential theories of the graph and the complex neutrosophic hesitant fuzzy set. The proposed theory is named a Complex neutrosophic hesitant fuzzy graph which has further been explained with the help of various algebraic operations and properties. Later, an application in cellular networks has also been presented, proving the proposed theory's applicability.**

***Index Terms*—Complex neutrosophic hesitant fuzzy set, Complex neutrosophic graph, Algebraic operations and Cellular network.**

I. Introduction

Fuzzy sets [1] have proven to be a powerful mathematical tool for dealing with uncertainties and imprecise data in various domains. By allowing the representation of partial truths and degrees of membership, fuzzy sets enable more robust decision-making processes, enhanced control systems, and improved image-processing techniques. As technology and research progress, fuzzy sets will continue to play a significant role in tackling real-world complexities and uncertainties, making our systems and applications more efficient and adaptive. Later, in literature, various researchers worked in the area which resulted in the introduction of numerous new and useful theories like the intuitionistic fuzzy set (IFS) by Atanassov [2] added the non-membership function to the Zadeh theory. Similarly, the neutrality membership function has been introduced as an independent function by Smarandache [3] and named a neutrosophic set. In the neutrosophic set theory, three independent functions (truth, falsity and neutrality) have been introduced which is a generalization of many theories such as classical, fuzzy, intuitionistic and many more. The three independent functions of neutrosophy made it easy to apply the concept to practical life problems related to science and engineering. These all sets are advanced extensions of fuzzy sets on the real plane but an important aspect of phase is ignored in all these kinds of extensions.

This aspect has been first noticed by Ramot [4] and the author extended the theory of uncertainty to the complex plane of the unit disk named the theory ad Complex fuzzy set. While a regular fuzzy set represents uncertainty using a single membership function, the proposed theory employs multiple membership functions to capture diverse aspects of uncertainty simultaneously. Similarly, several researchers have worked to extend multiple theories (IFS, NS and so on) [5], [6] have been extended to the complex plane. This property of mentioned theories made it easy for the researchers to apply them to problems with real-life applications.

The graphical representation makes it easier to present the pictorial representation of the problem for a better understanding of the issue. Therefore, graph theory proves to be very efficient in solving problems based on real-life issues. In graph theory, the representation of crisp set theory is mainly represented by two vertices because of its property of dealing with the relationship of either 0 or 1. Therefore, it has been very difficult to give a graphical representation of the case of uncertainties. Thus, Rosenfeld presented the theory of fuzzy graphs [8] to increase the graphical representation of imprecise pieces of information. Later, numerous theories of Intuitionistic fuzzy [7], hesitant [9] and neutrosophic [10] graphs have been presented which widen the concept to solve a large number of problems. The concept of a Complex neutrosophic graph and the complex hesitant fuzzy graph has been contributed to the literature by Samrandache and AbuHijleh with some basic information related to preliminaries and multiplication properties.

A flowchart related to the proposed work and the advancement in the described field has been presented to depict the detailed idea behind the presented research gap. The flow chart has been presented in Fig.1. shows the advancements in the field with years and authors respectively. This also proves that researchers have shown keen interest in complex graph theory due to its ability on solving problems based on real-life applications.

The present work aims to extend the concept of graph theory from the real plane to the complex plane to add the component of phase or time term which has been ignored by various authors in the literature. The present works combine the two

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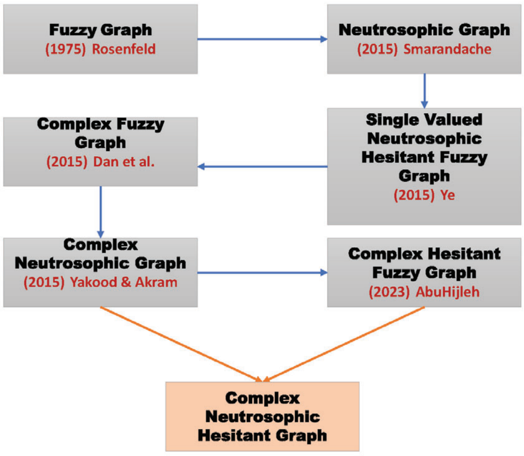

Fig. 1. Flowchart of extensions of the fuzzy graph.

important theories of hesitant and neutrosophic graphs. This combination leads to the introduction of the Complex neutrosophic hesitant graph (Cnhf-Graph) theory. The importance of the presented concept of Cnif-Graph has been listed below:

1) The proposed concept has the ability of both neutrosophic hesitant fuzzy graphs on the complex plane.
2) The phase component adds the benefits of tracking the cycle or pattern of the problem.
3) The concept of Cnhf-Graph has been reduced to Cnhf-Graph when the component of the non-membership function is taken to zero.
4) The Cnhf-Graph has been reduced to a single-valued Cnhf-Graph when the single value for all three independent functions has been considered.

