Kybernetes

An introduction to alysidal algebra (V): phenomenological components

Antonio Nescolarde-Selva Josué, Usó-Domenech Jose-Luis,

Received 27 September 2012
Revised 2 October 2013
Accepted 3 October 2013

An introduction to alysidal algebra (V): phenomenological components

Josué Antonio Nescolarde-Selva
University of Alicante, Alicante, Spain, and
Jose-Luis Usó-Domenech
Universidad Jaume I, Castellón, Spain

Abstract

Purpose – This paper aims to refer to a subjective approach to a type of complex system: human ecosystems, referred to as deontical impure systems (DIS) to capture a set of properties fundamental to the distinction between human and natural ecosystems. There are four main phenomenological components: directionality, intensity, connection energy and volume. The paper establishes thermodynamics of deontical systems based on the Law of Zipf and the temperature of information.

Design/methodology/approach – Mathematical and logical development of human society structure.

Findings – A fundamental question in this approach to DIS is the intensity or forces of a relation. Concepts are introduced as the system volume and propose a system thermodynamic theory. It hints at the possibility of adapting the fractal theory by introducing the fractal dimension of the system.

Originality/value – This paper is a continuation of other previous papers and developing the theory of DIS.

Keywords Energy, Entropy, Alysidal algebra, Heat of information, Intensity, Relations, Temperature of information, Volume

Paper type Research paper

1. Introduction: the deontical system

In this work, continuation of previous (Nescolarde-Selva *et al.*, 2012a, b; Nescolarde-Selva and Usó-Doménech, 2012) we are going to deal with a very special type of systems: deontical impure systems (DIS). They are systems because objects and relations among them exist. They are impure because these objects are formed by material and/or energy beings. They are deontical because between its relations it has, at least, one that fulfills at least one of the deontical modalities: obligation, permission, prohibition, faculty and analogy. We are talking about the human societies. Not a particular society, but to any human society, at any time and place:

(1) A system is an organization of the knowledge on the part of the subject S that fulfils the following conditions (Nescolarde-Selva and Usó-Doménech, 2013b):
- The subjective condition.
- The condition of rationality.

To Dr Daniel Berend.

Kybernetes
Vol. 42 No. 8, 2013
pp. 1248-1264

0368-492X
DOI 10.1108/K-09-2012-0055

- The external condition.
- S knows what is a system[1].
  The vision of a system interpreted by the set formed by different subjects within the system is determined by the belief system (Nescolarde-Selva and Usó-Doménech, 2013a, b; Usó-Domènech and Nescolarde-Selva, 2012) that the subjects conceived as true, about themselves, the system and its environment.

(2) An impure set is whose referential elements (absolute beings) are not counted as abstract objects and have the following conditions:
- They are real (material or energetic absolute beings).
- They exist independently of subject.
- Subject develops perceptual significances.
- True things can be said about them.
- Subject can know these true things about them.
- They have properties that support a robust notion of mathematical truth.

(3) An impure system is one whose set of elements is an impure set.

(4) A deontical system is an organization of the knowledge on the part of the subject S that fulfils aforementioned conditions and the following others:
- Subjects are the human beings. We can distinguish the subject as observer (subjectively outside the system) and, by definition, is the subject itself, or within the system. In this case, acquires category of object.
- Objects (relative beings) are significances Nescolarde-Selva and Usó-Doménech, 2013a, b), which are consequence of the perceptual beliefs on the part of the subject of a material or energetic objects (absolute beings) with certain characteristics.
- Some existing relations between elements have deontical modalities[2].
- There is purpose (purposes).

(5) A DIS is a system that meets the conditions of being both impure and deontic system.

The DIS approach is the following one (Nescolarde-Selva, 2010; Nescolarde-Selva *et al.*, 2012a, b; Nescolarde-Selva and Usó-Doménech, 2012, 2013b; Usó-Domènech and Nescolarde-Selva, 2012):

- Objects are perceptual significances (relative beings) of material or energetic objects (absolute beings).
- The set of these objects will form an impure set of first order.
- The existing relations between these relative objects will be of two classes: transactions of matter and/or energy and inferential relations.
- Transactions have alethic modality: necessity, possibility, impossibility and contingency. They are ontic relations.
- Ontic existence of possibility causes that inferential relations have deontical modality: obligation, permission, prohibition and faculty. They are human relations.

- We distinguished between theorems (natural laws) and norms (ethical, legislative and customary rules of conduct).
- Each relation has intensity and direction.
- Between these relative objects it exists, not an only relation, but sheaves of relations and going in both directions, clockwise and no clockwise.
- The sheaves also have intensity.
- An inferential relation has modal and neutrosophic components.
- In each sheaf there will be generating and generated relations.
- Sheaves of both directions between two relative objects form a freeway. Relative objects united by freeways form a chain (network).
- This network is a chain of transmission of direct or indirect causality. Therefore, in our approach network will be denominated chain.
- Being all the DIS' objects directly or an indirectly related to each other, it will be formed by a single chain with multiple ramifications.
- An alysidal set is one whose elements are chains.
- Coupling functions between alysidal sets can be established.
- Nodes are subject. These may be individuals or group of individuals (corporations, regions, states, etc.).
- Special coupling function of recognition denominated gnorpsic function can be established.
- Gnorpsic function allows operations of connection between systems. Gnorpsic functions involve knowledge and decision.
- The system has two semiotic environments (Lloret-Climent *et al.*, 2002; Patten 1978, 1982; Usó-Domènech *et al.*, 2002a, b; Usó-Domènech *et al.*, 2002; Usó-Domènech and Nescolarde-Selva, 2012): a stimulus (input) environment $\mathbf{H}'$ and response (output) environment $\mathbf{H}''$. The system in itself consists of a succession of states through a temporary interval (Figure 1).
- Both semiotic environments $\mathbf{H}'$ and $\mathbf{H}''$ are also systems (Figure 1).
- Clockwise sheaves Z form stimuli from $\mathbf{H}'$ semiotic environment to DIS.
- Clockwise sheaves Y form response from DIS to $\mathbf{H}''$ semiotic environment.
- Conditions of permission and prohibition of stimuli and responses are established.
- It is demonstrate that DIS is inconsistent or incomplete whenever the sheaf of stimuli is allowed by the system.

