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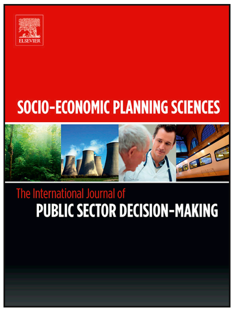

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Vladimir Simic: Methodology, Software, Validation, Visualization, Writing - Original Draft, Review & Editing, Supervision. **Ilgin Gokasar**: Conceptualization, Data acquisition, Writing - Original Draft, Review & Editing, Investigation, Supervision. **Muhammet Deveci**: Methodology, Validation, Visualization, Writing - Original Draft, Review & Editing, Supervision. **Ahmet Karakurt**: Data acquisition, Writing - Original Draft, Review & Editing, Investigation.

Title Page

An Integrated CRITIC and MABAC based Type-2 Neutrosophic Model for Public Transportation Pricing System Selection

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An Integrated CRITIC and MABAC based Type-2 Neutrosophic Model for Public Transportation Pricing System Selection

Abstract

The pricing of public transportation services is a complex task that authorities deal with because many criteria should be considered while deciding on the pricing system. Some include decentralization of the cities due to lower rents in outer-city regions and operating costs of longer transportation lines. Hence, four alternative public transportation pricing systems are defined, which are flat fare, distance-based, zonal, and rent-based fare. To prioritize these alternatives, four aspects are determined, namely cost, transportation, social and political, and there are 13 criteria present under these aspects. A two-stage hybrid multi-criteria decision-making model based on type-2 neutrosophic numbers (T2NNs) is introduced to provide a straightforward and flexible decision-making tool for researchers and practitioners. In its first stage, the reputation of the experts is determined under the T2NN environment. Second, the novel T2NN-based CRiteria Importance Through Intercriteria Correlation (CRITIC) method is employed to evaluate the criteria importance, while the new T2NN-based Multi-Attributive Border Approximation area Comparison (MABAC) method is used to rank the public transportation pricing systems. The results show that rent-based fare pricing is the most advantageous alternative. The high reliability and robustness of the integrated CRITIC and MABAC based type-2 neutrosophic model are demonstrated with the comparative and sensitivity analyses.

Keywords: Public Transport, Transportation Pricing, Type-2 Neutrosophic Number, Multi-Criteria Decision-Making, CRITIC, MABAC.

1. Introduction

Public transportation is one of the most effective solutions for dealing with road traffic congestion, which has become a severe problem especially for developing countries all around the world (Jain et al., 2012). The continuing increase in world population has increased and is still increasing the number of private vehicles in traffic (Simić et al., 2021). Environmental pollution associated with this increase is a growing problem of humanity overall (Švadlenka et al., 2020). The incentivization of public transportation is an important aspect to mitigate the increase in private vehicles, the levels of congestion, and the pollution caused by private vehicle usage. However, while promoting the usage of public transportation vehicles, there are things that authorities must decide on such as the pricing of the public transportation systems.

While deciding on the pricing system, the decentralization of the cities must be taken into consideration. If the public transportation network is covering the outer-city parts, people work in the inner-city and live at lower rent houses in the outer-city, since transportation between home and work is available via public transportation. This creates a decentralization problem by expanding the city to outer ranges. As the city expands, the lengths of the public transportation lines increase, which puts an additional operating cost on operators due to frequent maintenance needs and increases air pollution because of the larger travel times of the vehicles. However, a distance-based, rent-based (Karakurt, 2021), or zone-based pricing system increases the financial burden of transportation on the people traveling for longer lines, which eventually motivates them to choose their residence close to their work and social environment. This mitigates the decentralization of cities and shortens the lines of public transportation trips, which reduces the operating costs of the vehicles and also is an environmentally friendly outcome (Gaigné et al., 2012).

Determination of the pricing system must be dealt with carefully because public acceptance and welfare of the public transportation commuters is an important aspect for municipalities. If a distance-based pricing system is applied and demands are lowered, the frequency of the public transportation lines is to be lowered to compensate for the operating costs. This increases the waiting times of the commuters and reduces the level of satisfaction (Cantwell et al., 2009). If a flat fare pricing system is applied, decentralization of the city is a potential problem to be faced and longer lines are to be increased. This has the potential to increase the crowdedness of the longer lines, which again reduces the satisfaction level of the commuters and also decreases sustainability due to more greenhouse gas emissions.

1.1. Objectives of the Study

In this study, the critical worldwide problem of public transportation pricing system selection is for the first time comprehensively addressed and solved in this study. Flat fare, distance-based, zonal, and rent-based pricing systems are defined. To prioritize these alternatives, four aspects are determined, namely cost, transportation, social and political. Thirteen evaluation criteria are identified and briefly defined under these aspects. Studies in the literature were used to determine the criteria. A case scenario is constructed to create an environment for experts to efficiently assess the public transportation pricing systems.

This study aims to help municipalities to compare and improve their public transportation pricing systems by providing a straightforward and flexible decision-making tool. Thus, the study suggests a hybrid two-stage type-2 neutrosophic model is developed to solve the addressed multi-criteria decision-making (MCDM) problem. The first stage determines the reputation of experts under the type-2 neutrosophic number (T2NN) environment by making the trade-off between their experience and expertise. In the second stage, the novel T2NN-based CRiteria Importance Through Intercriteria Correlation (CRITIC) method is employed to evaluate the criteria importance, while the new T2NN-based Multi-Attributive Border Approximation area Comparison (MABAC) method is used to rank the public transportation pricing systems.

1.2. The Motivation for Using Type-2 Neutrosophic Numbers

The fuzzy set theory is based on the concept of membership function value by considering linguistic variables. This theory tries to evaluate ambiguous data with expert opinions and matches evaluations with membership function values. However, it is not always the case that expert opinions focus on membership values, especially when there is a lack of experience and expertise. Besides, the experts can be confident about the falsity of their opinions rather than their truthiness (Tiwari and Kumar, 2020). Smarandache (1998, 2003) introduced the neutrosophic set theory to resolve these real-life decision-making situations. Neutrosophic sets are an extension of the fuzzy theory and intuitionistic fuzzy set (IFS). Neutrosophic numbers have proven to be a valid field of study for identifying incompatible and indefinite information. Type-1 neutrosophic numbers are represented in the form of a triplet (T, I, F), where all T, I, and F are from a range [0, 1]. They are called a membership or truth value (T), neutral value or indeterminacy (I), and non-membership or falsehood value (F), respectively. T2NN is represented in the form $<(T_T, T_I, T_F), (I_T, I_I, I_F), (F_T, F_I, F_F)>$; i.e., each neutrosophic component is split into its truth, indeterminacy, and falsehood subparts. T2NN represents an advanced type of neutrosophic technique. It is an efficient tool to deal with experts' impreciseness or incompleteness (Abdel-Basset et al., 2019). Unfortunately, no previous research provided a T2NN-based model to evaluate transportation pricing systems.

1.3. The Motivation for Using CRITIC and MABAC Methods

The CRITIC and MABAC methods are extended under the T2NN environment to calculate the importance of evaluation criteria and rank the alternative public transportation pricing systems, respectively. The CRITIC method was developed by Diakoulaki et al. (1995) to determine objective weights considering the differences and correlations among various criteria simultaneously (Lai and Liao, 2021). Although many MCDM approaches used the CRITIC method in recent years, its extension into the T2NN environment is still missing.

On the other hand, the MABAC method is an exceptional MCDM method introduced by Pamučar and Ćirović (2015). The MABAC method is a reliable tool for rational decision-making (Gigović et al., 2017). It comprehensively considers the potential values of gains and losses (Büyüközkan et al., 2021). One of the main principles of the MABAC method is the distance between each alternative and the border approximation area (Büyüközkan et al., 2021; Liu et al., 2020b). The calculation process of this method is forthright, logical, and methodical (Luo and Liang, 2019). The MABAC method has been extended under various uncertain environments, such as interval 2-tuple linguistic (Shi et al., 2017), the hesitant fuzzy linguistic term (Sun et al., 2018), single-valued neutrosophic (Peng and Dai, 2018), single-valued neutrosophic linguistic (Ji et al., 2018), 2-tuple linguistic neutrosophic (Wang et al., 2019b), linguistic neutrosophic (Luo and Liang, 2019; Luo and Xing, 2019), bipolar fuzzy (Liu et al., 2020b), bipolar neutrosophic (Rahim et al., 2020), fuzzy neutrosophic (Pamucar et al., 2020), linguistic distribution assessment (Liang et al., 2020), normal wiggly hesitant fuzzy linguistic term (Liu et al., 2020a), picture 2-tuple linguistic (Zhang et al., 2020), probabilistic single-valued neutrosophic hesitant fuzzy (Şahin and Altun, 2020), the probabilistic uncertain linguistic term (Wei et al., 2020), Z number (Fan et al., 2020; Shen et al., 2020), normal wiggly hesitant fuzzy (Liu and Zhang, 2021), and rough plithogenic (Abdel-Basset et al., 2021). Unfortunately, this innovative MCDM has not been extended before under the T2NN environment.

This research is structured: Section 2 provides a review of related state-of-the-art research. Section 3 defines the investigated problem. Section 4 presents the developed hybrid type-2 neutrosophic model for public transportation pricing. Section 5 provides the results and discussion. Section 6 presents the conclusions and indicates possible future research directions.

2. Literature Review

The literature review is organized into three sub-sections. The first sub-section overviews decision-making approaches for transportation pricing. The second sub-section surveys the MABAC method. The third sub-section highlights major research gaps.

2.1. Decision-Making Approaches for Transportation Pricing

Public transportation is used by many people today. The widespread use of public transportation has been a priority for the administrations for people to have mobility. Today, global warming threatens the existence of all living things and affects the environment negatively because of the gases and pollutants released into the environment due to transportation activities. Encouraging people to use public transportation will play a role in reducing both the traffic density in cities and the gas emissions that cause global warming and air pollution. Besides these, it is more economical for people who take an active role in transportation to prefer public transportation. Public transportation pricing has different methods such as flat fare, distance-based fare, zonal pricing, and rent-based fare.

A flat fare is one of the most widely implemented pricing in public transportation. There is a fixed price for all users since the fee is not dependent on the duration of use and distance. Nuworsoo et al. (2009) analyzed the five options flat fare proposals of Alameda-Contra Costa Transit Districts including base fare reductions, fare hikes, discontinuation of periodic passes, and eliminations of free transfers. They evaluated the effects of individual flat fare offerings on different subsets of riders and estimated likely flat revenues. They found out that the five alternative proposals of Transit Districts that raised the cost of the transfers or removed unlimited-use passes caused inequity on certain riders. Flat fares proposals are found to be least fair since young commuters, minorities, and lower-income riders made more trips. Cheng et al. (2015) examined the unit fare, farebox recovery ratio, and unit cost in terms of indexes of fare equity. They used the Gini coefficient and theory of the Lorenz curve to implement the equity evaluation of unit fares distribution among the public transportation users. They used the established methods and indexes to assess fare equity and observed that flat fare is unfair. Brown (2018) analyzed the frequency of transit use via lower and higher-income transit riders based on six evaluated fare structures in Los Angeles. They found low-income public transportation users pay far higher per-mile transit fares compared to more wealthy users. Jin et al. (2019) examined the relationship between flat fares and the quality of public transportation services. They observed that with higher flat fares, operators have more sources to supply a better service focusing on flat fare structure. Rubensson et al. (2020) use the socio-economic statistics of census areas with public transportation modes data from a transportation estimation representation to evaluate the distributional and geographical fairness of alternative fare plans that are flat fare, zone-based, and distance-based in Stockholm Public

Transportation. They revealed high-income public transit riders benefit from all three fare schemes reviewed. These riders benefit the least from flat fares.

Distance-based fare determines the relationship between the amount of payment and trip distance. Since the operating costs of public transportation vehicles are directly proportional to the lengths of the routes, this pricing method becomes preferable by the authorities when the lengths of the routes are long, and hence operating costs are high. Tsai et al. (2008) offered an optimization perspective, maximizing total profit depending on fleet size and service capacity, for an intercity transportation system where demand for public transportation is sensitive to fare and travel time. They developed an efficient solution and applied it to the profit maximization problem. Farber et al. (2014) aimed to develop and apply a new procedure for evaluating the social equity effects of distance-based public transit fares. They showed that distance-based fares affect low-wealth, non-white and elderly populations. Hoshino and Beairsto (2018) proposed two-part optimal pricing for changing to distance-based transit fares. The first was to maximize the estimated revenue given a target ridership, and the second was to maximize the estimated ridership given a target revenue. They tested their model successfully on the SkyTrain mass transit network in Metro Vancouver, British Columbia, with over 400.000 public transportation users. Maadi and Schmöcker (2020) exemplified the effects of switching fares from a zonal to a distance-based structure. They took the zonal case as a base model and examined the effects of fare/km and consisting of non-additive fares. Chen et al. (2021) assessed a distance-based preferential fare project for park and ride services in a multichannel transportation network. They also examined the choice of passengers that prefer park and ride mode to get a discount. Their analysis proved that the proposed distance-based fare scheme might motivate public transportation travelers to use park-and-ride services and develop the performance of the transportation network.

Zonal pricing is one of the public transportation payment methods that can be implemented in big cities like London. The amount of payment is independent of the distance and depends on whether the trip changes a zone boundary. If the user has not changed the zone, he pays the standard fee, but if the user has changed the zone, the user pays a part of the transition fee to that zone besides the standard fee. Rouhani (2018) formulated the three basic extensions of the analysis of standard optimal road use: maximizing the social welfare outside and inside of the congestion zone, examining the fuel and emissions variable costs, and travel demand variations focusing on zonal pricing. They observed that the effects of emission costs are significant, the examination over predicts the optimal toll rate and may even cause a social income loss, and policymakers should keep away from implementing a flat daily charge. Yuan and Hesamzadeh

(2016) offered to design zonal pricing in a dispensation network based on the concept of pricing equivalence. They obtained the rules of zonal pricing and demonstrated the equivalent load move from demand response can be accessible via zonal pricing in case pricing equivalence is positioned. It is seen that that zonal pricing planned according to pricing equivalence may attain the same load shift effects and quite close consumer discharges.

Because of high rents in places of work, employees can stay in other locations where rents are lower. Since the places with lower rents are far from workplaces, they can reach their workplaces either by personal vehicles or by using public transportation. Commuters who use public transportation, for this reason, come from outside of the city, causing increased traffic in the city and larger travel time of vehicles. To prevent the city from decentralization, the rent-based fare can be applied and commuters coming from outside of the city are charged with more public transportation fees. Here, the increase in the cost of transportation may cause the person to move from outside of the city to the city, and therefore, the operation cost of public transportation may decrease. In Melbourne, Australia, one study applied a survey of participants with and without travel plans, and propensity score matching to see how their own housing choices could be assessed in the context of trip plans (De Gruyter et al., 2016). According to the findings, residents in settlements with travel plans own fewer cars as well as walk and cycle more frequently than residents in settlements without a travel plan. Zhang (2020) started a housing valuation model to discover the interface relation between transit rail and other transportation modes. He also got the family transportation impedance, forming a relationship between the distance of house to city and public transportation. Results showed that users of public transportation who have higher income would choose the residence close to districts. Results also showed that having urban rail transit has a key role effect on the low-income family.

Table 1 provides a widespread overview of the decision-making approaches for transportation pricing.

Table 1

Survey of the available decision-making approaches for transportation pricing.