Later, an application based on the network has also been presented which validated the proposed theory of solving problems to prove the ability of the proposed work in solving problems.

The present work is organized as follows: Section 2, consists of the preliminaries related to the proposed concept. In Section 3, the Cnhf-Graph has been proposed with some basic properties to increase the understanding of the concept. The application based on networks has been described in detail in Section 4. Finally, the work has been concluded in Section 5.

II. Preliminaries

In the following section, we have described the basic definitions and theories for the proposed work.

Definition 1: (Fuzzy Graph) [8]"A graph is an ordered pair $\mathbb{G} = (V, E)$, where the edges and vertices of the graph have been denoted by $E \,\&\, V$ respectively."

Definition 2: (Hesitant fuzzy graph) [9] 'Consider $\mathbb{G} = (V, E, \sigma, \mu)$ be an hesitant fuzzy graph, with $\sigma : V$ß$S_f[0,1]$, and $\mu : E$ß$S_f[0,1]$, where the finite sets in the interval $[0,1]$ are denoted by S_f, σ and μ are the membership functions of vertex and edge sets of the hesitant fuzzy graph, respectively."

Definition 3: (Single valued Neutrosophic Graph)"Consider a graph $\mathbb{G} = (V_{\mathbb{G}}, E_{\mathbb{G}})$ with vertex and edges be $V_{\mathbb{G}}$and $E\mathbb{G}$ respectively. Then, the single-valued neutrosophic graph of $\mathbb{G}$ is denoted by $\mathbb{G} = (V_{\mathbb{G}}, \hat{\sigma}, \hat{\mu})$, where set and relations have been denoted by $\hat{\sigma} = (T_{\hat{\sigma}}, I_{\hat{\sigma}}, F_{\hat{\sigma}})$ is an SVNS on $V_{\mathbb{G}}$ is a single-valued neutrosophic symmetric relation on $E_{\mathbb{G}} \subseteq V_{\mathbb{G}} \times V_{\mathbb{G}}\{T_{\hat{\sigma}} : V_{\mathbb{G}} \to [0,1], I_{\hat{\sigma}} : V_{\mathbb{G}} \to [0,1], F_{\hat{\sigma}} : V_{\mathbb{G}} \to [0,1]$, $T_{\hat{\mu}} : V_{\mathbb{G}} \times V_{\mathbb{G}} \to [0,1], I_{\hat{\mu}} : V_{\mathbb{G}} \times V_{\mathbb{G}} \to [0,1]$, $F_{\hat{\mu}} : V_{\mathbb{G}} \times V_{\mathbb{G}} \to [0,1]\}$, and defined in detail as follows:

- $T_{\mu}(x, y) \leq T_{\sigma}(x) \wedge T_{\sigma}(y), \forall (x, y) \in V_{\mathbb{G}} \times V_{\mathbb{G}}$;
- $I_{\mu}(x, y) \leq I_{\sigma}(x) \wedge I_{\sigma}(y), \forall (x, y) \in V_{\mathbb{G}} \times V_{\mathbb{G}}$;
- $F_{\mu}(x, y) \leq F_{\sigma}(x) \vee F_{\sigma}(y), \forall (x, y) \in V_{\mathbb{G}} \times V_{\mathbb{G}}$."

Definition 4: (Complex Fuzzy Graph) [11] Consider a complex fuzzy graph $\mathbb{G} = (V,\, E)$ where $V,\, E$ denotes the complex fuzzy set on $\mathbb{A}$ and the relation on set is given by$\mathbb{B} \subseteq \mathbb{A} \times \mathbb{A}$ such that

$$\mu_E(xy)e^{ia_E(xy)} \leq min\{\mu_V(x), \mu_E(y)\}e^{imin\{a_V(x), a_V(y)\}}$$

$\mu_V(x) \in [0,1]$, and $a_V(x) \in [0, 2\pi] \forall x, y \in \mathbb{A}$.

Definition 5: (Complex Neutrosophic Graph) [12]"Consider a complex neutrosophic graph $\mathbb{G} = (V, E)$ on $X \neq \phi$, where Set is denoted by V and Relation of the set is denoted by E such that

$$t_E(xy)e^{ia_E(xy)} \leq min\{t_V(x), t_V(y)\}e^{imin\{a_V(x), a_V(y)\}},$$

$$I_E(xy)e^{ib_E(xy)} \leq min\{I_V(x), I_V(y)\}e^{imin\{b_V(x), b_V(y)\}},$$

$$F_E(xy)e^{ic_E(xy)} \leq min\{F_V(x), F_V(y)\}e^{imin\{c_V(x), c_V(y)\}},$$

$\forall x, y \in X$. V and E are called the complex neutrosophic vertex set and the complex neutrosophic edge set of $\mathbb{G}$, respectively."