In the case of complex systems of the type human society, are the cultural evolution, with their corresponding knowledge acquisition, development of technologies and social advances, elevating the resistance threshold and diminishing the probability of inassimilable stimuli. On this class of relations the advantage is based that have the structures of greater size on those of so large minor, which is related to the attainment of a stability at a higher level, done of local instabilities. Great systems and subsystems in size have greater longevity, as if they had a prize at evolutionary level. They combine segments

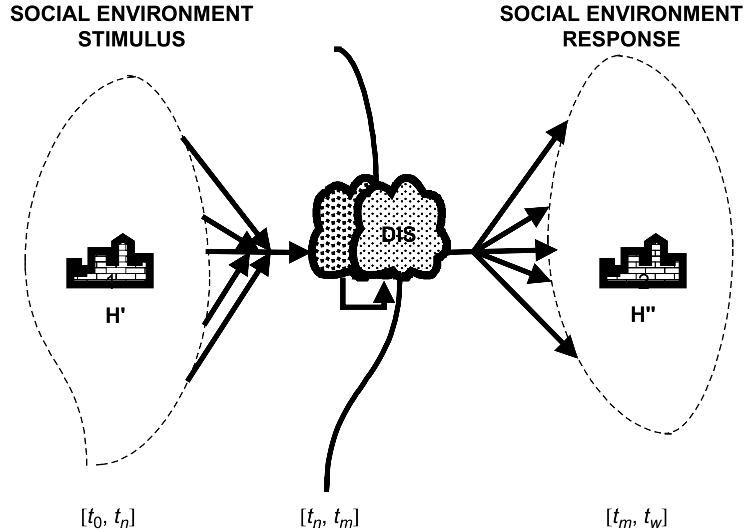

Figure 1.
DIS and the two environments

of the natural continuous (Margalef, 1980) extending in space and time. This is applied so much to the whole ecosystems, as human societies and its components.

2. Phenomenological components

In DIS we distinguished four main phenomenological components: directionality, intensity, connection energy and volume. With respect to the directionality of relations, sheaves and freeways, we have mentioned its characteristics in a previous work (Nescolarde-Selva, 2010; Nescolarde-Selva *et al.*, 2012a, b; Nescolarde-Selva and Usó-Doménech, 2012; Usó-Domènech and Nescolarde-Selva, 2012).

A fundamental question in this approach to DIS is the intensity or forces of a relation. We know by disciplines as physics, chemistry, sociology, psychology, etc. that a same class of relation can have different intensity. We thought advisable to introduce the concept of *degree of intensity of a relation*[3].

Definition 1. The degree of relational intensity of a relation r between two p-significances (relative beings), and denoted as I_r is the a real number $I_r \in [0,1]$ measuring, by direct or indirect procedures, the intensity or forces of a relation.

Note 1. If $I_r = 1$ we will call rigidity with respect to a relation r and if $I_r = 0$ we will call independence with respect to this relation.

Definition 2. The degree of intensity of a sheaf h_{ij} formed by n independent relations, and denoted as I_h, is the real number $I_r \in [0,1]$ so that $I_h = \frac{\sum_{i=1}^{n} (I_r)_i}{n}$.

Note 2. If $I_h = 1$ we will call rigidity of sheaf and if $I_h = 0$ we will call flexibility of sheaf.

We will consider as positive the degree of intensity of d-sheaf and as negative, the one of l-sheaf. In r-sheaf, the clockwise will be positive degree of intensity, and negative the no clockwise one. Therefore, if in a reciprocal relation its clockwise sense has a degree of intensity I_{ij}. Its no clockwise sense has a degree of intensity I_{ji}, the degree of intensity of the reciprocal relation will be:

$$\overleftrightarrow{I_{ij}} = I_{ij} + I_{ji}, \overleftrightarrow{I_{ij}} \leq, 0, \overleftrightarrow{I_{ij}} \geq 0.$$

The degree of intensity of r-sheaf will be, of course, the sum of the degrees of intensity of all the reciprocal relations that form it, divided by the number of them.

The degree of intensity of a highway will be:

$$I_{hw} = \frac{I_{d-b} + I_{l-b} + I_{r-b}}{3}$$

being I_{d-b} the degree of intensity of d-sheaf, I_{l-b} the degree of intensity of l-sheaf and I_{r-b} the degree of intensity of r-sheaf and $I_{hw} \geq 0, I_{hw} \leq 0$.

Note 3. The positivity and negativity have not a strict physical sense but of directionality:

If $h_{ij} > 0$ it means that sheaf has greater intensity in the direction x_i to x_j.

If $h_{ij} < 0$ it means that sheaf has greater intensity in the direction x_j to x_i.

If $h_{ij} = 0$ it means that sheaf has the same intensity in the two directions.

Note 4. Both senses of the same reciprocal relation $\overleftrightarrow{r}$ within a same sheaf can have different intensity, is to say $I_{r_{AB}} \neq I_{r_{BA}}$.

Definition 3. The degree of intensity of a chain $\wp_i^k$ formed by m sheaves, and denoted as $I_\wp$, is the real number $I_r \in [0,1]$ so that $I_\wp = \frac{\sum_{i=1}^{m} (I_h)_i}{m}$

Note 5. If $I_\wp = 1$ we will call linking and if $I_\wp = 0$ we will call no-not bound and we will represent it as $\neg \wp_i^k$.

Definition 4. The degree of intensity of a DIS formed by ω chains and denoted as I_Σ, is a real number $I_\Sigma \in [0,1]$ so that $I_\Sigma = \frac{\sum_{I=1}^{\omega} (I_\wp)_i}{\omega}$

Note 6. If $I_\Sigma = 1$ and we will call rigid DIS and if $I\Sigma = 0$ we will call incoherent DIS and we will represent it as $\neg \Sigma$.

An essential notion in the theory of impure systems is the one of Volume Λ. We are not talking about a physical volume, but to an abstract concept, that nothing has to do with a space occupation, in the same way that "color" in the theory of quarks, nothing has to do with the color defined by the Optics.

Definition 5. The maximum lexicon indicated by ш is an ideal vocabulary formed by all possible existing monochains both in the mind of subject S and outside of it, and such that $\Im \subset$ ш.

