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University of Management and Technology (Pakistan)
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Rana Muhammad Zulqarnain
University of Management and Technology (Pakistan)
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An MCDM Technique Using Cosine and Set-Theoretic Similarity Measures for Neutrosophic hypersoft set

Aiyared Iampan[1], Hamiden Abd El-Wahed Khalifa[2, 3], Imran Siddique[4], Rana Muhammad Zulqarnain[5*]

[1] Department of Mathematics, School of Science, University of Phayao, Mae Ka, Mueang, Phayao 56000, Thailand. E-mail: aiyared.ia@up.ac.th

[2] Department of Mathematics, College of Science and Arts , Al-Badaya, Qassim University E-mail: Ha.Ahmed@qu.edu.sa

[3] Operations Research Department, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt, Email: hamiden@cu.edu.eg

[4] Department of Mathematics, University of Management and Technology Lahore, Pakistan. E-mail: imransiddique@umt.edu.pk

[5] Department of Mathematics, University of Management and Technology Lahore, Sialkot Campus, Pakistan. E-mail: ranazulqarnain7777@gmail.com

Correspondence: E-mail: ranazulqarnain7777@gmail.com

Abstract:

A similarity measure is used to tackle many issues that include indistinct and blurred information, excluding is not able to deal with the general fuzziness and obscurity of the problems that have various information. The neutrosophic hypersoft set is the most generalized and advanced extension of neutrosophic sets, which deals with the multi sub-attributes of the considered parameters. In this paper, we study some basic concepts which are helpful to build the structure of the article, such as soft set, neutrosophic soft set, hypersoft set, and neutrosophic hypersoft set, etc. The main objective of the present research is to develop a cosine similarity measure and set-theoretic similarity measure for an NHSS with their necessary properties. A decision-making approach has been established by using cosine and set-theoretic similarity measures. Furthermore, we used to develop a technique to solve multi-criteria decision-making problems. Finally, the advantages, effectiveness, flexibility, and comparative analysis of the algorithms are given with prevailing methods.

Keywords: Neutrosophic set; hypersoft set; neutrosophic hypersoft set; similarity measures

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1. Introduction

Decision-making is an interesting concern to pick the perfect alternate for any specific persistence. Firstly, it is pretended that details about alternatives are accumulated in crisp numbers, but in real-life situations, collective farm information always conquers wrong and inaccurate information. Fuzzy sets are like sets having an element of membership (Mem) degree. In classical set theory, the Mem degree of the elements in a set is examined in binary form to see that the element is not entirely concomitant. In contrast, the fuzzy set theory enables advanced Mem categorization of the components in the set. The Mem function portrays it, and also the multipurpose unit interval of the Mem function is [0, 1]. In some circumstances, decision-makers consider objects' Mem and nonmember-ship (Nmem) values. Zadeh's FS cannot handle imprecise and vague information in such cases. Atanassov [2] developed the notion of intuitionistic fuzzy sets (IFS) to deal above mentioned difficulties. The IFS accommodates the imprecise and inaccurate information using Mem and Nmem values.

Atanassov IFS was unable to solve those problems in which decision-makers considered the membership degree (MD) and nonmembership degrees (NMD) such as MD = 0.5 and NDM = 0.8, then $0.5 + 0.8 \nleq 1$. Yager [3, 4] extended the notion of IFS to Pythagorean fuzzy sets (PFSs) to overcome above-discussed complications by modifying $MD + NMD \leq 1$ to $MD^2 + NMD^2 \leq 1$. After developing PFSs, Zhang and Xu [5] proposed operational laws for PFSs and established a DM approach to resolving the MCDM problem. Wei and Lu [6] planned some power aggregation operators (AOs) and proposed a DM technique to solve multi-attribute decision-making (MADM) issues under the Pythagorean fuzzy environment. Wang and Li [7] presented power Bonferroni mean operators for PFSs with their basic properties using interaction. Gao et al. [8] presented several aggregation operators by considering the interaction and proposed a DM approach to solving MADM difficulties utilizing the developed operators. Wei [9] developed the interaction operational laws for Pythagorean fuzzy numbers (PFNs) by considering interaction and established interaction aggregation operators by using the developed interaction operations. Zhang [10] developed the accuracy function and presented a DM approach to solving multiple criteria group decision-making (MCGDM) problems using PFNs. Wang et al. [11] extended the PFSs and introduced an interactive Hamacher operation with some novel AOs. They also established a DM method to solve MADM problems using their proposed operators. Wang and Li [12] developed some interval-valued PFSs and utilized their operators to resolve multi-attribute group decision-making (MAGDM) issues. Peng and Yuan [13] established novel operators such as Pythagorean fuzzy point operators and developed a DM technique using their proposed operators. Peng and Yang [14] introduced fundamental operations and their necessary possessions under PFSs and planned DM methodology. Garg [15] developed the logarithmic operational laws for PFSs and proposed some AOs. Arora and Garg [16] presented the operational laws for linguistic IFS and developed prioritized AOs. Ma and Xu [17] presented some innovative AOs for PFSs and proposed the score and accuracy functions for PFNs.

Above mentioned theories and their DM methodologies have been used in several fields of life. But, these theories cannot deal with the parametrization of the alternatives. Molodtsov [18] developed soft sets (SS) to overcome the complications above. Molodtsov's SS competently deals with imprecise, vague, and unclear objects considering their parametrization. Maji et al. [19] prolonged the notion of SS and introduced some necessary operators with their properties. Maji et al. [20] established a DM technique using their developed operations for SS. They also merged two well-known theories, such as FS and SS, and established the concept of fuzzy soft sets (FSS) [21]. They also proposed an intuitionistic fuzzy soft set (IFSS) [22] and discussed their basic operations. Garg and Arora [23] extended the idea of IFSS and presented a generalized form of IFSS with AOs. They also planned a DM technique to resolve undefined and inaccurate information under IFSS information. Garg and Arora [24] presented the correlation and weighted correlation coefficients for IFSS and developed the TOPSIS approach utilizing established correlation procedures. Zulqarnain et al. [25] introduced some AOs and correlation coefficients for interval-valued IFSS. They also extended the TOPSIS technique using their developed correlation measures to solve the MADM problem. Peng et al. [26] proposed the Pythagorean fuzzy soft sets (PFSSs) and presented fundamental operations of PFSSs with their desirable properties by merging PFS and SS. Zulqarnain et al. [27-28] proposed the Einstein weighted ordered average and geometric operators for PFSSs. Zulqarnain et al. [29] introduced operational laws for Pythagorean fuzzy soft numbers (PFSNs) and developed AOs utilizing defined operational laws for PFSNs. They also planned a DM approach to solve MADM problems with the help of presented operators. Riaz et al. [30] prolonged the idea of PFSSs and developed the m polar PFSSs. They also established the TOPSIS method under the considered hybrid structure and proposed a DM methodology to solve the MCGDM problem. Siddique et al. [31] introduced the score matrix for PFSS and established a DM approach using their developed concept. Zulqarnain et al. [32-34] planned the TOPSIS methodology in the PFSS environment based on the correlation coefficient. They also proposed some AOs and interaction AOs for PFSS.