Definition 6: (Complex Hesitant Graph) [13]"Consider a complex hesitant graph $\mathbb{G} = (V, E, \mu, \nu)$ with mapping $\mu : V \to S_g\{\zeta \in \mathbb{C} : |\zeta| \leq 1\}$ and $\nu : E \to S_g\{\zeta \in \mathbb{C} : |\zeta| \leq 1\}$, where $S_g$denotes the subset of all finite sets on complex plane of unit disc. The vertex and edges sets have been represented by μ and ν respectively."

III. Complex Neutrosophic Hesitant Fuzzy Graph (Cnhf-Graph)

In this section, we have proposed the theory of a Complex Neutrosophic Hesitant Fuzzy Graph (Cnhf-Graph) with some basic properties to increase the clarity of the concept.

Definition 7: (Cnhf-graph) A complex neutrosophic hesitant fuzzy graph can be defined as $\mathbb{G} = (V,\ E,\ \mu, \nu)$ with $\mu : V \to S_f\{\delta \in \mathbb{C} \mid |\delta| \leq 1\}\ \&\ \nu : E \to S_f\{\delta \in \mathbb{C} \mid |\delta| \leq 1\}$, where $S_f\{\delta \in \mathbb{C} \mid |\delta| \leq 1\}\ where\ \delta = (T_f,\ I_f,\ F_f)$ is a cluster of all subsets of the unit disc on a complex plane. The Cnhf-Graph set on V and relation $E = V \times V$ is denoted by μ and ν such that

$$T_E(x_\nu, y_\nu) \leq \min\left[T_f(x_\mu),\ T_f(y_\mu)\right].$$

Or in expanded form

$$r_E(x_\nu, y_\nu)\, e^{iQ_E(x_\nu, y_\nu)} \leq \min\{r_f(x_\mu), r_f(y_\mu)\}e^{i\ \min\{Q_f(x_\mu), Q_f(y_\mu)\}}.$$

$$\Rightarrow T_E\left(x_\nu, y_\nu\right) = r_E^T\left(x_\nu, y_\nu\right) e^{iQ_E^T\left(x_\nu, y_\nu\right)},$$

$$\Rightarrow T_f\left(x_\mu\right) = r_f^T\left(x_\mu\right) e^{iQ_f^T\left(x_\mu\right)}$$

and

$$\Rightarrow T_f\left(y_\mu\right) = r_f^T\left(y_\mu\right) e^{iQ_f^T\left(y_\mu\right)}.$$

Similarly, for Indeterminacy and Falsity membership functions.

$$I_E\left(x_\nu, y_\nu\right) \leq \max\left[I_f\left(x_\mu\right),\ I_f\left(y_\mu\right)\ \right];$$

$$F_E\left(x_\nu, y_\nu\right) \leq \max\left[T_f\left(x_\mu\right),\ T_f\left(y_\mu\right)\ \right];$$

such that $0 \leq |T_E| + |I_E| + |F_E| \leq 1$.
Again, both functions are complex-valued and can be written in the same manner (Truth membership function).

Definition 8: Consider Cnhf-Graph $\mathbb{G} = (V,\ E,\ \mu, \nu)$, then the degree of the vertex v_i The cnhf graph has been given below:

$$\deg\left(v_i\right) = \underset{e_{i,j} \in E}{\oplus} \nu\left(e_{i,j}\right).$$

$$= \begin{pmatrix} \left(\bigcup_{j_1 \neq j_2} \left(r_{i,j_1}^T \oplus r_{i,j_2}^T\right)\right) \exp\left(2\pi i \bigcup_{j_1 \neq j_2} \left(r_{i,j_1}^T \oplus r_{i,j_2}^T\right)\right), \\ \left(\bigcup_{j_1 \neq j_2} \left(r_{i,j_1}^I \oplus r_{i,j_2}^I\right)\right) \exp\left(2\pi i \bigcup_{j_1 \neq j_2} \left(r_{i,j_1}^I \oplus r_{i,j_2}^I\right)\right), \\ \left(\bigcup_{j_1 \neq j_2} \left(r_{i,j_1}^F \oplus r_{i,j_2}^F\right)\right) \exp\left(2\pi i \bigcup_{j_1 \neq j_2} \left(r_{i,j_1}^F \oplus r_{i,j_2}^F\right)\right) \end{pmatrix}.$$

