A Study of Neutrosophic Differential Equations By Using the One- Dimensional Geometric AH-Isometry Of NeutrosophicLaplace Transformation

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A. A. Salama
Port Said University

Rozina Ali
Cairo University

A Study of Neutrosophic Differential Equations By Using the One-Dimensional Geometric AH-Isometry Of NeutrosophicLaplace Transformation

Ahmad Salama[1], Malath Al Aswad[2], Rasha Dalla[3], Rozina Ali[4]

[1]Department of Mathematics, port said, Egypt
[2]PHD of Mathematics, GaziAntep, Turkey
[3]Departement of Mathematics, Albaath University, Homs, Syria
[4]Department Of Mathematics, Cairo University, Egypt

Email:drsalama44@gmail.com; Malaz.Aswad@yahoo.com; rasha.dallah20@gmail.com; rozyyy123n@gmail.com

Abstract

In this paper, we study the neutrosophic differential equation by using the one-dimensional geometric AH-Isometry of Neutrosophic Laplace Transformation.Where we use this AH-isometry to find the algebraic image of this transformation, and then to apply this image directly on the problem of finding the solutions of differential equations.

Keywords: One-Dimensional Geometric AH-Isometry; Neutrosophic Differential Equation; Neutrosophic Laplace Transformation.

1.Introduction

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability and alike, are recently creations of Smarandache, being characterized by having the indeterminacies component of their framework,and a notable feature of neutrosophic logic is that can be considered a generaliazationof fuzzy logic, encompassing the classical logic as well [1]. Also.F. Smarandache, has defined the concept of continuation of a neutrosophic function in year 2015 in [1],and neutrosophic mereo-limit[1], mereo-continuity.Moreover, in 2014, he has defined the concept of a neutro-oscillator differentialin [3], and mereo-derivative. Finallyin 2013 he introducedneutrosophic integration in [2], and mereo-integral, besides the classical defintions of limit, continuity, deverative, and integral respectively.
Among the recent applications there are: neutrosophic crisp set theory in image processing[4][5],neutrosophic sets medical field [6-10], in information geographic systems[11] and possible applications to database[12]. Also,neutrosophic triplet group application to physics[13].Morever Several researches have made multiple contributions to neutrosophic topological$[14-20,23-33,36-50]$, Also More researches have made multiple contributions to neutrosophic analysis[21]. Finally the neutrosophic integration may have application in calculus the areas between twoneutrosophic functions.

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2. Preliminaries

Definition 2.1.Neutrosophic Real Number:[1]

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represents the indeterminacy, where $0.I = 0$ and $I^n = I$ for all positive integers n.

For example:

$w = 1 + 2I, w = 3 = 3 + 0I.$

Definition 2.2. **Division of neutrosophic real numbers**:[2]

Suppose that w_1, w_2 are two neutrosophic number,where

$$w_1 = a_1 + b_1 I, w_2 = a_2 + b_2 I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$$

Definition 2.3[**21**]

Let $R(I) = \{a + bI\ ; a, b \in R\}$ where $I^2 = I$be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows: [21]

$$T: R(I) \to R \times R$$
$$T(a + bI) = (a, a + b)$$

Remark 2.4.[21]

T is an algebraic isomorphism between two rings, it has the following properties:

1) T is bijective.
2) T preserves addition and multiplication, i.e.:

$$T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$$

And

$$T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$$

3) Since T is bijective, then it is invertible by:

$$T^{-1}: R \times R \to R(I)$$
$$T^{-1}(a, b) = a + (b - a)I$$

4) T preserves distances, i.e.:

The distance on $R(I)$ can be defined as follows:

Let $A = a + bI, B = c + dI$ be two neutrosophic real numbers, then $L = \|\overrightarrow{AB}\| = d[(a + bI, c + dI)] = |a + bI - (c + dI)| = |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|].$

On the other hand, we have:

$T(\|\overrightarrow{AB}\|) = (|a - c|, |(a + b) - (c + d)|) = (d(a, c), d(a + b, c + d)) = d[(a, a + b), (c, c + d)]$=$d(T(a + bI), T(c + dI))$

=$\|T(\overrightarrow{AB})\|$.

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This implies that the distance is preserved up to isometry. i.e. $\|T(AB)\| = T(\|AB\|)$

Definition 2.5. []

Let $f: R(I) \to R(I); f = f(X)$ and $X = x + yI \in R(I)$ the f is called a neutrosophic real function with one neutrosophic variable.
a neutrosophic real function $f(X)$ written as follows:

$$f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$$

Theorem 2.6. any neutrosophic real function into two classical real functions, i.e., to the classical Euclidean plane $R \times R$.

Proof.

Let $f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$ a neutrosophic real function.
Now, Using the one-dimensional AH-isometry, we have.
$T\big(f(X)\big) = T(f(x) + I[f(x + y) - f(x)])$, then.
$(f_1, f_2) = \big(f(x), f(x + y)\big)$, then, we have.

$$\begin{cases} f_1 = f(x) \\ f_2 = f(x + y) \end{cases}$$

the functions $f(x), f(x + y)$ are a real functions.