All the above studies only deal the inadequate information because of membership and non-membership values. However, these theories cannot handle the overall incompatible and imprecise data. To address such inconsistent and inaccurate records, the idea of the neutrosophic set (NS) was developed by Smarandache [35]. Maji [36] offered the perception of a neutrosophic soft set (NSS) with necessary operations. Broumi [37] developed the generalized NSS with some operations and properties and used the projected concept for DM. Deli and Subas [38] developed the single-valued Neutrosophic numbers (SVNNs) to solve MCDM problems. They also established the cut sets for SVNNs. Wang et al. [39] proposed the correlation coefficient (CC) for SVNSs. Ye [40] introduced the simplified NSs with operational laws and AOs. Also, he presented an MCDM technique utilizing his planned AOs. Zulqarnain et al. [41-42] offered the generalized neutrosophic TOPSIS and an integrated model for neutrosophic TOPSIS. They used their developed techniques for supplier selection and MCDM problems.

All the above studies have some limitations. When any attribute from a set of attributes contains further sub-attributes, then the above-presented theories fail to solve such problems. To overcome the limitations mentioned above, Smarandache [43] protracted the idea of SS to hypersoft sets (HSS) by substituting the one-parameter function f to a multi-parameter (sub-attribute) function. Samarandache claimed that the established HSS competently deals with uncertain objects compared to SS. Several researchers explored the HSS and presented a lot of extensions for HSS [44, 45]. Zulqarnain et al. [46] presented the IFHSS, the generalized version of IFSS. They established the TOPSIS method utilizing the developed correlation coefficient. Zulqarnain et al. [47] proposed the Pythagorean fuzzy hypersoft sets with AOs and correlation coefficients. They also established the TOPSIS technique using their developed correlation coefficient and utilized the presented approach to select appropriate anti-virus face masks. Zulqarnain et al. [48, 49] presented some fundamental operations with their properties for interval-valued NHSS. Also, they proposed the CC and WCC for interval-valued NHSS and established a decision-making approach utilizing their developed CC. Several researchers extended the notion of HSS and introduced different extensions of HSS with their DM methodologies [50-58]. However, all existing studies only deal with the scenario by using MD and NMD of sub-attributes of the considered attributes. If any decision-maker considers the MD = 0.7 and NDM = 0.6, then $0.7 + 0.6 \leq 1$ of any sub-attribute of the alternatives. We can observe that the theories mentioned above cannot handle it. To overwhelm the above boundaries, we proposed some AOs for PFHSS such as PFHSWA and PFHSWG operators by modifying the condition $\boldsymbol{\mathcal{T}}_{\mathcal{F}(\breve{d})}(\boldsymbol{\delta})$ + $\boldsymbol{\mathcal{J}}_{\mathcal{F}(\breve{d})}(\boldsymbol{\delta}) \leq 1$ to $\left(\boldsymbol{\mathcal{T}}_{\mathcal{F}(\breve{d})}(\boldsymbol{\delta})\right)^2 + \left(\boldsymbol{\mathcal{J}}_{\mathcal{F}(\breve{d})}(\boldsymbol{\delta})\right)^2 \leq 1$.. The essential objective of the following scientific research is to grow novel AOs for the PFHSS environment and processing mechanism, which can also follow the assumptions of PFHSNs. Furthermore, I developed an algorithm to explain the MCGDM problem and presented a numerical illustration to justify the effectiveness of the proposed approach under the PFHSS environment.

The following research is organized: In section 2, we recollected some basic definitions used in the subsequent sequel, such as NS, SS, NSS, HSS, and NHSS. Section 3 proposes the similarity measures such as cosine and set-theoretic for NHSS with its properties. We also introduced some operational laws for NHSS in the same section and established a decision-making technique to solve decision-making complications utilizing our developed similarity measures. In section 4, we use the proposed similarity measures for decision-making. A brief comparative analysis has been conducted between proposed techniques with existing methodologies in section 5. Finally, the conclusion and future directions are presented in section 6.

2. Preliminaries

The following section recalled fundamental concepts that helped us develop the current article's structure, such as SS, NS, NSS, HSS, FHSS, and NHSS.

Definition 2.1 [18]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathsf{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathsf{A})$ is called a soft set over $\mathcal{U}$, and its mapping is given as

$$\mathcal{F}: \mathsf{A} \to \mathcal{P}(\mathcal{U})$$

It is also defined as:

$$(\mathcal{F}, \mathsf{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}): e \in \mathcal{E}, \mathcal{F}(e) = \emptyset \; if \; e \notin \mathsf{A}\}$$

Definition 2.2 [21] $\mathcal{U}$ and $\mathcal{E}$ be a universe of discourse and set of attributes respectively and $\mathcal{F}(\mathcal{U})$ be a power set of $\mathcal{U}$. Let $\mathcal{A} \subseteq \mathcal{E}$, then $(\mathcal{F}, \mathcal{A})$ is an FSS over $\mathcal{U}$, its mapping can be stated as follows: $\mathcal{F}: \mathcal{A} \to F(\mathcal{U})$

Definition 2.3 [35] Let $\boldsymbol{\mathcal{U}}$ be a universe and $\boldsymbol{\mathcal{A}}$ be an NS on $\boldsymbol{\mathcal{U}}$ is defined as $\boldsymbol{\mathcal{A}} = \{\boldsymbol{\delta}, (\boldsymbol{\sigma_{\mathcal{F}}(\delta)}, \boldsymbol{\tau_{\mathcal{F}}(\delta)}, \boldsymbol{\gamma_{\mathcal{F}}(\delta)}): \boldsymbol{\delta} \in \boldsymbol{\mathcal{U}}\}$, where $\boldsymbol{\sigma}, \boldsymbol{\tau}, \boldsymbol{\gamma}: \boldsymbol{\mathcal{U}} \to]\mathbf{0}^-, \mathbf{1}^+[$ and $\mathbf{0}^- \leq \boldsymbol{\sigma_{\mathcal{F}}(\delta)} + \boldsymbol{\tau_{\mathcal{F}}(\delta)} + \boldsymbol{\gamma_{\mathcal{F}}(\delta)} \leq \mathbf{3}^+$.

Definition 2.4 [36] Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the Neutrosophic values of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called a Neutrosophic soft set over $\mathcal{U}$ and its mapping is given as

$$\mathcal{F}: \mathcal{A} \to \mathcal{P}(\mathcal{U})$$

Definition 2.5 [43]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, \ldots, k_n\}$,(n ≥ 1) be a set of attributes and set K_i a set of corresponding sub-attributes of k_i respectively with $K_i \cap K_j$ = φ for $n \geq 1$ for each $i, j \in \{1,2,3 \ldots n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} = \{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of multi-attributes, where $1 \leq h \leq \alpha, 1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and α, β, and $\gamma \in \mathbb{N}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}})$ is said to be HSS over $\mathcal{U}$, and its mapping is defined as

$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}} \to \mathcal{P}(\mathcal{U})$.