Example 3.1: Consider a Cnhf graph $\mathbb{G}$ such that $V = \{v_1, v_2,\ v_3\}$, where

$$\mu\left(v_1\right) = \left(\left(\{.1, .2\}\, e^{\{.3,.2\}i2\pi}\right),\ \left(\{.5, .1\}\, e^{\{.1,.4\}i2\pi}\right), \left(\{.7, .3\}\, e^{\{.5,.4\}i2\pi}\right)\ \right),$$

$$\mu\left(v_2\right) = \left(\left(\{.1, .6\}\, e^{\{.3,.8\}i2\pi}\right),\ \left(\{.3, .7\}\, e^{\{.3,.5\}i2\pi}\right), \left(\{.5, .1\}\, e^{\{.2,.5\}i2\pi}\right)\ \right),$$

$$\mu\left(v_3\right) = \left(\left(\{.8, .4\}\, e^{\{.1,.8\}i2\pi}\right),\ \left(\{.3, .5\}\, e^{\{.3,.2\}i2\pi}\right), \left(\{.3, .4\}\, e^{\{.2,.3\}i2\pi}\right)\ \right)$$

and the edges of the vertices are

$$\nu\left(v_1, v_2\right) = \left(\left(\{.2, .4\}\, e^{\{.1,.8\}i2\pi}\right),\ \left(\{.1, .5\}\, e^{\{.3,.5\}i2\pi}\right), \left(\{.6, .1\}\, e^{\{.2,.4\}i2\pi}\right)\ \right),$$

$$\nu\left(v_1, v_3\right) = \left(\left(\{.5, .1\}\, e^{\{.4,.3\}i2\pi}\right),\ \left(\{.6, .5\}\, e^{\{.1,.2\}i2\pi}\right), \left(\{.5, .1\}\, e^{\{.3,.1\}i2\pi}\right)\ \right).$$

Then, the degree of vertex v_1 is

$$\deg\left(v_1\right) = \begin{pmatrix} \left(\{.7, .3, .9, .5\}\, e^{\{.5,.4,1.2,.1.1\}i2\pi}\right),\ \left(\{.7, .6, 1.1, 1\}\, e^{\{.4,.5,.6,.7\}i2\pi}\right), \\ \left(\{1.1, .7, .6, .2\}\, e^{\{.5,.3,.7\}i2\pi}\right) \end{pmatrix}.$$

A. Cartesian Product of two Cnhf Graph

In the current subsection, we have described the Cartesian product of two Cnhf-Graph in detail and an example proving the same has also been presented.

Definition 9: Consider $\mathbb{A} = (V_\mathbb{A}, E_\mathbb{A}, \mu, \nu)$ and $\mathbb{B} = (V_\mathbb{B}, E_\mathbb{B}, \mu, \nu)$ which is defined by $\mathbb{G} = \mathbb{A} \times \mathbb{B}$

- $$\left(\mu_\mathbb{A} \times \mu_\mathbb{B}\right)(x, y) = \begin{cases} \mu_\mathbb{A}^T(x) \wedge \mu_\mathbb{B}^T(y) \\ \mu_\mathbb{A}^I(x) \vee \mu_\mathbb{B}^I(y) \\ \mu_\mathbb{A}^F(x) \vee \mu_\mathbb{B}^F(y) \end{cases};$$

- For truth membership function,

$$\left(\nu_\mathbb{A}^T \times \nu_\mathbb{B}^T\right)\left(\left(x_1, y_1\right),\ \left(x_2, y_2\right)\right)$$

$$= \begin{cases} \mu_\mathbb{A}^T\left(x_1\right) \wedge \nu_\mathbb{B}^T\left(y_1, y_2\right) : x_1 = x_2, \left(y_1, y_2\right) \in E_\mathbb{B}, \\ \nu_\mathbb{A}^T\left(x_1, x_2\right) \wedge \mu_\mathbb{B}^T\left(y_1\right) : y_1 = y_2, \left(x_1, x_2\right) \in E_\mathbb{A}. \end{cases}$$

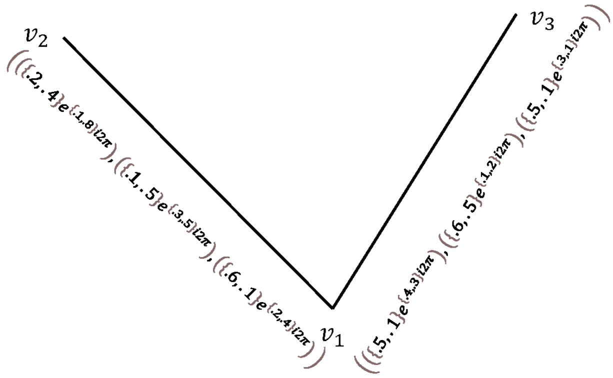

Fig. 2. Cnhf Graph for the presented example .

