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Article · August 2021
DOI: 10.3390/math9172047

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Commutative Ideals of BCI-Algebras Using MBJ-Neutrosophic Structures

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Abstract: As a generalization of a neutrosophic set, the notion of MBJ-neutrosophic sets is introduced by Mohseni Takallo, Borzooei and Jun, and it is applied to BCK/BCI-algebras. In this article, MBJ-neutrosophic set is used to study commutative ideal in BCI-algebras. The concept of closed MBJ-neutrosophic ideal and commutative MBJ-neutrosophic ideal is introduced and their properties and relationships are studied. The conditions for an MBJ-neutrosophic ideal to be a commutative MBJ-neutrosophic ideal are given. The conditions for an MBJ-neutrosophic ideal to be a closed MBJ-neutrosophic ideal are provided. Characterization of a commutative MBJ-neutrosophic ideal is established. Finally, the extension property for a commutative MBJ-neutrosophic ideal is founded.

Keywords: MBJ-neutrosophic set; MBJ-neutrosophic subalgebra; MBJ-neutrosophic ideal; commutative MBJ-neutrosophic ideal

MSC: 06F35; 03G25; 08A72

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Citation: Song, S.-Z.; Öztürk, M.A.; Jun, Y.-B. Commutative Ideals of BCI-Algebras Using MBJ-Neutrosophic Structures. *Mathematics* **2021**, *9*, 2047. https://doi.org/10.3390/math9172047

Academic Editor: Jan L. Cieśliński

Received: 29 June 2021
Accepted: 24 August 2021
Published: 25 August 2021

1. Introduction

Various types of uncertainty arise in many complex systems and/or real-world situations such as behavior, biology, chemistry, etc. The fuzzy set introduced by L.A. Zade [1] in 1965 is a useful tool for dealing with uncertainties in many of these real-world applications. One of the extended concepts of the fuzzy set, the intuitionistic fuzzy set was introduced by Atanassov in 1983 (see [2]), and it has been applied in several fields. Intuitionistic fuzzy set is very useful in providing a flexible model to elaborate uncertainty and vagueness involved in decision making, and it is a tool in modelling real life problems like sale analysis, new product marketing, financial services, negotiation process, psychological investigations etc. The concept of neutrosophic set has been introduced by Smarandache [3–5] and it is a generalization of classic set, (inconsistent) intuitionistic fuzzy set, interval valued (intuitionistic) fuzzy set, picture fuzzy set, ternary fuzzy set, Pythagorean fuzzy set, q-rung orthopair fuzzy set, spherical fuzzy set, and n-hyperspherical fuzzy set. Neutrosophic set is able to handle inconsistency, indeterminacy, and uncertainty for reasoning and computing. Therefore, we can see that neutrosophic set is widely applied to a variety of areas. It can be said that the generalization of the theory shows that the scope of application will be greatly expanded. In [6], the notion of MBJ-neutrosophic sets has been introduced as a little extended concept of neutrosophic set and it has been applied to BCK/BCI-algebras. Jun et al. [7] and Hur et al. [8] applied the concept of MBJ-neutrosophic sets to ideals and positive implicative ideals in BCK/BCI-algebras, respectively.

The purpose of this paper is to study commutative ideal in BCI-algebra using the MBJ-neutrosophic set. We introduce the notion of closed MBJ-neutrosophic ideal and commutative MBJ-neutrosophic ideal, and investigate their properties. We discuss the next items.

1. Using commutative ideal to set up commutative MBJ-neutrosophic ideal
2. Investigating the relationship between MBJ-neutrosophic ideal and commutative MBJ-neutrosophic ideal.
3. Presenting the conditions under which a commutative MBJ-neutrosophic ideal can be made from an MBJ-neutrosophic ideal.
4. Presenting a condition for an MBJ-neutrosophic set to be a closed MBJ-neutrosophic ideal.
5. Establishing characterization of a commutative MBJ-neutrosophic ideal (using the MBJ-level sets of an MBJ-neutrosophic set).
6. Constructing the extension property for a commutative MBJ-neutrosophic ideal.

In the second section, we list the well-known foundations for BCK-algebra and MBJ-neutrosophic set required in this paper. Commutative ideal in BCI-algebra using MBJ-neutrosophic set will be studied in the third section.

2. Preliminaries

2.1. Default Background for BCI-Algebras

In mathematics, BCK and BCI-algebras are algebraic structures in universal algebra, which were introduced by Y. Imai, K. Iséki and S. Tanaka in 1966, that describe fragments of the propositional calculus involving implication known as BCK and BCI logics.

By a *BCI-algebra* (It is a generalization of a BCK-algebra.),we mean a set X with a binary operation $*$ and a special element 0 that satisfies the following conditions:

(I_1) $((\tilde{x} * \tilde{y}) * (\tilde{x} * \tilde{z})) * (\tilde{z} * \tilde{y}) = 0$,
(I_2) $(\tilde{x} * (\tilde{x} * \tilde{y})) * \tilde{y} = 0$,
(I_3) $\tilde{x} * \tilde{x} = 0$,
(I_4) $\tilde{x} * \tilde{y} = 0, \tilde{y} * \tilde{x} = 0 \Rightarrow \tilde{x} = \tilde{y}$,

for all $\tilde{x}, \tilde{y}, \tilde{z} \in X$.

