



Bellman-Ford Algorithm Under Trapezoidal Interval Valued Neutrosophic Environment

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Abstract. The shortest path problem has been one of the most fundamental practical problems in network analysis. One of the good algorithms is Bellman-Ford, which has been applied in network, since the last some years. By virtue of complexity in the decision making process, the decision makers face complication to express their view and judgment with an exact number for single valued membership degrees under neutrosophic environment. Though the interval number is a special situation of the neutrosophic, it is not solved the shortest path problems in an absolute manner. Hence, in this work, we have proposed the trapezoidal interval valued neutrosophic version of Bellman's algorithm to solve the shortest path problem absolutely.

Keywords: Bellman's algorithm · Trapezoidal interval valued neutrosophic number · Ranking method · Shortest path problem · Network

1 Introduction

A set which is defined in terms of 'affiliation function' is called fuzzy set (FS) in 1965 [1] and mainly deals with which handles with numerous real world situations, where the data possesses some sort of uncertainty. The notion of FS is concern about only the membership value of each and every element not about the non-membership. This issue was sorted out by intuitionistic fuzzy set (IFS) introduced by Atannasov in 1975 [2] which allows the membership function and non-membership function as well. Since the real world situations may contain indeterminacy in the data, FS and IFS could not deal with indeterminacy. This problem was rectified by neutrosophic set (NS), generalization of FS and IFS, introduced by Smarandache [3]. NS is a set in which, all the elements

have degree of membership, indeterminate membership and non-membership and the sum these membership functions (MFs) should be less than or equal to 3. All the three MFs are independent to each other. Uncertainty can be captured for the elements using fuzzy numbers and intuitionistic fuzzy numbers. In the same way, neutrosophic numbers are useful in capturing uncertainty and indeterminacy of the elements. And hence it is a special case of the NS which enhances the domain of real numbers to neutrosophic numbers.

Shortest path problem has been solved by many researchers under fuzzy and intuitionistic fuzzy environments [5–8]. The concept of Bellman's algorithm has been applied in a fuzzy network [9] for solving shortest path problem and it is not applied in neutrosophic network so far. Distance measure can be obtained using single and interval valued trapezoidal neutrosophic numbers in a multi attribute decision making problem. Dijkstra algorithm is very useful and optimized one to solve the shortest path problem but cannot able to handle negative weights where as Bellman can able to deal negative weights. SPP also can be solved by using single valued neutrosophic graph [11–16].

Hence for the first time, the interval valued trapezoidal neutrosophic portrayal of Bellman's algorithm is applied here to solve neutrosophic shortest path problem [17, 18]. An extension of fuzzy sets called type-2 fuzzy sets and its special cases called interval type-2 fuzzy sets have growing applications in control systems, image processing and other medical fields. Shortest path problems can be solved using triangular and trapezoidal interval neutrosophic environments as an extension of neutrosophic sets. From the overview of solving shortest path problem under various sets environments one can understand the difference and capacity of handling uncertainty with various level [19–26].

In this paper, we are motivated to present a new version of Bellman's algorithm for solving the shortest path problem on a network where the edge weight is characterized by interval valued trapezoidal neutrosophic number. The rest of this paper is organized as follows. In Sect. 2, some concepts and theories are reviewed. Section 3 presents the neutrosophic version of Bellman algorithm. In Sect. 4, a numerical example is provided as an application of our proposed algorithm. Section 5, shows the advantages of the proposed algorithm. The last but not least the section, in which the conclusion is drawn and some hints for further research is given.

2 Overview on Trapezoidal Interval Valued Neutrosophic Number

In this section, we review some basic concepts regarding neutrosophic sets, single valued neutrosophic sets, trapezoidal neutrosophic sets and some existing ranking functions for trapezoidal neutrosophic numbers which are the background of this study and will help us to further research.