3. Neutrosophic Laplace Transformation.

In this section we define Neutrosophic Transformation of Laplace for a neutrosophic function $f(X)$.

Definition3.1.

Let $Y = y_1 + y_2 I$, $X = x_1 + x_2 I$, and $f(X) = f(x_1 + x_2 I) = f(x_1) + I[f(x_1 + x_2) - f(x_1)]$ a neutrosophic function on $R(I)$, then we define a NeutrosophicTransformation Laplace of $f(X)$ as forms:

$$F(P) = \int_0^{-\infty} e^{-PX} f(X) dX$$

Where $P = p_1 + p_2 I \in R(I)$.

Now, $F(P)$ can be written as follow:

$$F(p_1 + p_2 I) = \int_0^{-\infty} e^{-(p_1 + p_2 I)(x_1 + x_2 I)} (f(x_1) + I[f(x_1 + x_2) - f(x_1)]) d(x_1 + x_2 I)$$

$$F(p_1 + p_2 I) = \int_0^{-\infty} e^{-[p_1 x_1 + (p_1 x_2 + p_2 x_1 + p_2 x_2) I]} (f(x_1) + I[f(x_1 + x_2) - f(x_1)]) d(x_1 + x_2 I)$$

$$F(p_1 + p_2 I) = \int_0^{-\infty} \left[e^{-p_1 x_1} + \left(e^{-(p_1 x_1 + p_1 x_2 + p_2 x_1 + p_2 x_2)} - e^{-p_1 x_1}\right) I\right] (f(x_1) + I[f(x_1 + x_2) - f(x_1)]) d(x_1 + x_2 I)$$

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$$F(p_1 + p_2 I) = \int_0^{-\infty} \left[e^{-p_1 x_1} + \left(e^{-(p_1+p_2)(x_1+x_2)} - e^{-p_1 x_1}\right)I\right](f(x_1) + I[f(x_1 + x_2) - f(x_1)])d(x_1 + x_2 I)$$

$$F(p_1 + p_2 I) = \int_0^{-\infty} \Big[e^{-p_1 x_1} f(x_1) + I\Big(e^{-p_1 x_1} f(x_1 + x_2) - e^{-p_1 x_1} f(x_1) + e^{-(p_1+p_2)(x_1+x_2)} f(x_1) + e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) - e^{-(p_1+p_2)(x_1+x_2)} f(x_1) - e^{-p_1 x_1} f(x_1 + x_2) + e^{-p_1 x_1} f(x_1)\Big)\Big] d(x_1 + x_2 I)$$

$$F(p_1 + p_2 I) = \int_0^{-\infty} \left[e^{-p_1 x_1} f(x_1) + I\left(e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) - e^{-p_1 x_1} f(x_1)\right)\right] d(x_1 + x_2 I)$$

By Definition 2.9 , we have.

$$F(p_1 + p_2 I) = \int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1) + I\left[\int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2) - \int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1)\right] \ldots \ldots (1)$$

Method of solution.

1. Take AH-Isometry for equation(1), we have.

$$T\big(F(p_1 + p_2 I)\big) = T\left(\int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1) + I\left[\int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2) - \int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1)\right]\right)$$

$$[F(p_1), F(p_1 + p_2)] = \left[\int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1), \int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2)\right]$$

Then.

$$\begin{cases} F(p_1) = \int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1) \\ F(p_1 + p_2) = \int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2) \end{cases}$$

$F(p_1)$and $F(p_1 + p_2)$are two Lapalce transformation classical.

2. We find $F(p_1)$ and $F(p_1 + p_2)$.

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3. We Take invertibleAH-Isometry, then, we have aNeutrosophicLapalce transformation.

$$F(p_1+p_2I)=T^{-1}\big(F(p_1),F(p_1+p_2)\big)=F(p_1)+\big(F(p_1+p_2)-F(p_1)\big)I$$

Now,We show the laplace transform table for some analytical functions classical.

$f(x)$	$F(p)=L[f(x)]$
a	$\frac{a}{p}$
1	$\frac{1}{p}$
x^n	$\frac{n!}{p^{n+1}};n=1,2,3,\dots\dots$
$\sqrt{x}$	$\frac{\sqrt{\pi}}{2p^{\frac{3}{2}}}$
$\sin ax$	$\frac{a}{p^2+a^2}$
$\cos ax$	$\frac{p}{p^2+a^2}$
$x\sin ax$	$\frac{2ap}{(p^2+a^2)^2}$
$x\cos ax$	$\frac{p^2-a^2}{(p^2+a^2)^2}$
e^{ax}	$\frac{1}{p-a}$
$\sin(ax+b)$	$\frac{p\sin b+a\cos b}{p^2+a^2}$
$\cos(ax+b)$	$\frac{p\cos b-a\sin b}{p^2+a^2}$
$e^{ax}\sin bx$	$\frac{b}{(p-a)^2+b^2}$
$e^{ax}\cos bx$	$\frac{p-a}{(p-a)^2-b^2}$
$\sinh ax$	$\frac{a}{p^2-a^2}$
$\cosh ax$	$\frac{p}{p^2-a^2}$

Properties laplace transform classical.