Every BCI-algebra X satisfies the following conditions:

$$(\forall \tilde{x} \in X)(\tilde{x} * 0 = \tilde{x}), \tag{1}$$
$$(\forall \tilde{x}, \tilde{y}, \tilde{z} \in X)(\tilde{x} \leq \tilde{y} \Rightarrow \tilde{x} * \tilde{z} \leq \tilde{y} * \tilde{z}, \tilde{z} * \tilde{y} \leq \tilde{z} * \tilde{x}), \tag{2}$$
$$(\forall \tilde{x}, \tilde{y}, \tilde{z} \in X)((\tilde{x} * \tilde{y}) * \tilde{z} = (\tilde{x} * \tilde{z}) * \tilde{y}), \tag{3}$$
$$(\forall \tilde{x}, \tilde{y}, \tilde{z} \in X)((\tilde{x} * \tilde{z}) * (\tilde{y} * \tilde{z}) \leq \tilde{x} * \tilde{y}), \tag{4}$$
$$(\forall \tilde{x}, \tilde{y} \in X)(\tilde{x} * (\tilde{x} * (\tilde{x} * \tilde{y})) = \tilde{x} * \tilde{y}), \tag{5}$$
$$(\forall \tilde{x}, \tilde{y} \in X)(0 * (\tilde{x} * \tilde{y}) = (0 * \tilde{x}) * (0 * \tilde{y})). \tag{6}$$

where $\tilde{x} \leq \tilde{y}$ if and only if $\tilde{x} * \tilde{y} = 0$.

A BCI-algebra X is said to be *commutative* (see [9]) if it satisfies:

$$(\forall \tilde{x}, \tilde{y} \in X)(\tilde{x} \leq \tilde{y} \Rightarrow \tilde{x} = \tilde{y} * (\tilde{y} * \tilde{x})). \tag{7}$$

A nonempty subset S of a BCI-algebra X is called a *subalgebra* of X if $\tilde{x} * \tilde{y} \in S$ for all $\tilde{x}, \tilde{y} \in S$. A subset I of a BCI-algebra X is called an *ideal* of X if it satisfies:

$$0 \in I, \tag{8}$$
$$(\forall \tilde{x} \in X)(\forall \tilde{y} \in I)(\tilde{x} * \tilde{y} \in I \Rightarrow \tilde{x} \in I). \tag{9}$$

A subset I of a BCI-algebra X is called a *commutative ideal* of X (see [10]) if it satisfies (8) and

$$(\tilde{x} * \tilde{y}) * \tilde{z} \in I, \tilde{z} \in I \Rightarrow \tilde{x} * ((\tilde{y} * (\tilde{y} * \tilde{x})) * (0 * (0 * (\tilde{x} * \tilde{y})))) \in I \tag{10}$$

for all $\tilde{x}, \tilde{y}, \tilde{z} \in X$.

For more information on BCI-algebra and BCK-algebra, please refer to the books [11,12].

2.2. Default Background for MBJ-Neutrosophic Sets

Let X be a non-empty set. We consider three mappings $T_C : X \to [0,1]$, $I_C : X \to [0,1]$ and $F_C : X \to [0,1]$ which are called truth membership function, indeterminate membership function and false membership function, respectively. Then a *neutrosophic set* (NS) in X is defined to be a structure (see [4])

$$\mathcal{C} := \{\langle X; T_C(\tilde{x}), I_C(\tilde{x}), F_C(\tilde{x})\rangle \mid \tilde{x} \in X\}. \tag{11}$$

Let X be a non-empty set. By an *MBJ-neutrosophic set* in X (see [6]), we mean a structure of the form:

$$\mathcal{C} := \{\langle X; M_C(\tilde{x}), \tilde{B}_C(\tilde{x}), J_C(\tilde{x})\rangle \mid \tilde{x} \in X\}$$

where M_C and J_C are fuzzy sets in X, which are called a truth membership function and a false membership function, respectively, and $\tilde{B}_C$ is an interval-valued fuzzy set in X which is called an indeterminate interval-valued membership function.

For the sake of simplicity, we shall use the symbol $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ for the MBJ-neutrosophic set

$$\mathcal{C} := \{\langle X; M_C(\tilde{x}), \tilde{B}_C(\tilde{x}), J_C(\tilde{x})\rangle \mid \tilde{x} \in X\}.$$

The MBJ-neutrosophic set $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ in X can be represented as follows:

$$\begin{aligned} \mathcal{C} := (M_C, \tilde{B}_C, J_C) : X \to [0,1] \times [[0,1]] \times [0,1], \\ x \mapsto \big(M_C(x), \tilde{B}_C(x), J_C(x)\big) \end{aligned} \tag{12}$$

where $\tilde{B}_C(x) = [\tilde{B}_C^-(x), \tilde{B}_C^+(x)]$.

Given an MBJ-neutrosophic set $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ in X, we consider the following sets:

$$\Omega(\mathcal{C}) := \left\{ (\tilde{x}, \tilde{y}) \;\middle|\; \begin{array}{l} M_C(\tilde{x}) \geq M_C(\tilde{y}) \\ \tilde{B}_C(\tilde{x}) \succeq \tilde{B}_C(\tilde{y}) \\ J_C(\tilde{x}) \leq J_C(\tilde{y}) \end{array} \right\} \tag{13}$$

$$\mathcal{C}^{\min}_{\max} := \left\{ \tfrac{\tilde{x}}{\{\tilde{y},\tilde{z}\}} \;\middle|\; \begin{array}{l} M_C(\tilde{x}) \geq \min\{M_C(\tilde{y}), M_C(\tilde{z})\} \\ J_C(\tilde{x}) \leq \max\{J_C(\tilde{y}), J_C(\tilde{z})\} \end{array} \right\} \tag{14}$$

and consider the MBJ-level sets as follows:

$$\begin{aligned} &U(M_C; s) := \{\tilde{x} \in X \mid M_C(\tilde{x}) \geq s\}, \\ &U(\tilde{B}_C; [\gamma_1, \gamma_2]) := \{\tilde{x} \in X \mid \tilde{B}_C(\tilde{x}) \succeq [\gamma_1, \gamma_2]\}, \\ &L(J_C; t) := \{\tilde{x} \in X \mid J_C(\tilde{x}) \leq t\} \end{aligned}$$

where $(s, t) \in [0,1] \times [0,1]$ and $[\gamma_1, \gamma_2] \in [[0,1]]$.

