Comments on the paper "An Alternative Combination Rule for Evidential Reasoning" by Sebbak et al., published in Fusion 2014 Conference, Salamanca, Spain, July 2014

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Abstract.
In this note we want to show that the PCR6 fusion rule works and redistributes the conflicting mass properly, contrarily to the authors' assertion that "the focal element $\{v_3\}$ absorbs almost all of the conflicting mass (the majority)." We also question the validity of the new CREC rule of combination presented by the authors.

Keywords: Belief function; PCR6 rule; PCR5 rule; CREC rule.

Introduction.

Recently in [1], Sebbak et al. consider the following *Example 2*, where the frame of discernment is

$\Theta = \{v_1, v_2, v_3\}$, in Shafer's model (i.e. all intersections of the elements in Θ are empty), and three sources of information characterized by the basic belief assignments (BBA) m_1, m_2, and m_3:

	v_1	v_3	Θ
m_1	0.8	0	0.2
m_2	0.1	0.9	0
m_3	0.4	0	0.6

We verified by hand and with MatLab the PCR6 fusion of these BBAs and the authors correctly computed *PCR6* by combining these three sources altogether, and they got the correct result:

$m_{123|PCR6}(v_1) = 0.4340$, $m_{123|PCR6}(v_3) = 0.4437$, and $m_{123|PCR6}(\Theta) = 0.1223$.

The authors conclude that "the focal element $\{v_3\}$ absorbs almost all of the conflicting mass (the majority)" that we prove it is clearly untrue in the next section.

Reestablishing the truth.

In order to prove that the authors' claim "the focal element *{v3}* absorbs almost all of the conflicting mass (the majority)" in [1] is wrong, we detail the calculation of the PCR6 for the three sources. First, we apply the conjunctive rule:

	v_1	v_3	Θ
m_1	0.8	0	0.2
m_2	0.1	0.9	0
m_3	0.4	0	0.6
$m_{123conj}$	0.1000	0.1080	0

Now let's compute the total conflicting mass:
0.8(0.9)0.6 + 0.4(0.9)0.2 + 0.8(0.9)0.4 = 0.7920.

This total conflicting mass, 0.7920, was afterwards redistributed back to the elements involved in the conflict: v_1, v_3, and Θ proportionally respectively with their mass put in each partial conflict. There were three partial conflicts in this example. Hence:

The redistribution by *PCR6* of the conflicting mass for v_1 is passing from
$m_{123}(v_1) = 0.1000$ to $m_{123/PCR6}(v_1) = 0.4340$, or a gain of *0.3340.*
The redistribution by *PCR6* of the conflicting mass for v_3 is passing from
$m_{123}(v_3) = 0.1080$ to $m_{123/PCR6}(v_3) = 0.4437$, or a gain of *0.3357.*
The redistribution by *PCR6* of the conflicting mass for Θ is passing from
$m_{123}(\Theta) = 0.0000$ to $m_{123/PCR6}(\Theta) = 0.1223$, or a gain of *0.1223.*

Therefore v_3 actually gained from the total conflicting mass only *0.3357/0.7920 ≈ 42.3864%*, not even half of it, not "almost all of the conflicting mass (the majority)" as the authors wrongly claim in [1]. v_1 gained close to *v3*: *0.3340/0.7920 ≈ 42.1717%*, and Θ gained the difference (*15.4419%*). In our opinion *PCR6* did a pretty fair redistribution of the total conflicting mass.

Remark: If the authors applied *PCR5* instead of *PCR6*, the total conflicting mass will be redistributed in a bigger percentage to v_3, that's why we propose to use in the applications *PCR6* instead of *PCR5*. With MatLab, we got for *PCR5* the following result:
$m_{123/PCR5}(v_1) ≈ 0.3450$, $m_{123/PCR5}(v_3) ≈ 0.5327$, and $m_{123/PCR5}(\Theta) ≈ 0.1223$.