1- $L[e^{ax}f(x)]=F(p-a)$
2- $L[x^nf(x)]=(-1)^n\frac{d}{d^np}F(p)$
3- $L\left[\frac{f(x)}{x}\right]=\int_p^{-\infty}F(p)dp$
4- $L[y']=pL[y]-y(0)$
5- $L[y'']=p^2L[y]-py(0)-y'(0)$
6- $L[y''']=p^3L[y]-p^2y(0)-y''(0)+py'(0)$
7- $L\big[y^{(n)}\big]=p^nL[y]-p^{n-1}y(0)-p^{n-2}y'(0)-\cdots py^{(n-2)}(0)-y^{(n-1)}(0)$

Example3.2. Find a NeutrosophicLapalce transformation:

$$f(X)=2sin4(X)+X$$

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Solution.

Let $X = x_1 + x_2 I$. Then.

$f(X) = 2sin4(x_1 + x_2 I) + (x_1 + x_2 I) = 2sin4(x_1) + I[2sin4(x_1 + x_2) - 2sin4(x_1)] + x_1 + I[(x_1 + x_2) - (x_1)]$

$f(X) = 2sin4(x_1) + x_1 + I[2sin4(x_1 + x_2) + (x_1 + x_2) - 2sin4(x_1) - (x_1)]$

Now, we have.

$$F(p_1 + p_2 I) = \int_0^{-\infty} e^{-p_1 x_1}(2sin4(x_1) + x_1)d(x_1) + I\left[\int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)}\big(2sin4(x_1 + x_2) + (x_1 + x_2)\big)d(x_1 + x_2) - \int_0^{-\infty} e^{-p_1 x_1}(2sin4(x_1) + x_1)d(x_1)\right]$$

Now, Take AH-Isometry, we have.

$$\begin{cases} F(p_1) = \int_0^{-\infty} e^{-p_1 x_1}(2sin4(x_1) + x_1)d(x_1) \\ F(p_1 + p_2) = \int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)}\big(2sin4(x_1 + x_2) + (x_1 + x_2)\big)d(x_1 + x_2) \end{cases}$$

Then.

$$F(p_1) = L(2sin4(x_1) + x_1) = \frac{8}{{p_1}^2 + 16} + \frac{1}{{p_1}^2}$$

$$F(p_1 + p_2) = L\big(2sin4(x_1 + x_2) + (x_1 + x_2)\big) = \frac{8}{(p_1 + p_2)^2 + 16} + \frac{1}{(p_1 + p_2)^2}$$

Now, Take invertibleAH-Isometry, we have.

$$F(p_1 + p_2 I) = T^{-1}\big(F(p_1), F(p_1 + p_2)\big) = F(p_1) + \big(F(p_1 + p_2) - F(p_1)\big)I$$

$$F(p_1 + p_2 I) = \frac{8}{{p_1}^2 + 16} + \frac{1}{{p_1}^2} + \left(\frac{8}{(p_1 + p_2)^2 + 16} + \frac{1}{(p_1 + p_2)^2} - \frac{8}{{p_1}^2 + 16} - \frac{1}{{p_1}^2}\right)I$$

$$F(p_1 + p_2 I) = \frac{8}{(p_1 + p_2 I)^2 + 16} + \frac{1}{(p_1 + p_2 I)^2}$$

Example 3.3. Find a NeutrosophicLapalce transformation:

$$f(X) = e^{-X} sinX$$

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Solution.

Let $X = x_1 + x_2 I$. Then.

$$f(X) = e^{-(x_1+x_2 I)} sin(x_1 + x_2 I) = \left[e^{-(x_1)} + I\left(e^{-(x_1+x_2)} - e^{-(x_1)}\right)\right]. \left[sin(x_1) + I\left(sin\left((x_1 + x_2)\right) - sin(x_1)\right)\right]$$

$$f(X) = e^{-(x_1)} sin(x_1) + I\left[e^{-(x_1+x_2)} sin(x_1 + x_2) - e^{-(x_1)} sin(x_1)\right]$$

Now, we have.

$$F(p_1 + p_2 I) = \int_0^{-\infty} e^{-(p_1)(x_1)} e^{-(x_1)} sin(x_1) d(x_1) + I\left[\int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} e^{-(x_1+x_2)} sin(x_1 + x_2) d(x_1 + x_2) - \int_0^{-\infty} e^{-(p_1)(x_1)} e^{-(x_1)} sin(x_1) d(x_1)\right]$$

Now, Take AH-Isometry, we have.