Let X be a BCI-algebra. An MBJ-neutrosophic set $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ in X is called

- an *MBJ-neutrosophic subalgebra* of X (see [7]) if it satisfies:

$$(\forall \tilde{x}, \tilde{y} \in X)\Big(\tfrac{x*y}{\{x,y\}} \in \mathcal{C}^{\min}_{\max}\Big), \tag{15}$$

$$(\forall \tilde{x}, \tilde{y} \in X)\big(\tilde{B}_C(\tilde{x} * \tilde{y}) \succeq \mathrm{rmin}\{\tilde{B}_C(\tilde{x}), \tilde{B}_C(\tilde{y})\}\big). \tag{16}$$

- an *MBJ-neutrosophic ideal* of X (see [7]) if it satisfies:

$$(\forall \tilde{x} \in X)((0, \tilde{x}) \in \Omega(\mathcal{C})), \tag{17}$$

$$(\forall \tilde{x}, \tilde{y} \in X)\left(\frac{\tilde{x}}{\{\tilde{x} * \tilde{y}, \tilde{y}\}} \in \mathcal{C}_{\max}^{\min}\right), \tag{18}$$

$$(\forall \tilde{x}, \tilde{y} \in X)\left(\tilde{B}_C(\tilde{x}) \succeq \text{rmin}\{\tilde{B}_C(\tilde{x} * \tilde{y}), \tilde{B}_C(\tilde{y})\}\right). \tag{19}$$

3. Commutative MBJ-Neutrosophic Ideals of BCI-Algebras

In this section, let X denote a BCI-algebra unless otherwise specified.

Definition 1. *An MBJ-neutrosophic set $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ in X is called a commutative MBJ-neutrosophic ideal (briefly, cMBJ-neutrosophic ideal) of X if it satisfies* (17) *and*

$$\frac{x * ((y * (y * x)) * (0 * (0 * (x * y))))}{\{(x * y) * z, z\}} \in \mathcal{C}_{\max}^{\min}, \tag{20}$$

$$\tilde{B}_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \succeq \text{rmin}\{\tilde{B}_C((x * y) * z), \tilde{B}_C(z)\} \tag{21}$$

for all $x, y, z \in X$.

Example 1. *Consider a BCI-algebra $X = \{0, 1, a, b, c\}$ with the binary operation $*$ which is given in Table 1 (see [11]).*

Table 1. Cayley table for the binary operation "$*$".

$*$	**0**	**1**	***a***	***b***	***c***
0	0	0	c	b	a
1	1	0	c	b	a
a	a	a	0	c	b
b	b	b	a	0	c
c	c	c	b	a	0

Let $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ be an MBJ-neutrosophic set in X defined by Table 2.

Table 2. MBJ-neutrosophic set $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$.

X	$M_C(x)$	$\tilde{B}_C(x)$	$J_C(x)$
0	0.67	$[0.23, 0.73]$	0.12
1	0.45	$[0.35, 0.71]$	0.23
a	0.26	$[0.38, 0.65]$	0.54
b	0.26	$[0.38, 0.65]$	0.54
c	0.26	$[0.38, 0.65]$	0.54

It is routine to verify that $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is a cMBJ-neutrosophic ideal of X.

Proposition 1. *Every cMBJ-neutrosophic ideal $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ of X satisfies:*

$$(x * ((y * (y * x)) * (0 * (0 * (x * y)))), x * y) \in \Omega(\mathcal{C}) \tag{22}$$

for all $x, y \in X$.

Proof. The result (22) is obtained using (1) and (17) after choosing $z = 0$ in (20) and (21). □

Using a commutative ideal, we establish a cMBJ-neutrosophic ideal.

Theorem 1. *Given a commutative ideal I of X, consider an MBJ-neutrosophic set $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ in X as follows:*

$$\mathcal{C} := (M_C, \tilde{B}_C, J_C) : X \to [0,1] \times [[0,1]] \times [0,1],$$
$$x \mapsto \begin{cases} (t_1, \alpha, s_1) & \text{if } x \in I \\ (t_2, \beta, s_2) & \text{if } x \notin I \end{cases}$$

where $t_1 \geq t_2$, $s_1 \leq s_2$ in $[0,1]$ and $\alpha \succeq \beta$ in $[[0,1]]$. Then $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is a cMBJ-neutrosophic ideal of X.

Proof. It is clear that $(0, x) \in \Omega(\mathcal{C})$ for all $x \in X$. Let $x, y, z \in X$. If $(x * y) * z \in I$ and $z \in I$, then $x * ((y * (y * x)) * (0 * (0 * (x * y)))) \in I$ since I is a commutative ideal of X. Hence

$$M_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) = t_1 = \min\{M_C((x * y) * z), M_C(z)\},$$
$$J_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) = s_1 = \min\{J_C((x * y) * z), J_C(z)\},$$

that is, $\frac{x*((y*(y*x))*(0*(0*(x*y))))}{\{(x*y)*z, z\}} \in \mathcal{C}^{\min}_{\max}$, and

$$\tilde{B}_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) = \alpha = \text{rmin}\{\tilde{B}_C((x * y) * z), \tilde{B}_C(z)\}$$

Assume that $(x * y) * z \notin I$ or $z \notin I$. Then $\mathcal{C}((x * y) * z) = (t_2, \beta, s_2)$ or $\mathcal{C}(z) = (t_2, \beta, s_2)$. It follows that $\frac{x*((y*(y*x))*(0*(0*(x*y))))}{\{(x*y)*z, z\}} \in \mathcal{C}^{\min}_{\max}$ and

$$\tilde{B}_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \succeq \text{rmin}\{\tilde{B}_C((x * y) * z), \tilde{B}_C(z)\}.$$

Therefore $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is a cMBJ-neutrosophic ideal of X. □

We discuss the relationship between a cMBJ-neutrosophic ideal and an MBJ-neutrosophic ideal.