It is also defined as

$(\mathcal{F}, \ddot{\mathsf{A}}) = \{\breve{a}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a}): \breve{a} \in \ddot{\mathsf{A}}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a}) \in \mathcal{P}(\mathcal{U})\}$

Definition 2.6 [43] $\mathcal{U}$ be a universal set and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, \ldots, k_n\}$,(n ≥ 1) and K_i denoted the set of attributes and their corresponding sub-attributes like $K_i \cap K_j$ = φ, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1,2,3 \ldots n\}$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}} = \{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ is a collection of sub-attributes, where $1 \leq h \leq \alpha, 1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in \mathbb{N}$. Where $IFS^{\mathcal{U}}$ represents the intuitionistic fuzzy subsets of $\mathcal{U}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = (\mathcal{F}, \ddot{\mathsf{A}})$ is known as IFHSS defined as follows:

$$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}} \to IFS^{\mathcal{U}}.$$

It is also defined as

$(\mathcal{F}, \ddot{\mathsf{A}}) = \{(\breve{a}, \mathcal{F}_{\ddot{\mathsf{A}}}(\breve{a})): \breve{a} \in \ddot{\mathsf{A}}, \mathcal{F}_{\ddot{\mathsf{A}}}(\breve{a}) \in IFS^{\mathcal{U}} \in [0,1]\}$, where $\mathcal{F}_{\ddot{\mathsf{A}}}(\breve{a}) = \{\langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle: \delta \in \mathcal{U}\}$, where $\sigma_{\mathcal{F}(\breve{a})}(\delta)$ and $\tau_{\mathcal{F}(\breve{a})}(\delta)$ signifies the Mem and NMem values of the attributes: $\sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta) \in [0,1]$, and $0 \leq \sigma_{\mathcal{F}(\breve{a})}(\delta) + \tau_{\mathcal{F}(\breve{a})}(\delta) \leq 1$.

Definition 2.7 [47] Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, \ldots, k_n\}$,(n ≥ 1) and K_i represented the set of attributes and their corresponding sub-attributes such as $K_i \cap K_j$ = φ, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1,2,3 \ldots n\}$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}}$ =

$\{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ is a collection of sub-attributes, where $1 \leq h \leq \alpha, 1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in \mathbb{N}$. and $PFS^{\mathcal{U}}$ be a collection of all fuzzy subsets over $\mathcal{U}$. Then the pair ($\mathcal{F}$, $K_1 \times K_2 \times K_3 \times \ldots \times K_n$ = ($\mathcal{F}$, Ä) is known as PFHSS defined as follows:

$\mathcal{F}$: $K_1 \times K_2 \times K_3 \times \ldots \times K_n$ = Ä $\rightarrow PFS^{\mathcal{U}}$.

It is also defined as

$(\mathcal{F}, \ddot{A}) = \{(\breve{a}, \mathcal{F}_{\ddot{A}}(\breve{a})) : \breve{a} \in \ddot{A}, \mathcal{F}_{\ddot{A}}(\breve{a}) \in PFS^{\mathcal{U}} \in [0,1]\}$, where $\mathcal{F}_{\ddot{A}}(\breve{a}) = \{\langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}\}$, where $\sigma_{\mathcal{F}(\breve{a})}(\delta)$ and $\tau_{\mathcal{F}(\breve{a})}(\delta)$ signifies the Mem and NMem values of the attributes:

$\sigma_{\mathcal{F}(\breve{a})}(\delta)$, $\tau_{\mathcal{F}(\breve{a})}(\delta) \in [0,1]$, and $0 \leq \left(\sigma_{\mathcal{F}(\breve{a})}(\delta)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a})}(\delta)\right)^2 \leq 1$.

A Pythagorean fuzzy hypersoft number (PFHSN) can be stated as $\mathcal{F} = \left\{\left(\sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta)\right)\right\}$, where $0 \leq \left(\sigma_{\mathcal{F}(\breve{a})}(\delta)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a})}(\delta)\right)^2 \leq 1$.

Definition 2.8 [43]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, \ldots, k_n\}$,(n ≥ 1) be a set of attributes and set K_i a set of corresponding sub-attributes of k_i respectively with $K_i \cap K_j$ = φ for $n \geq 1$ for each $i, j \in \{1,2,3 \ldots n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n$ = Ä = $\{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and α, β, and $\gamma \in \mathbb{N}$ and $NS^{\mathcal{U}}$ be a collection of all neutrosophic subsets over $\mathcal{U}$. Then the pair ($\mathcal{F}$, $K_1 \times K_2 \times K_3 \times \ldots \times K_n$ = Ä) is said to be NHSS over $\mathcal{U}$, and its mapping is defined as

$\mathcal{F}$: $K_1 \times K_2 \times K_3 \times \ldots \times K_n$ = Ä $\rightarrow NS^{\mathcal{U}}$.

It is also defined as

$(\mathcal{F}, \ddot{A}) = \{(\breve{a}, \mathcal{F}_{\ddot{A}}(\breve{a})) : \breve{a} \in \ddot{A}, \mathcal{F}_{\ddot{A}}(\breve{a}) \in NS^{\mathcal{U}}\}$, where $\mathcal{F}_{\ddot{A}}(\breve{a}) = \{\langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta), \gamma_{\mathcal{F}(\breve{a})}(\delta)\rangle : \delta \in \mathcal{U}\}$, where $\sigma_{\mathcal{F}(\breve{a})}(\delta)$, $\tau_{\mathcal{F}(\breve{a})}(\delta)$, and $\gamma_{\mathcal{F}(\breve{a})}(\delta)$ represent the truth, indeterminacy, and falsity grades of the attributes such as $\sigma_{\mathcal{F}(\breve{a})}(\delta)$, $\tau_{\mathcal{F}(\breve{a})}(\delta)$, $\gamma_{\mathcal{F}(\breve{a})}(\delta) \in [0,1]$, and $0 \leq \sigma_{\mathcal{F}(\breve{a})}(\delta) + \tau_{\mathcal{F}(\breve{a})}(\delta) + \gamma_{\mathcal{F}(\breve{a})}(\delta) \leq 3$. Simply a neutrosophic hypersoft number (NHSN) can be expressed as $\mathcal{F}$ = $\left\{\left(\sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta), \gamma_{\mathcal{F}(\breve{a})}(\delta)\right)\right\}$, where $0 \leq \sigma_{\mathcal{F}(\breve{a})}(\delta) + \tau_{\mathcal{F}(\breve{a})}(\delta) + \gamma_{\mathcal{F}(\breve{a})}(\delta) \leq 3$.