$$\begin{cases} F(p_1) = \int_0^{-\infty} e^{-(p_1)(x_1)} e^{-(x_1)} sin(x_1) d(x_1) \\ F(p_1 + p_2) = \int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} e^{-(x_1+x_2)} sin(x_1 + x_2) d(x_1 + x_2) \end{cases}$$

Then.

$$F(p_1) = L\left(e^{-(x_1)} sin(x_1)\right) = \frac{1}{(p_1 + 1)^2 + 1}$$

$$F(p_1 + p_2) = L\left(e^{-(x_1+x_2)} sin(x_1 + x_2)\right) = \frac{1}{(p_1 + p_2)^2 + 1}$$

Now, Take invertibleAH-Isometry, we have.

$$F(p_1 + p_2 I) = T^{-1}\left(F(p_1), F(p_1 + p_2)\right) = F(p_1) + \left(F(p_1 + p_2) - F(p_1)\right)I$$

$$F(p_1 + p_2 I) = \frac{1}{(p_1 + 1)^2 + 1} + \left(\frac{1}{(p_1 + p_2)^2 + 1} - \frac{1}{(p_1 + 1)^2 + 1}\right)I$$

$$F(p_1 + p_2 I) = \frac{1}{(p_1 + p_2 I)^2 + 1}$$

4 Neutrosophic differential equation by using Laplace transformtion.

In this section is defined a Neutrosophicdifferentialequationby Using the One-Dimensional Geometric AH-Isometryof Laplace transformtionand solutions are found for this equation.

Definition 4.1 .

Let $Y = y_1 + y_2 I$, $X = x_1 + x_2 I$ We define the Neutrosophicdifferential equationas form:

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$$Y^{(n)} + (a_1 + a_2 I)Y^{(n-1)} + \cdots + (b_1 + b_2 I)Y' + (c_1 + c_2 I)Y = f(X) \ldots\ldots (2)$$

And takes a initial conditions.

Method of solution.

1- We take the laplace transform of both sides of the equation (2).
2- We substitute the initial conditions.
3- We take the inverse laplace transform.

Example 4.2.find the solution of equation.

$$Y'' + Y = X \ldots\ldots (3)$$

Where.

$$Y'(0) = 2\,, Y(0) = 1 \ldots\ldots (4)$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$y_1'' + y_1 + I[(y_1 + y_2)'' + (y_1 + y_2) - (y_1'' + y_1)] = x_1 + I[(x_1 + x_2) - x_1]$

And:

$y_1'(0) + I[(y_1 + y_2)'(0) - y_1'(0)] = 2, y_1(0) + I[(y_1 + y_2)(0) - y_1(0)] = 1$

Now, Take AH-Isometry for the differential equation, and Take AH-Isometry fora initial conditions, we have..

$$\begin{cases} y_1'' + y_1 = x_1 \ldots\ldots (5) \\ (y_1 + y_2)'' + (y_1 + y_2) = (x_1 + x_2) \ldots\ldots (6) \end{cases}$$

And:

$$\begin{cases} y_1'(0) = 2\,, (y_1 + y_2)'(0) = 2 \\ y_1(0) = 1\,, (y_1 + y_2)(0) = 1 \end{cases}$$

Now, we find solution of equation (5), then.

$L[y_1''] + L[y_1] = L[x_1]$

$$\Rightarrow (p_1{}^2)L[y_1] - p_1 y_1(0) - y_1'(0) + L[y] = [x_1] \Rightarrow (p_1{}^2)L[y_1] - p_1 y_1(0) - y_1'(0) + L[y] = \frac{1}{p_1{}^2}$$

$$\Rightarrow (p_1{}^2)L[y_1] - p_1[1] - [2] + L[y_1] = \frac{1}{p_1{}^2} \Rightarrow (p_1{}^2)L[y_1] - p_1 - 2 + L[y_1] = \frac{1}{p_1{}^2}$$

$$\Rightarrow (p_1{}^2 + 1)L[y_1] - 2 = \frac{1}{p_1{}^2} \Rightarrow (p_1{}^2 + 1)L[y_1] = \frac{1}{p_1{}^2} + p_1 + 2$$

$$\Rightarrow (p_1{}^2 + 1)L[y_1] = \frac{p_1{}^3 + 2p_1{}^2 + 1}{p_1{}^2} \Rightarrow L[y_1] = \frac{p_1{}^3 + 2p_1{}^2 + 1}{p_1{}^2(p_1{}^2 + 1)} \Rightarrow L[y_1] = \frac{p_1{}^3 + 2p_1{}^2 + 1}{p_1{}^2(p_1{}^2 + 1)}$$