Definition 2.1 [3]. Let ξ be a of points (objects) set and its generic elements denoted by x ; we define the neutrosophic set A (NS A) as the form $\tilde{A} = \{ \langle x : T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x) \rangle, x \in \xi \}$, where the functions $T, I, F: \xi \rightarrow]0, 1+[$ are called the

truth-membership function, an indeterminacy-membership function, and a falsity-membership function respectively and they satisfy the following condition:

$$^-0 \leq T_A^-(x) + I_A^-(x) + F_A^-(x) \leq 3^+. \quad (1)$$

The values of these three membership functions $T_A^-(x)$, $I_A^-(x)$ and $F_A^-(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$. As we have difficulty in applying NSs to practical problems. Wang et al. [4] proposes the concept of a SVN that represents the simplification of a NS and can be applied to real scientific and technical applications.

Definition 2.2 [4]. A single valued neutrosophic set \tilde{A} (SVNS \tilde{A}) in the universe set ξ is defined by the set

$$\tilde{A} = \left\{ \langle x : T_A^-(x), I_A^-(x), F_A^-(x) \rangle, x \in \xi \right\} \quad (2)$$

Where $T_A^-(x), I_A^-(x), F_A^-(x) \in [0, 1]$ satisfying the condition:

$$0 \leq T_A^-(x) + I_A^-(x) + F_A^-(x) \leq 3 \quad (3)$$

Definition 2.3 [10]. Let x be trapezoidal interval valued neutrosophic number (TrIVNN). Then its truth membership function are given by

$$T_x(z) = \begin{cases} \frac{(z-a)t_x}{(b-a)}, & a \leq z < b, \\ t_x, & b \leq z \leq c \\ \frac{(d-z)t_x}{(d-c)}, & c < z \leq d \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Its indeterminacy membership function is

$$I_x(z) = \begin{cases} \frac{(b-z) + (z-a)i_x}{(b-a)}, & a \leq z < b \\ i_x, & b \leq z \leq c \\ \frac{z-c + (d-z)i_x}{d-c}, & c < z \leq d \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

And its falsity membership function is

$$F_x(z) = \begin{cases} \frac{b-z + (z-a)f_x}{b-a}, & a \leq z < b \\ f_x, & b \leq z \leq c \\ \frac{z-c + (d-z)f_x}{d-c}, & c < z \leq d \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

Where $0 \leq T_x(z) \leq 1$, $0 \leq I_x(z) \leq 1$ and $0 \leq F_x(z) \leq 1$ and also t_x, i_x, f_x are subset of $[0, 1]$ and $0 \leq a \leq b \leq c \leq d \leq 1$. $0 \leq \sup(t_x) + \sup(i_x) + \sup(f_x) \leq 3$; Then x is called an interval trapezoidal neutrosophic number $x = ([a, b, c, d]; t_x, i_x, f_x)$ we take $t_x = [\underline{t}, \bar{t}]$, $i_x = [\underline{i}, \bar{i}]$ and $f_x = [\underline{f}, \bar{f}]$.

Definition 2.4 [10]. Let \tilde{a} and \tilde{r} be two TrIVNNs, the ranking of \tilde{a} and \tilde{r} by score function and accuracy function is described as follows:

1. if $s(\hat{r}^N) < s(\hat{s}^N)$ then $\hat{r}^N < \hat{s}^N$
2. if $s(\hat{r}^N) \approx s(\hat{s}^N)$ and if
 - a. $a(\hat{r}^N) < a(\hat{s}^N)$ then $\hat{r}^N < \hat{s}^N$
 - b. $a(\hat{r}^N) > a(\hat{s}^N)$ then $\hat{r}^N > \hat{s}^N$
 - c. $a(\hat{r}^N) \approx a(\hat{s}^N)$ then $\hat{r}^N \approx \hat{s}^N$

Definition 2.5 [17]. There are some advantages of using interval valued trapezoidal neutrosophic number as follow.