- For indeterminacy membership function,

$$\left(\nu_\mathbb{A}^I \times \nu_\mathbb{B}^I\right)\left(\left(x_1, y_1\right),\ \left(x_2, y_2\right)\right)$$

$$= \begin{cases} \mu_\mathbb{A}^I\left(x_1\right) \vee \nu_\mathbb{B}^I\left(y_1, y_2\right) : x_1 = x_2, \left(y_1, y_2\right) \in E_\mathbb{B}, \\ \nu_\mathbb{A}^I\left(x_1, x_2\right) \vee \mu_\mathbb{B}^I\left(y_1\right) : y_1 = y_2, \left(x_1, x_2\right) \in E_\mathbb{A}. \end{cases}$$

- For falsity membership function,

$$\left(\nu_\mathbb{A}^F \times \nu_\mathbb{B}^F\right)\left(\left(x_1, y_1\right),\ \left(x_2, y_2\right)\right)$$

$$= \begin{cases} \mu_\mathbb{A}^F\left(x_1\right) \vee \nu_\mathbb{B}^F\left(y_1, y_2\right) : x_1 = x_2, \left(y_1, y_2\right) \in E_\mathbb{B}, \\ \nu_\mathbb{A}^F\left(x_1, x_2\right) \vee \mu_\mathbb{B}^F\left(y_1\right) : y_1 = y_2, \left(x_1, x_2\right) \in E_\mathbb{A}. \end{cases}$$

Applying the above definition 9 to solve the following example.

Example 3.2: Consider $\mathbb{G} = \mathbb{A} \times \mathbb{B}$ be the Cartesian product of $\mathbb{A}$ and $\mathbb{B}$ and the figures of the three have been given by 2, 3 and 4 respectively. The score values for the edges and vertices have also been calculated for the labelling of the graph $\mathbb{G}$.

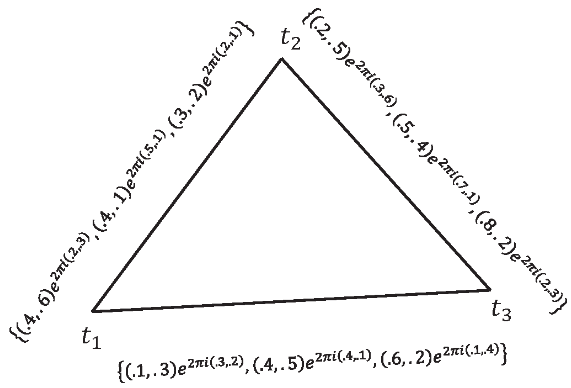

Fig. 3. Cnhf graph for the presented example.

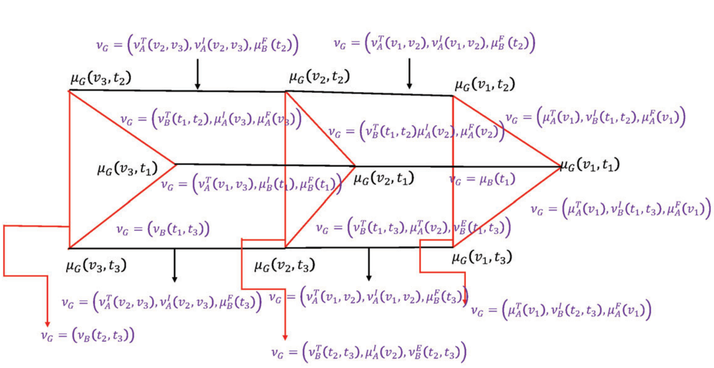

Fig. 4. Complex hesitant fuzzy graph $\mathbb{G}$ of Fig 2 and 3 .

The detailed information in Figure 4 has been presented below: The vertices of the Graph $\mathbb{G}$ in figure (4) have been given below:

$$\mu_{\mathbb{G}}(v_1,t_1) = \left(\mu_{\mathbb{B}}^T(t_1), \mu_{\mathbb{B}}^I(t_1), \mu_{\mathbb{B}}^F(t_1)\right);$$

$$\mu_{\mathbb{G}}(v_1,t_2) = \left(\mu_{\mathbb{A}}^T(v_1), \mu_{\mathbb{B}}^I(t_2), \mu_{\mathbb{B}}^F(t_2)\right);$$

$$\mu_{\mathbb{G}}(v_1,t_3) = \left(\mu_{\mathbb{A}}^T(v_1), \mu_{\mathbb{B}}^I(t_3), \mu_{\mathbb{A}}^F(v_1)\right);$$