Theorem 2. *Every cMBJ-neutrosophic ideal is an MBJ-neutrosophic ideal.*

Proof. Let $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ be a cMBJ-neutrosophic ideal of X. If we take $y = 0$ in (20) and (21) and use (1), then

$$\frac{x}{\{x*z, z\}} = \frac{x*0}{\{(x*0)*z, z\}} = \frac{x*((0*(0*x))*(0*(0*(x*0))))}{\{(x*0)*z, z\}} \in \mathcal{C}^{\min}_{\max}$$

and

$$\begin{aligned} \tilde{B}_C(x) &= \tilde{B}_C(x * 0) \\ &= \tilde{B}_C(x * ((0 * (0 * x)) * (0 * (0 * (x * 0))))) \\ &\succeq \text{rmin}\{\tilde{B}_C((x * 0) * z), \tilde{B}_C(z)\} \\ &= \text{rmin}\{\tilde{B}_C((x * z), \tilde{B}_C(z)\} \end{aligned}$$

for all $x, z \in X$. Therefore $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is an MBJ-neutrosophic ideal of X. □

The next example shows that the converse of Theorem 2 is not true.

Example 2. *Consider a BCI-algebra $X = \{0, 1, a, b, c\}$ with the binary operation $*$ which is given in Table 3 (see [12]).*

Table 3. Cayley table for the binary operation "$*$".

$*$	**0**	**1**	***a***	***b***	***c***
0	0	0	0	0	0
1	1	0	1	0	0
a	a	a	0	0	0
b	b	b	b	0	0
c	c	c	c	b	0

Let $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *be an MBJ-neutrosophic set in X defined by Table 4.*

Table 4. MBJ-neutrosophic set $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$.

X	$M_C(x)$	$\tilde{B}_C(x)$	$J_C(x)$
0	0.67	$[0.18, 0.77]$	0.15
1	0.52	$[0.25, 0.61]$	0.27
a	0.36	$[0.48, 0.55]$	0.64
b	0.36	$[0.48, 0.55]$	0.64
c	0.36	$[0.48, 0.55]$	0.64

It is routine to verify that $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *is an MBJ-neutrosophic ideal of X. We can observe that*

$$(a * ((b * (b * a)) * (0 * (0 * (a * b)))), a * b) = (a, 0) \notin \Omega(\mathcal{C}).$$

Hence $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *is not a cMBJ-neutrosophic ideal of X by Proposition 1.*

We find and present the conditions under which a cMBJ-neutrosophic ideal can be made from an MBJ-neutrosophic ideal.

Theorem 3. *Given an MBJ-neutrosophic set* $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *in X, the next assertions are equivalent.*

(i) $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *is a cMBJ-neutrosophic ideal of X.*
(ii) $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *is an MBJ-neutrosophic ideal of X that satisfies (22).*

Proof. The necessity is evident by Proposition 1 and Theorem 2. Let $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ be an MBJ-neutrosophic ideal of X that satisfies (22). Then $\frac{x*y}{\{(x*y)*z, z\}} \in \mathcal{C}^{\min}_{\max}$ and

$$\tilde{B}_C(x * y) \succeq \text{rmin}\{\tilde{B}_C((x * y) * z), \tilde{B}_C(z)\}.$$

It follows from (22) that $\frac{x*((y*(y*x))*(0*(0*(x*y))))}{\{(x*y)*z, z\}} \in \mathcal{C}^{\min}_{\max}$ and

$$\tilde{B}_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \succeq \text{rmin}\{\tilde{B}_C((x * y) * z), \tilde{B}_C(z)\}.$$

Therefore $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is a cMBJ-neutrosophic ideal of X. □

Given an MBJ-neutrosophic set $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ in X, we consider the next assertion.

$$(\forall x \in X)((0 * x, x) \in \Omega(\mathcal{C})). \tag{23}$$

In the following example, we know that there exists an MBJ-neutrosophic ideal $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ of X which does not satisfy the condition (23).

Example 3. *Consider the BCI-algebra* $(\mathbb{Z}, -, 0)$ *where* $\mathbb{Z}$ *is the set of integers and "−" is the minus operation in* $\mathbb{Z}$*. Let* $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *be an MBJ-neutrosophic set in* $\mathbb{Z}$ *defined by*

$$\mathcal{C} := (M_C, \tilde{B}_C, J_C) : \mathbb{Z} \to [0,1] \times [[0,1]] \times [0,1],$$
$$x \mapsto \begin{cases} (t, \alpha, s) & \text{if } x \in \mathbb{N} \\ (0, \beta, 1) & \text{otherwise} \end{cases}$$

where $\mathbb{N}$ *is the set of natural numbers,* $(t, s) \in (0, 1] \times [0, 1)$ *and* α *is the proper superset of* β *in* $[[0, 1]]$*. Then* $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *is an MBJ-neutrosophic ideal of* $\mathbb{Z}$*, but it does not satisfy the condition* (23) *since* $(0 - 3, 3) = (-3, 3) \notin \Omega(\mathcal{C})$*.*