The concept of ш equates to the Platonic idea of the existence of ideas outside of the context of the human mind. It would be the equivalent of imagining an ideal space containing all objects attributes, etc. of which the system's lexicon would be a simple subset. Before subject S prepares to create the DIS, in the subject's mind there exist the components formed by the very concept of system, the elements of the maximum lexicon and the languages $L(G_\wp)$ and $L(G_{DO})$, formed by the generative grammars $G_\wp$ and G_{DO} from chains and from the deontical system, respectively.

A DIS is formed by an s-impure set $M \subset \Im$ with $cardM = k$, a set of chains $\rho = \{\wp_i^k; k > 0\}$ with card $\rho = \omega$, a set of sheaves H with cardH = m, and a set of relations R with cardR = n.

Definition 6. The volume of a DIS and denoted as Λ is the product $\Lambda = k.\omega.m.n$.

Definition 7. The fundamental volume of a DIS and denoted as Λ_P is the product $\Lambda_P = k{\cdot}m{\cdot}n$.

Definition 8. The size of a DIS is the doublet formed by (Λ, u_Σ).

Definition 9. A system DIS_1 with (Λ_1, u_Σ^1) is greater than another system DIS_2 with (Λ_2, u_Σ^2) if $\Lambda_1 \geq \Lambda_2, u_\Sigma^1 > u_\Sigma^2$.One of the essential characteristics of a deontical system is its degree of organisation, which produces a certain distribution of specific abundance, by means of a certain spectrum of relative frequency, from the most abundant monochain to the rarest.

Definition 10. The abundance of an element (monochain) is the number of sheaves which leads to the formation of chains.

However, the relative frequency of a monochain, by which its abundance is indicated and as used by Zipf's law, is none other than its probability of appearance within a particular system, assuming that the appearance is random by means of a particular technique or device. Theory of said device is not selective (Usó and Mateu, 2004).

3. Thermodynamics of deontical system

We will develop a DIS' thermodynamic theory based on information theory.

Definition 11. The primitive state is an ideal space formed by the concepts of system and relation, the elements of the maximum lexicon ш and a language $L(S)$ formed by the two sublanguages $L(G_\wp)$ and $L(G_{DO})$, formed by the generative grammars $G_\wp$ and G_{D0} from chains and from the deontical system, respectively.

Definition 12. The temperature of information Θ, the temperature of language $L(G_{SD})$ (Mandelbort, 1954, 1961) or temperature of the deontical system, is the magnitude indicated by the system's degree of structure.

In comparison with physical temperature, a low temperature of information would indicate a high degree of organisation; as the temperature Θ increases, the sheaf become weaker (the sheaf's intensity decreases) between the monochains until any kind of structure disappears when $\Theta = \infty$, leading to the incoherent system.

Mandelbort (1954, 1961) proposes the following modification of Zipf's law (1949) of the range and frequency of signs in a text. We suppose that in our case the signs would be the elements of the lexicon $\Im$ formed by the monochains $\wp_i^0$ of the S_D system:

$$f(r) = P\Lambda(r+\rho)^{-\beta} \tag{1}$$

where ρ, β is the two dependent parameters of the S_D system and $\beta > 0$, and P is determined by:

$$P^{-1} = (1+\rho)^{-\beta} + (2+\rho)^{-\beta} + \cdots + (N+\rho)^{-\beta} \tag{2}$$

where N is the number of different elements (monochains) of the lexicon $\Im$, and r is the range of these elements or the order in which the frequency of these elements is placed. The frequency of an element or monochain is the number of times that a sheaf comes out of an element or monochain initiating a chain.

Example 1. We assume the following four cases (Figure 2).

Explanation of the figure: we can assume the existence of three initial monochains, x_1, x_2, x_3, elements of a lexicon of a specific deontical system:

- Case a: x_1 will have a frequency of 1, as only a single sheaf is emitted.
- Case b: x_2 will have a frequency of 2, due to two sheaves being emitted, i.e. two chains with the same origin.

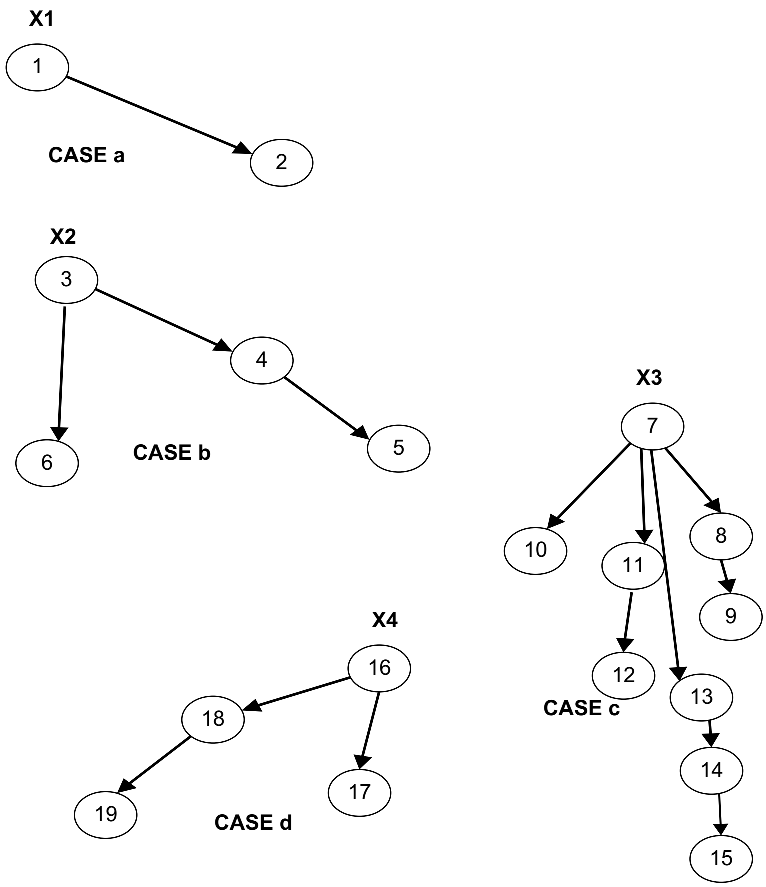

Figure 2.
Range and frequency

- Case c: x_3 will have a frequency of 4, due to four sheaves being emitted, leading to four chains with the same origin.
- Case d: x_4 will have a frequency of 2, due to two sheaves being emitted, leading to two chains with the same origin.