- Interval trapezoidal neutrosophic number is a generalized form of single valued trapezoidal neutrosophic number.
- In this number, the trapezoidal number is characterized with three independent membership degrees which are in interval form.
- The number can flexibly express neutrosophic information than the single valued neutrosophic trapezoidal number.
- Therefore the number can be employed to solve neutrosophic multiple attribute decision making problem, where the preference values cannot be expressed in terms of single valued trapezoidal neutrosophic number.

3 Proposed Concepts

Definition 3.1. Score Function of IVTrNN. Let $x = ([a, b, c, d]; [\underline{t}, \bar{t}], [\underline{i}, \bar{i}], [\underline{f}, \bar{f}])$ be a IVTrNN then its score function defined by,

$$S(x) = \frac{1}{16} (a + b + c + d)(2 + \underline{t} + \bar{t} - \underline{i} - \bar{i} - \underline{f} - \bar{f}), S(x) \in [0, 1]$$

Here we take $0 \leq a \leq b \leq c \leq d \leq 1$, t_x, i_x, f_x are subset of $[0, 1]$ where $t_x = [\underline{t}, \bar{t}]$, $i_x = [\underline{i}, \bar{i}]$ and $f_x = [\underline{f}, \bar{f}]$.

Property 3.2. Score function is bounded on $[0, 1]$.

Proof:

$$\begin{aligned} 0 &\leq a \leq b \leq c \leq d \leq 1 \\ \Rightarrow 0 &\leq a + b + c + d \leq 4 \end{aligned} \quad (7)$$

Now,

$$\begin{aligned}
 -4 &\leq \underline{t} + \bar{t} - \underline{i} - \bar{i} - \underline{f} - \bar{f} \leq 2 \\
 \Rightarrow 2 - 4 &\leq 2 + \underline{t} + \bar{t} - \underline{i} - \bar{i} - \underline{f} - \bar{f} \leq 4 \\
 \Rightarrow -2 &\leq 2 + \underline{t} + \bar{t} - \underline{i} - \bar{i} - \underline{f} - \bar{f} \leq 4
 \end{aligned} \tag{8}$$

Multiplying (7) and (8) we get,

$$\begin{aligned}
 0 &\leq (a + b + c + d)(2 + \underline{t} + \bar{t} - \underline{i} - \bar{i} - \underline{f} - \bar{f}) \leq 16 \\
 \Rightarrow 0 &\leq \frac{1}{16}(a + b + c + d)(2 + \underline{t} + \bar{t} - \underline{i} - \bar{i} - \underline{f} - \bar{f}) \leq 1
 \end{aligned}$$

Therefore, score function is bounded.

Example: Let $a = ([0.1, 0.2, 0.3, 0.4]; [0.1, 0.2], [0.2, 0.3], [0.4, 0.5])$

$$Sc(a) = \frac{1}{16}(0.1 + 0.2 + 0.3 + 0.4)(2 + .1 + .2 - .2 - .3 - .4 - .5) = 0.07875$$

Definition 3.3. Accuracy function. Let $x = ([a, b, c, d]; [\underline{t}, \bar{t}], [\underline{i}, \bar{i}], [\underline{f}, \bar{f}])$ be a IVTrNN then its accuracy function defined by,

$$Ac(x) = \frac{1}{8}(c + d - a - b)(2 + \underline{t} + \bar{t} - \underline{f} - \bar{f}), Ac(x) \in [0, 1]$$

Here we take $0 \leq a \leq b \leq c \leq d \leq 1$, t_x, i_x, f_x are subset of $[0, 1]$ where $t_x = [\underline{t}, \bar{t}]$, $i_x = [\underline{i}, \bar{i}]$ and $f_x = [\underline{f}, \bar{f}]$.

Property: Accuracy function is bounded on $[0, 1]$.