Author(s) and year	Research focus	GDM	Discussed pricing types	SA	CA	Method(s)	Application	
							Country	Type
Tsai et al. (2008)	Optimizing distance-based fares and headway of an intercity transportation system	No	DBF	Yes	Yes	Modified Gauss-Southwell method	Taiwan	Real-life
Nuworsoo et al. (2009)	Analyzing equity impacts of transit fare changes	No	FF	Yes	Yes	Alternative price elasticities	California, USA	Real-life
Farber et al. (2014)	Assessing social equity in distance-based transit fares	No	FF, DBF	No	Yes	Joint ordinal/continuous model for trip generation, GIS	Utah, USA	Real-life
Cheng et al. (2015)	Assessing equity of regular public transit fare policy	No	FF	No	No	Lorenz curve, Gini coefficient	-	Literature
De Gruyter et al. (2016)	Measuring self-selection effects	No	RBF	No	No	Case and control sites	Melbourne, Australia	Real-life
Yuan and Hesamzadeh (2016)	Implementing zonal pricing in the distribution network	No	ZF	No	Yes	Long-run incremental cost pricing, Forward cost pricing	-	Literature
Brown (2018)	How flat and variable fares affect transit equity	No	FF, DBF	No	Yes	Bonferroni correction	Los Angeles, USA	Real-life
Hoshino and Beairsto (2018)	Optimal pricing for distance-based transit fares	No	ZF, DBF	No	Yes	Two-part optimal pricing formula	Vancouver, Canada	Real-life
Rouhani (2018)	Standard zonal congestion pricing	No	ZF	Yes	No	Standard optimal road use	California, USA	Real-life
Jin et al. (2019)	The interaction between public transportation demand, service quality, and fare for social welfare optimization	No	FF	Yes	Yes	Daganzo's model	-	Literature
Maadi and Schmöcker (2020)	Route choice effects of changes from a zonal to a distance-based fare structure	No	ZF, DBF	No	Yes	Frequency-based assignment approach	-	Literature
Rubensson et al. (2020)	Equity impact of fare scheme change	No	FF, DBF, ZF	No	Yes	Transportation forecast model, socio-economic statistics	Stockholm, Sweden	Real-life
Zhang (2020)	Relationship between urban rail transit and residential location	No	RBF	No	No	Housing valuation model	-	Literature
Chen et al. (2021)	Distance-based preferential fare scheme for park-and-ride services in a multimodal transportation network	No	DBF	No	Yes	Transportation network equilibrium model, multinomial logit discrete choice model	-	Literature
Our study	*Public transportation pricing system selection*	*Yes*	*FF, DBF, ZF, RBF*	*Yes*	*Yes*	*Hybrid T2NN model, CRITIC, MABAC*	*Izmir, Turkey*	*Real-life*

Comparative Analysis: CA, CRiteria Importance Through Intercriteria Correlation: CRITIC, Distance-Based Fare: DBF, Flat Fare: FF, Geographic Information System: GIS, Group Decision-Making: GDM, Multi-Attributive Border Approximation area Comparison: MABAC, Rent-Based Fare: RBF, Sensitivity Analysis: SA, Type-2 Neutrosophic Number: T2NN, Zonal Fare: ZF.

2.2. MABAC Method

The MABAC method has become very influential in decision-making. Table 2 comprehensively surveys this well-known MCDM method.

Pamučar and Ćirović (2015) prioritized forklift types that are used in logistics centers to make investment decisions. Delice and Can (2017) examined potential failure modes of a high voltage assembly line in electro-mechanic system manufacturing firms. Gigović et al. (2017) ranked locations for the installation of wind farms. Shi et al. (2017) formulated an interval 2-tuple linguistic MABAC method to explore treatments for healthcare waste.

Chatterjee et al. (2018) developed a D number-based MABAC method to order construction risk response strategies for construction projects. Ji et al. (2018) provided a single-valued neutrosophic linguistic MABAC method to solve the outsourcing provider selection problem in the information technology (IT) industry. Peng and Dai (2018) introduced a single-valued neutrosophic MABAC method to compare computer books as well as air-conditioning systems. Sun et al. (2018) presented a hesitant fuzzy linguistic term MABAC method to prioritize hospital patients.

Adar and Delice (2019) studied treatment technologies for healthcare waste flow. Luo and Liang (2019) coped with the problem of ranking roadway support schemes in gold mines. Luo and Xing (2019) proposed a linguistic neutrosophic MABAC method to solve the personnel selection problem in IT companies. Wang et al. (2019a) examined collection modes for used parts of construction machinery. Wang et al. (2019b) proposed a 2-tuple linguistic neutrosophic MABAC method to perform the safety assessment of projects.

Fan et al. (2020) focused on the selection of the best third-party logistics (3PL) provider. Liang et al. (2020) formulated a linguistic distribution-based MABAC method to solve the emergency evacuation management problem. Liu et al. (2020a) suggested a normal wiggly hesitant fuzzy linguistic term MABAC method to evaluate the ecological security situation of coastal areas. Liu et al. (2020b) introduced a bipolar fuzzy MABAC method to prioritize occupational health and safety risks on construction sites. Pamucar et al. (2020) developed a fuzzy neutrosophic MABAC method to identify the most resilient supplier for construction companies. Rahim et al. (2020) presented a bipolar neutrosophic MABAC method to solve the sustainable energy management problem. Roy et al. (2020) studied the 3PL provider selection process in the food manufacturing industry. Shen et al. (2020) extended the MABAC method under the Z-information to rank regional development programs related to the circular economy. Şahin and Altun (2020) proposed a probabilistic single-valued neutrosophic hesitant fuzzy MABAC method to select the best investment company. Wei et al. (2020) developed a probabilistic uncertain linguistic term MABAC method to solve the green supplier selection problem in the furniture industry. Zhang et al. (2020) introduced a picture 2-tuple linguistic MABAC method for ranking renewable energy generation projects.

Recently, Abdel-Basset et al. (2021) formulated the MABAC method based on plithogenic sets and rough numbers to solve the supplier selection problem in the healthcare industry. Büyüközkan et al. (2021) investigated strategies for health tourism development. Hristov et al. (2021) compared automatic cannon weapon systems designed for integration into combat vehicles. Jana and Pal (2021) focused on the selection of road construction companies. Liu and Zhang (2021) stated a normal wiggly hesitant fuzzy MABAC method to assess suppliers for university libraries. Pamučar et al. (2021) compared equipment for disinfecting medical waste flow.

2.3. Research Gaps

According to the literature review, the research gaps are as follows:

a) This is the first work to address and solve the public transportation pricing system selection problem, as can be seen from Table 1. Previously, public transportation pricing systems have not been systematically explored and prioritized. Besides, a practical decision-making framework for municipalities to compare and improve their public transportation pricing systems is still missing since the earlier research efforts failed to comprehensively address and define viable alternatives as well as evaluation aspects and criteria. Also, the available decision-making (Table 1) could generate wrong pricing decisions since they are unable to deal with experts' impreciseness or incompleteness.

b) The T2NN-based technique for the order preference by similarity to ideal solution (TOPSIS) method introduced by Abdel-Basset et al. (2019) is the only available decision-making approach under the T2NN environment in the literature. This method is unable to differentiate expert reputation, determine criteria importance, and provide higher flexibility when making decisions.

c) The MABAC method has neither been extended under the T2NN environment nor been applied for transportation pricing, as can be seen from Table 2. Besides, no earlier work has integrated the CRITIC and MABAC methods into a single methodological framework (Table 2).

Table 2
Survey of the MABAC method.

Author(s) and year	Research focus	GDM	Parameter type	SA	CA	Combined method(s)	Application		Criteria	Alt.
							Country	Type		
Pamučar and Ćirović (2015)	Machine tool selection	Yes	Crisp	Yes	Yes	DEMATEL	–	IE	10	7
Delice and Can (2017)	Failure mode prioritization	Yes	Random, Crisp	No	Yes	FMEA, Brainstorming	–	IE	6	12
Gigović et al. (2017)	Wind farm location selection	Yes	Crisp, Fuzzy	Yes	Yes	GIS, DEMATEL, ANP	Serbia	Real-life	11	11
Shi et al. (2017)	Waste treatment technology selection	Yes	I2TL	Yes	No	-	China	Real-life	6	4
Chatterjee et al. (2018)	Construction project risk assessment	Yes	D number	Yes	Yes	ANP	–	IE	9	5
Ji et al. (2018)	3PL provider selection	Yes	SVNLT	No	Yes	MSD, ELECTRE	–	From lit.	6	5
Peng and Dai (2018)	-	No	SVN	No	Yes	-	–	IEs	5, 3	10, 5
Sun et al. (2018)	Hospital patient prioritization	No	HFLT	Yes	Yes	-	China	Real-life	6	8
Adar and Delice (2019)	Waste treatment technology selection	Yes	HFLT	Yes	Yes	MAIRCA	Turkey	Real-life	19	4
Luo and Liang (2019)	Roadway support scheme selection	Yes	LN	Yes	Yes	GST, Direct rating	China	Real-life	4	4
Luo and Xing (2019)	Personnel selection	Yes	LN	No	Yes	BWM, PROMETHEE II	–	IE	9	4
Wang et al. (2019a)	Take-back pattern selection	No	Crisp, Grey	Yes	No	AHP, Shannon entropy	China	Real-life	9	3
Wang et al. (2019b)	Construction project selection	Yes	2TLN	No	Yes	-	–	IE	4	5
Fan et al. (2020)	3PL provider selection	No	Z number	No	Yes	Shannon entropy	–	IE	5	4
Liang et al. (2020)	Flood evacuation planning	Yes	LDA	Yes	Yes	-	–	IE	5	4
Liu et al. (2020a)	Ecological security assessment	Yes	NWHFLT	Yes	Yes	Maximizing deviation	–	IE	6	5
Liu et al. (2020b)	Occupational hazard risk assessment	Yes	BF	No	Yes	SWARA	–	IE	3	12
Pamucar et al. (2020)	Resilient supplier selection	Yes	FN	Yes	Yes	Pairwise comparison	Spain	Real-life	12	8
Rahim et al. (2020)	Sustainable energy selection	Yes	BNN	No	Yes	-	Malaysia	Real-life	14	7
Roy et al. (2020)	3PL provider selection	Yes	IVFR	Yes	Yes	FARE	India	Real-life	15	6
Shen et al. (2020)	Development program selection	No	Z number	Yes	Yes	Standard deviation	–	IE	8	5
Şahin and Altun (2020)	Investment project selection	No	PSVNHFS	No	Yes	-	–	From lit.	3	4
Wei et al. (2020)	Supplier selection	Yes	PULT	No	Yes	Shannon entropy	–	IE	5	4
Zhang et al. (2020)	Power generation project selection	Yes	P2TL	No	Yes	-	–	IE	4	5
Abdel-Basset et al. (2021)	Supplier selection	Yes	RP	No	No	BWM	Malaysia	Real-life	9	5
Büyüközkan et al. (2021)	Tourism strategy selection	Yes	HFLT	No	Yes	SWOT, AHP	Turkey	Real-life	24	8
Hristov et al. (2021)	Weapon system selection	Yes	D number	Yes	No	LBWA	–	IE	7	9

Table 2 (continued)

Author(s) and year	Research focus	GDM (Yes/No)	Parameter type	SA (Yes/No)	CA (Yes/No)	Combined method(s)	Application		Criteria	Alt.
							Country	Type		
Jana and Pal (2021)	Construction company selection	No	SVN	Yes	Yes	-	India	Real-life	5	4
Liu and Zhang (2021)	Supplier selection	Yes	NWHF	Yes	Yes	CCSD	China	Real-life	5	4
Pamučar et al. (2021)	Disinfection facility selection	Yes	D number	Yes	No	BWM	B&H	Real-life	18	6
Our study	*Public transportation pricing system selection*	*Yes*	*T2NN*	*Yes*	*Yes*	*CRITIC, MABAC*	*Turkey*	*Real-life*	*13*	*4*

2-Tuple Linguistic Neutrosophic: 2TLN; Analytic Hierarchy Process: AHP; Analytic Network Process: ANP; Best Worst Method: BWM; Bipolar Fuzzy: BF; Bipolar Neutrosophic Number: BNN; Bosnia and Herzegovina: B&H; Comparative Analysis: CA; Correlation Coefficient and Standard Deviation: CCSD; CRiteria Importance Through Intercriteria Correlation: CRITIC; DEcision MAking Trial and Evaluation Laboratory: DEMATEL; ELimination Et Choix Traduisant la Realite: ELECTRE; FActor RElationship: FARE; Failure Mode and Effect Analysis: FMEA; Fuzzy Neutrosophic: FN; Geographic Information System: GIS; Gray System Theory: GST; Group Decision-Making: GDM; Hesitant Fuzzy Linguistic Term: HFLT; Illustrative Example: IE; Interval 2-Tuple Linguistic: I2TL; Interval-Valued Fuzzy-Rough: IVFR; Level Based Weight Assessment: LBWA; Linguistic Distribution Assessment: LDA; Linguistic Neutrosophic LN; Mean-Squared Deviation: MSD; Multi-Attributive Border Approximation area Comparison: MABAC; Multi-Attributive Ideal-Real Comparative Analysis: MAIRCA; Normal Wiggly Hesitant Fuzzy Linguistic Term: NWHFLT; Normal Wiggly Hesitant Fuzzy: NWHF; Picture 2-Tuple Linguistic: P2TL; Preference Ranking Organization METHod for Enrichment of Evaluations: PROMETHEE; Probabilistic Single-Valued Neutrosophic Hesitant Fuzzy: PSVNHFS; Probabilistic Uncertain Linguistic Term: PULT; Rough Plithogenic: RP; Sensitivity Analysis: SA; Single-Valued Neutrosophic Linguistic Term: SVNLT; Single-Valued Neutrosophic: SVN; Step-wise Weight Assessment Ratio Analysis: SWARA; Strengths, Weaknesses, Opportunities, and Threats: SWOT; Third-Party Logistics: 3PL, Type-2 Neutrosophic Number: T2NN.

3. Problem Definition

There are many systems for public transportation pricing. However, each of these systems has certain superiorities to others based on the situation that they are used in. There is still a gap for effectively prioritizing these systems systematically and objectively.

3.1. Case Scenario

Pricing of public transportation services needs to be considered regarding various aspects of a city such as mobility, decentralization, and environmental issues. Since this study concentrates on the pricing of public transportation, a case scenario is to be formed to illustrate the conceptual framework, so that solid results and policy implications can be achieved.

In choosing our case scenario, we select a big metropolitan city in a developing country in which a new pricing system for public transportation is planned. The municipality of the city is ready to take action; however, it is required to prioritize the pricing systems. By doing thorough research on the literature, decentralization of cities, compact cities, satisfaction of commuters, improvements of shorter public transportation lines, and environmentally friendly public transportation are seen to be the concepts that should be included in the created case scenario. That is why the alternatives present in the following section are determined as the pricing methods to be assessed on the created case scenario. Experts, whose reputations are also integrated into the analysis, are gathered to observe the relative importance of each criterion and assess the alternatives.

3.2. Alternatives

Alternative 1 (A_1): Flat fare pricing

One of the public transportation pricing systems handled in this study is flat fare pricing. This pricing alternative makes up for all passengers, regardless of distance traveled, zone, and rent, to pay the same price for public transportation. Implementation and control of flat fare pricing are relatively easier and do not create confusion among passengers because of the simplicity of the alternative.