Total conflict is naturally the same: *0.7920.*

The redistribution by *PCR5* of the conflicting mass for v_1 is passing from
$m_{123}(v_1) = 0.1000$ to $m_{123/PCR5}(v_1) = 0.3450$, or a gain of *0.2450.*
The redistribution by *PCR5* of the conflicting mass for v_3 is passing from

$m_{123}(v_3) = 0.1080$ to $m_{123/PCR5}(v_3) = 0.5327$, or a gain of *0.4247*.
The redistribution by *PCR5* of the conflicting mass for Θ is passing from
$m_{123}(\Theta) = 0.0000$ to $m_{123/PCR5}(\Theta) = 0.1223$, or a gain of *0.1223*.

With *PCR5* one gets *0.4247/0.7920* ≈ *53.62%* of the total conflicting mass redistributed to v_3, but not "almost all conflicting mass" either.

On the CREC rule of combination.

In [1], the authors proposed a new rule of combination called the CREC (Combination Rule of Evidences with Cardinalities), based on the introduction of "hidden conflict" which is mathematically problematic and doesn't make sense from the point of view of the conjunctive operator.

For example, if one considers two BBAs $m_1(.)$ *and* $m_2(.)$ and two elements X and Y of the power-set of a given frame of discernment, with X included in Y, then the product mass $m_1(X)m_2(Y)$ is not entirely transferred back to X with CREC rule (as it should be done in the conjunctive way), but it is split as (see example given in [1] page *4*)

$m_{12}(X) = |X \cap Y|/|Y|m_1(X)m_2(Y)$ (called *effective conjunctive rule*)

and

$\overline{m}_{12}(X) = (1-|X \cap Y|/|Y|)m_1(X)m_2(Y)$ (called *hidden conflict*);

of course

$m_1(X)m_2(Y) = m_{12}(X) + \overline{m}_{12}(X),$

but we disagree that $\overline{m}_{12}(X)$ has something to do with a "hidden conflict", and we question why such decomposition makes sense, or could be useful.

In fact, there are many ways to make decompositions of $m_1(X)m_2(Y)$, as already proposed in Chap. 8 (An In-Depth Look at Quantitative Information Fusion Rules) of [2], Vol. 2, pp. 205-236, or in [2], Vol. 3, Chap. 5, pp. 161-183. These books are freely available respectively on the web at http://fs.gallup.unm.edu/DSmT-book2.pdf, and http://fs.gallup.unm.edu/DSmT-book3.pdf.

For example, instead of using the *Cardinality Atomicity* of a focal element X with respect to another focal element Y in the combination fusion rules, defined by the authors as:

$$ca(X \uparrow Y) = \frac{|X \cap Y|}{|Y|}$$

we believe it would be more interesting of using the *degree of intersection* of the elements X and Y, defined as:

$$d(X \cap Y) = \frac{|X \cap Y|}{|X \cup Y|}$$

or the *degree of union* of the focal elements X and Y, defined as:

$$d(X \cup Y) = \frac{|X \cup Y| - |X \cap Y|}{|X \cup Y|}$$

or the *degree on inclusion* of a focal element X into another focal element Y, defined as:

$$d(X \subseteq Y) = \frac{|X|}{|Y|}.$$

Conclusion.

In this short note, we have reestablished the truth about the real behavior of *PCR6* (and even *PCR5*) for Example 2 given by Sebbak et al., and we point out a problem with the mathematical definition of the "hidden conflict" introduced by the authors, which has clearly nothing to do with conflicting evidences.

References

[1] Faouzi Sebbak, Farid Benhammadi, M'hamed Mataoui, Sofiane Bouznad and Yacine Amirat, *An Alternative Combination Rule for Evidential Reasoning*, Proceedings of the 17th International Conference on Information Fusion, Salamanca, Spain, 7-10 July 2014.

[2] Florentin Smarandache, Jean Dezert, *Advances and Applications of DSmT in Information Fusion*, Am. Res. Press, Rehoboth, Vol. I, II, and III, 2004, 2006, and 2009.

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