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A STUDY ON GAME THEORY IN NEUTROSOPHIC ENVIRONMENT

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Abstract

Objective of this paper, a new method is developed to solve the game matrix in neutrosophic environment and evaluation matrix is constructed followed by Score functions have been defined and applied Max-Min principle in the score function to find the value of the game and numerical example has been given in support of the solution method.

Keywords: Neutrosophic Environment, Game Matrix, Evaluation Matrix, Score Functions, Max-Min Principle.

1. Introduction

In every competitive situation, it is often required to take the decision where there are two or more opposite parties with conflicting of interests and the action of one depends upon the action which is taken by the opponent. A variety of competitive situation is seen in real life society like, in political campaign, elections, advertisement, marketing, etc. Game theory is a mathematical way out for describing the strategic interactions among multiple players who select several strategies from the set of admissible strategies. In 1944, Von Neumann and Oscar Morgenstern [1] introduced game theory in their most pioneer work "Theory of Games and Economic Behavior". Since then many diverse kinds of mathematical games have been defined and different types of solution methodologies have been proposed. The participants in the game are called the players. During the past, it is assumed that all the information about game is known precisely by players. But in traditional game theory, the precise information about the game is more difficult to collect due to the lack of information about the exact values of certain parameters and uncertain measuring of several situations by players. To overcome these types of situation, the problem can be formulated using the concept of uncertainty theory and the domain of payoffs are considered from uncertain environment like fuzzy, interval, stochastic, fuzzy-stochastic environment etc. In such cases fuzzy set theory is a vital tool to handle such situation. Fuzzy set Zadeh (1965) introduced an effective way to model data uncertainty by defining membership function in the range of [0,1]. the fuzzy set has demonstrated a good performance in handling data uncertainty by using membership function, they failed to handle non-membership degree and indeterminacy membership degree.

Intuitionistic fuzzy set (IFs) introduced by Atanassov (1986) is an extension of fuzzy set in order to overcome the lack of knowledge about the non-membership degree. It is characterized by a membership degree and a non-membership degree functions. TOPSIS has been extended to solve MCGDM problems with intuitionistic fuzzy data. Pramanik (2011) studied the teacher selection in intuitionistic fuzzy environment. Intuitionistic fuzzy TOPSIS has been used for the employee performance appraisal by Yinghui (2015). However, the intuitionistic fuzzy set can handle the membership degree and the non-membership degree, it can't handle problems involving indeterminate and inconsistent information.

Neutrosophic set (NS) first introduced by Smarandache (1999) in order to handle the problems with indeterminate and inconsistent information. NS is a generalization of crisp sets, fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and interval-valued intuitionistic fuzzy sets. NS is characterized by three membership functions, Truth membership function (T), Indeterminacy membership function (I), and Falsity membership function (F). NS is difficult to apply in real problems, so the single-valued neutrosophic set was introduced by Wang (2010) to be applied to real scientific and engineering situations. Biswas (2016) extended TOPSIS method to solve multi-attribute group decision making problem under SVN environment. MCGDM problems can include more than one participant, each of them needs to take the best decision, so competition may appear between these participants. Game theory is a powerful tool which used to handle the competition situations between two or more participants. A game is a formal description of a strategic situation.

This paper is organized as follows: Section. 2 briefly introduces some basic preliminaries related with neutrosophic sets. In Section. 3 Principles of game theory are included in Section. 4. The proposed approach for solving the problem with uncertainty and competition is introduced in Section. 5. A numerical example is illustrated in Section.6. Finally, conclusions and future work are pointed out at the end of this paper.

2. Preliminaries

2.1. Neutrosophic Set

Let U be the space of points (or objects) with generic element x. A neutrosophic set A in U is characterized by a truth membership function T_A, and indeterminacy function I_A and a falsity membership function F_A, where T_A, I_A and F_A are real standard or non- standard subsets of $]^{-}0, 1^{+}[$, i.e., $\sup T_A : X \to]^{-}0, 1^{+}[$, $\sup F_A : X \to]^{-}0, 1^{+}[$, $\sup I_A : X \to]^{-}0, 1^{+}[$.

A neutrosophic set A upon U as an object is defined as $\frac{x}{T_A(x), I_A(x), F_A(x)} = \left\{ \frac{x}{T_A, I_A, F_A} : x \in U \right\}$, where $T_A(x), I_A(x)$ and $F_A(x)$ are subintervals or union of subintervals of $[0, 1]$.

2.2. Algebraic Operations with Neutrosophic Set

For two neutrosophic sets A and B,

(a) Complement of A

$$A' = \left\{ \frac{x}{T, I, F} \mid T = 1 - T_A, I = 1 - I_A, F = 1 - F_A \right\}.$$

(b) Intersection of A and B

$$A \cap B = \left\{ \frac{x}{T, I, F} \mid T = T_A T_B, I = I_A I_B, F = F_A F_B \right\}.$$

(c) Union of A and B

$$A \cup B = \left\{ \frac{x}{T, I, F} \mid T = T_A + T_B - T_A T_B, I = I_A + I_B - I_A I_B, F = F_A + F_B - F_A F_B \right\}.$$

(d) Cartesian Product of A and B

$$A \times B = \left\{ \left(\frac{x}{T_A, I_A, F_A}, \frac{y}{T_B, I_B, F_B} \right) \mid \frac{x}{T_{A,}, I_A, F_A} \in A, \frac{y}{T_B, I_B, F_B} \in B \right\}.$$

(e) A is a subset of B

$$A \subseteq B, \forall \frac{x}{T_A, I_A, F_A} \in A \text{ and } \frac{y}{T_B, I_B, F_B} \in B, T_A \leqslant T_B, I_A \geqslant I_B, F_A \geqslant F_B.$$

(f) Difference of A and B

$$A \backslash B = \left\{ \frac{x}{T, I, F} \mid T = T_A - T_A T_B, I = I_A - I_A I_B, F = F_A - F_A F_B \right\}.$$

3. Neutrosophic Matrix Game

Let A_i $(i = 1, 2, 3, \ldots, n)$ and B_j $(j = 1, 2, 3, \ldots, m)$ be pure strategies for players A and B, respectively. If player A adopts the pure strategy A_i (i.e., the row) and player B adopts pure strategy B_j (i.e., the column), then the pay-off for players is expressed with the Neutrosophic number ($T_A(x)$, $I_A(x)$ and $F_A(x)$).