Alternative 2 (A_2): Distance-based fare pricing

Implementation of a distance-based fare pricing system aims to make passengers who travel for longer distances pay more since these passengers use the transits more than others. Also, public transportation vehicles, which travel for longer lines, need more frequent maintenance and their operational costs increase. Charging passengers, who travel for longer distances, provides equality between public transportation users (Bandegani and Akbarzadeh, 2016).

Alternative 3 (A_3): Zonal pricing

In this alternative, public transportation passengers are charged according to the number of zones that they travel through. The application of zonal pricing has the potential to provide equality in terms of meeting the operational expenses accompanied to longer lines by charging passengers of long public transportation lines higher because of more zones traveled. One disadvantage of this alternative is that if the prices of zones are different, decentralization of the city can take place.

Alternative 4 (A_4): Rent-based fare pricing

The rent-based fare proposed by Karakurt (2021) is a rail mass public transit pricing system developed by using a nonlinear optimization model which includes rental costs. Avoiding the decentralization due to people working in cities, but living off-cities to pay less rent is the primary reason behind this pricing system. Decentralization of a city increases the length of public transportation lines, expands the city, and brings more traffic load to urban roads, which is unwanted. By implementing this system, people would live closer to their social or work destinations and travel short distances. Operating costs of public transportation would be less because of having shorter lines. Unlike flat fare, the rent-based fare is based on the distance traveled and the rental prices. Clustered stations in rent-based fare differ from distance-based pricing in that ticket price in clusters is reliant on rent. In contrast to zonal pricing, rent-based pricing considers travel distance and rent.

Fig. 1. illustrates the different types of public transportation pricing systems.

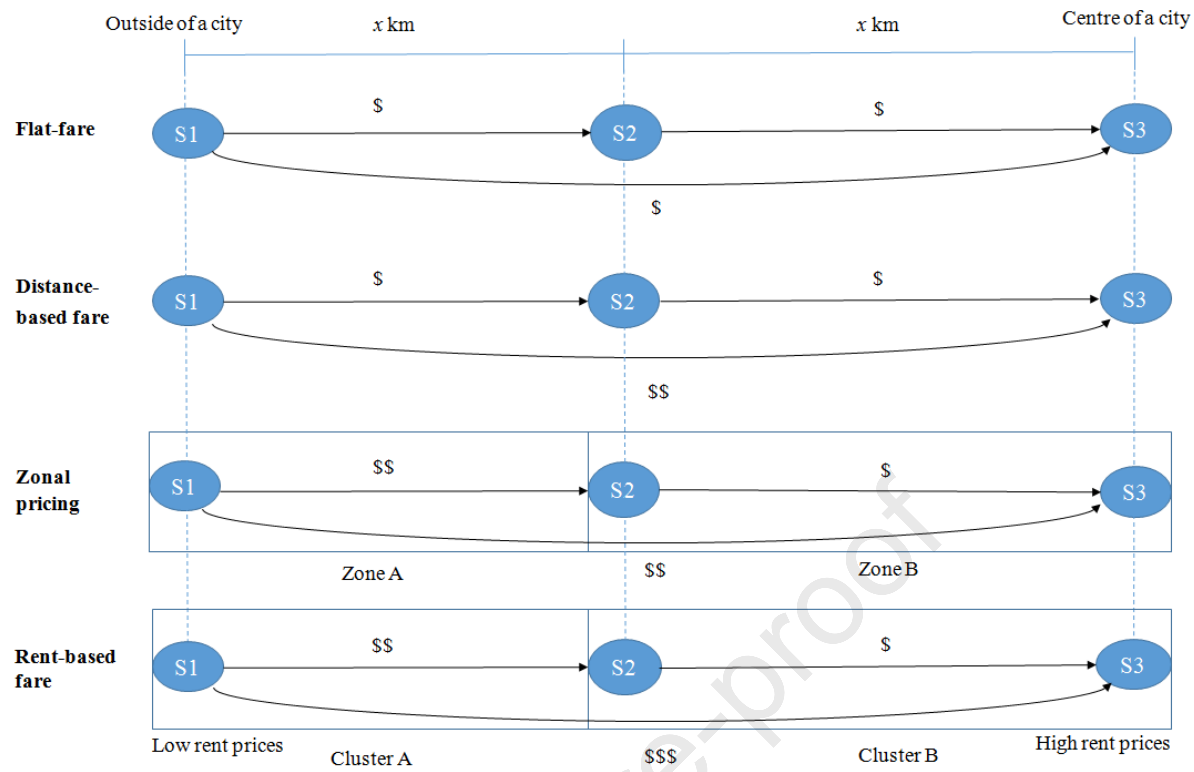

Fig. 1. Public transportation pricing types.

3.3. Definition of Criteria

i) Cost Aspect

Operating cost (C_1): Operating costs are the set of expenses that includes some variable and fixed costs that cover operating the transportation system. Operating costs include a wide range of components from the energy consumption cost to labor, maintenance, and repair costs. Factors such as the frequency of public transportation trips and the lengths of lines directly affect the operating cost. As the number of trips and lengths of these trips increase, operating costs also increase.

User cost (C_2): User cost is defined as the cost to which passengers are exposed. User costs are divided into in-vehicle travel time cost, waiting time cost, access, egress time, and ticket price (Chien and Schonfeld, 1998; Samanta and Jha, 2011; Almujibah and Preston, 2019). User costs and operating costs are always in conflict. Reducing user costs increase operating cost, the acceptance of the fare system, the demand to public transportation and the life quality of people. However, ensuring this situation entirely with low-ticket prices may reduce in-vehicle comfort, depending on high demand. The low user cost makes the system more attractive.

Expert cost (C_3): Some of the public transportation pricing systems require special research. In this sense, it is necessary to get support from expert teams. Especially complex fare systems are in this scope. Decision-makers can observe which pricing system brings additional costs.

ii) Transportation Aspect

Travel time (C_4): Travel time of public transportation vehicles increases directly proportional to the lengths of the lines. As the travel times increase, operating costs of the vehicles also increase by more frequent maintenance needs and fuel consumption. Travel time of passengers comprises in-vehicle travel time, vehicle waiting time, and access and egress time. High-ticket tariffs can reduce the demand for public transportation. Here, public transportation frequency is reduced to meet the operational costs. Thus, as the waiting time increases, the travel time and the user cost increase. Less travel time is more attractive for users (Lisco, 1968).

Travel comfort (C_5): The travel comfort of passengers is a factor that highly affects the usage of public transportation (İmre and Çelebi, 2017). Travel comfort is affected notably by the demand for public transportation systems. Increasing demand for public transportation by reducing ticket prices can negatively affect travel comfort. In systems that charge the passengers more, comfortable travel may become more possible with a decrease in demand. Therefore, the demand sourced by the transportation pricing system is very effective in terms of travel comfort.

Switch to a personal vehicle (C_6): When public transportation is not accessible and ticket prices are high, people switch to personal vehicles. This situation causes an increase in the number of vehicles in traffic and thus an increase in environmental pollution.

Personal mobility (C_7): Mobility is the ability of people to move from one place to another easily and freely. As the ticket prices increase, the demand for commuters towards public transportation decreases, which eventually decreases the frequency of public transportation trips. This will increase waiting times and negatively affect personal mobility. Providing better mobility to passengers provides a positive effect in terms of the preferability of the public transportation systems.

iii) Social Aspect

Environmental cost (C_8): This criterion deals with the environmental pollution associated with the public transportation system. Many factors are affecting pollution, such as public transportation trip frequency and lengths of the trips. As the public transportation trips concentrate on shorter lines, the environmental cost of the system reduces. The pricing system of public transportation is very effective to adjust the traveling behaviors of the commuters by increasing the prices of long lines of travel. A pricing system that causes less environmental pollution is always more attractive.

Life quality (C_9): The quality of life of public transportation commuters is highly affected by accessibility, travel times, travel comfort, and the price that they pay for tickets since it is a constant expense of public transportation users.

Decentralization of cities (C_{10}): The centralization of a city provides a good number of benefits in terms of both public transportation systems and general living conditions. As centralization-sensitive pricing is used in public transportation systems such as distance-based, rent-based, and zonal pricing, people live closer to their work and social environment. This shortens the lengths of transportation lines and reduces the operating costs of vehicles. Therefore, a pricing system that creates centralization in cities is more beneficial.

iv) Political Aspect

Public acceptability (C_{11}): This criterion is about the acceptability of the applied pricing system by the commuters. Passengers view high prices negatively since they increase the financial burden of transportation. Low pricing of tickets increases the demand, which eventually decreases the level of in-vehicle service, and passengers are most likely not to accept it. Therefore, the pricing system must be selected carefully.

Political success (C_{12}): This criterion refers to the success that municipalities will achieve for the sustainability of the implemented public transportation pricing policy and its acceptance by the public.

Deployment of resources (C_{13}): A system whose operating costs can be covered by the pricing system will no longer be a burden to the municipality and will even provide profit. Thus, both sustainability will be provided and resources for other investments will be created or existing resources will remain sufficient. A system where resources are used properly, and that makes a profit, is more attractive to both the operators and the public.

4. Methodology

This section gives some preliminaries and presents the hybrid type-2 neutrosophic model for public transportation pricing.

4.1. Preliminaries

Definition 1 (Abdel-Basset et al., 2019). Consider Y as the limited universe of discourse and $D[0, 1]$ as the set of all triangular neutrosophic sets on $D[0, 1]$. A type-2 neutrosophic number set (T2NNS) represented by $\tilde{M}$ can be defined in Y as an object having the form:

$$\tilde{M} = \left\{ \left\langle y, \tilde{T}_{\tilde{M}}(y), \tilde{I}_{\tilde{M}}(y), \tilde{F}_{\tilde{M}}(y) \mid y \in Y \right\rangle \right\}, \tag{1}$$

where $\tilde{T}_{\tilde{M}}(y): Y \to D[0,1]$, $\tilde{I}_{\tilde{M}}(y): Y \to D[0,1]$, and $\tilde{F}_{\tilde{M}}(y): Y \to D[0,1]$. A T2NNS $\tilde{T}_{\tilde{M}}(y) = \left(T_{T_{\tilde{M}}}(y), T_{I_{\tilde{M}}}(y), T_{F_{\tilde{M}}}(y) \right)$, $\tilde{I}_{\tilde{M}}(y) = \left(I_{T_{\tilde{M}}}(y), I_{I_{\tilde{M}}}(y), I_{F_{\tilde{M}}}(y) \right)$, and $\tilde{F}_{\tilde{M}}(y) = \left(F_{T_{\tilde{M}}}(y), F_{I_{\tilde{M}}}(y), F_{F_{\tilde{M}}}(y) \right)$, denote the truth, indeterminacy, and falsity memberships of y in $\tilde{M}$, respectively. The membership parameters satisfy the condition:

$$0 \le \tilde{T}_{\tilde{M}}(y)^3 + \tilde{I}_{\tilde{M}}(y)^3 + \tilde{F}_{\tilde{M}}(y)^3 \le 3, \quad \forall y \in Y. \tag{2}$$

For ease of simplicity, we consider $\tilde{M} = \left\langle \left(T_{T_{\tilde{M}}}(y), T_{I_{\tilde{M}}}(y), T_{F_{\tilde{M}}}(y) \right), \left(I_{T_{\tilde{M}}}(y), I_{I_{\tilde{M}}}(y), I_{F_{\tilde{M}}}(y) \right), \left(F_{T_{\tilde{M}}}(y), F_{I_{\tilde{M}}}(y), F_{F_{\tilde{M}}}(y) \right) \right\rangle$, as a T2NN.

Definition 2 (Abdel-Basset et al., 2019). Let $\tilde{M} = \left\langle \left(T_{T_{\tilde{M}}}(y), T_{I_{\tilde{M}}}(y), T_{F_{\tilde{M}}}(y) \right), \left(I_{T_{\tilde{M}}}(y), I_{I_{\tilde{M}}}(y), I_{F_{\tilde{M}}}(y) \right), \left(F_{T_{\tilde{M}}}(y), F_{I_{\tilde{M}}}(y), F_{F_{\tilde{M}}}(y) \right) \right\rangle$, $\tilde{M}_1 = \left\langle \left(T_{T_{\tilde{M}_1}}(y), T_{I_{\tilde{M}_1}}(y), T_{F_{\tilde{M}_1}}(y) \right), \left(I_{T_{\tilde{M}_1}}(y), I_{I_{\tilde{M}_1}}(y), I_{F_{\tilde{M}_1}}(y) \right), \left(F_{T_{\tilde{M}_1}}(y), F_{I_{\tilde{M}_1}}(y), F_{F_{\tilde{M}_1}}(y) \right) \right\rangle$, and $\tilde{M}_2 = \left\langle \left(T_{T_{\tilde{M}_2}}(y), T_{I_{\tilde{M}_2}}(y), T_{F_{\tilde{M}_2}}(y) \right), \left(I_{T_{\tilde{M}_2}}(y), I_{I_{\tilde{M}_2}}(y), I_{F_{\tilde{M}_2}}(y) \right), \left(F_{T_{\tilde{M}_2}}(y), F_{I_{\tilde{M}_2}}(y), F_{F_{\tilde{M}_2}}(y) \right) \right\rangle$ be three T2NNs and $\lambda > 0$. Their operations are defined as follows:

(a) Addition "⊕"

$$\begin{aligned} \tilde{M}_1 \oplus \tilde{M}_2 = \Big\langle \Big(& T_{T_{\tilde{M}_1}}(y) + T_{T_{\tilde{M}_2}}(y) - T_{T_{\tilde{M}_1}}(y) \times T_{T_{\tilde{M}_2}}(y), T_{I_{\tilde{M}_1}}(y) + T_{I_{\tilde{M}_2}}(y) \\ & - T_{I_{\tilde{M}_1}}(y) \times T_{I_{\tilde{M}_2}}(y), T_{F_{\tilde{M}_1}}(y) + T_{F_{\tilde{M}_2}}(y) - T_{F_{\tilde{M}_1}}(y) \times T_{F_{\tilde{M}_2}}(y) \Big), \\ & \Big(I_{T_{\tilde{M}_1}}(y) \times I_{T_{\tilde{M}_2}}(y), I_{I_{\tilde{M}_1}}(y) \times I_{I_{\tilde{M}_2}}(y), I_{F_{\tilde{M}_1}}(y) \times I_{F_{\tilde{M}_2}}(y) \Big), \\ & \Big(F_{T_{\tilde{M}_1}}(y) \times F_{T_{\tilde{M}_2}}(y), F_{I_{\tilde{M}_1}}(y) \times F_{I_{\tilde{M}_2}}(y), F_{F_{\tilde{M}_1}}(y) \times F_{F_{\tilde{M}_2}}(y) \Big) \Big\rangle, \end{aligned} \tag{3}$$