Definition 2 ([7])**.** *An MBJ-neutrosophic ideal* $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *of X is said to be closed if it satisfies* (23)*.*

Example 4. *Let* $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *be an MBJ-neutrosophic set in X defined by*

$$\mathcal{C} := (M_C, \tilde{B}_C, J_C) : X \to [0,1] \times [[0,1]] \times [0,1],$$
$$x \mapsto \begin{cases} (t, \alpha, s) & \text{if } x \in X_+ \\ (0, \beta, 1) & \text{if } x \notin X_+ \end{cases}$$

where $X_+ := \{x \in X \mid 0 \leq x\}$*,* $(t, s) \in (0, 1] \times [0, 1)$ *and* α *is the proper superset of* β *in* $[[0, 1]]$*. It is routine to verify that* $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ *is a closed MBJ-neutrosophic ideal of X.*

We provide a condition for an MBJ-neutrosophic set to be a closed MBJ-neutrosophic ideal.

Theorem 4. *Given an element* $a \in X$*, let* $\mathcal{C}^a := (M_C^a, \tilde{B}_C^a, J_C^a)$ *be an MBJ-neutrosophic set in X defined by*

$$\mathcal{C}^a := (M_C^a, \tilde{B}_C^a, J_C^a) : X \to [0,1] \times [[0,1]] \times [0,1],$$
$$y \mapsto \begin{cases} (t, \alpha, s) & \text{if } y \in I_a := \{x \in X \mid a * x = a\} \\ (0, \beta, 1) & \text{otherwise} \end{cases}$$

where $(t, s) \in (0, 1] \times [0, 1)$ *and* α *is the proper superset of* β *in* $[[0, 1]]$*. Then* $\mathcal{C}^a := (M_C^a, \tilde{B}_C^a, J_C^a)$ *is a closed MBJ-neutrosophic ideal of X.*

Proof. Since $0 \in I_a$, it is clear that $(0, x) \in \Omega(\mathcal{C}^a)$ for all $x \in X$. For any $x \in X$, if $x \notin I_a$, then $\mathcal{C}^a(x) = (0, \beta, 1)$ and so $(0 * x, x) \in \Omega(\mathcal{C}^a)$. If $x \in I_a$, then $a * x = a$ and thus

$$0 * x = (a * a) * x = (a * x) * a = a * a = 0 \in I_a. \tag{24}$$

Hence $(0 * x, x) \in \Omega(\mathcal{C}^a)$ for all $x \in X$, and therefore $\mathcal{C}^a := (M_C^a, \tilde{B}_C^a, J_C^a)$ satisfies the condition (23). Let $x, y \in X$. Assume that $x \in I_a$ and $y * x \in I_a$. Then

$$(a * y) * a = (a * a) * y = 0 * y = (0 * y) * (0 * x) = 0 * (y * x) = 0$$

by (I_3), (1), (3), (6) and (24). On the other hand, we have

$$a = a * (y * x) = (a * x) * (y * x) \leq a * y$$

by (I_1) and (3). Hence $a * y = a$, that is, $y \in I_a$. Therefore

$$M_C^a(y) = \min\{M_C^a(y * x), M_C^a(x)\},$$

$$\tilde{B}_C^a(y) = \text{rmin}\{\tilde{B}_C^a(y * x), \tilde{B}_C^a(x)\}$$

and $J_C^a(y) = \max\{J_C^a(y * x), J_C^a(x)\}$. If $x \notin I_a$ or $y * x \notin I_a$, then $\mathcal{C}^a(x) = (0, \beta, 1)$ or $\mathcal{C}^a(y * x) = (0, \beta, 1)$. Therefore $\frac{y}{\{y*x, x\}} \in \mathcal{C}^{a\min}_{\max}$ and

$$\tilde{B}_C^a(y) \succeq \text{rmin}\{\tilde{B}_C^a(y * x), \tilde{B}_C^a(x)\}.$$

As a result, $\mathcal{C}^a := (M_C^a, \tilde{B}_C^a, J_C^a)$ is a closed MBJ-neutrosophic ideal of X. □

Lemma 1 ([7]). *Every MBJ-neutrosophic ideal $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ of X satisfies:*

$$(\forall x, y, z \in X)\left(x * y \le z \;\Rightarrow \left\{ \begin{array}{l} \frac{x}{\{y,z\}} \in \mathcal{C}^{\min}_{\max} \\ \tilde{B}_C(x) \succeq \text{rmin}\{\tilde{B}_C(y), \tilde{B}_C(z)\} \end{array} \right. \right). \tag{25}$$

Theorem 5. *A closed MBJ-neutrosophic ideal $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ of X is commutative if and only if it satisfies:*

$$(\forall x, y \in X)((x * (y * (y * x)), x * y) \in \Omega(\mathcal{C})). \tag{26}$$

Proof. Assume that a closed MBJ-neutrosophic ideal $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ of X is commutative, and let $x, y, z \in X$. Note that

$$\begin{aligned} &(x * (y * (y * x))) * (x * ((y * (y * x)) * (0 * (0 * (x * y))))) \\ &\le ((y * (y * x)) * (0 * (0 * (x * y)))) * (y * (y * x)) \\ &= ((y * (y * x)) * (y * (y * x))) * (0 * (0 * (x * y))) \\ &= 0 * (0 * (0 * (x * y))) \\ &= 0 * (x * y). \end{aligned}$$

It follows from Proposition 1 and Lemma 1 that $\frac{x*(y*(y*x))}{\{x*y, 0*(x*y)\}} \in \mathcal{C}^{\min}_{\max}$ and

$$\tilde{B}_C(x * (y * (y * x))) \succeq \text{rmin}\{\tilde{B}_C(x * y), \tilde{B}_C(0 * (x * y))\}$$