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$$\frac{p_1^{\,3}+2p_1^{\,2}+1}{p_1^{\,2}(p_1^{\,2}+1)}=\frac{A}{p_1}+\frac{B}{p_1^{\,2}}+\frac{Cp_1+D}{p_1^{\,2}+16}\Rightarrow \mathrm{A}=0,\mathrm{B}=\mathrm{C}=\mathrm{D}=1$$

$$\Rightarrow \frac{p_1^{\,3}+2p_1^{\,2}+1}{p_1^{\,2}(p_1^{\,2}+1)}=\frac{1}{p_1^{\,2}}+\frac{p_1}{p_1^{\,2}+16}+\frac{1}{p_1^{\,2}+16}$$

$$\Rightarrow L[y_1]=\left(\frac{1}{p_1^{\,2}}+\frac{p_1}{p_1^{\,2}+16}+\frac{1}{p_1^{\,2}+16}\right)$$

$$L^{-1}[y_1]=L^{-1}\left(\frac{1}{p_1^{\,2}}\right)+L^{-1}\left(\frac{p_1}{(p_1^{\,2}+16)}\right)+L^{-1}\left(\frac{1}{p_1^{\,2}+16}\right)$$

$$\Rightarrow y_1=x_1+cosx_1+sinx_1$$

By the method same, The solution of equation (6) written as follow:

$$(y_1+y_2)=(x_1+x_2)+cos(x_1+x_2)+sin(x_1+x_2).$$

Now, Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic differential equation .

$$Y=y_1+y_2I=T^{-1}\big(x_1+cosx_1+sinx_1,(x_1+x_2)+cos(x_1+x_2)+sin(x_1+x_2)\big)$$

$$Y=y_1+y_2I=x_1+cosx_1+sinx_1+I[(x_1+x_2)+cos(x_1+x_2)+sin(x_1+x_2)-(x_1+cosx_1+sinx_1)]$$

$$Y=y_1+y_2I=x_1+I\big((x_1+x_2)-x_1\big)+cosx_1+I[cos(x_1+x_2)-cosx_1]+sinx_1+I[sin(x_1+x_2)-sinx_1]$$

$$Y=y_1+y_2I=(x_1+x_2I)+cos(x_1+x_2I)+sin(x_1+x_2I)$$

$$Y=y_1+y_2I=X+cosX+sinX$$

Example 4.3.find the solution of equation.

$$Y''+2Y'+5Y=e^{-X}sinX\ \dots\dots\ (7)$$

Where.

$$Y'(0)=1\,,Y(0)=0\ \dots\dots(8)$$

Solution.

Let $Y=y_1+y_2I, X=x_1+x_2I$. Then.

$$y_1''+2y_1'+5y_1+I[(y_1+y_2)''+2(y_1+y_2)'+5(y_1+y_2)-(y_1''+2y_1'+5y_1)]$$
$$=e^{-x_1}sinx_1+I\left[e^{-(x_1+x_2)}sin(x_1+x_2)-e^{-x_1}sinx_1\right]$$

And:

$$y_1'(0)+I[(y_1+y_2)'(0)-y_1'(0)]=1, y_1(0)+I[(y_1+y_2)(0)-y_1(0)]=0$$

Now, Take AH-Isometry for the differential equation, and Take AH-Isometry fora initial conditions, we have..

$$\begin{cases} y_1''+2y_1'+5y_1=e^{-x_1}sinx_1\ \dots\dots(9) \\ (y_1+y_2)''+2(y_1+y_2)'+5(y_1+y_2)=e^{-(x_1+x_2)}sin(x_1+x_2)\ \dots\dots(10)\end{cases}$$

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And:

$$\begin{cases} y_1{}'(0) = 1\,, (y_1 + y_2)'(0) = 1 \\ y_1(0) = 0\,, (y_1 + y_2)(0) = 0 \end{cases}$$

Now, we find solution of equation (9), then.

$$L[y_1{}''] + 2L[y_1{}'] + 5L[y_1] = L[e^{-x_1} sinx_1]$$

$$\Rightarrow (p_1{}^2)L[y_1] - p_1 y_1(0) - y_1{}'(0) + 2p_1 L[y_1] - 2y_1(0) + 5L[y_1] = \frac{1}{(p_1+1)^2+1}$$

$$\Rightarrow (p_1{}^2)L[y_1] - 1 + 2p_1 L[y_1] + 5L[y_1] = \frac{1}{(p_1+1)^2+1}$$

$$\Rightarrow (p_1{}^2 + 2p_1 + 5)L[y_1] = 1 + \frac{1}{p_1{}^2 + 2p_1 + 3} \Rightarrow (p_1{}^2 + 2p_1 + 5)L[y_1] = \frac{p_1{}^2 + 2p_1 + 4}{p_1{}^2 + 2p_1 + 3}$$

$$L[y_1] = \frac{p_1{}^2 + 2p_1 + 4}{(p_1{}^2 + 2p_1 + 3)(p_1{}^2 + 2p_1 + 5)}$$

$$\frac{p_1{}^2 + 2p_1 + 4}{(p_1{}^2 + 2p_1 + 3)(p_1{}^2 + 2p_1 + 5)} = \frac{Ap_1 + B}{p_1{}^2 + 2p_1 + 5} + \frac{Cp_1 + D}{p_1{}^2 + 2p_1 + 3}$$

$$\Rightarrow \mathrm{A} = 0, \mathrm{B} = \frac{1}{2}, \mathrm{C} = 0, \mathrm{D} = \frac{1}{2}$$