(b) Multiplication "⊗"

$$\tilde{M}_1 \otimes \tilde{M}_2 = \Big\langle \Big(T_{T_{\tilde{M}_1}}(y) \times T_{T_{\tilde{M}_2}}(y), T_{I_{\tilde{M}_1}}(y) \times T_{I_{\tilde{M}_2}}(y), T_{F_{\tilde{M}_1}}(y) \times T_{F_{\tilde{M}_2}}(y) \Big),$$
$$\Big(T_{T_{\tilde{M}_1}}(y) + T_{T_{\tilde{M}_2}}(y) - T_{T_{\tilde{M}_1}}(y) \times T_{T_{\tilde{M}_2}}(y), T_{I_{\tilde{M}_1}}(y) + T_{I_{\tilde{M}_2}}(y)$$
$$- T_{I_{\tilde{M}_1}}(y) \times T_{I_{\tilde{M}_2}}(y), T_{F_{\tilde{M}_1}}(y) + T_{F_{\tilde{M}_2}}(y) - T_{F_{\tilde{M}_1}}(y) \times T_{F_{\tilde{M}_2}}(y) \Big), \tag{4}$$
$$\Big(T_{T_{\tilde{M}_1}}(y) + T_{T_{\tilde{M}_2}}(y) - T_{T_{\tilde{M}_1}}(y) \times T_{T_{\tilde{M}_2}}(y), T_{I_{\tilde{M}_1}}(y) + T_{I_{\tilde{M}_2}}(y)$$
$$- T_{I_{\tilde{M}_1}}(y) \times T_{I_{\tilde{M}_2}}(y), T_{F_{\tilde{M}_1}}(y) + T_{F_{\tilde{M}_2}}(y) - T_{F_{\tilde{M}_1}}(y) \times T_{F_{\tilde{M}_2}}(y) \Big) \Big\rangle,$$

(c) Scalar multiplication

$$\lambda \tilde{M} = \Big\langle \Big(1 - \big(1 - T_{T_{\tilde{M}}}(y)\big)^{\lambda}, 1 - \big(1 - T_{I_{\tilde{M}}}(y)\big)^{\lambda}, 1 - \big(1 - T_{F_{\tilde{M}}}(y)\big)^{\lambda} \Big),$$
$$\Big(\big(I_{T_{\tilde{M}}}(y)\big)^{\lambda}, \big(I_{I_{\tilde{M}}}(y)\big)^{\lambda}, \big(I_{F_{\tilde{M}}}(y)\big)^{\lambda} \Big), \tag{5}$$
$$\Big(\big(F_{T_{\tilde{M}}}(y)\big)^{\lambda}, \big(F_{I_{\tilde{M}}}(y)\big)^{\lambda}, \big(F_{F_{\tilde{M}}}(y)\big)^{\lambda} \Big) \Big\rangle,$$

(d) Power

$$\tilde{M}^{\lambda} = \Big\langle \Big(\big(T_{T_{\tilde{M}}}(y)\big)^{\lambda}, \big(T_{I_{\tilde{M}}}(y)\big)^{\lambda}, \big(T_{F_{\tilde{M}}}(y)\big)^{\lambda} \Big),$$
$$\Big(1 - \big(1 - I_{T_{\tilde{M}}}(y)\big)^{\lambda}, 1 - \big(1 - I_{I_{\tilde{M}}}(y)\big)^{\lambda}, 1 - \big(1 - I_{F_{\tilde{M}}}(y)\big)^{\lambda} \Big), \tag{6}$$
$$\Big(1 - \big(1 - F_{T_{\tilde{M}}}(y)\big)^{\lambda}, 1 - \big(1 - F_{I_{\tilde{M}}}(y)\big)^{\lambda}, 1 - \big(1 - F_{F_{\tilde{M}}}(y)\big)^{\lambda} \Big) \Big\rangle.$$

Definition 3 (Abdel-Basset et al., 2019). Suppose that $\tilde{M}_l = \Big\langle \Big(T_{T_{\tilde{M}_l}}(y), T_{I_{\tilde{M}_l}}(y), T_{F_{\tilde{M}_l}}(y) \Big),$ $\Big(I_{T_{\tilde{M}_l}}(y), I_{I_{\tilde{M}_l}}(y), I_{F_{\tilde{M}_l}}(y) \Big), \Big(F_{T_{\tilde{M}_l}}(y), F_{I_{\tilde{M}_l}}(y), F_{F_{\tilde{M}_l}}(y) \Big) \Big\rangle$ (l=1, …, p) is a collection of T2NNs, and $\gamma=(\gamma_1, ..., \gamma_p)^T$ be the weight vector of them, with $\gamma_l \in [0, 1]$ and $\sum_{l=1}^{p} \gamma_l = 1$. A type-2 neutrosophic number weighted averaging (T2NNWA) operator is defined as follows:

$$
\begin{aligned}
&T2NNWA_{\gamma}(\tilde{M}_1,...,\tilde{M}_l,...,\tilde{M}_p)=\gamma_1\tilde{M}_1\oplus\cdots\oplus\gamma_l\tilde{M}_l\oplus\cdots\oplus\gamma_p\tilde{M}_p=\overset{p}{\underset{l=1}{\oplus}}\gamma_l\tilde{M}_l\\
&=\Bigg\langle\Bigg(1-\prod_{l=1}^{p}\left(1-T_{T_{\tilde{M}_l}}(y)\right)^{\gamma_l},1-\prod_{l=1}^{p}\left(1-T_{I_{\tilde{M}_l}}(y)\right)^{\gamma_l},1-\prod_{l=1}^{p}\left(1-T_{F_{\tilde{M}_l}}(y)\right)^{\gamma_l}\Bigg),\\
&\quad\left(\prod_{l=1}^{p}\left(I_{T_{\tilde{M}_l}}(y)\right)^{\gamma_l},\prod_{l=1}^{p}\left(I_{I_{\tilde{M}_l}}(y)\right)^{\gamma_l},\prod_{l=1}^{p}\left(I_{F_{\tilde{M}_l}}(y)\right)^{\gamma_l}\right),\\
&\quad\left(\prod_{l=1}^{p}\left(F_{T_{\tilde{M}_l}}(y)\right)^{\gamma_l},\prod_{l=1}^{p}\left(F_{I_{\tilde{M}_l}}(y)\right)^{\gamma_l},\prod_{l=1}^{p}\left(F_{F_{\tilde{M}_l}}(y)\right)^{\gamma_l}\right)\Bigg\rangle.
\end{aligned}
\tag{7}
$$

Definition 4 (Abdel-Basset et al., 2019). Let $\tilde{M}=\Big\langle\left(T_{T_{\tilde{M}}}(y),T_{I_{\tilde{M}}}(y),T_{F_{\tilde{M}}}(y)\right),$ $\left(I_{T_{\tilde{M}}}(y),I_{I_{\tilde{M}}}(y),I_{F_{\tilde{M}}}(y)\right),\left(F_{T_{\tilde{M}}}(y),F_{I_{\tilde{M}}}(y),F_{F_{\tilde{M}}}(y)\right)\Big\rangle$ be a T2NN. The score function of $\tilde{M}$ is defined as follows:

$$
\begin{aligned}
S(\tilde{M})=\frac{1}{12}\Big\langle 8&+\left(T_{T_{\tilde{M}}}(y)+2\left(T_{I_{\tilde{M}}}(y)\right)+T_{F_{\tilde{M}}}(y)\right)-\left(I_{T_{\tilde{M}}}(y)+2\left(I_{I_{\tilde{M}}}(y)\right)+I_{F_{\tilde{M}}}(y)\right)\\
&-\left(F_{T_{\tilde{M}}}(y)+2\left(F_{I_{\tilde{M}}}(y)\right)+F_{F_{\tilde{M}}}(y)\right)\Big\rangle.
\end{aligned}
\tag{8}
$$

4.2 The Hybrid Type-2 Neutrosophic Model for Public Transportation Pricing

Fig. 2 presents the flowchart of the hybrid type-2 neutrosophic model for public transportation pricing. The model has two stages. In the first stage, the reputation of the invited experts is determined under the T2NN environment by making the trade-off between their experience and expertise. In the second stage, the T2NN-CRITIC-MABAC approach is formulated to solve the investigated MCDM problem. More detailed, the novel T2NN-CRITIC method is employed to evaluate the criteria importance, while the new T2NN-MABAC method is used to rank the alternatives.

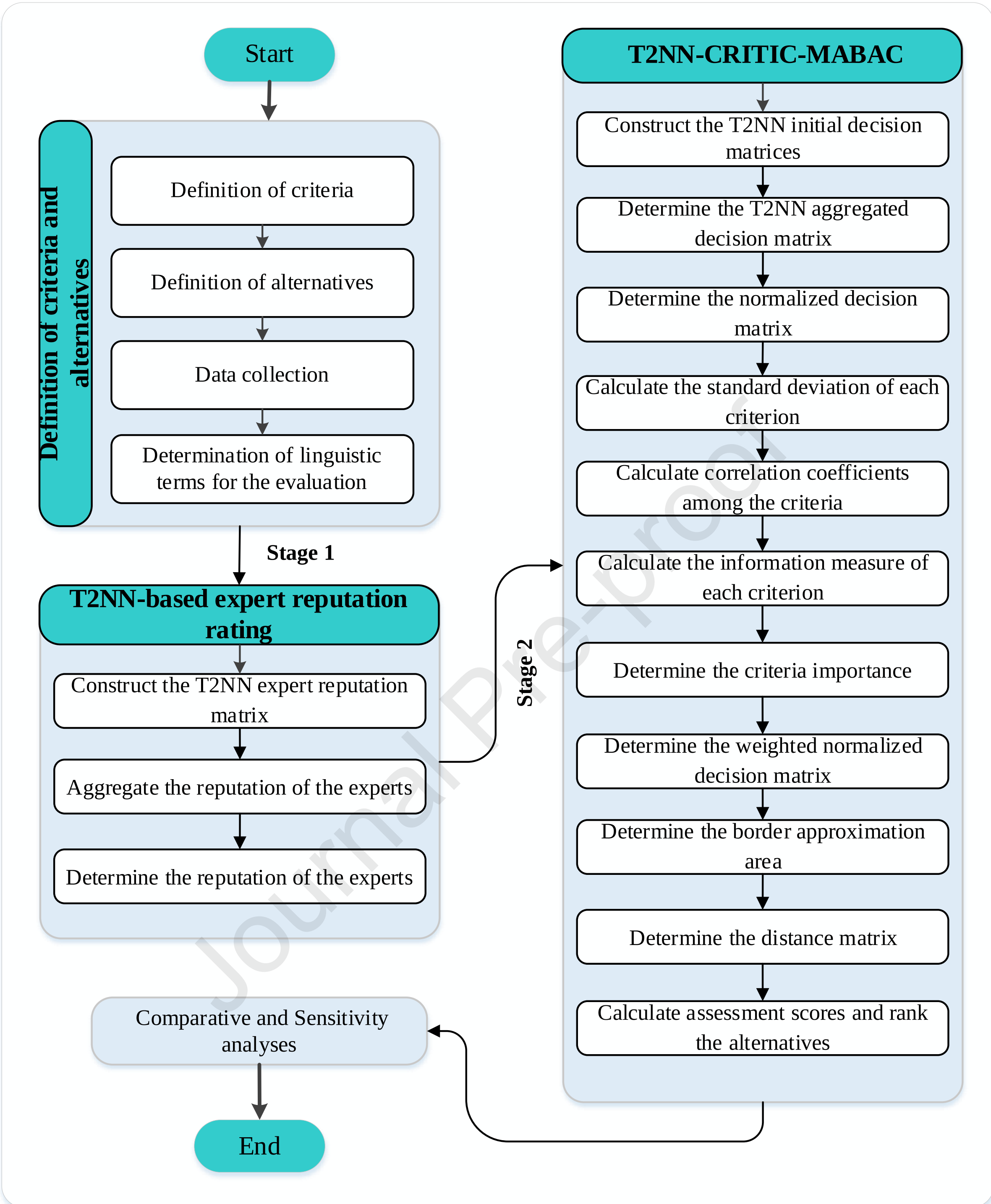

Fig. 2. The flowchart of the hybrid type-2 neutrosophic model for public transportation pricing.

Let A={A_1, …, A_i, …, A_m} (m≥2) be a finite set of alternatives, C={C_1, …, C_j, …, C_n} (n≥2) be a finite set of criteria, and D={D_1, …, D_e, …, D_k} (k≥2) is a set of invited experts. The stages and encompassed steps of the introduced hybrid type-2 neutrosophic model for public transportation pricing are given in the following:

Stage 1: T2NN-based expert reputation rating.

Step 1.1. *Construct the T2NN expert reputation matrix* $\tilde{\mathrm{V}} = \left[\tilde{V}_e^{(l)}\right]_{2\times k}$:

$$\tilde{\mathrm{V}} = \begin{array}{c} \begin{array}{ccc} D_1 & \cdots & D_k \end{array} \\ \left[\begin{array}{ccc} \left\langle \begin{array}{l} \left(T_{T_{\tilde{V}_1^{(1)}}}(y), T_{I_{\tilde{V}_1^{(1)}}}(y), T_{F_{\tilde{V}_1^{(1)}}}(y)\right), \\ \left(I_{T_{\tilde{V}_1^{(1)}}}(y), I_{I_{\tilde{V}_1^{(1)}}}(y), I_{F_{\tilde{V}_1^{(1)}}}(y)\right), \\ \left(F_{T_{\tilde{V}_1^{(1)}}}(y), F_{I_{\tilde{V}_1^{(1)}}}(y), F_{F_{\tilde{V}_1^{(1)}}}(y)\right) \end{array} \right\rangle & \cdots & \left\langle \begin{array}{l} \left(T_{T_{\tilde{V}_k^{(1)}}}(y), T_{I_{\tilde{V}_k^{(1)}}}(y), T_{F_{\tilde{V}_k^{(1)}}}(y)\right), \\ \left(I_{T_{\tilde{V}_k^{(1)}}}(y), I_{I_{\tilde{V}_k^{(1)}}}(y), I_{F_{\tilde{V}_k^{(1)}}}(y)\right), \\ \left(F_{T_{\tilde{V}_k^{(1)}}}(y), F_{I_{\tilde{V}_k^{(1)}}}(y), F_{F_{\tilde{V}_k^{(1)}}}(y)\right) \end{array} \right\rangle \\ \left\langle \begin{array}{l} \left(T_{T_{\tilde{V}_1^{(2)}}}(y), T_{I_{\tilde{V}_1^{(2)}}}(y), T_{F_{\tilde{V}_1^{(2)}}}(y)\right), \\ \left(I_{T_{\tilde{V}_1^{(2)}}}(y), I_{I_{\tilde{V}_1^{(2)}}}(y), I_{F_{\tilde{V}_1^{(2)}}}(y)\right), \\ \left(F_{T_{\tilde{V}_1^{(2)}}}(y), F_{I_{\tilde{V}_1^{(2)}}}(y), F_{F_{\tilde{V}_1^{(2)}}}(y)\right) \end{array} \right\rangle & \cdots & \left\langle \begin{array}{l} \left(T_{T_{\tilde{V}_k^{(2)}}}(y), T_{I_{\tilde{V}_k^{(2)}}}(y), T_{F_{\tilde{V}_k^{(2)}}}(y)\right), \\ \left(I_{T_{\tilde{V}_k^{(2)}}}(y), I_{I_{\tilde{V}_k^{(2)}}}(y), I_{F_{\tilde{V}_k^{(2)}}}(y)\right), \\ \left(F_{T_{\tilde{V}_k^{(2)}}}(y), F_{I_{\tilde{V}_k^{(2)}}}(y), F_{F_{\tilde{V}_k^{(2)}}}(y)\right) \end{array} \right\rangle \end{array}\right] \end{array}, \quad (9)$$

where $\tilde{V}_e^{(1)} = \left\langle \left(T_{T_{\tilde{V}_e^{(1)}}}(y), T_{I_{\tilde{V}_e^{(1)}}}(y), T_{F_{\tilde{V}_e^{(1)}}}(y)\right), \left(I_{T_{\tilde{V}_e^{(1)}}}(y), I_{I_{\tilde{V}_e^{(1)}}}(y), I_{F_{\tilde{V}_e^{(1)}}}(y)\right), \left(F_{T_{\tilde{V}_e^{(1)}}}(y), F_{I_{\tilde{V}_e^{(1)}}}(y), F_{F_{\tilde{V}_e^{(1)}}}(y)\right) \right\rangle$

and $\tilde{V}_e^{(2)} = \left\langle \left(T_{T_{\tilde{V}_e^{(2)}}}(y), T_{I_{\tilde{V}_e^{(2)}}}(y), T_{F_{\tilde{V}_e^{(2)}}}(y)\right), \left(I_{T_{\tilde{V}_e^{(2)}}}(y), I_{I_{\tilde{V}_e^{(2)}}}(y), I_{F_{\tilde{V}_e^{(2)}}}(y)\right), \left(F_{T_{\tilde{V}_e^{(2)}}}(y), F_{I_{\tilde{V}_e^{(2)}}}(y), F_{F_{\tilde{V}_e^{(2)}}}(y)\right) \right\rangle$,

are T2NNs that represent the (self-)appraisal of the experience and the expertise of the invited expert D_e (e=1, ..., k), respectively. The seven-point T2NN linguistic scale presented in Table 3 can be used to distinct experts in accordance with their experience and expertise.