The range and frequency can then be established as follows (Table I).

We will assume that in the primitive state, we consider the existence of a singularity formed by an infinite number of elements and comprising the maximum lexicon, which are in a state of minimum energy and having a form such that it belongs to the

Table I.
Element, range and frequency

Element	Range	Frequency
x_3	4	4
x_2	3	2
x_4	2	2
x_1	1	1

language structure, but without any relation to the other elements to form a system. In this primitive state there will be an infinite temperature of information, an infinite primitive volume, as $k = \infty$, and an infinite H entropy. In this state, it is possible to apply Hawking's Principle of Absolute Ignorance, and therefore an absolute lack of information, i.e. $I = 0$ and $H = \infty$. This primitive state is in a condition of maximum disorder or thermodynamic equilibrium. If it is compared with a perfect gas, the most probable state in a certain amount of gas enclosed in a locked recipient is of a uniform density, and the position of the individual molecules is random, with any configuration from among a high number of possible configurations being equally probable.

This would be the situation of the deontical system before using the language L(S), when only this primitive state exists and is formed by all the elements of the maximum lexicon. This is the limit of our measure of information, the situation of maximum statistical entropy.

Formula 2 can be written in probabilistic form as follows:

$$p_r = P\Lambda(r+\rho)^{-\beta} \tag{3}$$

Parameter β is the inverse of the temperature of information $\Theta = (1/\beta)$. The system's entropy H will be determined by the Shannon formula:

$$H = -\sum p_r \log_2 p_r \quad \frac{\partial H}{\partial \beta} < 0, \frac{\partial H}{\partial \Theta} > 0$$

H grows continuously from 0 to $\log \Lambda$ when Θ goes from 0 to $+\infty$. H determines Θ for a given Λ. The Mandelbrot criterion consists of transforming into 0 the free variation of $A = E(\delta) - \Theta H$, i.e. the excess of energy if the synergy per element in the Shannon formula is $(1/\Theta)$.

A is the Helmholtz free energy, i.e. the energy available for dissipation, where $E(\delta)$ is the enthalpy or calorie content of the information. Therefore, Λ, Θ can be assimilated as variables of state. In an initial approach, the volume of the system would be equal to the number of elements in the system, i.e. $\Lambda = N$.

Temperature of discourse is defined by Mandelbort (1975) as $\Theta = (\log_2 N)/(\log_2(1/r))$, which constitutes a fractal dimension. The larger the dimension, the greater the range r, i.e. the greater the wealth of the lexicon or the greater the number of elements in the system (chains), which does not mean that it is more structured. Entropy H measures how much information is missing to understand which structure has a system that is disordered for the system observer. Therefore:

$$\begin{aligned}
H &= -\sum p_r \log_2 p_r = -\sum P\Lambda\left(r+\rho\right)^{-1/\Theta} \log_2 P\Lambda\left(r+\rho\right)^{-1/\Theta} \\
&= -\sum P\Lambda(r+\rho)^{-1/\Theta}\left[\log_2 P\Lambda + \log_2\left(r+\rho\right)^{-1/\Theta}\right] \\
&= -\sum P\Lambda(r+\rho)^{-1/\Theta}\left[\log_2 P\Lambda - \frac{1}{\Theta}\log_2(r+\rho)\right] \\
&= -\sum P\Lambda(r+\rho)^{-1/\Theta}\log_2 P\Lambda + \frac{1}{\Theta}\sum P\Lambda(r+\rho)^{-1/\Theta}\log_2(r+\rho) \\
&= \frac{1}{\Theta}\sum P\Lambda(r+\rho)\log_2 P\Lambda + \frac{1}{\Theta}\sum \frac{P\Lambda}{(r+\rho)^{1/\Theta}}\log_2(r+\rho)
\end{aligned} \tag{4}$$

from (3):

$$\Lambda = p_r P^{-1}(r+\rho)^{1/\Theta} = \frac{p_r(r+\rho)^{1/\Theta}}{P} \quad (5)$$

from which we can deduce that:

$$k \cdot \omega \cdot n \cdot m = \frac{p_r(r+\rho)^{1/\Theta}}{P} \quad (6)$$

and:

$$k \cdot \omega \cdot n \cdot m = \frac{p_r(r+\rho)^{(1/\log_2 N)/(\log_2(1/r))}}{P} = \frac{p_r(r+\rho)^{(\log_2(1/r)/(\log_2 N))}}{P} \quad (7)$$

Given a number k of elements (monochains), a number of sheaves and a number of relations n, the number of chains that can be formed would be:

$$\omega = \frac{p_r(r+\rho)(1/\log_2 N)/(\log_2(1/r))}{P}\left(\frac{1}{k \cdot n \cdot m}\right) \quad (8)$$

therefore:

$$\omega = \frac{p_r(r+\rho)^{(1/\log_2 N)/(\log_2(1/r))}}{P}\left(\frac{1}{\Lambda_P}\right) \quad (9)$$

We will call the quotient $(1/\Lambda_P)$ the deontical system chain factor, and it indicates the inverse of the system's primary volume.

Let a deontical system be S_D. We will have the following thermodynamic conditions:

- Entropy H is a magnitude that ranges from 0 to $-\sum p_r \log_2 p_r$.
- The temperature of information ranges from 0 to infinity, $0 < \Theta \leq \infty$.
- The temperature of information 0 is considered the absolute zero of discourse. Just as with absolute physical zero, absolute zero of information is an ideal magnitude, and means absolute rigidity, supreme order.
- For a fixed volume Λ, if $\Theta = 1$, then $H = -\sum \frac{P\Lambda}{(r+\rho)} \log_2 \frac{P\Lambda}{(r+\rho)}$.
- For a fixed volume Λ, if $\Theta \geq 1$, the series $\sum (r+\rho)^{-1/\Theta}$ is divergent, which is only avoided if r is delimited, which means that the system has a finite number of elements.
- For a fixed volume Λ, if $\Theta < 1$, the number of elements may well be infinite. In this case, and following the simile established by Mandelbrot, we would establish an infinite succession of statistically independent blank (empty) elements and spaces, and would use the empty spaces to break said series down into related elements (the equivalent to words in a text). The probabilities of the related elements would follow Zipf's generalised law.
- If $0 < \Theta \leq \infty$ and Θ are close to 0, then the term:

$$\frac{1}{\Theta}\sum \frac{P\Lambda}{(r+\rho)^{1/\Theta}} \log_2(r+\rho) \gg \frac{1}{\Theta}\sum \frac{P\Lambda}{(r+\rho)^{1/\Theta}} \log_2 P\Lambda$$

therefore $H \rightarrow 0$ and the information will be close to zero. The system will tend to be more and more structured.