$$\Rightarrow \frac{p_1{}^2 + 2p_1 + 4}{(p_1{}^2 + 2p_1 + 3)(p_1{}^2 + 2p_1 + 5)} = \frac{1}{2}\left(\frac{1}{p_1{}^2 + 2p_1 + 5}\right) + \frac{1}{2}\left(\frac{1}{p_1{}^2 + 2p_1 + 3}\right)$$

$$\Rightarrow L[y_1] = \frac{1}{2}\left(\frac{1}{p_1{}^2 + 2p_1 + 5}\right) + \frac{1}{2}\left(\frac{1}{p_1{}^2 + 2p_1 + 3}\right)$$

$$L^{-1}[y_1] = L^{-1}\left[\frac{1}{2}\left(\frac{1}{p_1{}^2 + 2p_1 + 5}\right)\right] + L^{-1}\left[\frac{1}{2}\left(\frac{1}{p_1{}^2 + 2p_1 + 3}\right)\right]$$

$$\Rightarrow L^{-1}[y_1] = \frac{1}{2}\left(\frac{1}{p_1{}^2 + 2p_1 + 5}\right) + \frac{1}{2}L^{-1}\left(\frac{1}{p_1{}^2 + 2p_1 + 3}\right)$$

$$\Rightarrow L^{-1}[y_1] = \frac{1}{2}L^{-1}\left\{\frac{1}{(p_1+1)^2+4}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{(p_1+1)^2+2}\right\}$$

$$\Rightarrow y_1 = \frac{1}{4}e^{-x_1} sin2x_1 + \frac{1}{2\sqrt{2}}e^{-x_1} sin\sqrt{2}x_1$$

By the method same, The solution of equation (10) written as follow:

$$(y_1 + y_2) = \tfrac{1}{4}e^{-(x_1+x_2)} sin2(x_1 + x_2) + \tfrac{1}{2\sqrt{2}}e^{-(x_1+x_2)} sin\sqrt{2}(x_1 + x_2).$$

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Now, Take the inverse AH-Isometry, then, we have the solution of the Neutrosophic differential equation .

$$Y = y_1 + y_2 I = T^{-1}\left(\frac{1}{4}e^{-x_1}sin2x_1 + \frac{1}{2\sqrt{2}}e^{-x_1}sin\sqrt{2}x_1, \frac{1}{4}e^{-(x_1+x_2)}sin2(x_1+x_2) + \frac{1}{2\sqrt{2}}e^{-(x_1+x_2)}sin\sqrt{2}(x_1+x_2)\right)$$

$$Y = y_1 + y_2 I = \frac{1}{4}e^{-x_1}sin2x_1 + \frac{1}{2\sqrt{2}}e^{-x_1}sin\sqrt{2}x_1 + I\left[\frac{1}{4}e^{-(x_1+x_2)}sin2(x_1+x_2) + \frac{1}{2\sqrt{2}}e^{-(x_1+x_2)}sin\sqrt{2}(x_1+x_2) - \left(\frac{1}{4}e^{-x_1}sin2x_1 + \frac{1}{2\sqrt{2}}e^{-x_1}sin\sqrt{2}x_1\right)\right]$$

$$Y = y_1 + y_2 I = \frac{1}{4}e^{-x_1}sin2x_1 + I\left(\frac{1}{4}e^{-(x_1+x_2)}sin2(x_1+x_2) - \frac{1}{4}e^{-x_1}sin2x_1\right) + \frac{1}{2\sqrt{2}}e^{-x_1}sin\sqrt{2}x_1 + I\left[\frac{1}{2\sqrt{2}}e^{-(x_1+x_2)}sin\sqrt{2}(x_1+x_2) - \frac{1}{2\sqrt{2}}e^{-x_1}sin\sqrt{2}x_1\right]$$

$$Y = y_1 + y_2 I = \frac{1}{4}e^{-(x_1+x_2 I)}sin2(x_1+x_2 I) + \frac{1}{2\sqrt{2}}e^{-(x_1+x_2 I)}sin\sqrt{2}(x_1+x_2 I)$$

$$Y = y_1 + y_2 I = \frac{1}{4}e^{-X}sin2X + \frac{1}{2\sqrt{2}}e^{-X}sin\sqrt{2}X$$

5. Conclusion

In this paper, we have applied the one dimensional neutrosophic AH-isometry to study the algebraic direct image of Laplace transformation. Then, we have used this image to find solutions for some neutrosophic differential equations.

In the future, we aim to find more applications of AH-isometry in the study of neutrosophic structures.