Example 2.7

Consider the universe of discourse $\mathcal{U} = \{\delta_1, \delta_2\}$ and $\mathfrak{L} = \{\ell_1 = Teaching\ methdology, \ell_2 = Subjects, \ell_3 = Classes\}$ be a collection of attributes with following their corresponding attribute values are given as teaching methodology = $L_1 = \{a_{11} = project\ base, a_{12} = class\ discussion\}$, Subjects = $L_2 = \{a_{21} = Mathematics, a_{22} = Computer\ Science, a_{23} = Statistics\}$, and Classes = $L_3 = \{a_{31} = Masters, a_{32} = Doctorol\}$. Let Ä = $L_1 \times L_2 \times L_3$ be a set of attributes

$$\ddot{A} = L_1 \times L_2 \times L_3 = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}, a_{23}\} \times \{a_{31}, a_{32}\}$$

$$= \left\{\begin{matrix}(a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), (a_{11}, a_{23}, a_{31}), (a_{11}, a_{23}, a_{32}), \\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32}), (a_{12}, a_{23}, a_{31}), (a_{12}, a_{23}, a_{32}),\end{matrix}\right\}$$

$$\ddot{A} = \{\breve{a}_1, \breve{a}_2, \breve{a}_3, \breve{a}_4, \breve{a}_5, \breve{a}_6, \breve{a}_7, \breve{a}_8, \breve{a}_9, \breve{a}_{10}, \breve{a}_{11}, \breve{a}_{12}\}$$

Then the NHSS over $\mathcal{U}$ is given as follows

$(\mathcal{F}, \ddot{A})$ =

$$\left\{\begin{matrix} (\breve{a}_1, (\delta_1, (.6, .3, .8)), (\delta_2, (.9, .3, .5))), (\breve{a}_2, (\delta_1, (.5, .2, .7)), (\delta_2, (.7, .1, .5))), (\breve{a}_3, (\delta_1, (.5, .2, .8)), (\delta_2, (.4, .3, .4))), \\ (\breve{a}_4, (\delta_1, (.2, .5, .6)), (\delta_2, (.5, .1, .6))), (\breve{a}_5, (\delta_1, (.8, .4, .3)), (\delta_2, (.2, .3, .5))), (\breve{a}_6, (\delta_1, (.9, .6, .4)), (\delta_2, (.7, .6, .8))), \\ (\breve{a}_7, (\delta_1, (.6..5, .3)), (\delta_2, (.4, .2, .8))), (\breve{a}_8, (\delta_1, (.8, .2, .5)), (\delta_2, (.6, .8, .4))), (\breve{a}_9, (\delta_1, (.7, .4, .9)), (\delta_2, (.7..3, .5))), \\ (\breve{a}_{10}, (\delta_1, (.8, .4, .6)), (\delta_2, (.7, .2, .9))), (\breve{a}_{11}, (\delta_1, (.8, .4, .5)), (\delta_2, (.4, .2, .5))), (\breve{a}_5, (\delta_1, (.7, .5, .8)), (\delta_2, (.7, .5, .9))) \end{matrix}\right\}$$

3. Similarity Measures and Their Decision-Making Approaches

Many mathematicians developed various methodologies to solve MCDM problems in the past few years, such as aggregation operators for different hybrid structures, CC, similarity measures, and decision-making applications. The following section develops the cosine and set-theoretic similarity measure for NHSS.

Definition 3.1

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs defined over a universe of discourse $\mathcal{U}$. Then, the then cosine similarity measure of $(\mathcal{F},\ddot{A})$ and $(\mathcal{G},\ddot{\mathbb{A}})$ can be described as follows:

$\mathcal{S}^1_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) =$

$$\frac{1}{mn}\sum_{k=1}^{m}\sum_{i=1}^{n}\frac{\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)\right)}{\left(\sqrt{\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}\sqrt{\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)}\right)}$$

Proposition 3.2

Let $(\mathcal{F},\ddot{A})$, $(\mathcal{G},\ddot{\mathbb{A}})$, and $(\mathcal{H},\breve{C}) \in$ NHSS, then the following properties hold

1. $0 \le \mathcal{S}^1_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) \le 1$
2. $\mathcal{S}^1_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) = \mathcal{S}^1_{NHSS}((\mathcal{G},\ddot{\mathbb{A}}), (\mathcal{F},\ddot{A}))$
3. If $(\mathcal{F},\ddot{A}) \subseteq (\mathcal{G},\ddot{\mathbb{A}}) \subseteq (\mathcal{H},\breve{C})$, then $\mathcal{S}^1_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{H},\breve{C})) \le \mathcal{S}^1_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}}))$ and $\mathcal{S}^1_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{H},\breve{C})) \le \mathcal{S}^1_{NHSS}((\mathcal{G},\ddot{\mathbb{A}}), (\mathcal{H},\breve{C}))$.

Proof: **U**sing the above definition, the proof of these properties can be done easily.

Definition 3.3

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs defined over a universe of discourse $\mathcal{U}$. Then, the then set-theoretic similarity measure of $(\mathcal{F},\ddot{A})$ and $(\mathcal{G},\ddot{\mathbb{A}})$ can be described as follows:

$\mathcal{S}^2_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) =$

$$\frac{1}{mn}\sum_{k=1}^{m}\sum_{i=1}^{n}\frac{\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)\right)}{max\left\{\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right),\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\right\}}$$

Proposition 3.4

Let $(\mathcal{F},\ddot{A})$, $(\mathcal{G},\ddot{\mathbb{A}})$, and $(\mathcal{H},\breve{C}) \in$ NHSS, then the following properties hold

1. $0 \le \mathcal{S}^2_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) \le 1$
2. $\mathcal{S}^2_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) = \mathcal{S}^2_{NHSS}((\mathcal{G},\ddot{\mathbb{A}}), (\mathcal{F},\ddot{A}))$
3. If $(\mathcal{F},\ddot{A}) \subseteq (\mathcal{G},\ddot{\mathbb{A}}) \subseteq (\mathcal{H},\breve{C})$, then $\mathcal{S}^2_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{H},\breve{C})) \le \mathcal{S}^2_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}}))$ and $\mathcal{S}^2_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{H},\breve{C})) \le \mathcal{S}^2_{NHSS}((\mathcal{G},\ddot{\mathbb{A}}), (\mathcal{H},\breve{C}))$.

Proof: **U**sing the above definition, the proof of these properties can be done easily.

3.5 Algorithm 1 for Similarity Measures of NHSS

Step 1. Pick out the set containing parameters.

Step 2. Construct the NHSS according to experts.

Step 3. Compute the cosine similarity measure by using definition 3.1.

Step 4. Compute the set-theoretic similarity measure for NHSS by utilizing definition 3.3.

Step 5. An alternative with a maximum value with cosine similarity measure has the maximum rank according to considered numerical illustration.

Step 6. An alternative with a maximum value with a set-theoretic similarity measure has the maximum rank according to considered numerical illustration.

Step 7. Analyze the ranking.