- If $0 < \Theta \leq \infty$ and $\Theta \gg 1$, then the term:

$$\frac{1}{\Theta}\sum\frac{P\Lambda}{(r+\rho)^{1/\Theta}}\log_2(r+\rho) \ll \frac{1}{\Theta}\sum\frac{P\Lambda}{(r+\rho)^{1/\Theta}}\log_2 P\Lambda$$

and $H \rightarrow -\sum p_r \log_2 p_r$. The system will begin to deconstruct.

- In the extreme case that $\Theta = \infty$ and $\Lambda = (p_r/P)$, then entropy will be:

$$H = -\sum P\Lambda \log_2 P\Lambda = -\sum p_r \log_2 p_r$$

and information will be $I = \sum p_r \log_2 p_r$. This will correspond to a source of information (system or medium) the elements of which are emitted (appear) with equal probability or in an equally divided whole. No structure will therefore exist.

- Taking information by the system means making the system's structure more complete, stronger and logically make the system's temperature of information approach absolute zero. In contrast, giving information by the medium means making the structure of the medium weaker and the temperature of information approach infinity.
- The system is informationally colder with regard to another system or to its medium when its temperature of information is closer to absolute zero than the other system or its medium, which will be considered as informationally warmer.
- If an informationally cold system comes into contact with another system or with its informationally warmer medium, the system will cool down more, at the same time as the second system or its medium warms up, while it increases its temperature of information.
- A system takes information from another system or from its medium when it makes its structure more complex, and therefore reduces its temperature of information.

Definition 13. The heat of information Q is the information in circulation, in which the form is how the information is transmitted from one system to another as the result of the difference of temperatures of information.

Definition 14. The calorific capacity of information C is the property which, multiplied by the variation in the temperature of information, means the amount of information that the system takes or gives when it enters into contact with another system or a part of the system enters into contact with another, with a different temperature of information.

If the system's temperature of information drops from Θ_1 to Θ_2 (increasing the structure and thus the information) when taking an amount of heat of information Q, then the calorific capacity of the system's information C will be given by:

$$Q = C(\Theta_1 - \Theta_2) \tag{10}$$

therefore:

$$C = \frac{Q}{\Theta_1 - \Theta_2} \tag{11}$$

The value of C will always be negative when the complexity of the system increases. If the variation in temperature of information $\Theta_1 - \Theta_2$ is represented by $\Delta\Theta < 0$, the calorific capacity of true information will be given by the expression:

$$C = \lim_{\Theta_2 \to \Theta_1} \frac{Q}{\Delta\Theta} \tag{12}$$

If in an initial approach we consider system pressure Π on the system's hypothetical frontiers, then $(\partial H/\partial\Theta)$ we will consider as $(\partial H/\partial\Theta) = C/\Theta$, by which:

$$C = \Theta\left(\frac{\partial H}{\partial\Theta}\right) \tag{13}$$

which will be expressed as:

$$dH = \frac{C}{\Theta}\, d\Theta = Cd\log\Theta \tag{14}$$

and integrating:

$$(H - H_0) = H = \int_{\Theta}^{1} \frac{C}{\Theta}\, d\Theta = \int_{\Theta}^{1} Cd\log\Theta \tag{15}$$

where H_0 is the hypothetical value of the entropy in absolute zero of information with the same pressure of information Π in each case, which as we have seen previously will be 0. For the variation in entropy that accompanies a change of temperature of Θ_1 to Θ_2, $\Theta_2 < \Theta_1$, the entropy gradients formula is obtained as:

$$\Delta H = (H_2 - H_1) = \int_{\Theta_1}^{\Theta_2} \frac{C}{\Theta}\, d\Theta = \int_{\Theta_1}^{\Theta_2} Cd\log\Theta \tag{16}$$

The entropy gradient must help to sharpen all the gradients of properties that can be interpreted as carriers of information, which can contribute to the progressive compartmentalisation of a hypothetical systemic volume. According to Margalef (1980), these effects are particularly important in the creation of discontinuous blocks that function as pieces of open systems, far from a situation of equilibrium. It is an example of how the structure can grow by feeding on the function.

If after recognising the two halves of a space the mental direction can be further in one half than in the opposite half, it is said that more information exists in the first half. It represents a centre or nucleus of convergent information, which in the figure is represented by Ω. However, the information is not a resource or a product, and neither is it entropy. In any exchange between two subsystems of the same system, the information increases more quickly or is conserved better in the subsystem that had already reached more complexity than the other subsystem. However, to talk of information flow is to do so in a figurative sense, i.e. it is a mental construct, as the production of entropy. On the other hand, it is clear that the construct of a systemic space results from the progressive recognition of information.

Let DIS be a deontical system. Let $\wp_1^0, \wp_2^0 \in \Im$ be two elements (monochains) belonging to the lexicon of the deontical system.

Definition 15. The informational heat of information of a deontical system DIS is the increase in calorific capacity of information ΔC caused when a system is formed from its elements to a specific pressure of information Π and temperature of information Θ.

The value of ΔC depends on the state and conditions of the elements, and it is thus to be supposed that the elements of the system are found in what is called their typical states.

Definition 16. The breaking energy of a relation is the energy needed to destroy the relation between two $\wp_1^0, \wp_2^0 \in \Im$ elements of the deontical system.