Proof:

$$\begin{aligned}
 0 &\leq a \leq b \leq c \leq d \leq 1 \\
 &\Rightarrow -2 \leq c + d - a - b \leq 2
 \end{aligned} \tag{9}$$

$$\begin{aligned}
 -2 &\leq \underline{t} + \bar{t} - \underline{f} - \bar{f} \leq 2 \\
 \Rightarrow 0 &\leq 2 + \underline{t} + \bar{t} - \underline{f} - \bar{f} \leq 4
 \end{aligned} \tag{10}$$

Multiplying (9) and (10)

$$\begin{aligned} 0 &\leq (c + d - a - b)(2 + \underline{t} + \bar{t} - \underline{f} - \bar{f}) \leq 8 \\ \Rightarrow 0 &\leq \frac{1}{8}(a + b + c + d)(2 + \underline{t} + \bar{t} - \underline{f} - \bar{f}) \leq 1 \end{aligned}$$

Therefore, accuracy function is bounded.

Example: Let $x = ([0.1, 0.2, 0.3, 0.4]; [0.1, 0.2], [0.2, 0.3], [0.4, 0.5])$

$$Ac(x) = \frac{1}{8}(0.1 + 0.2 + 0.3 + 0.4)(2 + .1 + .2 - .4 - .5) = 0.175$$

4 Computation of Shortest Path Based on Interval Valued Trapezoidal Neutrosophic Number

This section introduces an algorithmic approach to solve NSPP. Consider a network with ‘n’ nodes where the node ‘1’ is the source node and the node ‘n’ is the destination node. The neutrosophic distance between the nodes is denoted by d_{ij} (node ‘i’ to node ‘j’). Here $M_{N(i)}$ denotes the set of all nodes having a relation with the node ‘i’.

We begin with the following definition.

Bellman Dynamic Programming

Given an acyclic directed connected graph $G = (V, E)$ with ‘n’ vertices where node ‘1’ is the source node and ‘n’ is the destination node. The nodes of the given network are organized with the topological ordering (E_{ij} : $i < j$). Now for the given network the shortest path can be obtained based on the formulation of Bellman dynamic programming by forward pass computation method.

The formulation of Bellman dynamic programming is described as follows:

$$\begin{aligned} f(1) &= 0 \\ f(i) &= \min_{i < j} \{f(i) + d_{ij}\} \end{aligned}$$

where d_{ij} = weight of the directed edge E_{ij}

$f(i)$ = length of the shortest path of i^{th} node from the source node 1.

Applying the concept of Bellman's algorithm in neutrosophic environment, we have

Neutrosophic Bellman-Ford Algorithm:

1. $nranks[s] \leftarrow 0$
2. $ndist[s] \leftarrow$ Empty neutrosophic number.
3. Add s into Q
4. **For** each node i (except the s) in the neutrosophic graph G
5. $rank[i] \leftarrow \infty$
6. Add i into Q
7. **End For**
8. $u \leftarrow s$
9. **While**(Q is not empty)
10. remove the vertex u from Q
11. **For** each adjacent vertex v of vertex u
12. relaxed \leftarrow False
13. $temp_ndist[v] \leftarrow ndist[u] \oplus edge_weight(u,v)$ // \oplus represents the addition of neutrosophic//
14. $temp_nranks[v] \leftarrow rank_of_neutrosophic(temp_ndist[v])$
15. **If** $temp_nranks[v] < nranks[v]$ **then**
16. $ndist[v] \leftarrow temp_ndist[v]$
17. $nranks[v] \leftarrow temp_nranks[v]$
18. $prev[v] \leftarrow u$
19. **End If**
20. **End For**
21. **If** relaxed equals False **then**
22. exit the loop
23. **End If**
24. $u \leftarrow$ Node in Q with minimum rank value
25. **End While**
26. **For** each arc (u,v) in neutrosophic graph G do
27. **If** $nranks[v] > rank_of_neutrosophic(ndist[u] \oplus edge_weight(u,v))$
28. return false
29. **End If**
30. **End For**
31. The neutrosophic number $ndist[u]$ is a neutrosophic number and its represents the shortest path between source node s and node u .