Since $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is closed, the combination these with (23) induce

$$(x * (y * (y * x)), x * y) \in \Omega(\mathcal{C}).$$

Conversely, suppose that a closed MBJ-neutrosophic ideal $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ of X satisfies the condition (26). For every $x, y \in X$, we have

$$\begin{aligned} &(x * ((y * (y * x)) * (0 * (0 * (x * y))))) * (x * (y * (y * x))) \\ &\le (y * (y * x)) * ((y * (y * x)) * (0 * (0 * (x * y)))) \\ &\le 0 * (0 * (x * y)). \end{aligned}$$

It follows from Lemma 1 and (26) that $\frac{x*((y*(y*x))*(0*(0*(x*y))))}{\{x*y, 0*(0*(x*y))\}} \in \mathcal{C}^{\min}_{\max}$ and

$$\tilde{B}_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \succeq \text{rmin}\{\tilde{B}_C(x * y), \tilde{B}_C(0 * (0 * (x * y)))\}.$$

Since $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is closed, the combination these with (23) induce

$$(x * ((y * (y * x)) * (0 * (0 * (x * y)))), x * y) \in \Omega(\mathcal{C}).$$

Therefore $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is commutative by Theorem 3. □

Lemma 2 ([9]). *A BCI-algebra X is commutative if and only if it satisfies:*

$$(\forall x, y \in X)(x * (x * y) = y * (y * (x * (x * y)))). \tag{27}$$

Theorem 6. *In a commutative BCI-algebra, every closed MBJ-neutrosophic ideal is commutative.*

Proof. Let $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ be a closed MBJ-neutrosophic ideal of X. For every $x, y \in X$, we have

$$\begin{aligned} &(x*(y*(y*x)))*(x*y) = (x*(x*y))*(y*(y*x)) \\ &= (y*(y*(x*(x*y))))*(y*(y*x)) \\ &= (y*(y*(y*x)))*(y*(x*(x*y))) \\ &= (y*x)*(y*(x*(x*y))) \\ &\leq (x*(x*y))*x \\ &= (x*x)*(x*y) = 0*(x*y) \end{aligned}$$

by (I_1), (I_2), (3), (5) and Lemma 2. It follows from Lemma 1 and (23) that $(x*(y*(y*x)), x*y) \in \Omega(\mathcal{C})$. Consequently, $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is a cMBJ-neutrosophic ideal of X by Theorem 5. □

We form the characterization of a cMBJ-neutrosophic ideal using the MBJ-level sets of an MBJ-neutrosophic set $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ in X.

Lemma 3 ([7])**.** *An MBJ-neutrosophic set $\mathcal{C} = (M_C, \tilde{B}_C, J_C)$ in X is an MBJ-neutrosophic ideal of X if and only if the non-empty MBJ-level sets of $\mathcal{C} = (M_C, \tilde{B}_C, J_C)$ are ideals of X.*

Lemma 4 ([10])**.** *A subset I of X is a commutative ideal of X if and only if it is an ideal of X that satisfies:*

$$x*y \in I \;\Rightarrow\; x*((y*(y*x))*(0*(0*(x*y)))) \in I \tag{28}$$

for all $x, y \in X$.

Theorem 7. *An MBJ-neutrosophic set $\mathcal{C} = (M_C, \tilde{B}_C, J_C)$ in X is a cMBJ-neutrosophic ideal of X if and only if the non-empty MBJ-level sets of $\mathcal{C} = (M_C, \tilde{B}_C, J_C)$ are commutative ideals of X.*

Proof. Assume that $\mathcal{C} = (M_C, \tilde{B}_C, J_C)$ is a cMBJ-neutrosophic ideal of X. Then it is an MBJ-neutrosophic ideal of X. Let $(s,t) \in [0,1] \times [0,1]$ and $[\gamma_1, \gamma_2] \in [[0,1]]$ be such that $U(M_C; s)$, $U(\tilde{B}_C; [\gamma_1, \gamma_2])$ and $L(J_C; t)$ are non-empty. Then $U(M_C; s)$, $U(\tilde{B}_C; [\gamma_1, \gamma_2])$ and $L(J_C; t)$ are ideals of X for all $(s,t) \in [0,1] \times [0,1]$ and $[\gamma_1, \gamma_2] \in [[0,1]]$ by Lemma 3. Let $x, y, a, b, u, v \in X$ be such that $x*y \in U(M_C; s)$, $a*b \in U(\tilde{B}_C; [\gamma_1, \gamma_2])$ and $u*v \in L(J_C; t)$. Then $M_C(x*y) \geq s$, $\tilde{B}_C(a*b) \succeq [\gamma_1, \gamma_2]$ and $J_C(u*v) \leq t$. It follows from Proposition 1 that

$$M_C(x*((y*(y*x))*(0*(0*(x*y))))) \geq M_C(x*y) \geq s,$$

$$\tilde{B}_C(a*((b*(b*a))*(0*(0*(a*b))))) \succeq \tilde{B}_C(a*b) \succeq [\gamma_1, \gamma_2]$$

and $J_C(u*((v*(v*u))*(0*(0*(u*v))))) \leq J_C(u*v) \leq t$. It follows that

$$x*((y*(y*x))*(0*(0*(x*y)))) \in U(M_C; s),$$

$$a*((b*(b*a))*(0*(0*(a*b)))) \in U(\tilde{B}_C; [\gamma_1, \gamma_2])$$

and $u*((v*(v*u))*(0*(0*(u*v)))) \in L(J_C; t)$. Therefore $U(M_C; s)$, $U(\tilde{B}_C; [\gamma_1, \gamma_2])$ and $L(J_C; t)$ are commutative ideals of X by Lemma 4.