Definition 17. The energy of relations E_Σ is defined as the mean energy needed to maintain particular relations in a deontical system and not separate the elements that constitute the system from each other. Therefore:

$$E_\Sigma = \frac{\sum_{i=1}^{n} E_{ri}}{m} \tag{17}$$

Let m be the number of sheaves having breaking energy of connection E_r minor that the energy E_Σ of connection of impure system DIS is to say $E_r < E_\Sigma$. Elements of the impure system fulfilling this condition are denominated of weak connection. In the opposite case, they will be of strong connection being stronger whichever greater is its breaking energy in relation to the energy of connection of the impure system.

We considered all incident stimulus in an impure system DIS is a relation located within a d-sheaf, coming from an alysidal set located in the stimulus environment $\mathbf{H}'$ (Lloret-Climent *et al.*, 1998, 2001, 2002; Patten, 1978, 1980, 1982; Patten *et al.*, 1976, 1982; Patten and Auble, 1981; Usó-Doménech *et al.*, 2002a, b; Nescolarde-Selva, 2010; Nescolarde-Selva *et al.*, 2012a, b, c; Usó-Domenech *et al.*, 2012) and so that $A_{al} \subset \mathbf{H}'$, and affecting a certain node of a certain chain of the impure system DIS. Of course, this relation that unites a node n_i of a chain $\wp_i^k \in A_{al}$ of the alysidal set A_{al} with a node m_j of a chain $\wp_j^m \in DIS$ will have a certain energy of connection. However, for DIS, this energy of connection becomes stimulus energy E_Z. Let us suppose entering within the impure system DIS. And originating of the stimulus environment $\mathbf{H}'$, a set of stimuli Z, with an energy E_Z and so that $E_Z > E_r$ and $E_Z \leq E_\Sigma$. If the energy by group of stimuli Z entering the impure system coming from the stimulus environment $\mathbf{H}'$ is greater than the energy of rupture of the elements of weak connection, the system will experience a loss of order, and therefore of structuring and information. And to the inverse one, if the group of stimuli Z has an inferior energy to the energy of rupture of the elements of weak connection, impure system will experience a net gain of order, and therefore of structure and information.

Sagan's principle (1975): If the energy per group of stimuli Z entering the system from the exterior stimulus medium $\mathbf{H}'$ is greater than the relation energy of the elements of the system with a weak bond, system Σ will experience a loss of order and therefore of structure and information. And in inverse terms, if the group of stimuli Z has an energy lower than the rupture energy of the elements of the system with a weak bond, system DIS will experience a net gain of order and therefore of structure and information.

Definition 18. The threshold of resistance u_Σ of a system is marked by its bond energy E_Σ.

Definition 19. The size of a system is the cognate formed by (Λ, u_Σ).

Definition 20. It is said that a DIS_1 with $\left(\Lambda, u_\Sigma^1\right)$ is larger than another system DIS_2 with $\left(\Lambda, u_\Sigma^2\right)$ if $\Lambda_1 \geq \Lambda_2, u_\Sigma^1 > u_\Sigma^2$.

The energy needed to alter the system increases with the size of the system, i.e. with the increase in volume Λ of the system, but also with the number of sheaf. This can be checked in small physical systems, from the molecular to the subatomic level, where the energy of disruption decreases as the size of the system increases. However, this can present an apparent paradox in biotic systems.

4. Transmission of information

Let us suppose the stimulus environment $\mathbf{H}'$ as a discreet sign-generator nil memory source S and such that $S = \{S_1, S_2, \ldots, S_N\}$. This source therefore emits a sequence of symbols belonging to a fixed and finite alphabet (Abramson, 1980), the elements of which form a data structure. These symbols are chosen with a fixed law of probability and we will admit that they are statistically independent. The probabilities with which the symbols are presented are $p(S_1), p(S_2), \ldots, p(S_N)$. The amount of information generated by the occurrence of S_i is:

$$I(S_i) = \log \frac{1}{p(S_i)} = -\log p(S_i) \tag{18}$$

Definition 21. The amount of information $I(S_i)$ is called the surprise value of symbol S_i.

Definition 22. The mean amount of information $I(S_i)$ associated with source S *is:*

$$\sum_S p(S_i) I(S) \tag{19}$$

That is, the surprise values of each one of the possibilities of source S are taken and weighted according to a probability of occurrence $p(S_i)$. The sum of everything will be the amount of information generated by the source S. If the measure of $p(S_i)$ is close to 1, the amount of information associated with the occurrence of the symbol S_i tends toward 0. In the extreme case of the symbol's probability being 1, the occurrence of S_i generates no information. That is, the information is not generated by the occurrence of the symbols, so no alternative possibilities exist. We will designate as s a receptor of information in S. Receptor s is formed by the elements of lexicon $\Im$ that existed before the interaction in the primitive state.

How does *I*(*s*) receive information from source S? We will designate this new information as $I_S(s)$, with the subindex S indicating the part of *I*(*s*) that has received information from source S. The information transmitted from S to s is the total amount of information available in s, that is *I*(*s*) less an amount R or noise, which we will express as:

$$I_S(s) = I(s) - R \tag{20}$$

Similarly:

$$I_S(s) = I(s) - \varepsilon \tag{21}$$

where ε is the equivocality of the information generated by the source S that is not transmitted to s. The information generated in the source S is divided into two parts:

(1) The part $[I_S(s)]$, which is transmitted to s.

(2) The part ε, which is not transmitted, or equivocality.

If P is denoted as an operator of permission, and Ph the operator of prohibition, then:

$$PI(S) = [I_S(s)] \wedge PhI(S) = \varepsilon \tag{22}$$

At the same time, the information that is in s can be divided in the same way into two parts:

(1) Part $[I_S(s)]$, which is the information received from source S.

(2) The other part, the source of which is not S or noise R. An increase of R means that a part of the sign S_i is hidden, and $[I_S(s)]$ will thus decrease by means of an increase of the equivocality ε.

If the noise increases, an amount of information is lost, and the amount of information transmitted drops. Let S be a significant and s the significance (Nescolarde-Selva and Usó-Doménech. 2013a, b; Usó-Domènech and Nescolarde-Selva, 2012).

Definition 23. The existence of information is independent of the fact existence of a Subject able to decode the message. This objective information is termed significant S.

Definition 24. The information in a message acquires meaning if a subject decodes the message. This subjective information is termed significance s.