In the following, we will provide a simple example for a better understanding as follows.

5 Illustrative Example

For this purpose, a numerical problem from [11] is taken to prove the inherent application of the proposed algorithm.

Example 1: Consider a network (Fig. 1) with six nodes eight edges and the edge weights are characterized by IVTpNNs, where the first node is the source node and the sixth node is the destination node. Trapezoidal interval valued neutrosophic distance is given in Table 1.

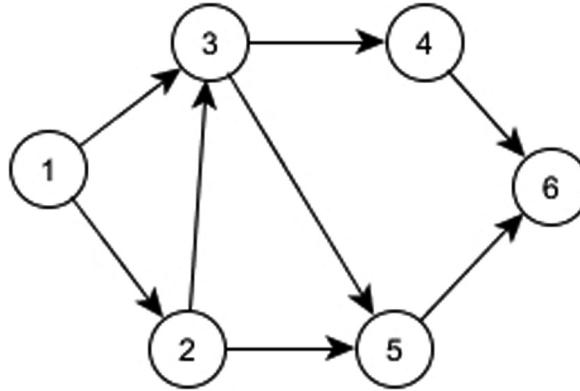


Fig. 1. A network with six vertices and eight edges (Broumi et al. [11])

In this situation, we need to evaluate the shortest distance from source node i.e. node 1 to destination node i.e. node 6 (Table 2)

Table 1. The details of edges information in terms of TrIVNNs

Edges	Trapezoidal interval valued neutrosophic distance
1–2 (e_1)	$\langle (0.1, 0.2, 0.3, 0.4); [0.1, 0.2], [0.2, 0.3], [0.4, 0.5] \rangle$
1–3 (e_2)	$\langle (0.2, 0.5, 0.7, 0.8); [0.2, 0.4], [0.3, 0.5], [0.1, 0.2] \rangle$
2–3 (e_3)	$\langle (0.3, 0.7, 0.8, 0.9); [0.3, 0.4], [0.1, 0.2], [0.3, 0.5] \rangle$
2–5 (e_4)	$\langle (0.1, 0.5, 0.7, 0.9); [0.1, 0.3], [0.3, 0.4], [0.2, 0.3] \rangle$
3–4 (e_5)	$\langle (0.2, 0.4, 0.8, 0.9); [0.2, 0.3], [0.2, 0.5], [0.4, 0.5] \rangle$
3–5 (e_6)	$\langle (0.3, 0.4, 0.5, 1); [0.3, 0.6], [0.1, 0.2], [0.1, 0.4] \rangle$
4–6 (e_7)	$\langle (0.7, 0.8, 0.9, 1); [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] \rangle$
5–6 (e_8)	$\langle (0.2, 0.4, 0.5, 0.7); [0.2, 0.3], [0.3, 0.4], [0.1, 0.5] \rangle$

Table 2. The details of deneutrosophication value of edge (i, j)

Edges	Score function	Edges	Score function
e_{12}	0.05625	e_{34}	0.416875
e_{13}	0.48125	e_{35}	0.56375
e_{23}	0.6075	e_{46}	0.85
e_{25}	0.44	e_{56}	0.36

According to the algorithm method proposed in Sect. 4, the shortest path from node one to node six can be computed as follows:

$$f(1) = 0$$

$$f(2) = \min_{i < 2} \{f(i) + c_{i2}\} = c_{12}^* = 0.05625$$

$$\begin{aligned} f(3) &= \min_{i < 3} \{f(i) + c_{i3}\} = \min\{f(1) + c_{13}; f(2) + c_{23}\} \\ &= \{0 + 0.48125, 0.05625 + 0.6075\} = \{0.48125; 0.66375\} = 0.48125 \end{aligned}$$