Conversely, suppose that the non-empty MBJ-level sets $U(M_C; s)$, $U(\tilde{B}_C; [\gamma_1, \gamma_2])$ and $L(J_C; t)$ are commutative ideals of X for all $(s,t) \in [0,1] \times [0,1]$ and $[\gamma_1, \gamma_2] \in [[0,1]]$.

Then they are ideals of X, and so $\mathcal{C} = (M_C, \tilde{B}_C, J_C)$ is an MBJ-neutrosophic ideal of X by Lemma 3. Assume that

$$(x * ((y * (y * x)) * (0 * (0 * (x * y)))), x * y) \notin \Omega(\mathcal{C})$$

for some $x, y \in X$. Then

$$M_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \ngeq M_C(x * y),$$

$$\tilde{B}_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \nsucceq \tilde{B}_C(x * y)$$

or $J_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \nleq J_C(x * y)$. If

$$M_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \ngeq M_C(x * y),$$

then $x * y \in U(M_C; s)$ and $x * ((y * (y * x)) * (0 * (0 * (x * y)))) \notin U(M_C; s)$ for $s = M_C(x * y)$. This is a contradiction. If

$$\tilde{B}_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \nsucceq \tilde{B}_C(x * y),$$

then $x * y \in U(\tilde{B}_C; [\gamma_1, \gamma_2])$ and $x * ((y * (y * x)) * (0 * (0 * (x * y)))) \notin U(\tilde{B}_C; [\gamma_1, \gamma_2])$ for $[\gamma_1, \gamma_2] = \tilde{B}_C(x * y)$, which is impossible. If

$$J_C(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \nleq J_C(x * y),$$

then $x * y \in L(J_C; t)$ and $x * ((y * (y * x)) * (0 * (0 * (x * y)))) \notin L(J_C; t)$ for some $t = J_C(x * y)$. This is also a contradiction. As a result, we know that

$$(x * ((y * (y * x)) * (0 * (0 * (x * y)))), x * y) \in \Omega(\mathcal{C})$$

for all $x, y \in X$. Therefore $\mathcal{C} = (M_C, \tilde{B}_C, J_C)$ is a cMBJ-neutrosophic ideal of X by Theorem 3. □

Note that any MBJ-neutrosophic ideal might not be a cMBJ-neutrosophic ideal (see Example 2). But we have the following extension property for a cMBJ-neutrosophic ideal.

Theorem 8. *Let $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ and $\mathcal{D} := (M_D, \tilde{B}_D, J_D)$ be MBJ-neutrosophic ideals of X such that*

(i) $M_C(0) = M_D(0)$, $\tilde{B}_C(0) = \tilde{B}_D(0)$ *and* $J_C(0) = J_D(0)$.
(ii) $M_C(x) \leq M_D(x)$, $\tilde{B}_C(x) \preceq \tilde{B}_D(x)$ *and* $J_C(x) \geq J_D(x)$ *for all* $x \in X$.

If $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is a cMBJ-neutrosophic ideal and $\mathcal{D} := (M_D, \tilde{B}_D, J_D)$ is a closed MBJ-neutrosophic ideal of X, then $\mathcal{D} := (M_D, \tilde{B}_D, J_D)$ is a cMBJ-neutrosophic ideal of X.

Proof. Assume that $\mathcal{C} := (M_C, \tilde{B}_C, J_C)$ is a cMBJ-neutrosophic ideal and $\mathcal{D} := (M_D, \tilde{B}_D, J_D)$ is a closed MBJ-neutrosophic ideal of X. Then

$$(\forall x, y \in X)((0 * (x * y), x * y) \in \Omega(\mathcal{D})) \tag{29}$$

by (23). Using (I_3), (1), (3), Proposition 1 and the given conditions, we have

$$\begin{aligned}
& M_D((x * (y * (y * (x * (x * y))))) * (x * y)) \\
&= M_D((x * (x * y)) * (y * (y * (x * (x * y))))) \\
&= M_D((x * (x * y)) * ((y * (y * (x * (x * y)))) * 0)) \\
&= M_D((x * (x * y)) * ((y * (y * (x * (x * y)))) * (0 * (0 * ((x * (x * y)) * y))))) \\
&\geq M_C((x * (x * y)) * ((y * (y * (x * (x * y)))) * (0 * (0 * ((x * (x * y)) * y))))) \\
&\geq M_C((x * (x * y)) * y) = M_C((x * y) * (x * y)) = M_C(0) = M_D(0),
\end{aligned}$$

$$\begin{aligned}
&\tilde{B}_D((x*(y*(y*(x*(x*y)))))*(x*y)) \\
&= \tilde{B}_D((x*(x*y))*(y*(y*(x*(x*y))))) \\
&= \tilde{B}_D((x*(x*y))*((y*(y*(x*(x*y))))*0)) \\
&= \tilde{B}_D((x*(x*y))*((y*(y*(x*(x*y))))*(0*(0*((x*(x*y))*y))))) \\
&\succeq \tilde{B}_C((x*(x*y))*((y*(y*(x*(x*y))))*(0*(0*((x*(x*y))*y))))) \\
&\succeq \tilde{B}_C((x*(x*y))*y) = \tilde{B}_C((x*y)*(x*y)) = \tilde{B}_C(0) = \tilde{B}_D(0)
\end{aligned}$$

and

$$\begin{aligned}
&J_D((x*(y*(y*(x*(x*y)))))*(x*y)) \\
&= J_D((x*(x*y))*(y*(y*(x*(x*y))))) \\
&= J_D((x*(x*y))*((y*(y*(x*(x*y))))*0)) \\
&= J_D((x*(x*y))*((y*(y*(x*(x*y))))*(0*(0*((x*(x*y))*y))))) \\
&\leq J_C((x*(x*y))*((y*(y*(x*(x*y))))*(0*(0*((x*(x*y))*y))))) \\
&\leq J_C((x*(x*y))*y) = J_C((x*y)*(x*y)) = J_C(0) = J_D(0).
\end{aligned}$$