$$\mu_{\mathbb{G}}(v_2,t_1) = (\mu_{\mathbb{B}}(t_1));$$

$$\mu_{\mathbb{G}}(v_2,t_2) = \left(\mu_{\mathbb{A}}^T(v_2), \mu_{\mathbb{A}}^I(v_2), \mu_{\mathbb{B}}^F(t_2)\right);$$

$$\mu_{\mathbb{G}}(v_2,t_3) = \left(\mu_{\mathbb{A}}^T(v_2), \mu_{\mathbb{A}}^I(v_2), \mu_{\mathbb{B}}^F(t_3)\right);$$

$$\mu_{\mathbb{G}}(v_3,t_1) = (\mu_{\mathbb{B}}(t_1));$$

$$\mu_{\mathbb{G}}(v_3,t_2) = \left(\mu_{\mathbb{A}}^T(v_3), \mu_{\mathbb{B}}^I(t_2), \mu_{\mathbb{B}}^F(t_2)\right);$$

$$\mu_{\mathbb{G}}(v_3,t_3) = (\mu_{\mathbb{B}}(t_3)).$$

The calculation behind the above values has been described below:

$$\mu_{\mathbb{G}}(v_1,t_1) = \begin{cases} \mu_{\mathbb{A}}^T(v_1) \wedge \mu_{\mathbb{B}}^T(t_1); \\ \mu_{\mathbb{A}}^I(v_1) \vee \mu_{\mathbb{B}}^I(t_1); \\ \mu_{\mathbb{A}}^F(v_1) \vee \mu_{\mathbb{B}}^F(t_1). \end{cases}$$

$$\mu_{\mathbb{G}}(v_1,t_1) = \left(\mu_{\mathbb{B}}^T(t_1), \mu_{\mathbb{B}}^I(t_1), \mu_{\mathbb{A}}^F(v_1)\right).$$

$$\nu_{\mathbb{G}}((v_1,t_1),(v_1,t_2)) = \begin{cases} \mu_{\mathbb{A}}^T(v_1) \wedge \nu_{\mathbb{B}}^T(t_1,t_2); \\ \mu_{\mathbb{A}}^I(v_1) \vee \mu_{\mathbb{B}}^I(t_1,t_2); \\ \mu_{\mathbb{A}}^F(v_1) \vee \mu_{\mathbb{B}}^F(t_1,t_2). \end{cases}$$

$$\mu_{\mathbb{G}}(v_1,t_1) = \left(\mu_{\mathbb{A}}^T(v_1), \nu_{\mathbb{B}}^I(t_1,t_2), \mu_{\mathbb{A}}^F(v_1)\right).$$

Theorem 1: Consider the Cartesian product of $\mathbb{A}$ and $\mathbb{B}$ graph has been denoted by $\mathbb{G} = \mathbb{A} \times \mathbb{B}$ where $\mathbb{A} = (V_{\mathbb{A}}, E_{\mathbb{A}}, \mu_{\mathbb{A}}, \nu_{\mathbb{A}}), \mathbb{B} = (V_{\mathbb{B}}, E_{\mathbb{B}}, \mu_{\mathbb{B}}, \nu_{\mathbb{B}})$ and $\mathbb{G} = \mathbb{A} \times \mathbb{B} = (V \times E, \mu_{\mathbb{A}} \times \mu_{\mathbb{B}}, \nu_{\mathbb{A}} \times \nu_{\mathbb{B}})$ such that $\max(\mu_{\mathbb{A}}) \leq \min(\nu_{\mathbb{B}}) \,\&\, \max(\nu_{\mathbb{A}}) \leq \min(\mu_{\mathbb{B}})$, then, $deg_{\mathbb{G}}(v_1,t_1) = deg_{\mathbb{A}}(v_1) \oplus deg_{\mathbb{B}}(t_1)$.
Proof: Now, the theorem will be proved for the truth membership function and in a similar manner, the indeterminacy and falsity membership function can be proved.

$$deg_{\mathbb{G}}^T(v_1,t_1) = \bigoplus_{((v_1,t_1),(v_2,t_2)) \in E} \left(\nu_{\mathbb{A}}^T \times \nu_{\mathbb{B}}^T\right)((v_1,t_1),(v_2,t_2))$$
$$= \bigoplus_{\{v_1 = v_2, (t_1,t_2) \in E} \mu_{\mathbb{A}}^T(v_1) \wedge \nu_{\mathbb{B}}^T(t_1,t_2)\} \oplus \bigoplus_{\{t_1 = t_2, (v_1,v_2) \in E} \nu_{\mathbb{A}}^T(v_1,v_2) \wedge \nu_{\mathbb{B}}^T(t_1)\},$$