In classic information theory (Abramson, 1980), an equivalence is established between the noise R and the equivocality ε, as a result of having chosen the set of possibilities from the source S and from the receptor s, so that $I(S) = I(s)$. If we imagine changes in the set of possibilities that define $I(s)$ without the corresponding changes in the set of possibilities that it defines $I(s)$, and vice versa, a necessary equivalence between them will not exist. In the event of there being a maximum dependency between what occurs in S and what occurs in s, then $R = \varepsilon = 0$ and the amount of information transmitted $[I_s(s)]$ will be higher and in this case $[I_s(s)] = I(s)$.

Let $p(s_i/S_i)$ be the conditional probability of a significance s_i, generated in s based on a significant S_i transmitted from the source S. We must be able to calculate the contribution of S_i to the noise R by means of:

$$R = -\sum_S p\left(\frac{s_i}{S_i}\right) \log\left(p\left(\frac{s_i}{S_i}\right)\right) \tag{23}$$

The equivocality ε will be calculated in a similar way:

$$\varepsilon = -\sum_S p\left(\frac{s_i}{S_i}\right) \log\left(p\left(\frac{s_i}{S_i}\right)\right) \tag{24}$$

The flow of information is closely related to causal processes. However, it will be necessary to distinguish between the causal relations and informational relations that exist between the course (the $\mathbf{H}'$ stimulus environment) and the receptor, which in this case would be the deontical system. If the data were always the same, i.e. if it had experimental perseverance, which does not normally occur, a strong causal dependence would exist between the source and receptor.

It occurs that $\{S_i\}$ is the cause of $\{s_i\}$. The significance s_i should tell us that it occurs in the source, as s knows this. From the informational point of view, s takes more information than about what has occurred in S. Although, for a particular data structure, each symbol has a particular significant or specific adjustment, and the temporal change of this structure can determine a change in the symbol that represents it, a change made in its significant or adjustment of the symbol and in its significance or decoding by the receiving subject.

5. Conclusions

The development of the structures receiving stimuli of the stimulus environment $\mathbf{H}'$, constitutes a developed historical process in the time interval $T = [t_0, t_n]$. It represents the accumulation of information making a more efficient use of the energy in a later while $[t_{n+1}, t_\omega]$. The necessary energy for the alteration of the system increases with the size of this one. We can verify it in the small physical systems, of molecular to subatomic, in where the disruption energy diminishes when increasing the size of the system. The certainty, that sooner or later some nonassimilable group of stimuli will arrive $E_Z \geq E_\Sigma$. During the evolutionary development of the system, as the threshold rises describing the form to deal with environment $\mathbf{H}'$, it seems as if the nonassimilable stimuli were less and less frequent and therefore, less probability exists in the disintegration of the system. This argument is logical. If we suppose the rank of energies as a statistically normal distribution, as the system structure becoming that their energy of connection is every greater time, the external energy necessary to hit negatively also will be every greater time, happening to the end of the rank of smaller frequency and therefore, of smaller probability of appearance. This phenomenon must consider understanding the asymmetries, as much in succession as in evolution.

Within this context it is understood that the survival of the DIS will depend on its capacity to combine certain stimuli coming from $\mathbf{H}'$ calls resources (material and energetic or population transactions (emigration flows), combined with stimuli of nonmaterial order: beliefs, values, ideas, etc.) or to combine the variation of select resources and inferential relations on the space or the time.

Notes

1. Knowing that a system does not necessarily imply know the laws that govern it and the knowledge of the general systems theory. The word is commonly used system. It is common in the everyday language use "ecosystem", "systemic effect", "anti-system", etc.
2. As a branch of symbolic logic, deontic logic is of theoretical interest for some of the same reasons that modal logic is of theoretical interest. However, despite the fact that we need to be cautious about making too easy a link between deontic logic and practicality, many of the notions listed are typically employed in attempting to regulate and coordinate our lives together (but also to evaluate states of affairs). For these reasons, deontic logics often directly involve topics of considerable practical significance such as morality, law, social and business organizations (their norms, as well as their normative constitution), and security systems. These concepts and their logical relationships to one another are distinguished from value concepts such as goodness and badness (or evil), as well as from such

agent-based concepts as act, choice, decision, desire, freedom, and will. Then and in conclusion: there is only one known species having deontic own of free will, in turn a result of alethic modality of possibility, and this is the human species. Therefore, if conceived different human societies as systems (not ceases to be a mental model), it is legitimate to define these societies as deontic systems (DIS).

3. We are not talking of vector components. In vector theory, a direction vector that describes a line segment D is any vector $\overrightarrow{AB}$ where A and B are two distinct points on the line D. If $\overrightarrow{v}$ is a direction vector for D, so is $k\overrightarrow{v}$ for any nonzero scalar k; and these are in fact all of the direction vectors for the line D. Under some definitions, the direction vector is required to be a unit vector, in which case each line has exactly two direction vectors, which are negatives of each other (equal in magnitude, opposite in direction). Sheaves could be considered vectors if we make the abstraction that the "vector" is formed by a bundle of parallel relationships in both directions. The concept of intensity of a vector is associated with the presence of a gravitational or electromagnetic field. This is not the case. Conclusion: We are not in a vector space.

References

Abramson, N. (1980), *Information Theory and Coding*, McGraw-Hill, New York, NY.

Lloret-Climent, M., Villacampa, Y. and Usó, J.L. (1998), "System-linkage: structural functions and hierarchies", *Cybernetics and Systems: An International Journal*, Vol. 29, pp. 29-39.

Lloret-Climent, M., Usó-Doménech, J.L., Patten, B.C. and Vives-Maciá, F. (2002), "Causality in H-semiotic systems of ecosystems: routes and tours", *International Journal of General Systems*, Vol. 31 No. 2, pp. 119-130.

Lloret-Climent, M., Usó-Domènech, J.L., Sastre-Vazquez, P., Vives-Maciá, F. and Patten, B.C. (2001), "Epistemological and mathematical considerations on the structure of H-semiotic systems", *Cybernetics and Systems*, Vol. 33 No. 5, pp. 507-535.

Mandelbort, B. (1954), "Structure formelle des texts et communication. Deux études", *Word*, Vol. 10, pp. 1-27.