Table 3
T2NN linguistic variables to distinct experts.

Experience (years)	**Expertise**	**Type-2 neutrosophic number**
5<	Very poor	<(0.20, 0.20, 0.10), (0.65, 0.80, 0.85), (0.45, 0.80, 0.70)>
[5, 10)	Poor	<(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)>
[10, 15)	Medium poor	<(0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60)>
[15, 20)	Medium	<(0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45)>
[20, 25)	Medium good	<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>
[25, 30)	Good	<(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)>
≥30	Very good	<(0.95, 0.90, 0.95), (0.10, 0.10, 0.05), (0.05, 0.05, 0.05)>

Step 1.2. *Aggregate the reputation of the experts*:

$$\tilde{Q}_e = T2NNWA_{\xi}(\tilde{V}_e^{(1)}, \tilde{V}_e^{(2)}) = \xi_1\tilde{V}_e^{(1)} \oplus \xi_2\tilde{V}_e^{(2)} = \mathop{\oplus}_{l=1}^{2} \xi_l\tilde{V}_e^{(l)}$$

$$= \left\langle \left(1-\prod_{l=1}^{2}\left(1-T_{T_{\tilde{V}_e^{(l)}}}(y)\right)^{\xi_l}, 1-\prod_{l=1}^{2}\left(1-T_{I_{\tilde{V}_e^{(l)}}}(y)\right)^{\xi_l}, 1-\prod_{l=1}^{2}\left(1-T_{F_{\tilde{V}_e^{(l)}}}(y)\right)^{\xi_l} \right), \right.$$
$$\left(\prod_{l=1}^{2}\left(I_{T_{\tilde{V}_e^{(l)}}}(y)\right)^{\xi_l}, \prod_{l=1}^{2}\left(I_{I_{\tilde{V}_e^{(l)}}}(y)\right)^{\xi_l}, \prod_{l=1}^{2}\left(I_{F_{\tilde{V}_e^{(l)}}}(y)\right)^{\xi_l} \right), \tag{10}$$
$$\left. \left(\prod_{l=1}^{2}\left(F_{T_{\tilde{V}_e^{(l)}}}(y)\right)^{\xi_l}, \prod_{l=1}^{2}\left(F_{I_{\tilde{V}_e^{(l)}}}(y)\right)^{\xi_l}, \prod_{l=1}^{2}\left(F_{F_{\tilde{V}_e^{(l)}}}(y)\right)^{\xi_l} \right) \right\rangle, \ e=1,\ldots,k,$$

where $\tilde{Q}_e = \left\langle \left(T_{T_{\tilde{Q}_e}}(y), T_{I_{\tilde{Q}_e}}(y), T_{F_{\tilde{Q}_e}}(y)\right), \left(I_{T_{\tilde{Q}_e}}(y), I_{I_{\tilde{Q}_e}}(y), I_{F_{\tilde{Q}_e}}(y)\right), \left(F_{T_{\tilde{Q}_e}}(y), F_{I_{\tilde{Q}_e}}(y), F_{F_{\tilde{Q}_e}}(y)\right) \right\rangle$ is the T2NN aggregated reputation of the invited expert D_e based on the T2NNWA operator. In Eq. (10), ξ_1 and ξ_2 denotes the trade-off parameters of the reputation of the invited experts, with $\xi_1, \xi_2 \in [0,1]$ and $\xi_1+\xi_2=1$. If the trade-off parameter ξ_1 is 1 (i.e., ξ_2=0), then the reputation of the invited experts is determined according to their experience. If the value of ξ_2 is set to 1, then the reputation of the invited experts is in line with their expertise. If $\xi_1=\xi_2=0.5$, then the experience and expertise of the invited experts are equally appraised in the hybrid type-2 neutrosophic model.

Step 1.3. *Determine the reputation of the experts*:

$$\delta_e = \frac{S\left(\tilde{Q}_e\right)}{\sum_{l=1}^{k} S\left(\tilde{Q}_l\right)}, \ e=1,\ldots,k, \tag{11}$$

where $\delta = (\delta_1,\ldots,\delta_e,\ldots,\delta_k)^T$ is the reputation vector of the invited experts, with $\delta_e \in [0,1]$ (e=1, …, k), and $\sum_{e=1}^{k}\delta_e = 1$. The score function of the T2NN aggregated reputation for each invited expert is computed as follows:

$$S\left(\tilde{Q}_e\right) = \frac{1}{12}\left\langle 8 + \left(T_{T_{\tilde{Q}_e}}(y) + 2\left(T_{I_{\tilde{Q}_e}}(y)\right) + T_{F_{\tilde{Q}_e}}(y)\right) - \left(I_{T_{\tilde{Q}_e}}(y) + 2\left(I_{I_{\tilde{Q}_e}}(y)\right) + I_{F_{\tilde{Q}_e}}(y)\right) \right.$$
$$\left. - \left(F_{T_{\tilde{Q}_e}}(y) + 2\left(F_{I_{\tilde{Q}_e}}(y)\right) + F_{F_{\tilde{Q}_e}}(y)\right) \right\rangle, \ e=1,\ldots,k. \tag{12}$$

Stage 2: T2NN-CRITIC-MABAC.

Step 2.1. *Construct the T2NN initial decision matrices* $\tilde{\mathrm{B}}^e = \left[\tilde{B}_{ij}^e\right]_{m \times n}$:

$$\tilde{\mathrm{B}}^e = \begin{matrix} \\ A_1 \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} C_1 & \cdots & C_n \\ \left\langle \begin{matrix} \left(T_{T_{\tilde{B}_{11}^e}}(y), T_{I_{\tilde{B}_{11}^e}}(y), T_{F_{\tilde{B}_{11}^e}}(y)\right), \\ \left(I_{T_{\tilde{B}_{11}^e}}(y), I_{I_{\tilde{B}_{11}^e}}(y), I_{F_{\tilde{B}_{11}^e}}(y)\right), \\ \left(F_{T_{\tilde{B}_{11}^e}}(y), F_{I_{\tilde{B}_{11}^e}}(y), F_{F_{\tilde{B}_{11}^e}}(y)\right) \end{matrix} \right\rangle & \cdots & \left\langle \begin{matrix} \left(T_{T_{\tilde{B}_{1n}^e}}(y), T_{I_{\tilde{B}_{1n}^e}}(y), T_{F_{\tilde{B}_{1n}^e}}(y)\right), \\ \left(I_{T_{\tilde{B}_{1n}^e}}(y), I_{I_{\tilde{B}_{1n}^e}}(y), I_{F_{\tilde{B}_{1n}^e}}(y)\right), \\ \left(F_{T_{\tilde{B}_{1n}^e}}(y), F_{I_{\tilde{B}_{1n}^e}}(y), F_{F_{\tilde{B}_{1n}^e}}(y)\right) \end{matrix} \right\rangle \\ \vdots & \ddots & \vdots \\ \left\langle \begin{matrix} \left(T_{T_{\tilde{B}_{m1}^e}}(y), T_{I_{\tilde{B}_{m1}^e}}(y), T_{F_{\tilde{B}_{m1}^e}}(y)\right), \\ \left(I_{T_{\tilde{B}_{m1}^e}}(y), I_{I_{\tilde{B}_{m1}^e}}(y), I_{F_{\tilde{B}_{m1}^e}}(y)\right), \\ \left(F_{T_{\tilde{B}_{m1}^e}}(y), F_{I_{\tilde{B}_{m1}^e}}(y), F_{F_{\tilde{B}_{m1}^e}}(y)\right) \end{matrix} \right\rangle & \cdots & \left\langle \begin{matrix} \left(T_{T_{\tilde{B}_{mn}^e}}(y), T_{I_{\tilde{B}_{mn}^e}}(y), T_{F_{\tilde{B}_{mn}^e}}(y)\right), \\ \left(I_{T_{\tilde{B}_{mn}^e}}(y), I_{I_{\tilde{B}_{mn}^e}}(y), I_{F_{\tilde{B}_{mn}^e}}(y)\right), \\ \left(F_{T_{\tilde{B}_{mn}^e}}(y), F_{I_{\tilde{B}_{mn}^e}}(y), F_{F_{\tilde{B}_{mn}^e}}(y)\right) \end{matrix} \right\rangle \end{bmatrix}, \quad (13)$$

$e = 1, \ldots, k,$

where $\tilde{B}_{ij}^e = \left\langle \left(T_{T_{\tilde{B}_{ij}^e}}(y), T_{I_{\tilde{B}_{ij}^e}}(y), T_{F_{\tilde{B}_{ij}^e}}(y)\right), \left(I_{T_{\tilde{B}_{ij}^e}}(y), I_{I_{\tilde{B}_{ij}^e}}(y), I_{F_{\tilde{B}_{ij}^e}}(y)\right), \left(F_{T_{\tilde{B}_{ij}^e}}(y), F_{I_{\tilde{B}_{ij}^e}}(y), F_{F_{\tilde{B}_{ij}^e}}(y)\right) \right\rangle,$

(i=1, ..., m; j=1, ..., n; e=1, ..., k) is a T2NN that represents the evaluation of the alternative A_i under the criterion C_j given by the invited expert D_e. The initial decision matrices are structured by using a T2NN linguistic scale. The seven-point T2NN linguistic scale presented in Table 4 can be used to present alternative evaluation preferences of the invited experts. In this study, T2NN linguistic scales are utilized to help experts to express their opinions, since some of them may not have sufficient experience with neutrosophic numbers. As a result, the experts do not give direct values of truth, indeterminacy, and falsity memberships. Instead, they provide linguistic evaluations to make this approach more practical in real-life applications.

Table 4
T2NN linguistic variables for evaluating alternatives.

Linguistic variable	Type-2 neutrosophic number
Very low	<(0.20, 0.20, 0.10), (0.65, 0.80, 0.85), (0.45, 0.80, 0.70)>
Low	<(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)>
Medium low	<(0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60)>
Medium	<(0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45)>
Medium high	<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>
High	<(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)>

Very high	<(0.95, 0.90, 0.95), (0.10, 0.10, 0.05), (0.05, 0.05, 0.05)>

Step 2.2. *Determine the T2NN aggregated decision matrix* $\tilde{Z} = \left[\tilde{Z}_{ij}\right]_{m \times n}$:

$$\tilde{Z}_{ij} = T2NNWA_{\delta}(\tilde{B}_{ij}^{1}, ..., \tilde{B}_{ij}^{e}, ..., \tilde{B}_{ij}^{k}) = \mathop{\oplus}_{e=1}^{k} \delta_e \tilde{B}_{ij}^{e}$$

$$= \left\langle \left(1 - \prod_{e=1}^{k}\left(1 - T_{T_{\tilde{B}_{ij}^{e}}}(y)\right)^{\delta_e}, 1 - \prod_{e=1}^{k}\left(1 - T_{I_{\tilde{B}_{ij}^{e}}}(y)\right)^{\delta_e}, 1 - \prod_{e=1}^{k}\left(1 - T_{F_{\tilde{B}_{ij}^{e}}}(y)\right)^{\delta_e} \right), \right.$$
$$\left(\prod_{e=1}^{k}\left(I_{T_{\tilde{B}_{ij}^{e}}}(y)\right)^{\delta_e}, \prod_{e=1}^{k}\left(I_{I_{\tilde{B}_{ij}^{e}}}(y)\right)^{\delta_e}, \prod_{e=1}^{k}\left(I_{F_{\tilde{B}_{ij}^{e}}}(y)\right)^{\delta_e} \right), \tag{14}$$
$$\left. \left(\prod_{e=1}^{k}\left(F_{T_{\tilde{B}_{ij}^{e}}}(y)\right)^{\delta_e}, \prod_{e=1}^{k}\left(F_{I_{\tilde{B}_{ij}^{e}}}(y)\right)^{\delta_e}, \prod_{e=1}^{k}\left(F_{F_{\tilde{B}_{ij}^{e}}}(y)\right)^{\delta_e} \right) \right\rangle, \; i = 1, \ldots, m; \; j = 1, \ldots, n,$$

where the aggregation is determined by applying the T2NNWA operator (Definition 3), $\tilde{Z}_{ij} = \left\langle \left(T_{T_{\tilde{Z}_{ij}}}(y), T_{I_{\tilde{Z}_{ij}}}(y), T_{F_{\tilde{Z}_{ij}}}(y) \right), \left(I_{T_{\tilde{Z}_{ij}}}(y), I_{I_{\tilde{Z}_{ij}}}(y), I_{F_{\tilde{Z}_{ij}}}(y) \right), \left(F_{T_{\tilde{Z}_{ij}}}(y), F_{I_{\tilde{Z}_{ij}}}(y), F_{F_{\tilde{Z}_{ij}}}(y) \right) \right\rangle$ is the T2NN aggregated evaluation of the alternative A_i under the criterion C_j given by the experts.