Above equations verifies the definitions 8 and 9.
According to the theorem, consider $\max\{\mu_{\mathbb{A}}^T\} \leq \min\{\nu_{\mathbb{B}}^T\}$ or $\max\{\mu_{\mathbb{B}}^T\} \leq \min\{\nu_{\mathbb{A}}^T\}$, then

$$deg_{\mathbb{G}}^T(v_1,t_1) = \bigoplus_{\{v_1 = v_2, (t_1,t_2) \in E} \nu_{\mathbb{B}}^T(t_1,t_2)\} \oplus \bigoplus_{\{t_1 = t_2, (v_1,v_2) \in E} \nu_{\mathbb{A}}^T(v_1,v_2)\},$$

$$\implies deg_{\mathbb{G}}^T(v_1,t_1) = deg_{\mathbb{B}}^T(t_1) \oplus deg_{\mathbb{A}}^T(v_1).$$

Similarly, it can be proved for the falsity and indeterminacy functions. Hence, the theorem has been successfully satisfied.

IV. Application in Case of Cellular Networks

In this section applications in the case of cellular companies have been stated in detail and later, the solution of the given problem has been successfully obtained using the proposed theory.

Let us assume a cellular company need to solve an issue related to the signals by planning to fix towers at various places in the city, but this must satisfy the following conditions for maximum benefit.

- Places with a maximum range of signals and users
- Minimum requirement of transportation and distance with the main server for better connectivity
- Information of already existing tower and the area (hilly or plain)
- Minimum economical investment

Let us consider the team of five selectors who will take care of the above-instructed points and select five locations $\mathbb{V}((V) = \{S_1, S_2, S_3, S_4, S_5\})$ for fixing the tower. In other words, these will be considered as the set of vertices. The amplitude and the phase term of the vertex have been selected according to the belief of selectors for the location $S_1, 50\%$ of the selectors give a positive response and 20% gives a negative review and 30% selectors have mixed review. Similarly, the phase term has been selected by collecting the information on the period of a maximum number of signals again by experts' choice. Like, for vertex S_1, 30% of the experts have favourable responses whereas $40\%\&30\%$ have negative and mixed responses. Then, the information collected through the responses has been presented in the form of $< S_1 : 0.5e^{i}0.3\pi, 0.2e^{i}0.4\pi, 0.3e^{i}0.3\pi >$ for location S_1.
Similarly, the rest of the locations have been modelled below:

$$\mathbb{A} = \begin{cases} < S_1 : 0.5e^{i0.3\pi}, 0.2e^{i0.4\pi}, 0.3e^{i0.3}\pi >, \\ < S_2 : 0.6e^{i0.1\pi}, 0.1e^{i0.5\pi}, 0.1e^{i0.2\pi} >, \\ < S_3 : 0.4e^{i0.2\pi}, 0.6e^{i0.4\pi}, 0.0 >, \\ < S_4 : 0.6e^{i0.4\pi}, 0.3e^{i0.2\pi}, 0.3e^{i0.1\pi} >, \\ < S_5 : 0.7e^{i0.5\pi}, 0.1e^{i0.4\pi}, 0.2e^{i0.7\pi} >. \end{cases}$$

Next, the absolute values of the above-mentioned information have been obtained

$$|S_1| = (0.5, 0.2, 0.3),$$
$$|S_2| = (0.6, 0.1., 0.1),$$
$$|S_3| = (0.4, 0.6, 0.0),$$
$$|S_4| = (0.6, 0.3, 0.1),$$
$$|S_5| = (0.7, 0.1, 0.2).$$

To find the optimum result, the score values of all locations have been calculated.

$$s(S_1) = 0.5 - 0.2 - 0.3 = 0,$$
$$s(S_2) = 0.6 - 0.1 - 0.1 = 0.4,$$

$$s(S_3) = 0.4 - 0.6 = -0.2,$$

$$s(S_4) = 0.6 - 0.3 - 0.1 = 0.2,$$

$$s(S_5) = 0.7 - 0.1 - 0.2 = 0.4.$$

This implies that S_5 & S_2 are the most suitable places for placing a tower, the maximum favourable among the two places have been selected with the help of accuracy both places $A(S_2) = 0.6 + 0.1 = 0.7 \leq A(S_5) = 0.8$ and this also forms a graph [5] with no edges.

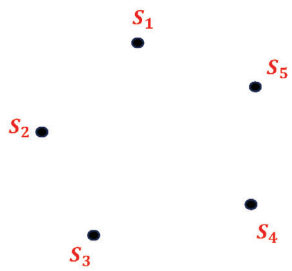

Fig. 5. Cnhf graph with no edges.