Definition 3.6

Let $(\mathcal{F},\ddot{\mathrm{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$, $(\mathcal{G},\dddot{\mathrm{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$, and $(\mathcal{H},\check{C}) = \left\{\left(\delta_i, \sigma_{\mathcal{H}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{H}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{H}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be three NHSSs defined over a universe of discourse $\mathcal{U}$ when $\delta > 0$, then the following laws hold.

$$(\mathcal{F},\ddot{\mathrm{A}}) \oplus (\mathcal{G},\dddot{\mathrm{A}}) = \left\langle \begin{matrix} \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i) + \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i) - \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i), \\ \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i) \end{matrix} \right\rangle$$

$$(\mathcal{F},\ddot{\mathrm{A}}) \otimes (\mathcal{G},\dddot{\mathrm{A}}) = \left\langle \begin{matrix} \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) + \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i) - \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i), \\ \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) + \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i) - \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i) \end{matrix} \right\rangle$$

$$\delta(\mathcal{F},\ddot{\mathrm{A}}) = \left\langle 1 - \left(1 - \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^{\delta}, \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^{\delta}, \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^{\delta} \right\rangle$$

$$((\boldsymbol{\mathcal{F}},\ddot{\mathrm{A}}))^{\boldsymbol{\delta}} = \left\langle \left(\boldsymbol{\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^{\boldsymbol{\delta}}, \mathbf{1} - \left(\mathbf{1} - \boldsymbol{\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^{\boldsymbol{\delta}}, \mathbf{1} - \left(\mathbf{1} - \boldsymbol{\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^{\boldsymbol{\delta}} \right\rangle.$$

Proposition 3.7

Let $(\mathcal{F},\ddot{\mathrm{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$, $(\mathcal{G},\dddot{\mathrm{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$, and $(\mathcal{H},\check{C}) = \left\{\left(\delta_i, \sigma_{\mathcal{H}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{H}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{H}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be three NHSSs defined over a universe of discourse $\mathcal{U}$ and $\delta,\ \delta_1,\ \delta_2 > 0$, then the following laws hold

1. $(\mathcal{F},\ddot{\mathrm{A}}) \oplus (\mathcal{G},\dddot{\mathrm{A}}) = (\mathcal{G},\dddot{\mathrm{A}}) \oplus (\mathcal{F},\ddot{\mathrm{A}})$
2. $(\mathcal{F},\ddot{\mathrm{A}}) \otimes (\mathcal{G},\dddot{\mathrm{A}}) = (\mathcal{G},\dddot{\mathrm{A}}) \otimes (\mathcal{F},\ddot{\mathrm{A}})$
3. $\delta((\mathcal{F},\ddot{\mathrm{A}}) \oplus (\mathcal{G},\dddot{\mathrm{A}}))= \delta(\mathcal{G},\dddot{\mathrm{A}}) \oplus \delta(\mathcal{F},\ddot{\mathrm{A}})$
4. $((\mathcal{F},\ddot{\mathrm{A}}) \otimes (\mathcal{G},\dddot{\mathrm{A}}))^{\delta} = ((\mathcal{F},\ddot{\mathrm{A}}))^{\delta} \otimes ((\mathcal{G},\dddot{\mathrm{A}}))^{\delta}$
5. $\delta_1(\mathcal{F},\ddot{\mathrm{A}}) \oplus \delta_2(\mathcal{F},\ddot{\mathrm{A}})= (\delta_1 \oplus \delta_2)(\mathcal{F},\ddot{\mathrm{A}})$
6. $((\mathcal{F},\ddot{\mathrm{A}}))^{\delta_1} \otimes ((\mathcal{F},\ddot{\mathrm{A}}))^{\delta_2} = ((\mathcal{F},\ddot{\mathrm{A}}))^{\delta_1+\delta_2}$
7. $((\mathcal{F},\ddot{\mathrm{A}}) \oplus (\mathcal{G},\dddot{\mathrm{A}})) \oplus (\mathcal{H},\check{C}) = (\mathcal{F},\ddot{\mathrm{A}}) \oplus \left((\mathcal{G},\dddot{\mathrm{A}}) \oplus (\mathcal{H},\check{C})\right)$
8. $((\mathcal{F},\ddot{\mathrm{A}}) \otimes (\mathcal{G},\dddot{\mathrm{A}})) \otimes (\mathcal{H},\check{C}) = (\mathcal{F},\ddot{\mathrm{A}}) \otimes \left((\mathcal{G},\dddot{\mathrm{A}}) \otimes (\mathcal{H},\check{C})\right)$

Proof. The proof of the above laws is straightforward by using definition 4.6.

Definition 3.8

Let $(\mathcal{F},\ddot{\mathrm{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be a collection of NHSNs, Ω_i and γ_k are weight vector for expert's and parameters respectively with given conditions $\Omega_i > 0$, $\sum_{i=1}^{n}\Omega_i = 1$, $\gamma_k > 0$, $\sum_{k=1}^{m}\gamma_k = 1$, where $(i = 1,2,\dots,n, and\ k = 1,2,\dots,m)$. Then NHSWA operator defined as NHSWA: $\Delta^n \rightarrow \Delta$ defined as follows

$$NHSWA\left(\mathcal{F}_{\ddot{\mathrm{A}}}(\delta_{11}), \mathcal{F}_{\ddot{\mathrm{A}}}(\delta_{12}), \dots, \mathcal{F}_{\ddot{\mathrm{A}}}(\delta_{nm})\right) = \oplus_{k=1}^{m} \gamma_j\left(\oplus_{i=1}^{n} \Omega_i \mathcal{F}_{\ddot{\mathrm{A}}}(\delta_i)\right).$$

Proposition 3.9

Let $(\mathcal{F},\ddot{\mathrm{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be a collection of NHSNs, the aggregated value is also an NHSN, such as

$$NHSWA\left(\mathcal{F}_{\ddot{\mathrm{A}}}(\delta_{11}), \mathcal{F}_{\ddot{\mathrm{A}}}(\delta_{12}), \dots, \mathcal{F}_{\ddot{\mathrm{A}}}(\delta_{nm})\right)$$

$$= \left\langle \begin{matrix} \mathbf{1} - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\mathbf{1} - \boldsymbol{\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^{\Omega_i}\right)^{\gamma_k}, \mathbf{1} - \left(\mathbf{1} - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\mathbf{1} - \boldsymbol{\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^{\Omega_i}\right)^{\gamma_k}\right), \\ \mathbf{1} - \left(\mathbf{1} - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\mathbf{1} - \boldsymbol{\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^{\Omega_i}\right)^{\gamma_k}\right) \end{matrix} \right\rangle$$

Definition 3.10

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be an NHSNs, then the score, accuracy, and certainty functions for NHSN respectively defined as follows:

$$\mathbb{S}((\mathcal{F},\ddot{A})) = \frac{1}{6m}\sum_{\alpha=1}^{m}\left(6 + \sigma^{\alpha}_{\mathcal{F}(\breve{a}_k)}(\delta_i) - \tau^{\alpha}_{\mathcal{F}(\breve{a}_k)}(\delta_i) - \gamma^{\alpha}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)$$

$$\mathbb{A}((\mathcal{F},\ddot{A})) = \frac{1}{4m}\left(4 + \sigma^{\alpha}_{\mathcal{F}(\breve{a}_k)}(\delta_i) - \gamma^{\alpha}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)$$

$$\mathbb{C}((\mathcal{F},\ddot{A})) = \frac{1}{2m}\left(2 + \sigma^{\alpha}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)$$

where $\alpha = 1, 2, \cdots, m$.