Mandelbort, B. (1961), "Word frequencies and Markovian models of discourse", *Structure of Language and Its Mathematical Aspects. Procedings of Symposia in Applied Mathematics. American Mathematical Society, Providence, Rhode Island*, Vol. 12, pp. 190-219.

Mandelbort, B. (1975), *Les objects fractals: forme, harsard et determination*, Flammarion, Paris.

Margalef, R. (1980), *La Biosfera entre la termodinámica y el juego*, Ediciones Omega SA, Barcelona.

Nescolarde-Selva, J.A. (2010), "A systemic vision of beliefs systems and ideologies", doctoral thesis, Universidad de Alicante, Alicante.

Nescolarde-Selva, J.A. and Usó-Doménech, J.L. (2012), "An introduction to alysidal algebra III", *Kybernetes*, Vol. 41 No. 10, pp. 1638-1649.

Nescolarde-Selva, J.A. and Usó-Doménech, J. (2013b), "Reality, system and impure systems", *Foundations of Science*, June.

Nescolarde-Selva, J.A. and Usó-Doménech, J. (2013a), "Semiotic vision of ideologies", *Foundations of Science*, April.

Nescolarde-Selva, J.A., Vives Maciá, F., Usó-Doménech, J.L. and Berend, D. (2012a), "An introduction to alysidal algebra I", *Kybernetes*, Vol. 41 Nos 1/2, pp. 21-34.

Nescolarde-Selva, J.A., Vives Maciá, F., Usó-Doménech, J.L. and Berend, D. (2012b), "An introduction to alysidal algebra II", *Kybernetes*, Vol. 41 Nos 5/6, pp. 780-793.

Patten, B.C. (1978), "Systems approach to the concept of environment", *Ohio J. Sci.*, Vol. 78 No. 4, pp. 206-222.

Patten, B.C. (1980), "Systems approach to the concept of niche", *Synthese*, Vol. 43, pp. 155-181.

Patten, B.C. (1982), "Environs: relativistic elementary particles for ecology", *The American Naturalist*, Vol. 119, pp. 179-219.

Patten, B.C. and Auble, G.T. (1981), "System theory of the ecological niche", *The American Naturalist*, Vol. 117, pp. 893-922.

Patten, B.C., Richardson, T.H. and Barberm, G. (1982), "Path analysis of a reservoir ecosystem model", *Canadian Water Resources Journal*, Vol. 7 No. 1, pp. 252-282.

Patten, B.C., Bosserman, R.W., Finn, J.T. and Cale, W.G. (1976), "Propagation of cause in ecosystems", in Patten, B.C. (Ed.), *Systems Analysis and Simulation in Ecology*, Vol. 4, Academic Press, New York, NY, pp. 457-579.

Sagan, C. (1975), "Nota distribuida por", *The Coevolution Quart*.

Usó, J.L. and Mateu, J. (2004), *Teoría del medio Ambiente. Modelización*, Publicacions de la Universitat Jaime I, Castelló de la Plana.

Usó-Domènech, J.L. and Nescolarde-Selva, J. (2012), *Mathematics and Semiotical Theory of Ideological Systems*, Lambert Academic Publishing, Saarbrucken.

Usó-Domènech, J.L., Mateu, J. and Patten, B.C. (2002a), "Mathematical approach to the concept of environment: open systems and processes", *International Journal of General Systems*, Vol. 31 No. 3, pp. 213-223.

Usó-Domènech, J.L., Lloret-Climent, M., Vives-Maciá, F., Patten, B.C. and Sastre-Vazquez, P. (2002b), "Epistemological and mathematical considerations on the structure of H-semiotic systems", *Cybernetics and Systems: An International Journal*, Vol. 33 No. 5, pp. 507-535.

Zipf, G.K. (1949), *Human Behavior and the Principle of Last Effort*, Addison-Wesley, Cambridge, MA.

About the authors

Josué Antonio Nescolarde-Selva has a doctorate in mathematics from the University of Alicante, Spain. He is a Professor of applied mathematics in the University of Alicante. He is devoted to research on the theory of systems. Josué Antonio Nescolarde-Selva is the corresponding author and can be contacted at: josue.selva@ua.es

Jose-Luis Usó-Domenech graduated in sciences and doctor in mathematics from the University of Valencia, Spain. He made his doctoral thesis on a mathematical model of a terrestrial mediterranean ecosystem. He has directed many doctoral theses in theory of systems. He is author and co-author of several papers in journals, proceedings and books. He has obtained scholarships of research in many universities (University of Joensuu, Finland; University of Ben Gurion of Negev, Israel and the University of Georgia, USA). He has collaborated in numerous occasions with the Wessex Institute of Technology, UK and the fruit of this collaboration was the creation of the international congress, ECOSUD.

This article has been cited by:

1. J.L. Usó-Doménech, J.A. Nescolarde-Selva, M. Lloret-Climent, H. Gash. 2016. Semantics of language for ecosystems modelling: A model case. *Ecological Modelling* **328**, 85-94. [CrossRef]
2. J. Nescolarde-Selva, J. L. Usó-Doménech. 2016. Textual Theory and Complex Belief Systems: Topological Theory. *Foundations of Science* **21**:1, 153-175. [CrossRef]
3. Josué Antonio Nescolarde-Selva, José Luis Usó-Doménech, Miguel Lloret-Climent. 2015. Ideological Complex Systems: Mathematical Theory. *Complexity* **21**:2, 47-65. [CrossRef]
4. J.L. Usó-Doménech, J. Nescolarde-Selva, M. Lloret-Climent. 2015. "Unintended effects": A theorem for complex systems. *Complexity* **21**:2, 342-354. [CrossRef]
5. J. L. Usó-Doménech, J. Nescolarde-Selva, H. Gash. 2015. Guest Editorial: Belief Systems and Science. *Cybernetics and Systems* **46**:6-7, 379-389. [CrossRef]
6. J. Nescolarde-Selva, J. L. Usó-Doménech, M. J. Sabán. 2015. Linguistic Knowledge of Reality: A Metaphysical Impossibility?. *Foundations of Science* **20**:1, 27-58. [CrossRef]
7. Josué Antonio Nescolarde-Selva, Josep-Lluis Usó-Doménech. 2014. Myth, language, and complex ideologies. *Complexity* **20**:2, 63-81. [CrossRef]