In the second situation, the tower can be placed between the locations S_2 & S_5, then it will represent the edge S_2S_5. Therefore, all the edges of the graph will be obtained by using the definition[7]. That is

$$\mathbb{B} = \left\{ \begin{array}{l} < S_1S_2 : 0.5e^{i0.1\pi}, 0.2e^{i0.5\pi}, 0.3e^{i0.3}\pi >, \\ < S_1S_3 : 0.4e^{i0.2\pi}, 0.6e^{i0.4\pi}, 0.3e^{i0.3\pi} >, \\ < S_1S_4 : 0.5e^{i0.3\pi}, 0.3e^{i0.4\pi}, 0.3e^{i0.3\pi} >, \\ < S_1S_5 : 0.5e^{i0.3\pi}, 0.2e^{i0.4\pi}, 0.3e^{i0.3\pi} >, \\ < S_2S_3 : 0.4e^{i0.1\pi}, 0.6e^{i0.5\pi}, 0.1e^{i0.2\pi} >, \\ < S_2S_4 : 0.6e^{i0.1\pi}, 0.6e^{i0.5\pi}, 0.1e^{i0.2}\pi >, \\ < S_2S_5 : 0.6e^{i0.1\pi}, 0.1e^{i0.5\pi}, 0.3e^{i0.7\pi} >, \\ < S_3S_4 : 0.4e^{i0.2\pi}, 0.6e^{i0.4\pi}, 0.3e^{i0.1\pi} >, \\ < S_3S_5 : 0.4e^{i0.2\pi}, 0.6e^{i0.4\pi}, 0.2e^{i0.7\pi} >, \\ < S_4S_5 : 0.6e^{i0.4\pi}, 0.3e^{i0.4\pi}, 0.3e^{i0.7\pi} >, \end{array} \right.$$

Again, the absolute values for the edges have to be obtained and given below:

$$|S_1S_2| = 0.5 - 0.2 - 0.3 = 0$$

$$|S_1S_3| = 0.4 - 0.6 - 0.3 = -0.5$$

$$|S_1S_4| = 0.5 - 0.3 - 0.3 = -0.1$$

$$|S_1S_5| = 0.5 - 0.2 - 0.3 = 0$$

$$|S_2S_3| = 0.4 - 0.6 - 0.1 = -0.3$$

$$|S_2S_4| = 0.6 - 0.6 - 0.1 = -0.1$$

$$|S_2S_5| = 0.6 - 0.1 - 0.3 = 0.3$$

$$|S_3S_4 = 0.4 - 0.6 - 0.3 = -0.5$$

$$S_3S_5 = 0.4 - 0.6 - 0.2 = -0.4$$

$$S_4S_5 = 0.6 - 0.3 - 0.3 = 0$$

Hence, the maximum absolute value is of edge $S_2S_5 = 0.3$. Therefore, the tower can be placed on the S_2S_5 edge of the graph for maximum benefit. In this case, we have a connected graph and its figure is given below:

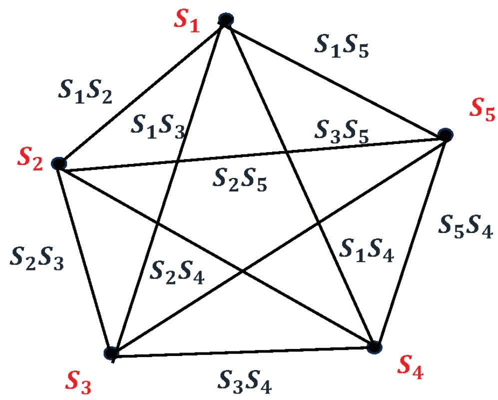

Fig. 6. Cnhf graph with edges.

V. Comparison Tabel of CNHF-Graph

Theories	HFG	CNG
Interval	$[0, 1]$	Unit disk in complex plane
Uncertainty Factor Measurement	✓	✓
Phase Term	×	✓
Amplitude Term	×	✓
Advantage	Finite value sets have been considered on the real plane	The independent uncertain membership functions have been considered
Theories	CHG	CNHFG
Uncertainty Factor Measurement	✓	✓
Interval	Unit disk in complex plane	Unit disk in complex plane
Phase Term	✓	✓
Amplitude Term	✓	✓
Advantage	Finite favourable sets have been considered	Finite favourable sets with three independent unceratin membership functions have been considered.

VI. Conclusion

In the presented work, the novel concept of a Complex neutrosophic hesitant graph has been defined in detail. The score and divergence values of the vertices and edges compare the labelling process and this also helps to provide a new kind of pictorial representation of complex neutrosophic hesitant graphs. To increase the basic understanding of the proposed concept, the Cartesian product of the graphs has been explained with the help of an example and a theorem. Later, an application related to the problem of fixing the cellular tower to improve the signal transfer in the city has also been presented, which validates the theory.

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