Definition 3.11

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$, and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSNs. The comparison approach is present as follows:

1. If $\mathbb{S}(\mathcal{F},\ddot{A}) > \mathbb{S}(\mathcal{G},\ddot{\mathbb{A}})$, then $(\mathcal{F},\ddot{A})$ is superior to $(\mathcal{G},\ddot{\mathbb{A}})$.
2. If $\mathbb{S}(\mathcal{F},\ddot{A}) = \mathbb{S}(\mathcal{G},\ddot{\mathbb{A}})$ and $\mathbb{A}(\mathcal{F},\ddot{A}) > \mathbb{A}(\mathcal{G},\ddot{\mathbb{A}})$, then $(\mathcal{F},\ddot{A})$ is superior to $(\mathcal{G},\ddot{\mathbb{A}})$.
3. If $\mathbb{S}(\mathcal{F},\ddot{A}) = \mathbb{S}(\mathcal{G},\ddot{\mathbb{A}})$, $\mathbb{A}(\mathcal{F},\ddot{A}) = \mathbb{A}(\mathcal{G},\ddot{\mathbb{A}})$, and $\mathbb{C}(\mathcal{F},\ddot{A}) > \mathbb{C}(\mathcal{G},\ddot{\mathbb{A}})$, then $(\mathcal{F},\ddot{A})$ is superior to $(\mathcal{G},\ddot{\mathbb{A}})$.
4. If $\mathbb{S}(\mathcal{F},\ddot{A}) = \mathbb{S}(\mathcal{G},\ddot{\mathbb{A}})$, $\mathbb{A}(\mathcal{F},\ddot{A}) > \mathbb{A}(\mathcal{G},\ddot{\mathbb{A}})$, and $\mathbb{C}(\mathcal{F},\ddot{A}) = \mathbb{C}(\mathcal{G},\ddot{\mathbb{A}})$, then $(\mathcal{F},\ddot{A})$ is indifferent to $(\mathcal{G},\ddot{\mathbb{A}})$, can be denoted as $(\mathcal{F},\ddot{A}) \sim (\mathcal{G},\ddot{\mathbb{A}})$.

4. Application of Similarity Measures in Decision Making

In this section, we proposed the algorithm for NHSS by using developed similarity measures. We also used the proposed methods for decision-making in real-life problems.

4.1. Problem Formulation and Application of NHSS For Decision Making

A construction company calls for the appointment of a civil engineer to supervise the workers. Several engineers apply for the civil engineer post, simply four engineers call for an interview based on experience for undervaluation such as $S = \{S_1, S_2, S_3, S_4\}$ be a set of selected engineers call for the interview. The managing director hires a committee of four experts $\mathcal{U} = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}\}$ for the selection of civil engineer. The group of experts chooses the set of attributes for the selection of an appropriate civil engineer such as $\mathfrak{L} = \{\ell_1 = personality, \ell_2 = communication\ skills, \ell_3 = qualification\}$ with their corresponding sub-attribute: personality $= \ell_1 = \{d_{11} = \text{attractive}\}$, communication skills $= \ell_2 = \{d_{21} = \text{normal}, d_{22} = \text{excellent}\}$, and qualification $= \ell_3 = \{d_{31} = \text{masters}, d_{32} = \text{doctor}\}$. The experts evaluate the applicants under defined parameters and forward the evaluation performa to the company's managing director. Finally, the director scrutinizes the best applicant based on the expert's evaluation report.

4.1.1. Application of NHSS For Decision Making

Let $S = \{S_1, S_2, S_3, S_4\}$ be a set of civil engineers who are shortlisted for interviews (alternatives) such as. The managing director hires a team of four experts such as $\mathcal{U} = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$. The group of experts chooses the set of attributes for the selection of an appropriate civil engineer such as $\mathfrak{L}$ = $\{\ell_1 = personality, \ell_2 = communication\ skills, \ell_3 = qualification\}$ with their corresponding sub-attribute: personality = ℓ_1 = $\{d_{11}$ = attractive$\}$, communication skills = ℓ_2 = $\{d_{21}$ = normal, d_{22} = excellent$\}$, and qualification = ℓ_3 = $\{d_{31}$ = masters, d_{32} = doctor$\}$. Let $\mathfrak{L}'$ = $\ell_1 \times \ell_2 \times \ell_3$ shows the multi sub-attributes

$\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3 = \{d_{11}\} \times \{d_{21}, d_{22}\} \times \{d_{31}, d_{32}\}$

$= \{(d_{11}, d_{21}, d_{31}),\ (d_{11}, d_{22}, d_{31}), (d_{11}, d_{21}, d_{32}), (d_{11}, d_{22}, d_{32})\}$,

$\mathfrak{L}' = \{\breve{d}_1, \breve{d}_2, \breve{d}_3, \breve{d}_4\}$ with weights $(0.2, 0.1, 0.4, 0.3)^T$. Experts' opinion in the form of NHSNs following multi sub-attributes of considered attributes.

Step 2. Construct the NHSS according to experts.

Table 1. Construction of NHSS of all Applicants According to Company Requirement

S	$\breve{d}_1$	$\breve{d}_2$	$\breve{d}_3$	$\breve{d}_4$
κ_1	(0.7,0.2,0.4)	(0.4, 0.3, 0.7)	(0.9, 0.7, 0.4)	(0.6,0.3,0.7)
κ_2	(0.5, 0.6,0.2)	(0.8,0.5, 0.6)	(0.8, 0.2, 0.5)	(0.7, 0.5, 0.9)
κ_3	(0.6,0.6,0.2)	(0.5,0.8,0.3)	(0.4, 0.7, 0.3)	(0.6, 0.7, 0.4)
κ_4	(0.8, 0.7, 0.5)	(0.2,0.4,0.9)	(0.7, 0.5, 0.1)	(0.6,0.8,0.2)

Now we will construct the NHSS S_t according to four experts, where t = 1, 2, 3, 4.