3.1. Pure Strategy

Pure strategy is a decision making rule in which one particular course of action is selected. For fuzzy games the min - max principle is described by Nishizaki [2]. The course of the fuzzy game is determined by the desire of to maximize his gain and that of restrict his loss to a minimum.

3.1.1 Definition (Saddle Point):

The concept of saddle point in classical form is introduced by Neumann [12]. The position of the pay-off matrix will be called a saddle point, if and only if, We call the position of entry a saddle point, the entry itself, the value of the game (denoted by) and the pair of pure strategies leading to it are optimal pure strategies.

3.2. Solution Procedure of Neutrosophic Game:

To solve Neutrosophic game, first we have calculated the Evaluation Matrix for each alternative. Using the elements of Evaluation Matrix for alternatives Score function (S_{ij}) matrix has been calculated. By applying Max-Min principle method is described by Nishizaki [2] in the Score function matrix (S_{ij}), then find the value of the game.

4. Algorithm for Neutrosophic Game:

Step 1. Construct the Neutrosophic game matrix $D = (C_{ij})_{m\times n}$.

Step 2. Determine the Evaluation Matrix of the job J_i as $E(J_i) = \left[T^l_{j_i}, T^u_{j_i}\right]$, where

$$T^l_{j_i}, T^u_{j_i} = \begin{bmatrix} \min\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \\ \max\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \end{bmatrix}$$

Step 3. Compute the Score function $S(J_{ij})$ of an alternative

$$S(J_{ij}) = 2\left(T^u_{J_{ij}} - T^l_{J_{ij}}\right) = 2\begin{bmatrix} \max\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \\ \min\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \end{bmatrix},$$

where $0 \leqslant S(J_{ij}) \leqslant 1$.

Step 4. By applying Max-Min principle method in the Score function matrix (S_{ij}),

Step 5. Find the value of the game.

5. Numerical Example

Let us consider a Neutrosophic game, the matrix contains neutrosophic elements denoting pure strategies for players A and B respectively.

$$\begin{matrix} B_1 & B_2 & B_3 \\ [[0.75, 0.39, 0.1] & [0.8, 0.6, 0.5] & [0.4, 0.8, 0.45] \\ [0.6, 0.5, 0.25] & [0.75, 0.9, 0.05] & [0.68, 0.46, 0.2] \\ [0.8, 0.4, 0.2] & [0.45, 0.1, 0.5] & [0.1, 0.5, 1.0] \\ [0.4, 0.6, 0.3] & [0.5, 0.4, 0.8] & [0.5, 0.6, 0.9] \end{matrix} \Bigg]$$

Solution:
Evaluate $E(J_i)$ as the evaluation function of the job J_i as

$$E(J_i) = \left[T^l_{J_i}, T^u_{J_i}\right]$$

where

$$\left[T^l_{j_i} T^u_{j_i}\right] = \begin{bmatrix} \min\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \\ \max\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \end{bmatrix}.$$

Therefore elements of the Evaluation matrix for alternatives.

$$\left[T^l_{j_i} T^u_{j_i}\right] = \begin{bmatrix} [0.57, 0.645] & [0.7, 0.725] & [0.6, 0.675] \\ [0.55, 0.625] & [0.825, 0.925] & [0.57, 0.63] \\ [0.6, 0.6] & [0.275, 0.3] & [0.25, 0.75] \\ [0.5, 0.65] & [0.3, 0.45] & [0.35, 0.55] \end{bmatrix}.$$

Compute the Score function $S(J_{ij})$ of an alternative

$$S(J_{ij}) = 2\left(T^u_{J_{ij}} - T^l_{J_{ij}}\right) = 2\begin{bmatrix} \max\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \\ \min\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \end{bmatrix},$$

where $0 \leqslant S(J_{ij}) \leqslant 1$.

Therefore elements of Score function matrix will be as follows-

$$S(J_{ij}) = \begin{bmatrix} 0.15 & 0.05 & 0.15 \\ 0.15 & 0.2 & 0.12 \\ 0.0 & 0.05 & 1.0 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}.$$

Solving $S(J_{ij})$ by Max-Min principle,

	B_1	B_2	B_3	(Row Min)
A_1	0.15	0.05	0.15	0.05
A_2	0.15	0.2	0.12	0.12
A_3	0.0	0.05	1.0	0.0
A_4	0.3	0.3	0.4	0.3
(Column Max)	0.3	0.3	0.4	

Min (Column Max) = Max (Column Min) = 0.3.
The Saddle points are (A_4, B_1) and (A_4, B_2).
The value of the game is 0.3.

6. Conclusion

In this paper, a matrix game has been considered with pay-off elements as Neutrosophic set. There is a scope to try different type defuzzication like ranking method to find the score function. Our new approach gives a strategic solution and value of the game as an Neutrosophic set. The example given, establishes the theory on strong ground. It has strong impact on modern socio economic structure where conflicting interests exist.

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