It follows from (17)–(19) that

$$\begin{aligned}
&M_D(x*(y*(y*(x*(x*y))))) \\
&\geq \min\{M_D((x*(y*(y*(x*(x*y)))))*(x*y)), M_D(x*y)\} \\
&\geq \min\{M_D(0), M_D(x*y)\} = M_D(x*y),
\end{aligned}$$

$$\begin{aligned}
&\tilde{B}_D(x*(y*(y*(x*(x*y))))) \\
&\succeq \operatorname{rmin}\{\tilde{B}_D((x*(y*(y*(x*(x*y)))))*(x*y)), \tilde{B}_D(x*y)\} \\
&\succeq \operatorname{rmin}\{\tilde{B}_D(0), \tilde{B}_D(x*y)\} = \tilde{B}_D(x*y)
\end{aligned}$$

and

$$\begin{aligned}
&J_D(x*(y*(y*(x*(x*y))))) \\
&\leq \max\{J_D((x*(y*(y*(x*(x*y)))))*(x*y)), J_D(x*y)\} \\
&\leq \max\{J_D(0), J_D(x*y)\} = J_D(x*y).
\end{aligned}$$

Since

$$\begin{aligned}
&(x*(y*(y*x)))*(x*(y*(y*(x*(x*y))))) \\
&\leq (y*(y*(x*(x*y))))*(y*(y*x)) \\
&\leq (y*x)*(y*(x*(x*y))) \\
&\leq (x*(x*y))*x = (x*x)*(x*y) = 0*(x*y),
\end{aligned}$$

we have

$$\begin{aligned}
&M_D(x*(y*(y*x))) \\
&\geq \min\{M_D(x*(y*(y*(x*(x*y))))), M_D(0*(x*y))\} \\
&\geq \min\{M_D(x*y), M_D(0*(x*y))\} = M_D(x*y),
\end{aligned}$$

$$\begin{aligned}
&\tilde{B}_D(x*(y*(y*x))) \\
&\succeq \operatorname{rmin}\{\tilde{B}_D(x*(y*(y*(x*(x*y))))), \tilde{B}_D(0*(x*y))\} \\
&\succeq \operatorname{rmin}\{\tilde{B}_D(x*y), \tilde{B}_D(0*(x*y))\} = \tilde{B}_D(x*y)
\end{aligned}$$

and

$$\begin{aligned}&J_D(x*(y*(y*x)))\\&\le \max\{J_D(x*(y*(y*(x*(x*y))))), J_D(0*(x*y))\}\\&\le \max\{J_D(x*y), J_D(0*(x*y))\} = J_D(x*y)\end{aligned}$$

by Lemma 1. This shows that $(x*(y*(y*x)), x*y) \in \Omega(M_D)$. Consequently, $\mathcal{D} := (M_D, \tilde{B}_D, J_D)$ is a cMBJ-neutrosophic ideal of X by Theorem 5. □

4. Conclusions

Neutrosophic set, which is introduced by Smarandache, is a generalization of (inconsistent) intuitionistic fuzzy set, picture fuzzy set, ternary fuzzy set, Pythagorean fuzzy set, q-rung orthopair fuzzy set, spherical fuzzy set, and n-hyperspherical fuzzy set. Neutrosophic set is able to handle inconsistency, indeterminacy, and uncertainty for reasoning and computing. Therefore, we can see that neutrosophic set is widely applied to a variety of areas. The generalization of the theory shows that the scope of application will be greatly expanded. From this point of view, Mohseni Takallo et al. tried to introduce the notion of MBJ-neutrosophic sets as a little extended concept of neutrosophic set. The aim of this manuscript was to conduct a study that applied the MBJ-neutrosophic set to commutative ideal in BCI-algebra. We introduced the notion of closed MBJ-neutrosophic ideal and commutative MBJ-neutrosophic ideal, and investigated their related properties. We used commutative ideal to set up commutative MBJ-neutrosophic ideal, and discussed the relationship between MBJ-neutrosophic ideal and commutative MBJ-neutrosophic ideal. We presented the conditions under which a commutative MBJ-neutrosophic ideal can be made from an MBJ-neutrosophic ideal, and presented a condition for an MBJ-neutrosophic set to be a closed MBJ-neutrosophic ideal. We established characterizations of a commutative MBJ-neutrosophic ideal by using the MBJ-level sets of an MBJ-neutrosophic set, and constructed the extension property for a commutative MBJ-neutrosophic ideal. The ideas and results of this paper are expected to be applicable in related algebraic structures in the future, such as MV-algebra, BL-algebra, EQ-algebra, hoop, equality algebra, etc., so we hope that many mathematicians will proceed with the study and achieve good results.

Author Contributions: Conceptualization, Y.-B.J.; Funding acquisition, S.-Z.S.; Investigation, S.-Z.S., M.A.Ö. and Y.-B.J. All the authors were actively involved in the research and writing of this paper. All the authors have read and approved the final manuscript for submission.

Funding: This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2016R1D1A1B02006812).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors wish to thank the anonymous reviewers for their valuable suggestions.

Conflicts of Interest: The authors declare no conflict of interest.

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