Table 2. Decision Matrix for alternative S_1

S_1	$\breve{d}_1$	$\breve{d}_2$	$\breve{d}_3$	$\breve{d}_4$
κ_1	(0.9,0.2,0.1)	(0.3, 0.3, 0.7)	(0.6, 0.4, 0.2)	(0.7,0.1,0.3)
κ_2	(0.8, 0.3,0.2)	(0.6,0.2, 0.6)	(0.8, 0.3, 0.1)	(0.2, 0.6, 0.8)
κ_3	(0.6,0.1,0.3)	(0.6,0.1,0.3)	(0.8, 0.2, 0.1)	(0.6, 0.3, 0.4)
κ_4	(0.9, 0.1, 0.1)	(0.9,0.1,0.1)	(0.8, 0.1, 0.1)	(0.9,0.1,0.2)

Table 3. Decision Matrix for alternative S_2

S_2	$\breve{d}_1$	$\breve{d}_2$	$\breve{d}_3$	$\breve{d}_4$
κ_1	(0.3,0.3,0.7)	(0.9, 0.2, 0.1)	(0.6, 0.1, 0.3)	(0.3,0.6,0.2)
κ_2	(0.8, 0.2,0.1)	(0.8,0.3, 0.2)	(0.9, 0.1, 0.1)	(0.8, 0.3, 0.1)
κ_3	(0.6,0.3,0.4)	(0.8,0.1,0.2)	(0.9, 0.1, 0.1)	(0.2, 0.3, 0.8)
κ_4	(0.9, 0.1, 0.2)	(0.8,0.1,0.1)	(0.7, 0.1, 0.3)	(0.6,0.3,0.4)

Table 4. Decision Matrix for alternative S_3

S_3	$\breve{d}_1$	$\breve{d}_2$	$\breve{d}_3$	$\breve{d}_4$
κ_1	(0.6,0.3,0.4)	(0.2, 0.3, 0.8)	(0.3, 0.6, 0.2)	(0.3,0.6,0.2)
κ_2	(0.9, 0.1,0.1)	(0.9,0.1, 0.1)	(0.9, 0.1, 0.1)	(0.8, 0.3, 0.2)
κ_3	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9, 0.1, 0.1)	(0.2, 0.3, 0.8)
κ_4	(0.3, 0.3, 0.7)	(0.9,0.1,0.2)	(0.7, 0.1, 0.3)	(0.6,0.3,0.4)

Table 5. Decision Matrix for alternative S_4

S_4	$\breve{d}_1$	$\breve{d}_2$	$\breve{d}_3$	$\breve{d}_4$
κ_1	(0.9,0.1,0.1)	(0.9, 0.1, 0.2)	(0.8, 0.2, 0.1)	(0.3,0.6,0.2)
κ_2	(0.8, 0.2,0.1)	(0.8,0.2, 0.1)	(0.6, 0.3, 0.4)	(0.8, 0.3, 0.2)
κ_3	(0.8,0.1,0.1)	(0.8,0.1,0.2)	(0.9, 0.1, 0.1)	(0.3, 0.6, 0.2)
κ_4	(0.9, 0.1, 0.2)	(0.3,0.3,0.7)	(0.8, 0.3, 0.2)	(0.9,0.1,0.1)

Step 3. Compute the cosine similarity measure by using definition 3.1.

By using Tables 1-5, compute the cosine similarity measure between $\mathcal{S}^1_{NHSS}(S,S_1)$, $\mathcal{S}^1_{NHSS}(S,S_2)$, $\mathcal{S}^1_{NHSS}(S,S_3)$, and $\mathcal{S}^1_{NHSS}(S,S_t)$ by using equation 3.1, such as

$$\mathcal{S}^1_{NHSS}(S,S_1) = \frac{1}{3\times4}\left\{\frac{(.8)(.3)+(.5)(.5)+(.6)(.2)}{\sqrt{(.8)^2+(.5)^2+(.6)^2}\sqrt{(.3)^2+(.5)^2+(.2)^2}} + \frac{(.5)(.8)+(.4)(.7)+(.2)(.3)}{\sqrt{(.5)^2+(.4)^2+(.2)^2}\sqrt{(.8)^2+(.7)^2+(.3)^2}} + \cdots + \frac{(.4)(.7)+(.7)(.7)+(.6)(.9)}{\sqrt{(.4)^2+(.7)^2+(.6)^2}\sqrt{(.7)^2+(.7)^2+(.9)^2}}\right\} = \frac{1}{12}\left(\frac{28.99}{34.4799}\right) = 0.07007.$$

Similarly, we can find the cosine similarity measure between $\mathcal{S}^1_{NHSS}(S,S_2)$, $\mathcal{S}^1_{NHSS}(S,S_3)$, and $\mathcal{S}^1_{NHSS}(S,S_4)$ given as

$\mathcal{S}^1_{NHSS}(S,S_2) = \frac{1}{12}\left(\frac{26.32}{32.3767}\right) = 0.06771$, $\mathcal{S}^1_{NHSS}(S,S_3) = \frac{1}{12}\left(\frac{25.4}{29.4056}\right) = 0.06943$, and $\mathcal{S}^1_{NHSS}(S,S_4) = \frac{1}{12}\left(\frac{25.48}{30.88764}\right) = 0.06874$. This shows that $\mathcal{S}^1_{NHSS}(S,S_1) > \mathcal{S}^1_{NHSS}(S,S_3) > \mathcal{S}^1_{NHSS}(S,S_4) > \mathcal{S}^1_{NHSS}(S,S_2)$. It can be seen from this ranking alternative S_1 is most relevant and similar to S. Therefore S_1 is the best alternative for the civil engineer, the ranking of other alternatives given as $S_1 > S_3 > S_4 > S_2$.

Now we compute the set-theoretic similarity measure by using Definition 4.3 between $\mathcal{S}^2_{NHSS}(S,S_1)$, $\mathcal{S}^2_{NHSS}(S,S_2)$, $\mathcal{S}^2_{NHSS}(S,S_3)$, and $\mathcal{S}^2_{NHSS}(S,S_4)$ given as From Tables 1-5, we can find the set-theoretic similarity measure for each alternative by using definition 4.3 given as $\mathcal{S}^2_{NHSS}(S,S_1)$ = 0.06986, $\mathcal{S}^2_{NHSS}(S,S_2)$ = 0.06379, $\mathcal{S}^2_{NHSS}(S,S_3)$ = 0.06157, and $\mathcal{S}^2_{NHSS}(S,S_4)$ = 0.06176. $\mathcal{S}^1_{NHSS}(S,S_1) > \mathcal{S}^1_{NHSS}(S,S_2) > \mathcal{S}^1_{NHSS}(S,S_4) > \mathcal{S}^1_{NHSS}(S,S_3)$. It can be seen from this ranking alternative S_1 is most relevant and similar to S. Therefore S_1 is the best alternative for the civil engineer, the ranking of other alternatives given as $S_1 > S_2 > S_4 > S_3$.

5. Discussion and Comparative Analysis

In the subsequent section, we will talk over the usefulness, easiness, manageability, and assistance of the planned method. We also performed an ephemeral evaluation of the undermentioned: the planned technique along with some prevailing methodologies.