Step 2.3. *Determine the normalized decision matrix* $\mathrm{R} = [R_{ij}]_{m \times n}$:

$$R_{ij} = \begin{cases} \dfrac{S\left(\tilde{Z}_{ij}\right) - \min\limits_{1 \le l \le m} S\left(\tilde{Z}_{lj}\right)}{\max\limits_{1 \le l \le m} S\left(\tilde{Z}_{lj}\right) - \min\limits_{1 \le l \le m} S\left(\tilde{Z}_{lj}\right)} & \Big| \; C_j \in \mathrm{C}^{+} \\ \dfrac{\max\limits_{1 \le l \le m} S\left(\tilde{Z}_{lj}\right) - S\left(\tilde{Z}_{ij}\right)}{\max\limits_{1 \le l \le m} S\left(\tilde{Z}_{lj}\right) - \min\limits_{1 \le l \le m} S\left(\tilde{Z}_{lj}\right)} & \Big| \; C_j \in \mathrm{C}^{-} \end{cases}, \; i = 1, \ldots, m; \; j = 1, \ldots, n, \tag{15}$$

where R_{ij} denotes a normalized evaluation of the alternative A_i under the criterion C_j given by the experts, $\mathrm{C}^{+} \subseteq \mathrm{C}$ is the set of benefit criteria, $\mathrm{C}^{-} \subseteq \mathrm{C}$ is the set of cost criteria, and $\mathrm{C}^{+} \cup \mathrm{C}^{-} = \mathrm{C}$. The score function of the T2NN aggregated evaluation of the alternative A_i under the criterion C_j given by the experts is calculated as follows:

$$S\left(\tilde{Z}_{ij}\right) = \frac{1}{12} \left\langle 8 + \left(T_{T_{\tilde{Z}_{ij}}}(y) + 2\left(T_{I_{\tilde{Z}_{ij}}}(y)\right) + T_{F_{\tilde{Z}_{ij}}}(y) \right) - \left(I_{T_{\tilde{Z}_{ij}}}(y) + 2\left(I_{I_{\tilde{Z}_{ij}}}(y)\right) + I_{F_{\tilde{Z}_{ij}}}(y) \right) \right.$$
$$\left. - \left(F_{T_{\tilde{Z}_{ij}}}(y) + 2\left(F_{I_{\tilde{Z}_{ij}}}(y)\right) + F_{F_{\tilde{Z}_{ij}}}(y) \right) \right\rangle, \; i = 1, \ldots, m; \; j = 1, \ldots, n. \tag{16}$$

Step 2.4. *Calculate the standard deviation of each criterion*:

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{m}\left(R_{ij} - \bar{R}_j\right)^2}{m}}, \; j = 1, \ldots, n, \quad (17)$$

where $\bar{R}_j = \frac{1}{m}\sum_{i=1}^{m} R_{ij}$ (j=1, …, n) presents an average normalized evaluation of the alternatives under the criterion C_j given by the experts.

Step 2.5. *Calculate correlation coefficients among the criteria*:

$$\rho_{jk} = \frac{\sum_{i=1}^{m}\left[\left(R_{ij} - \bar{R}_j\right)\left(R_{ik} - \bar{R}_k\right)\right]}{\sqrt{\sum_{i=1}^{m}\left(R_{ij} - \bar{R}_j\right)^2 \sum_{i=1}^{m}\left(R_{ik} - \bar{R}_k\right)^2}}, \; j,k = 1, \ldots, n. \quad (18)$$

Step 2.6. *Calculate the information measure of each criterion*:

$$U_j = \sigma_j \sum_{l=1}^{n}\left(1 - \rho_{jl}\right), \; j = 1, \ldots, n, \quad (19)$$

where criteria of greater importance have higher values of the information measure and vice versa.

Step 2.7. *Determine the criteria importance*:

$$\omega_j = \frac{U_j}{\sum_{l=1}^{n} U_l}, \; j = 1, \ldots, n, \quad (20)$$

where $\omega = \left(\omega_1, \ldots, \omega_j, \ldots, \omega_n\right)^T$ is the importance vector of the criteria, with $\omega_j \in [0, 1]$ (j=1, …, n), and $\sum_{j=1}^{n} \omega_j = 1$.

Step 2.8. *Determine the weighted normalized decision matrix* $\mathrm{G} = \left[G_{ij}\right]_{m \times n}$:

$$G_{ij} = \omega_j\left(R_{ij} + 1\right), \; i = 1, \ldots, m; \; j = 1, \ldots, n, \quad (21)$$

where G_{ij} denotes a weighted normalized evaluation of the alternative A_i under the criterion C_j given by the experts.

Step 2.9. *Determine the border approximation area*:

$$O_j = \left(\prod_{i=1}^{m} G_{ij}\right)^{1/m}, \; j = 1, \ldots, n, \quad (22)$$

where $O=\{O_1, ..., O_j, ..., O_n\}$ represents the border approximation area given by the experts.

Step 2.10. *Determine the distance matrix* $P=[P_{ij}]_{m\times n}$:

$$P_{ij}=G_{ij}-O_j,\ i=1, \dots, m;\ j=1, \dots, n, \tag{23}$$

where P_{ij} presents a distance of the alternative A_i from the border approximation area under the criterion C_j given by the experts. The alternative A_i under the criterion C_j can belong to the border approximation area O, an upper approximation area O^+, or a lower approximation area O^-. If $P_{ij}>0$ then it belongs to O^+, if $P_{ij}=0$ then it belongs to O, and if $P_{ij}<0$ then it belongs to O^-.

Step 2.11. *Calculate assessment scores and rank the alternatives*:

$$B_i=\sum_{j=1}^{n} P_{ij},\ \ i=1, \dots, m, \tag{24}$$

where B_i represents the assessment score of the alternative A_i. The alternatives are ranked according to the decreasing values of their assessment score. The best alternative has the highest assessment score. The detailed nomenclatures for the indices, parameters, sets, matrices, and variables are provided in Appendix A.

5. Results and Discussion

5.1. Experimental Results

Stage 1: T2NN-based expert reputation rating.

Step 1.1. Five experts are invited to participate in the investigated real-life case study (Table 5). Due to the COVID-19 outbreak, the online questionnaire approach via Google Forms is used to collect information about the experts and their evaluations of the pricing systems for IZBAN.

Table 5
The information about the experts.

Expert	**Experience** (years)	**Expertise**	**Occupation**	**Profession**	**Gender**
D_1	6	Medium	Industry	Urban management	Male
D_2	13	Good	Industry	Transportation engineering	Male
D_3	9	Medium	Industry	Urban planning	Male
D_4	4	Very good	Industry	Transportation engineering	Male
D_5	10	Medium good	Industry	Urban planning	Male

The T2NN expert reputation matrix is constructed with the help of Eq. (9) and presented in Table 6. It is based on the experts' self-appraisal of their experience and expertise (Table 5) as well as the seven-point T2NN linguistic scale (Table 3).

Table 6
The T2NN expert reputation matrix.

Expert	Experience	Expertise
D_1	<(0.35, 0.35, 0.1), (0.5, 0.75, 0.8), (0.5, 0.75, 0.65)>	<(0.5, 0.45, 0.5), (0.4, 0.35, 0.5), (0.35, 0.3, 0.45)>
D_2	<(0.4, 0.3, 0.35), (0.5, 0.45, 0.6), (0.45, 0.4, 0.6)>	<(0.7, 0.75, 0.8), (0.15, 0.2, 0.25), (0.1, 0.15, 0.2)>
D_3	<(0.35, 0.35, 0.1), (0.5, 0.75, 0.8), (0.5, 0.75, 0.65)>	<(0.5, 0.45, 0.5), (0.4, 0.35, 0.5), (0.35, 0.3, 0.45)>
D_4	<(0.2, 0.2, 0.1), (0.65, 0.8, 0.85), (0.45, 0.8, 0.7)>	<(0.95, 0.9, 0.95), (0.1, 0.1, 0.05), (0.05, 0.05, 0.05)>
D_5	<(0.4, 0.3, 0.35), (0.5, 0.45, 0.6), (0.45, 0.4, 0.6)>	<(0.6, 0.45, 0.5), (0.2, 0.15, 0.25), (0.1, 0.25, 0.15)>

Step 1.2. In the base case scenario, the experience and expertise are equally appraised; i.e., the trade-off parameters ξ_1 and ξ_2 are 0.5. T2NNs that represent the experience and the expertise of the invited experts from Table 6 are aggregated with the help of the T2NNWA operator defined in Eq. (10). As a result, aggregated reputations of the experts are given in Table 7. For example, the T2NN aggregated reputation of the first expert is calculated as follows:

$$\tilde{Q}_1 = \Big\langle \Big(1-(1-0.35)^{0.5}\cdot(1-0.5)^{0.5}, 1-(1-0.35)^{0.5}\cdot(1-0.45)^{0.5}, 1-(1-0.1)^{0.5}\cdot(1-0.5)^{0.5}\Big),$$

$$\Big(0.5^{0.5}\cdot 0.4^{0.5}, 0.75^{0.5}\cdot 0.35^{0.5}, 0.8^{0.5}\cdot 0.5^{0.5}\Big), \Big(0.5^{0.5}\cdot 0.35^{0.5}, 0.75^{0.5}\cdot 0.3^{0.5}, 0.65^{0.5}\cdot 0.45^{0.5}\Big)\Big\rangle$$

=<(0.430, 0.402, 0.329), (0.447, 0.512, 0.632), (0.418, 0.474, 0.541)>.

Step 1.3. Score function values of the T2NN aggregated reputations are obtained by using Eq. (12). For example, the score function of the T2NN aggregated reputation of the first expert is computed as follows:

$$S(\tilde{Q}_1) = \frac{1}{12}\Big\langle 8 + (0.430 + 2\cdot 0.402 + 0.329) - (0.447 + 2\cdot 0.512 + 0.632) - (0.418 + 2\cdot 0.474 + 0.541)\Big\rangle = 0.463.$$

The score function values (see Table 7, Column 3) are normalized with the help of Eq. (11) to determine the reputation of the invited experts. The obtained reputation vector is $\delta = (0.155, 0.226, 0.155, 0.259, 0.205)^T$.

Table 7
The reputation of the experts.

Expert	T2NN aggregated	Score function	Reputation
D_1	<(0.430, 0.402, 0.329), (0.447, 0.512, 0.632), (0.418, 0.474, 0.541)>	0.463	0.155
D_2	<(0.576, 0.582, 0.639), (0.274, 0.300, 0.387), (0.212, 0.245, 0.346)>	0.672	0.226
D_3	<(0.430, 0.402, 0.329), (0.447, 0.512, 0.632), (0.418, 0.474, 0.541)>	0.463	0.155
D_4	<(0.800, 0.717, 0.788), (0.255, 0.283, 0.206), (0.150, 0.200, 0.187)>	0.772	0.259
D_5	<(0.510, 0.380, 0.430), (0.316, 0.260, 0.387), (0.212, 0.316, 0.300)>	0.611	0.205

Stage 2: T2NN-CRITIC-MABAC.

Step 2.1. Five invited experts used T2NN linguistic variables from Table 4 to evaluate four public transportation pricing systems under 13 criteria. Their evaluations of the pricing systems for IZBAN are provided in Table 8. The T2NN initial decision matrices are constructed based on the experts' input and with the help of Eq. (13). They are given in Table B.1 (Appendix B).

Table 8
Experts' evaluations of the public transportation pricing systems under the criteria.

Alternative	Expert	Criterion												
		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}
A_1: Flat fare	D_1	M	M	ML	MH	L	L	H	H	M	L	MH	M	L
	D_2	MH	L	L	H	L	L	H	ML	M	VH	H	H	ML
	D_3	H	ML	VL	H	L	M	VH	VH	ML	L	MH	M	L
	D_4	MH	M	ML	H	L	ML	H	H	VL	L	ML	L	VL
	D_5	MH	M	L	H	L	M	H	H	ML	L	MH	M	ML
A_2: Distance-based fare	D_1	M	M	M	M	L	ML	MH	MH	M	L	MH	MH	ML
	D_2	L	MH	H	ML	MH	VH	L	VH	MH	L	ML	M	VH
	D_3	VL	ML	M	L	H	L	H	ML	H	M	MH	MH	VH
	D_4	ML	L	M	ML	M	M	M	MH	M	M	MH	M	MH
	D_5	ML	M	M	M	MH	MH	M	M	MH	H	MH	MH	MH
A_3: Zonal pricing	D_1	L	MH	MH	M	MH	MH	M	ML	MH	H	L	L	H
	D_2	M	H	ML	ML	M	MH	VH	M	M	VH	L	L	M
	D_3	L	MH	H	M	MH	H	M	MH	MH	H	L	ML	MH
	D_4	ML	ML	H	ML	H	MH	ML	M	MH	VH	VL	L	H
	D_5	M	MH	MH	M	M	MH	ML	M	M	VH	L	ML	M
A_4: Rent-based fare	D_1	VL	MH	MH	ML	MH	MH	M	ML	MH	H	L	ML	VH
	D_2	L	VL	VL	M	ML	MH	VH	VL	VH	L	M	H	H
	D_3	VL	ML	VH	VL	VH	L	H	ML	H	M	L	M	H
	D_4	L	ML	H	L	VH	M	ML	M	MH	VH	L	L	H
	D_5	ML	ML	MH	L	H	VL	MH	M	H	VH	M	MH	MH

Very Low: VL, Low: L, Medium Low: ML, Medium: M, Medium High: MH, High: H, Very High: VH.

Step 2.2. The T2NN aggregated decision matrix is presented in Table B.2 (Appendix B). It is determined with the help of Eq. (14) by taking into account the reputation of the experts (Table 7, Column 4) and four T2NN initial decision matrices (Table B.1). For example, the T2NN evaluations of the public transportation pricing system "flat fare" (A_1) under the criterion "operation cost" (C_1) are (see Table B.1):

- expert 1 – $\tilde{B}_{11}^{1}$ =<(0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45)>,
- expert 2 – $\tilde{B}_{11}^{2}$ =<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>,
- expert 3 – $\tilde{B}_{11}^{3}$ =<(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)>,
- expert 4 – $\tilde{B}_{11}^{4}$ =<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>, and
- expert 5 – $\tilde{B}_{11}^{5}$ =<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>.

The T2NN aggregated evaluation of A_1 under C_1 given by five experts is computed as follows:

$$\tilde{Z}_{11}=\left\langle \begin{pmatrix} 1-(1-0.50)^{0.155}\cdot(1-0.60)^{0.226}\cdot(1-0.70)^{0.155}\cdot(1-0.60)^{0.259}\cdot(1-0.60)^{0.205}, \\ 1-(1-0.45)^{0.155}\cdot(1-0.45)^{0.226}\cdot(1-0.75)^{0.155}\cdot(1-0.45)^{0.259}\cdot(1-0.45)^{0.205}, \\ 1-(1-0.50)^{0.155}\cdot(1-0.50)^{0.226}\cdot(1-0.80)^{0.155}\cdot(1-0.50)^{0.259}\cdot(1-0.50)^{0.205} \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 0.40^{0.155}\cdot0.20^{0.226}\cdot0.15^{0.155}\cdot0.20^{0.259}\cdot0.20^{0.205}, \\ 0.35^{0.155}\cdot0.15^{0.226}\cdot0.20^{0.155}\cdot0.15^{0.259}\cdot0.15^{0.205}, \\ 0.50^{0.155}\cdot0.25^{0.226}\cdot0.25^{0.155}\cdot0.25^{0.259}\cdot0.25^{0.205} \end{pmatrix}, \begin{pmatrix} 0.35^{0.155}\cdot0.10^{0.226}\cdot0.10^{0.155}\cdot0.10^{0.259}\cdot0.10^{0.205}, \\ 0.30^{0.155}\cdot0.25^{0.226}\cdot0.15^{0.155}\cdot0.25^{0.259}\cdot0.25^{0.205}, \\ 0.45^{0.155}\cdot0.15^{0.226}\cdot0.20^{0.155}\cdot0.15^{0.259}\cdot0.15^{0.205} \end{pmatrix} \right\rangle$$

=<(0.604, 0.513, 0.566), (0.213, 0.179, 0.278), (0.121, 0.238, 0.186)>.

Step 2.3. Firstly, score function values of the T2NN aggregated evaluations (Table B.3) are calculated by using Eq. (16). Then, the normalized decision matrix (Table 9) is determined with the help of Eq. (15) by taking into account the score decision matrix, the extreme values (i.e., minimum and maximum), and the types of evaluation criteria (Table B.3). Cost criteria are "operation cost" (C_1), "user cost" (C_2), "expert cost" (C_3), "travel time" (C_4), "switch to a personal vehicle" (C_6), "environmental cost" (C_8), and "decentralization of cities" (C_{10}). The other six criteria are in the set of benefit criteria; i.e., $C^+=\{C_5, C_7, C_9, C_{11}, C_{12}, C_{13}\}$.