$$f(4) = \min_{i < 4} \{f(i) + c_{i4}\} = \min\{f(3) + c_{34}\} = \{0.48125 + 0.416875\} = 0.89$$

$$\begin{aligned} f(5) &= \min_{i < 5} \{f(i) + c_{i5}\} = \min\{f(2) + c_{25}; f(3) + c_{35}\} \\ &= \{0.05625 + 0.44; 0.48125 + 0.56375\} = \{0.49, 1.045\} = 0.49 \end{aligned}$$

$$\begin{aligned} f(6) &= \min_{i < 6} \{f(i) + c_{i6}\} = \min\{f(4) + c_{46}; f(5) + c_{56}\} \\ &= \{0.89 + 0.85; 0.49 + 0.36\} = \{1.74, 0.85\} = 0.85 \end{aligned}$$

thus,

$$\begin{aligned} f(6) &= f(5) + c_{56} = f(2) + c_{25} + c_{56} = f(1) + c_{12} + c_{25} + c_{56} \\ &= c_{12} + c_{25} + c_{56}. \end{aligned}$$

Therefore, the path P: $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$ is identified as the neutrosophic shortest path, and the crisp shortest path is 0.85.

6 Advantages of the Proposed Algorithm

The proposed Bellman algorithm under trapezoidal interval valued neutrosophic environment has the following advantages.

- (i) indeterminacy of the information can be dealt effectively
- (ii) cost of the neutrosophic shortest path can be minimized
- (iii) the performance of the network can be maximized though the data have Indeterminacy
- (iv) Indeterminacy can be captured and shortest path can be obtained by splitting the various paths and hence performance of the system can be increased.

7 Conclusion

Here in this work, proposed the definitions of score function and accuracy functions and their properties. Also proposed the neutrosophic version of Bellman's algorithm based on the trapezoidal interval valued neutrosophic number (TrIVNN) which expresses the flexibility of the neutrosophic information absolutely under trapezoidal interval valued neutrosophic environment. Also, one numeric example is presented. In future, bipolar neutrosophic version of Bellman algorithm can be introduced.