5.1. Superiority of the Proposed Approach

Through this study and comparison, it could be determined that the consequences acquired by the suggested approach have been more common than either available method. Overall, the DM procedure associated with the prevailing DM methods accommodates extra information to address hesitation. Also, FS's various hybrid structures are becoming a particular feature of NHSS, along with some appropriate circumstances added. The general info associated with the object could be stated precisely and analytically, see Table 6. Therefore, it is a suitable technique to syndicate inaccurate and ambiguous information in the DM process. Hence, the suggested approach is practical, modest, and in advance of fuzzy sets' distinctive hybrid structures.

Table 6. Comparison between NHSS and some existing techniques

	Set	Truthiness	Indeterminacy	Falsity	Parametrization	Attributes	Sub-attributes
Zadeh [1]	FS	✓	×	×	×	✓	×
Atanassov [2]	IFS	✓	×	✓	×	✓	×
Smarandache [35]	NS	✓	✓	✓	×	✓	×
Maji et al. [21]	FSS	✓	×	×	✓	✓	×
Maji et al. [22]	IFSS	✓	×	✓	✓	✓	×
Peng et al. [26]	PFSS	✓	×	✓	✓	✓	×
Maji [36]	NSS	✓	✓	✓	✓	✓	×
Zulqarnain et al. [46]	IFHSS	✓	×	✓	✓	✓	✓

Zulqarnain et al. [47]	PFHSS	✓	×	✓	✓	✓	✓
Proposed approach	NHSS	✓	✓	✓	✓	✓	✓

It turns out that this is a contemporary issue. Why do we have to embody novel algorithms based on the proposed novel structure? Many indications compared with other existing methods; the recommended method may be an exception. We remember the following fact: the mixed structure limits IFS, picture fuzzy sets, FS, hesitation fuzzy sets, NS, and other fuzzy sets and cannot provide complete information about the situation. But our m-polar model GmPNSS can deal with truthiness, indeterminacy, and falsity, so it is most suitable for MCDM. Due to the exaggerated multipolar neutrosophy, these three degrees are independent and provide a lot of information about alternative norms. Other similarity measures of available hybrid structures are converted into exceptional cases of GmPNSS. A comparative analysis of some already existing techniques is listed above in Table 6. Therefore, this model has more versatility and can efficiently resolve complications than intuitionistic, neutrosophic, hesitant, image, and ambiguity substitution. The similarity measure established for GmPNSS becomes better than the existing similarity measure for MCDM.

5.2. Comparative Analysis

In the following section, we recommend another algorithmic rule under NHSS by utilizing the progressed cosine similarity measure and set-theoretic similarity measure. Subsequently, we use the suggested algorithm to a realistic problem, namely the appropriate civil engineer in a company. The overall outcomes prove that the algorithmic rule is valuable and practical. It can be observed that S_1 is the most acceptable alternative for the civil engineer position. The recommended approach may be compared to other available methods. From the research findings, it has been concluded that the outcomes acquired by the planned approach exceed the consequences of the prevailing ideas. Therefore, compared to existing techniques, the established similarity measures handled the uncertain and ambiguous information competently. However, under current DM strategies, the core advantage of the planned method is that it can accommodate extra info in data comparative to existing techniques. It is also a beneficial tool to solve inaccurate and imprecise information in DM procedures. The benefit of the planned approach and related measures over present methods is evading conclusions grounded on adverse reasons.

5.3. Discussion

Zadeh's [1] FS handled the inaccurate and imprecise information using MD of sub-attributes of considered attributes for each alternative. But the FS has no evidence around the NMD of the considered parameters. Atanassov's [2] IFS accommodates the unclear and undefined objects using MD and NMD. However, IFS cannot handle the circumstances when MD + NMD ≥ 1, conversely, is presented notion competently deals with such difficulties. Meanwhile, these theories have no information about the indeterminacy of the attributes. To overcome such problems, Smarandache [35] proposed the idea of NS. Maji et al. [21] presented the notion of FSS to deal with the parametrization of the objects, which contains uncertainty by considering the MD of the attributes. But, the presented FSS provides no information about the NMD of the object. To overcome the presented drawback, Maji et al. [22] offered the concept of IFSS. The proposed notion handles the uncertain object more accurately by using the MD and NMD of the attributes with their parametrization. The sum of MD and NMD does not exceed 1. To handle this scenario, Peng et al. [26] proposed the notion of PFSS by modifying the condition $MD + NMD \leq 1$ to $MD^2 + NMD^2 \leq 1$ with their parametrization. The PFSS is unable to deal with the indeterminacy of the attributes. Maji

[36] introduced the concept of NSS, in which decision-makers competently solve the DM problems comparative to the above-studied theories using truthiness, falsity, and indeterminacy of the object. But all the studies mentioned above have no information about the sub-attributes of the considered attributes. So the theories discussed above cannot handle the scenario when attributes have their corresponding sub-attributes. Utilizing the MD and NMD, Zulqarnain et al. [46] extended the IFSS to IFHSS and proposed the CC and WCC for IFHSS in which $MD + NMD \leq 1$ for each sub-attribute. But IFHSS cannot provide any information on the NMem values of the sub-attribute of the considered attribute. Zulqarnain et al. [47] proposed the more generalized notion of PFHSS comparative to IFHSS. The PFHSS accommodates more uncertainty compared to IFHSS by updating the condition $MD + NMD \leq 1$ to $\left(\sigma_{\mathcal{F}(\breve{\alpha})}(\delta)\right)^2 + \left(\tau_{\mathcal{F}(\breve{\alpha})}(\delta)\right)^2 \leq 1$. All existing hybrid structures of FS cannot handle the indeterminacy of sub-attributes of considered n-tuple attributes. On the other hand, developed aggregation operators can accommodate the sub-attributes of considered attributes using truthness, indeterminacy, and falsity objects of sub-attributes with the following condition $0 \leq \sigma_{\mathcal{F}(\breve{\alpha})}(\delta)$, $\tau_{\mathcal{F}(\breve{\alpha})}(\delta)$, $\gamma_{\mathcal{F}(\breve{\alpha})}(\delta) \leq 3$. It may be seen that the best selection of the suggested approach is to resemble the verbalized own method, and that ensures the liableness along with the effectiveness of the recommended approach.

6. Conclusion

This paper studies some basic concepts such as soft set, NSS, HSS, IFHSS, PFHSS, and NHSS. We developed the idea of cosine similarity measure and set-theoretic similarity measure for NHSS and described their desirable properties. Some operational laws have been established for NHSS. The concept of score function, accuracy function, and certainty function is developed to compare NHSNs. Furthermore, a decision-making approach has been developed for NHSS based on the proposed technique. To verify the effectiveness of our developed techniques, we presented an illustration to solve MCDM problems. We presented a comprehensive comparative analysis of proposed techniques with existing methods. In the future, the concept of NHSS will be extended to interval-valued NHSS. It will solve different real-life problems such as medical diagnoses, decision-making, etc.

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