References

1. Smarandache F. NeutrosophicPrecalculus and Neutrosophiccalclus, University of New Mexico, 705 Gurley Ave. Gallup,NM 87301, USA, 2015.
2. Smarandache F. Neutrosophic Measure and NeutrosophicIntegral , In Neutrosophic Sets and Systems, 3 – 7, Vol. 1, 2013.
3. A. A Salama, Smarandache F. Kroumov,Neutrosophic Closed Set and Continuous Functions, in Neutrosophic Sets and Systems, Vol.4, 4- 8, 2014.
4. A. A Salama; I. M Hanafy; HewaydaElghawalbyDabash M.S, Neutrosophic Crisp Closed RRegion and Neutrosophic Crisp Continuous Functions, New Trends in Neutrosophic Theory and Applications.
5. A. A Salama; HewaydaElghawalby; M.S,Dabash; A.M. NASR, RetracNeutrosophic Crisp System For Gray Scale Image, Asian Journal Of Mathematics and Computer Research, Vol 24, 104-117, (2018).

DOI: https://doi.org/10.54216/JNFS.040201
Received: April 20, 2022 Accepted: August 25, 2022

6. F. smarandache. "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, neutrosophic Logic, Set, Probability, and Statistics" University of New Mexico, Gallup, NM87301, USA 2002.
7. M. Abdel-Basset; E. Mai. Mohamed; C. Francisco;H. Z. Abd EL-Nasser. "Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases" Artificial Intelligence in Medicine Vol. 101, 101735, (2019).
8. Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", International journal of neutrosophic science, 2022.
9. Smarandache F., and Abobala, M., "*n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.
10. Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021
11. Abobala, M., A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings, Journal of Fuzzy Extension and Applications, Vol. 2, pp. 16-22, 2021.
12. M. Abdel-Basset; E. Mohamed; G. Abdullah; and S. Florentin. "A novel model for evaluation Hospital medical care systems based on plithogenic sets" Artificial Intelligence in Medicine 100 (2019), 101710.
13. M. Abdel-Basset; G. Gunasekaran Mohamed; G. Abdullah. C. Victor, "A Novel Intelligent Medical Decision Support Model Based on soft Computing and Iot" IEEE Internet of Things Journal, Vol. 7, (2019).
14. M. Abdel-Basset; E. Mohamed; G. Abdullah; G. Gunasekaran; L. Hooang Viet." A novel group decision making model based on neutrosophic sets for heart disease diagnosis" Multimedia Tools and Applications, 1-26, (2019).
15. A. A Salama. Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets and Possible Application to GIS Topology. Neutrosophic Sets and Systems, Vol. 7, 18-22, (2015).
16. A. A Salama; F. Smarandache. Neutrosophic Set Theory, Neutrosophic Sets and Systems, Vol. 5, 1-9, (2014).
17. F. Smarandache, The Neutrosophic Triplet Group and its Application to physics, Seminar Universidad National de Quilmes , Department of science and Technology, Beunos Aires, Argentina, 20 June 2014.
18. A. B.AL-Nafee; R.K. Al-Hamido; F.Smarandache. "Separation Axioms In Neutrosophic Crisp Topological Spaces", Neutrosophic Sets and Systems, vol. 25, 25-32, (2019).
19. Abobala, M., Bal, M., and Hatip, A.," A Review On Recent Advantages In Algebraic Theory Of Neutrosophic Matrices**",** International Journal of Neutrosophic Science, Vol. 17, 2021.
20. R.K. Al-Hamido, Q. H. Imran, K. A. Alghurabi, T. Gharibah, "On Neutrosophic Crisp Semi Alpha Closed Sets", Neutrosophic Sets and Systems", vol. 21, 28-35, (2018).

DOI: https://doi.org/10.54216/JNFS.040201
Received: April 20, 2022 Accepted: August 25, 2022

21. Sankari, H., and Abobala, M., "Neutrosophic Linear Diophantine Equations With two Variables", Neutrosophic Sets and Systems, Vol. 38, pp. 22-30, 2020.
22. Sankari, H., and Abobala, M." *n*-Refined Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 36, pp. 1-11. 2020.
23. Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.
24. Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.
25. Agboola, A.A.A., "On Refined Neutrosophic Algebraic Structures," Neutrosophic Sets and Systems,Vol.10, pp. 99-101. 2015.
26. Abobala, M., "Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 72-75. 2020.
27. Q. H. Imran, F. Smarandache, R.K. Al-Hamido, R. Dhavasselan, “On Neutrosophic Semi Alpha open Sets”, Neutrosophic Sets and Systems, vol. 18, 37-42, (2017).
28. Al-Hamido, R. K.; “A study of multi-Topological Spaces”, PhD Theses, AlBaath university , Syria, (2019).
29. Al-Hamido, R. K.; “Neutrosophic Crisp Supra Bi-Topological Spaces”, International Journal of Neutrosophic Science, Vol. 1, 66-73, (2018).
30. R.K. Al-Hamido, “Neutrosophic Crisp Bi-Topological Spaces”, Neutrosophic Sets and Systems, vol. 21, 66-73, (2018).
31. R.K. Al-Hamido, T. Gharibah, S. JafariF.Smarandache, “On Neutrosophic Crisp Topology via N-Topology”, Neutrosophic Sets and Systems, vol. 21, 96-109, (2018).
32. A. Hatip, "The Special Neutrosophic Functions," International Journal of Neutrosophic Science (IJNS), p. 13, 12 May 2020.\
33. Kandasamy, I., Kandasamy, V., and Smarandache, F., "Algebraic structure of Neutrosophic Duplets in Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 18, pp. 85-95. 2018.
34. Sankari, H., and Abobala, M.," AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.
35. Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol. 16, pp. 72-79, 2021.
36. Aswad, F, M., " A Study of Neutrosophic Complex Number and Applications", Neutrosophic Knowledge, Vol. 1, 2020.
37. Aswad, M., " A Study of The Integration Of Neutrosophic Thick Function", International journal of neutrosophic Science, 2020.
38. Abobala, M, "*n*-Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95 . 2020.