References

1. Zadeh, L.A.: Fuzzy sets. *Inf. Control* **8**(3), 338–353 (1965)
2. Atanassov, K.T.: Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **20**(1), 87–96 (1986)
3. Smarandache, F.: Neutrosophy: neutrosophic probability, set, and logic. In: *ProQuest Information and Learning*, Ann Arbor, Michigan, USA, p. 105 (1998)
4. Wang, H., Smarandache, F., Zhang, Y., Sunderraman, R.: Single valued neutrosophic sets. *Multispace Multistruct.* **4**, 410–413 (2010)
5. De, P.K., Bhincher, A.: Dynamic programming and multi objective linear programming approaches. *Appl. Math. Inf. Sci.* **5**, 253–263 (2011)
6. Kumar, G., Bajaj, R.K., Gandotra, N.: Algorithm for shortest path problem in a network with interval-valued intuitionistic trapezoidal fuzzy number. *Procedia Comput. Sci.* **70**, 123–129 (2015)
7. Meenakshi, A.R., Kaliraja, M.: Determination of the shortest path in interval valued fuzzy networks. *Int. J. Math. Arch.* **3**(6), 2377–2384 (2012)
8. Elizabeth, S., Sujatha, L.: Fuzzy shortest path problem based on interval valued fuzzy number matrices. *Int. J. Math. Sci. Eng. Appl.* **8**(I), 325–335 (2014)
9. Das, D., De, P.K.: Shortest path problem under intuitionistic fuzzy setting. *Int. J. Comput. Appl.* **105**(1), 1–4 (2014)
10. Biswas, P., Pramanik, S., Giri, B.C.: Distance measure based MADM strategy with interval trapezoidal neutrosophic numbers. *Neutrosophic Sets Syst.* **19**, 40–46 (2018)
11. Broumi, S., Bakali, A., Talea, M., Smarandache, F., Vladareanu, L.: Computation of shortest path problem in a network with SV-trapezoidal neutrosophic numbers. In: *Proceedings of the 2016 International Conference on Advanced Mechatronic Systems*, Melbourne, Australia, pp. 417–422 (2016)
12. Broumi, S., Bakali, A., Talea, M., Smarandache, F., Vladareanu, L.: Applying Dijkstra algorithm for solving neutrosophic shortest path problem. In: *Proceedings of the 2016 International Conference on Advanced Mechatronic Systems*, Melbourne, Australia, 30 November–3 December, pp. 412–416 (2016)
13. Bellman, E.: On a routing problem. *Q. Appl. Math.* **16**(1), 87–90 (1958)
14. Wikipedia article. https://en.wikipedia.org/wiki/Bellman%E2%80%93Ford_algorithm
15. Broumi, S., Bakali, A., Talea, M., Smarandache, F., Kumar, P.K.: Shortest path problem on single valued neutrosophic graphs. In: *2017 International Symposium on Networks, Computers and Communications (ISNCC)* (2017)
16. Broumi, S., Singh, P.K., Talea, M., Bakali, A., Smarandache, F., Venkateswara Rao, V.: Single-valued neutrosophic techniques for analysis of WIFI connection. In: Ezziyyani, M. (ed.) *AI2SD 2018. AISC*, vol. 915, pp. 405–412. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-11928-7_36
17. Broumi, S., et al.: Shortest path problem in fuzzy, intuitionistic fuzzy and neutrosophic environment: an overview. *Complex Intell. Syst.* **5**, 371–378 (2019)
18. Broumi, S., Nagarajan, D., Bakali, A., Talea, M., Smarandache, F., Lathamaheswari, M.: The shortest path problem in interval valued trapezoidal and triangular neutrosophic environment. *Complex Intell. Syst.* **5**, 1–12 (2019). <https://doi.org/10.1007/s40747-019-0092-5>
19. Deli, I.: Some operators with IVGSVTrN-numbers and their applications to multiple criteria group decision making. *Neutrosophic Sets Syst.* **25**, 33 (2019)
20. Giri, B.C., Molla, M.U., Biswas, P.: TOPSIS method for MADM based on interval trapezoidal neutrosophic number. *Neutrosophic Sets Syst.* **22**, 151–167 (2018)

21. Deli, I., Subas, Y., Cagranan, N.: Single valued interval valued trapezoidal neutrosophic numbers and SVIVTN multi attribute decision-making method. In: International Conference on Mathematics and Mathematics Education (ICMME-2016), 19–20 May 2016 (2016)
22. Nagarajan, D., Lathamaheswari, M., Sujatha, R., Kavikumar, J.: Edge detection on DICOM image using triangular norms in type-2 fuzzy. *Int. J. Adv. Comput. Sci. Appl.* **9**(11), 462–475 (2018)
23. Lathamaheswari, M., Nagarajan, D., Udayakumar, A., Kavikumar, J.: Review on type-2 fuzzy in biomedicine. *Indian J. Public Health Res. Dev.* **9**(12), 322–326 (2018)
24. Nagarajan, D., Lathamaheswari, M., Kavikumar, J., Hamzha: A type-2 fuzzy in image extraction for DICOM image. *Int. J. Adv. Comput. Sci. Appl.* **9**(12), 352–362 (2018)
25. Lathamaheswari, M., Nagarajan, D., Kavikumar, J., Phang, C.: A review on type-2 fuzzy controller on control system. *J. Adv. Res. Dyn. Control Syst.* **10**(11), 430–435 (2018)
26. Sellappan, N., Nagarajan, D., Palanikumar, K.: Evaluation of risk priority number (RPN) in design failure modes and effects analysis (DFMEA) using factor analysis. *Int. J. Appl. Eng. Res.* **10**(14), 34194–34198 (2015)