Table 9
The normalized decision matrix and average normalized evaluations.

Alternative	Criterion												
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}
A_1	0.0	0.844	1.0	0.0	0.0	1.0	1.0	0.0	0.0	0.921	1.0	0.789	0.0
A_2	0.749	0.658	0.274	0.776	0.676	0.048	0.0	0.106	0.620	1.0	0.942	1.0	0.999
A_3	0.609	0.0	0.069	0.686	0.738	0.0	0.346	0.648	0.560	0.0	0.0	0.0	0.758
A_4	1.0	1.0	0.0	1.0	1.0	0.609	0.671	1.0	1.0	0.247	0.381	0.862	1.0
Average	0.590	0.626	0.336	0.615	0.603	0.414	0.504	0.439	0.545	0.542	0.581	0.663	0.689

Steps 2.4–2.7. Firstly, the standard deviations and the correlation coefficients among the criteria are calculated by employing Eq. (17) and Eq. (18), respectively. They are determined based on the normalized decision matrix and the average normalized evaluations (Table 9). Secondly, the information measures are computed to simultaneously consider the differences and correlations among the criteria by using Eq. (19). Lastly, Eq. (20) is applied to normalize the information measure of each criterion and elucidate their objective importance. The obtained importance vector of the evaluation criteria is presented in Table 10.

Table 10
The correlation coefficient matrix, standard deviations, information measures, and criteria importance.

Criterion	C_1	C_2	C_3	C_4	C_5	···	C_{13}	Standard deviation	Information measure	Importance
C_1	1.000	0.033	–0.932	0.997	0.980	···	0.969	0.368	3.741	0.067
C_2	0.033	1.000	0.305	–0.051	–0.115	···	–0.107	0.381	3.326	0.059
C_3	–0.932	0.305	1.000	–0.957	–0.981	···	–0.922	0.397	5.644	0.100
C_4	0.997	–0.051	–0.957	1.000	0.989	···	0.977	0.373	3.927	0.070
C_5	0.980	–0.115	–0.981	0.989	1.000	···	0.943	0.369	4.003	0.071
C_6	–0.544	0.709	0.695	–0.604	–0.586	···	–0.714	0.414	4.786	0.085
C_7	–0.562	0.446	0.588	–0.600	–0.530	···	–0.750	0.372	4.730	0.084
C_8	0.734	–0.044	–0.798	0.736	0.818	···	0.581	0.407	4.586	0.082
C_9	0.990	0.081	–0.924	0.982	0.981	···	0.925	0.357	3.570	0.064
C_{10}	–0.447	0.500	0.687	–0.487	–0.610	···	–0.353	0.428	5.159	0.092
C_{11}	–0.464	0.622	0.729	–0.515	–0.630	···	–0.412	0.413	4.793	0.085
C_{12}	0.085	0.888	0.279	0.012	–0.107	···	0.053	0.390	3.515	0.063
C_{13}	0.969	–0.107	–0.922	0.977	0.943	···	1.000	0.410	4.433	0.079

Steps 2.8–2.9. Firstly, the normalized evaluations of the public transportation pricing systems (Table 9) and the criteria importance (Table 10) are taken into account to determine the weighted normalized decision matrix by employing Eq. (21). This matrix is given in Table 11. Then, the border approximation area (Table 11, Column 6) is determined with the help of Eq. (22).

Table 11
The weighted normalized decision matrix and the border approximation area.

Criterion	Alternative				Border approximation area
	A_1: Flat fare	A_2: Distance-based fare	A_3: Zonal pricing	A_4: Rent-based fare	
C_1: Operation cost	0.067	0.116	0.107	0.133	0.103
C_2: User cost	0.109	0.098	0.059	0.118	0.093
C_3: Expert cost	0.201	0.128	0.107	0.100	0.129
C_4: Travel time	0.070	0.124	0.118	0.140	0.109
C_5: Travel comfort	0.071	0.119	0.124	0.142	0.111
C_6: Switch to a personal veh.	0.170	0.089	0.085	0.137	0.115
C_7: Personal mobility	0.168	0.084	0.113	0.141	0.123
C_8: Environmental cost	0.082	0.090	0.134	0.163	0.113
C_9: Life quality	0.064	0.103	0.099	0.127	0.095
C_{10}: Decentralization of cities	0.176	0.184	0.092	0.114	0.136
C_{11}: Public acceptability	0.171	0.166	0.085	0.118	0.130
C_{12}: Political success	0.112	0.125	0.063	0.116	0.100
C_{13}: Deployment of resources	0.079	0.158	0.139	0.158	0.128

Step 2.10. The distance matrix is provided in Table 12. It is constructed based on the weighted normalized decision matrix and the border approximation area (Table 11) with the help of Eq. (23).

Step 2.11. The assessment scores of the investigated public transportation pricing systems for IZBAN are calculated by employing Eq. (24) and presented in Table 12. Four pricing systems are ranked according to the decreasing values of their assessment scores. The obtained ordering is $A_4 \succ A_2 \succ A_1 \succ A_3$. Based on the results from Table 12, the most appropriate pricing system for the analyzed suburban system in Izmir Province is “rent-based fare” (A_4) since it has the highest assessment score.

Table 12
The distance matrix, assessment scores, and ranks of the public transportation pricing systems.

Alternative	Criterion								Assessment score	Rank
	C_1	C_2	C_3	C_4	C_5	C_6	···	C_{13}		
A_1: Flat fare	–0.036	0.016	0.072	–0.039	–0.039	0.055	···	–0.050	0.054	3
A_2: Distance-based fare	0.014	0.005	–0.001	0.015	0.009	–0.026	···	0.029	0.099	2
A_3: Zonal pricing	0.005	–0.034	–0.022	0.008	0.013	–0.030	···	0.010	–0.159	4
A_4: Rent-based fare	0.031	0.025	–0.029	0.030	0.032	0.022	···	0.029	0.223	1

5.2. Ranking Discussion

After each alternative and criteria are defined, the experts are consulted to gather their scorings of each alternative under each criterion. Gathered results from the experts are then used to sort the alternatives. According to the prioritization results, zonal pricing is seen to be the least advantageous alternative. A flat fare is observed to be the second least advantageous pricing system, and distance-based pricing is the third. According to the experts, a rent-based fare pricing alternative, which is a public transportation fare system proposed in a different study by Karakurt (2021), is seen to be the most advantageous alternative.

The zonal pricing is the least advantageous alternative since the price increases as commuters change the zone, aiming to make this pricing alternative travel distance sensitive and providing equity among the short-distance and long-distance travelers. However, a notable unfairness aspect is still present in this alternative. The price difference between traveling for long distances in a zone and traveling a short distance, but changing zones is an issue that creates unfairness in pricing. Zonal fees being different create zonal distinctions. These distinctions have the potential to eventually create decentralization in terms of lower public transportation zone fees. Two highly interconnected choice problems must be handled when developing a zone-based tariff: the zone and the pricing (Otto and Boysen, 2017). Expert labor may be required to establish the boundaries fairly. All these reasons have made this alternative the least advantageous one.

Flat fare pricing is the second least advantageous method. The fact that all commuters are charged the same amount regardless of the distance and time traveled creates unfairness for commuters. The public frequently rejects flat fare that charges the same for a short trip between two nearby stations as it would for a long trip throughout the entire system (Hamacher and Schöbel, 2004). Even though some commuters travel for very short distances using public transportation vehicles, they must meet the operating costs of the public transportation vehicles at the same rate compared to the commuters, who travel for longer distances. The flat fare usually cannot meet the long-haul operating costs (Yamada, 1985). Short-haul customers heavily subsidize long-haul passengers (Cervero, 1981). Therefore, a flat fare system is suitable for compact cities and short travel distances. Implementing a flat fare price has the potential to create a decentralization problem. As the price of public transportation is fixed regardless of the distance, low- and medium-income residents live in the outer parts of the city since rents are lower in those regions. This increases the lengths of the public transportation lines, which eventually increases the operating costs because of the more frequent maintenance needs of the vehicles. The unfairness aspect of the flat fare pricing alternative and the potential of creating a decentralization issue has made this alternative a disadvantageous one. However, compared to the zonal pricing method, the unfairness issues in the pricing mechanism of the zonal pricing alternative such as the mentioned price difference in traveling for long distances in a zone and short distances, but changing a zone makes the flat fare pricing system the second least advantageous alternative.

Distance-based fare pricing alternative is the second most advantageous pricing method. In this pricing alternative, commuters pay as much as they travel. Therefore, the unfair aspect of the public transportation pricing methods is eliminated. However, it may require the use of specialized equipment since it is complex to implement and manage (Fleishman et al., 1996). The fact that passengers pay for the distance that they travel also removes the decentralization possibility faced in flat fare and zonal pricing alternatives, which puts distance-based fare pricing in front as means of advantage. Therefore, the two reasons specified have made the distance-based fare pricing alternative the second most advantageous one.

A rent-based fare system (Karakurt, 2021) is selected to be the most advantageous alternative among all other alternatives. The most significant reason for the decentralization of the cities is the lower rent accommodation present in outer regions of the cities. Low- and medium-income residents of the city pay lower rents and use public transportation to travel to their work, which is in inner-city regions. The rent-based fare method aims to eliminate the decentralization possibility by charging commuters, which pay lower rents the more. As this pricing method is implemented, people live in inner-city regions, which shortens the public transportation lines. This also reduces the operating costs of the vehicles because of less frequent maintenance needs. As people move their homes closer to their work and social environment, long traveling needs are mitigated, which removes the unfairness aspect from the beginning. All these reasons have made this alternative the most advantageous one.

5.3. Comparative Analysis

The T2NN-TOPSIS method introduced by Abdel-Basset et al. (2019) is the only available decision-making approach under the T2NN environment in the literature. It computes a positive-ideal solution (PIS) and a negative ideal solution (NIS) to find an alternative with the shortest Euclidean distance from PIS and the farthest Euclidean distance from NIS. The differences between our hybrid type-2 neutrosophic model and T2NN-TOPSIS are as follows (Table 13):

i. Our model can not only take into account the experience and expertise of experts that contribute to the MCDM process but also make a trade-off between these important reputation indicators.

ii. The only available T2NN-based approach ranks alternatives with the help of the TOPSIS method while our model assesses public transportation pricing systems by using the new T2NN-MABAC method.

iii. The only available T2NN-based decision-making approach assumes that criteria weights are known in advance while our model utilizes the novel T2NN-CRITIC method to determine the objective importance of the evaluation criteria.

iv. Our model has two build-in parameters that provide higher flexibility when selecting public transportation pricing systems while the T2NN-based TOPSIS method has no intrinsic parameters.

Table 13
The characteristic of different type-2 neutrosophic approaches.

Characteristic	Approach	
	Our study	T2NN-TOPSIS
Uncertain environment	Type-2 neutrosophic	Type-2 neutrosophic
Group decision-making	Yes	Yes
Expert reputation rating	Yes	No
Criteria weighting	CRITIC	No
Alternative ranking	MABAC	TOPSIS
Build-in parameter(s)	Two	No
Flexibility	High	Low

CRiteria Importance Through Intercriteria Correlation: CRITIC, Multi-Attributive Border Approximation area Comparison: MABAC, Technique for the Order Preference by Similarity to Ideal Solution: TOPSIS, Type-2 Neutrosophic Number: T2NN.

The comparative analysis with the T2NN-based TOPSIS method is performed to investigate the reliability of the hybrid type-2 neutrosophic model for public transportation pricing. Table 14 gives the comparison results. According to this table, both T2NN-based approaches identify "rent-based fare" (A_4) as the best public transportation pricing system for the investigated suburban system. Moreover, our model and the T2NN-based TOPSIS method generate the same ordering of four evaluated pricing systems for IZBAN. As a result, it can be concluded that the hybrid type-2 neutrosophic model for public transportation pricing is highly reliable.

Table 14
The comparison of different T2NN-based approaches.

Alternative	Our study		T2NN-TOPSIS	
	Assessment score	Rank	Closeness coefficient	Rank
A_1: Flat fare	0.054	3	0.484	3
A_2: Distance-based fare	0.099	2	0.610	2
A_3: Zonal pricing	–0.159	4	0.410	4
A_4: Rent-based fare	0.223	1	0.615	1

5.4. Sensitivity Analysis

The sensitivity analysis to changes in the trade-off parameter ξ_1 is performed to check the robustness of the hybrid type-2 neutrosophic model for public transportation pricing (Fig. 3). The values of ξ_1 are varied in the interval [0, 1] with an increment value of 0.1 to comprehensively analyze its influence as well as the quality of the proposed pricing system for IZBAN; i.e., switching from the distance-based to rent-based fare system.

In the base case scenario, the trade-off parameter ξ_1 is set to 0.5 to equally appraise the experience and expertise of the invited experts (Fig. 3). In real-life applications, such an assumption may not be reasonable since the municipal body (IZBAN) in charge of selecting a public transportation pricing system may decide to have more trust in experienced experts or more prefer those with greater expertise. Values of the trade-off parameter ξ_1 that are in the interval $0.5<\xi_1\leq 1$ favor the experience. They present scenarios when the municipal body has more trust in experienced experts. On the other hand, values of the parameter ξ_1 that are in the interval $0\leq\xi_1<0.5$ favor the expertise. These values represent scenarios when the municipal body more prefers expertise in the pricing system selection process for IZBAN. Also, only the experience or expertise of the invited experts may be taken into account in some specific conditions. The reputation is determined according to the experience of the invited experts when $\xi_1=1$ (denoted as "experience only" in Fig. 3). On the other hand, the expert reputation rating is fully in line with the expertise when $\xi_1=0$ (denoted as "expertise only" in Fig. 3).

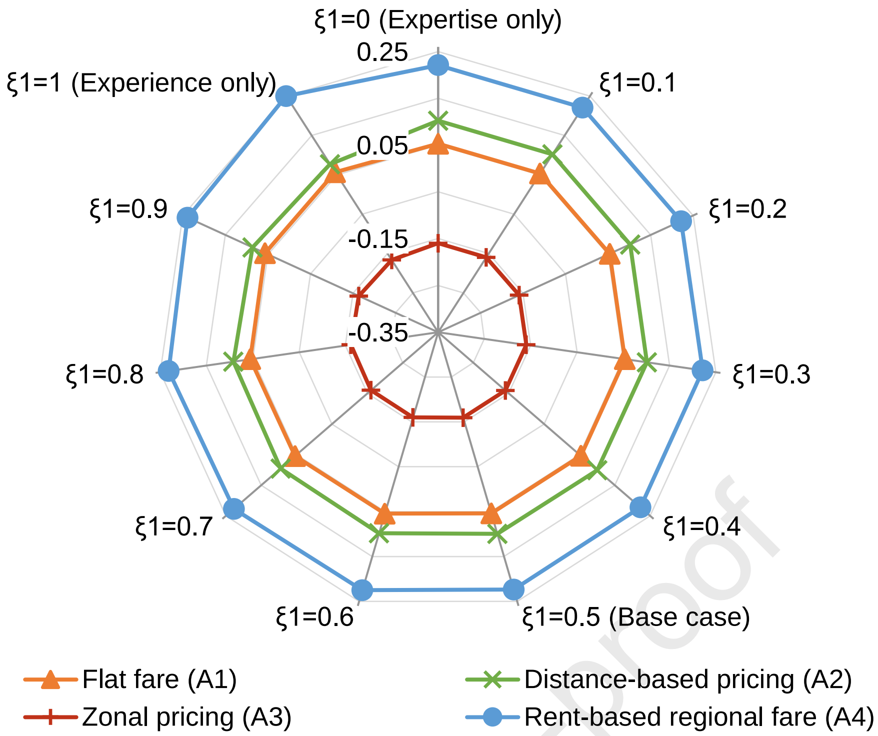

Fig. 3. The sensitivity analysis of the hybrid type-2 neutrosophic model.