DOI: https://doi.org/10.54216/JNFS.040201
Received: April 20, 2022 Accepted: August 25, 2022

39. M. Ali, F. Smarandache, M. Shabir and L. Vladareanu., "Generalization of Neutrosophic Rings and Neutrosophic Fields", Neutrosophic Sets and Systems, vol. 5, pp. 9-14, 2014.
40. Smarandache, F., and Abobala, M., n-Refined neutrosophic Rings, International Journal of Neutrosophic Science, Vol. 5 , pp. 83-90, 2020.
41. Abobala, M., " Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 39, 2021.
42. Abobala, M., "On Some Neutrosophic Algebraic Equations", Journal of New Theory, Vol. 33, 2020.
43. Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, Hindawi, 2021.
44. Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021
45. Kandasamy V, Smarandache F., and Kandasamy I., Special Fuzzy Matrices for Social Scientists . Printed in the United States of America,2007, book, 99 pages.
46. Abobala, M., Partial Foundation of Neutrosophic Number Theory, Neutrosophic Sets and Systems, Vol. 39 , 2021.
47. Giorgio, N, Mehmood, A., and Broumi, S.," Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.
48. Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., The algebraic creativity In The Neutrosophic Square Matrices, Neutrosophic Sets and Systems, Vol. 40, pp. 1-11, 2021.
49. Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, Vol. 43, 2021.
50. Abobala, M., "On Some Special Elements In Neutrosophic Rings and Refined Neutrosophic Rings", Journal of New Theory, vol. 33, 2020.
51. Chellamani, P., and Ajay, D., "Pythagorean neutrosophic Fuzzy Graphs", International Journal of Neutrosophic Science, Vol. 11, 2021.
52. Prem Kumar Singh, Fourth dimension data representation and its analysis using Turiyam Context, Journal of Computer and Communications, 2021, Vol. 9, no. 6, pp. 222-229,DOI: 10.4236/jcc.2021.96014, https://www.scirp.org/journal/paperinformation.aspx?paperid=110694
53. Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 45, 2021.
54. Abobala, M., Hatip, A., Bal,M.," A Study Of Some Neutrosophic Clean Rings", International journal of neutrosophic science, 2022.
55. Prem Kumar Singh, AntiGeometry and NeutroGeometry Characterization of Non-Euclidean Data Sets, Journal of Neutrosophic and Fuzzy Systems, Nov 2021, Volume 1, Issue 1, pp. 24-33, ISSN: , DOI: https://doi.org/10.54216/JNFS.0101012 [76] Singh, P,K., " Anti-geometry and NeutroGeometry Characterization of Non-Euclidean Data", Journal of Neutrosophic and Fuzzy Systems, vol. 1, 2022
56. Abobala, M., and Hatip, A., "An Algebraic Approch To Neutrosophic Euclidean Geometry",

DOI: https://doi.org/10.54216/JNFS.040201
Received: April 20, 2022 Accepted: August 25, 2022

57. Singh, P,K., " Data With Turiyam Set for Fourth Dimension Quantum Information Processing", Journal of Neutrosophic and Fuzzy Systems, vol.1, 2022.
58. Ibrahim, M., and Abobala, M., "An Introduction To Refined Neutrosophic Number Theory", Neutrosophic Sets and Systems, Vol. 45, 2021.
59. Singh, P, K., Ahmad, K., Bal, M., Aswad, M.," On The Symbolic Turiyam Rings", Journal of Neutrosophic and Fuzzy Systems, 2022.
60. Bal, M.,, Singh, P., Ahmad, K., and Aswad, M., " A Short Introduction To The Concept Of Symbolic Turiyam Matrix", Neutrosophic and Fuzzy Systems, 2022.
61. Bal, M., Singh, P., and Ahmad, K., " An Introduction To The Symbolic Turiyam R-Modules and Turiyam Modulo Integers", Neutrosophic and Fuzzy Systems, 2022.
62. Bal, M., Singh, P., and Ahmad, K., " An Introduction To The Symbolic Turiyam Vector Spaces and Complex Numbers", Neutrosophic and Fuzzy Systems, 2022.

DOI: https://doi.org/10.54216/JNFS.040201
Received: April 20, 2022 Accepted: August 25, 2022