As can be seen from Fig. 3, in both extreme cases (i.e., for $\xi_1 \in \{0, 1\}$) the order of the investigated public transportation pricing systems is $A_4 \succ A_2 \succ A_1 \succ A_3$. From Fig. 3 it can be outlined that alternative A_4 (i.e., rent-based fare) is the best pricing system for IZBAN. The current distance-based fare system (A_2) holds the second position in all investigated scenarios. The second worst pricing system is “flat fare” (A_1) that was applied till 2018, while by far the worst solution is “zonal pricing” (A_3). From the sensitivity analysis, it can be concluded that the introduced model for public transportation pricing is highly robust. Besides, switching IZBAN pricing to the rent-based regional fare system is strongly suggested.

5.5. Managerial Implications

There are various advantages and disadvantages to each alternative present in this study. Zonal pricing is the most disadvantageous in means of decentralization of the city and the issues regarding the pricing mechanism. Flat fare pricing is the second least disadvantageous regarding both fairness aspects because of fixing the price of public transportation and creating decentralization problem in the city. Distance-based fare pricing alternative is seen to be an advantageous pricing alternative in means of both fairness and decentralization by making commuters pay as much as they travel. However, the rent-based pricing method is the most advantageous one since it attacks the idea of decentralization. It makes the cities eventually more compact and so the need for long lines of public transportation becomes eliminated, which also solves the problem of unfairness.

This paper fills a gap concerning public transportation pricing methods. Rent-based fare pricing is a unique public transportation pricing method proposed by Karakurt (2021) and a rent-based fare pricing alternative is evaluated by the experts as a successful method. This paper also creates a consensus for municipalities to compare different transportation pricing systems. Municipalities have the chance to use transportation and rent cost dates, to confirm the need for rent-based fare pricing and implement it easily.

6. Conclusions

This study proposes the integrated CRITIC and MABAC based type-2 neutrosophic model for public transportation pricing system selection. A case scenario of Izmir, Turkey, confirms the effectiveness of the proposed model in the real-world context. Its findings indicate the rent-based fare pricing system as the most advantageous alternative. This unique pricing system makes cities eventually more compact and solves the problem of unfairness by eliminating the need for long public transportation lines.

The main contributions of this study are as follows:

i) Defined alternatives and evaluation criteria offer a practical decision-making framework for municipalities to compare and improve their public transportation pricing systems.

ii) Unlike other decision-making approaches for transportation pricing, this study uses advanced T2NNs to improve the recognition of information uncertainties in selecting public transportation pricing systems thus providing a more accurate prioritization.

iii) This is the first study that proposes an integrated methodological framework under T2NNs. Also, the CRITIC and MABAC methods are for the first time extended under the T2NN environment.

iv) Unlike the majority of the available group MCDM approaches, which assume that expert weights are equal or known in advance, this study introduces the straightforward expert reputation rating approach for determining the reputation of invited experts.

v) Even though this study is devoted to transportation pricing, our integrated CRITIC and MABAC based type-2 neutrosophic model can be used to solve various decision-making problems such as supply chain management, business management, construction management, healthcare management, and so on.

The introduced hybrid type-2 neutrosophic model for public transportation pricing provides a new perspective as well as an effective tool for researchers and practitioners. Its advantages are as follows:

1) Real-life preferences of municipal bodies (i.e., public transportation pricing actors) to favor experience over expertise and vice versa can be successfully represented.
2) Differences and correlations among evaluation criteria can be simultaneously considered to elucidate their objective importance.
3) Two build-in parameters provide higher flexibility when selecting public transportation pricing systems.

Alongside the advantages, there are also some limitations of this study. Firstly, four prioritized alternatives may yield different results according to the implementation area. For example, implementing flat fare pricing may provide more benefits for smaller cities. Secondly, the proposed type-2 neutrosophic model can be focused on different aggregation operators to increase its flexibility, such as t-norm and t-conorm based Dombi or Einstein operators. Thirdly, the T2NN-CRITIC method is developed and solely employed to assign weights to the criteria. The integration of some other weight assigning method (e.g., Shannon entropy (Shannon, 1948), cross-entropy (Kullback and Leibler, 1951), Tsallis-Havrda-Charvát entropy (Havdra and Charvat, 1967; Tsallis, 1988), nonprobabilistic entropy (De Luca and Termini, 1972), R-norm entropy (Boekee and Van der Lubbe, 1980), exponential entropy (Pal and Pal, 1989), etc.) into the proposed methodological framework can generate hybrid criteria weights and additionally improve its robustness. However, extensions of the aforementioned entropy-based methods under the T2NN environment are not yet available. This can be seen as another important avenue for future research.

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Appendix A. Notation

Indices

i	Index of alternatives, i=1, …, m
j	Index of criteria, j=1, …, n
e	Index of invited experts, e=1, …, k
l	Auxiliary index

Parameters

m	Number of alternatives
n	Number of criteria
k	Number of invited experts
ξ_1, ξ_2	Trade-off parameters of the reputation of invited experts, $\xi_1, \xi_2 \in [0, 1]$

Sets

$\tilde{\mathrm{M}}$	Type-2 neutrosophic number set
A	Set of alternatives
C	Set of criteria
D	Set of invited experts
C^+	Set of benefit criteria
C^-	Set of cost criteria

Matrices

$\tilde{\mathrm{V}}$	T2NN expert reputation matrix
$\tilde{\mathrm{B}}^e,\ e \in \mathrm{D}$	T2NN initial decision matrix by the invited expert D_e
$\tilde{\mathrm{Z}}$	T2NN aggregated decision matrix
R	Normalized decision matrix
G	Weighted normalized decision matrix
O	Border approximation area
O^+	Upper approximation area
O^-	Lower approximation area

Variables

$\tilde{T}_{\tilde{\mathrm{M}}}(y)$	Truth membership of y in $\tilde{\mathrm{M}}$
$\tilde{I}_{\tilde{\mathrm{M}}}(y)$	Indeterminacy membership of y in $\tilde{\mathrm{M}}$
$\tilde{F}_{\tilde{\mathrm{M}}}(y)$	Falsity membership of y in $\tilde{\mathrm{M}}$
$S(\tilde{\mathrm{M}})$	Score function of $\tilde{\mathrm{M}}$
$\tilde{V}_e^{(1)},\ e \in \mathrm{D}$	T2NN (self-)appraisal of the experience of the invited expert D_e
$\tilde{V}_e^{(2)},\ e \in \mathrm{D}$	T2NN (self-)appraisal of the expertise of the invited expert D_e
$\tilde{Q}_e,\ e \in \mathrm{D}$	T2NN aggregated reputation of the invited expert D_e

δ_e, $e \in D$	Reputation of the invited expert D_e, $\delta_e \in [0, 1]$
$\tilde{B}_{ij}^{e}$, $i \in A$, $j \in C$, $e \in D$	T2NN evaluation of the alternative A_i under the criterion C_j given by the invited expert D_e
$\tilde{Z}_{ij}$, $i \in A$, $j \in C$	T2NN aggregated evaluation of the alternative A_i under the criterion C_j given by the experts
R_{ij}, $i \in A$, $j \in C$	Normalized evaluation of the alternative A_i under the criterion C_j given by the experts
$\bar{R}_j$, $j \in C$	Average normalized evaluation of the alternatives under the criterion C_j given by the experts
σ_j, $j \in C$	Standard deviation of the criterion C_j
ρ_{jk}, $j, k \in C$	Correlation coefficient among the criteria C_j and C_k
U_j, $j \in C$	Information measure of the criterion C_j
ω_j, $j \in C$	Importance of the criterion C_j, $\omega_j \in [0, 1]$
G_{ij}, $i \in A$, $j \in C$	Weighted normalized evaluation of the alternative A_i under the criterion C_j
O_j, $j \in C$	Border approximation area under the criterion C_j given by the experts
P_{ij}, $i \in A$, $j \in C$	Distance of the alternative A_i from the border approximation area under the criterion C_j
B_i, $i \in A$	Assessment score of the alternative A_i

Appendix B

Table B.1
The T2NN initial decision matrices.

Alternative	Expert	Criterion		
		C_1	···	C_{13}
A_1: Flat fare	D_1	<(0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45)>	···	<(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)>
	D_2	<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>	···	<(0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60)>
	D_3	<(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)>	···	<(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)>
	D_4	<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>	···	<(0.20, 0.20, 0.10), (0.65, 0.80, 0.85), (0.45, 0.80, 0.70)>
	D_5	<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>	···	<(0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60)>
A_2: Distance-based fare	D_1	<(0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45)>	···	<(0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60)>
	D_2	<(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)>	···	<(0.95, 0.90, 0.95), (0.10, 0.10, 0.05), (0.05, 0.05, 0.05)>
	D_3	<(0.20, 0.20, 0.10), (0.65, 0.80, 0.85), (0.45, 0.80, 0.70)>	···	<(0.95, 0.90, 0.95), (0.10, 0.10, 0.05), (0.05, 0.05, 0.05)>
	D_4	<(0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60)>	···	<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>
	D_5	<(0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60)>	···	<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>
A_3: Zonal pricing	D_1	<(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)>	···	<(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)>
	D_2	<(0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45)>	···	<(0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45)>
	D_3	<(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)>	···	<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>
	D_4	<(0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60)>	···	<(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)>
	D_5	<(0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45)>	···	<(0.50, 0.45, 0.50), (0.40, 0.35, 0.50), (0.35, 0.30, 0.45)>
A_4: Rent-based fare	D_1	<(0.20, 0.20, 0.10), (0.65, 0.80, 0.85), (0.45, 0.80, 0.70)>	···	<(0.95, 0.90, 0.95), (0.10, 0.10, 0.05), (0.05, 0.05, 0.05)>
	D_2	<(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)>	···	<(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)>
	D_3	<(0.20, 0.20, 0.10), (0.65, 0.80, 0.85), (0.45, 0.80, 0.70)>	···	<(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)>
	D_4	<(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)>	···	<(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)>
	D_5	<(0.40, 0.30, 0.35), (0.50, 0.45, 0.60), (0.45, 0.40, 0.60)>	···	<(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)>

Table B.2
The T2NN aggregated decision matrix.

Criterion	Alternative		
	A_1: Flat fare	···	A_4: Rent-based fare
C_1	<(0.604, 0.513, 0.566), (0.213, 0.179, 0.278), (0.121, 0.238, 0.186)>	···	<(0.318, 0.296, 0.158), (0.542, 0.689, 0.769), (0.474, 0.673, 0.654)>
C_2	<(0.454, 0.407, 0.405), (0.435, 0.432, 0.572), (0.394, 0.386, 0.511)>	···	<(0.399, 0.305, 0.328), (0.460, 0.432, 0.567), (0.356, 0.435, 0.501)>
C_3	<(0.351, 0.308, 0.213), (0.521, 0.613, 0.717), (0.471, 0.584, 0.636)>	···	<(0.686, 0.625, 0.685), (0.217, 0.221, 0.257), (0.126, 0.222, 0.193)>
C_4	<(0.686, 0.717, 0.769), (0.157, 0.191, 0.250), (0.100, 0.162, 0.191)>	···	<(0.375, 0.346, 0.251), (0.495, 0.589, 0.695), (0.446, 0.559, 0.598)>
C_5	<(0.350, 0.350, 0.100), (0.500, 0.750, 0.800), (0.500, 0.750, 0.650)>	···	<(0.825, 0.756, 0.831), (0.174, 0.172, 0.156), (0.105, 0.129, 0.138)>
C_6	<(0.421, 0.376, 0.331), (0.461, 0.499, 0.627), (0.428, 0.458, 0.558)>	···	<(0.473, 0.390, 0.382), (0.351, 0.338, 0.461), (0.242, 0.395, 0.343)>
C_7	<(0.773, 0.783, 0.839), (0.141, 0.180, 0.195), (0.090, 0.126, 0.161)>	···	<(0.725, 0.647, 0.724), (0.231, 0.217, 0.243), (0.153, 0.187, 0.208)>
C_8	<(0.734, 0.726, 0.790), (0.185, 0.216, 0.237), (0.126, 0.158, 0.207)>	···	<(0.412, 0.355, 0.381), (0.478, 0.456, 0.596), (0.400, 0.409, 0.544)>
C_9	<(0.397, 0.339, 0.360), (0.492, 0.475, 0.613), (0.409, 0.429, 0.560)>	···	<(0.774, 0.718, 0.786), (0.154, 0.152, 0.174), (0.086, 0.145, 0.130)>
C_{10}	<(0.636, 0.574, 0.531), (0.348, 0.476, 0.428), (0.297, 0.407, 0.364)>	···	<(0.832, 0.771, 0.830), (0.190, 0.213, 0.172), (0.127, 0.144, 0.156)>
C_{11}	<(0.584, 0.510, 0.565), (0.238, 0.213, 0.314), (0.148, 0.252, 0.229)>	···	<(0.419, 0.395, 0.301), (0.454, 0.540, 0.653), (0.429, 0.505, 0.555)>
C_{12}	<(0.523, 0.519, 0.527), (0.340, 0.376, 0.483), (0.289, 0.325, 0.412)>	···	<(0.531, 0.501, 0.507), (0.305, 0.328, 0.431), (0.233, 0.328, 0.344)>
C_{13}	<(0.337, 0.292, 0.218), (0.535, 0.612, 0.718), (0.465, 0.582, 0.640)>	···	<(0.759, 0.745, 0.805), (0.149, 0.169, 0.195), (0.090, 0.140, 0.152)>

Table B.3

The score decision matrix, extreme values, and criteria types.

Alternative	Criterion												
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}
A_1	0.714	0.510	0.370	0.790	0.308	0.460	0.832	0.790	0.463	0.593	0.693	0.597	0.366
A_2	0.423	0.550	0.643	0.487	0.663	0.715	0.610	0.757	0.686	0.566	0.669	0.657	0.820
A_3	0.477	0.693	0.719	0.522	0.695	0.728	0.686	0.589	0.664	0.904	0.291	0.371	0.711
A_4	0.325	0.477	0.745	0.399	0.833	0.565	0.758	0.479	0.822	0.820	0.444	0.618	0.821
Minimum	0.325	0.477	0.370	0.399	0.308	0.460	0.610	0.479	0.463	0.566	0.291	0.371	0.366
Maximum	0.714	0.693	0.745	0.790	0.833	0.728	0.832	0.790	0.822	0.904	0.693	0.657	0.821
Type	Min	Min	Min	Min	Max	Min	Max	Min	Max	Min	Max	Max	Max

Highlights

> Public transportation pricing system selection problem is addressed and solved.

> Practical and methodological frameworks for public transportation pricing are given.

> Integrated CRITIC and MABAC based type-2 neutrosophic model is introduced.

> The presented hybrid T2NN-based model is highly robust, reliable, and flexible.

> The most advantageous pricing system is the rent-based fare.

UNIVERSITY OF BELGRADE

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Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: