A Review on Natural Reality with Physical Equations

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A natural behavior is used to characterize by differential equation established on human observations, which is assumed to be on one particle or one field complied with reproducibility. However, the multilateral property of a particle P and the mathematical consistence determine that such an understanding is only local, not the whole reality on P, which leads to a central thesis for knowing the nature, i.e. *how to establish a physical equation with a proper interpretation on a thing*. As it is well-known, a thing consists of parts. Reviewing on observations, we classify them into two categories, i.e. *out-observation* and *in-observation* for discussion. The former is such an observation that the observer is out of the particle or the field P, which is in fact a macroscopic observation and its dynamic equation characterizes the coherent behavior of all parts in P, but the later is asked into the particle or the field by arranging observers simultaneously on different subparticles or subfields in P and respectively establishing physical equations, which are contradictory and given up in classical because there are not applicable conclusions on contradictory systems in mathematics. However, the existence naturally implies the necessity of the nature. Applying a combinatorial notion, i.e. G^L-solutions on non-solvable equations, a new notion for holding on the reality of nature is suggested in this paper, which makes it clear that the knowing on the nature by solvable equations is macro, only holding on these coherent behaviors of particles, but the non-coherent naturally induces non-solvable equations, which implies that the knowing by G^L-solution of equations is the effective, includes the classical characterizing as a special case by solvable equations, i.e. mathematical combinatorics.

1 Introduction

An observation on a physical phenomenon, or characters of a thing in the nature is the received information via hearing, sight, smell, taste or touch, i.e. sensory organs of the observer himself, little by little for human beings fulfilled with the reproducibility. However, it is difficult to hold the true face of a thing for human beings because he is analogous to a blind man in "*the blind men with an elephant*", a famous fable for knowing the nature. For example, let $\mu_1, \mu_2, \cdots, \mu_n$ be all observed and $\nu_i, i \geq 1$ unobserved characters on a particle P at time t. Then, P should be understood by

$$P = \left(\bigcup_{i=1}^{n}\{\mu_i\}\right)\bigcup\left(\bigcup_{k\geq 1}\{\nu_k\}\right) \tag{1.1}$$

in logic with an approximation $P^\circ = \bigcup\limits_{i=1}^{n}\{\mu_i\}$ for P at time t. All of them are nothing else but Smarandache multispaces ([17]). Thus, $P \approx P^\circ$ is only an approximation for its true face of P, and it will never be ended in this way for knowing P as Lao Zi claimed "*Name named is not the eternal Name*" in the first chapter of his *TAO TEH KING* ([3]), a famous Chinese book.

A physical phenomenon of particle P is usually characterized by differential equation

$$\mathscr{F}\left(t, x_1, x_2, x_3, \psi_t, \psi_{x_1}, \psi_{x_2}, \cdots, \psi_{x_1x_2}, \cdots\right) = 0 \tag{1.2}$$

in physics established on observed characters of $\mu_1, \mu_2, \cdots, \mu_n$ for its state function $\psi(t, x)$ in $\mathbb{R}^4$. Usually, these physical phenomenons of a thing is complex, and hybrid with other things. *Is the reality of particle P all solutions of (1.2) in general?* Certainly not because (1.2) only characterizes the behavior of P on some characters of $\mu_1, \mu_2, \cdots, \mu_n$ at time t abstractly, not the whole in philosophy. For example, the behavior of a particle is characterized by the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + U\psi \tag{1.3}$$

in quantum mechanics but observation shows it in two or more possible states of being, i.e. superposition. We can not even say which solution of the Schrödinger equation (1.3) is the particle because each solution is only for one determined state. Even so, the understanding of all things is inexhaustible by (1.1).

Furthermore, *can we conclude (1.2) is absolutely right for a particle P?* Certainly not also because the dynamic equation (1.2) is always established with an additional assumption, i.e. the geometry on a particle P is a point in classical mechanics or a field in quantum mechanics and dependent on the observer is out or in the particle. For example, a water molecule H_2O consists of 2 Hydrogen atoms and 1 Oxygen atom such as those shown in Fig. 1. If an observer receives information on the behaviors of Hydrogen or Oxygen atom but stands out of the water molecule H_2O by viewing it a geometrical point, then such an observation is an out-observation because it only receives such coherent information on atoms H and O with the water molecule H_2O.

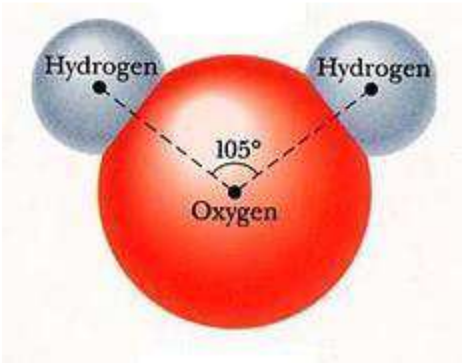

Fig. 1

If an observer is out the water molecule H_2O, his all observations on the Hydrogen atom H and Oxygen atom O are the same, but if he enters the interior of the molecule, he will view a different sceneries for atom H and atom O, which are respectively called *out-observation* and *in-observation*, and establishes 1 or 3 dynamic equations on the water molecule H_2O.

The main purpose of this paper is to clarify the natural reality of a particle with that of differential equations, and conclude that a solvable one characterizes only the reality of elementary particles but non-solvable system of differential equations essentially describe particles, such as those of baryons or mesons in the nature.

For terminologies and notations not mentioned here, we follow references [1] for mechanics, [5] for combinatorial geometry, [15] for elementary particles, and [17] for Smarandache systems and multispaces, and all phenomenons discussed in this paper are assumed to be true in the nature.

2 Out-observations

An *out-observation* observes on the external, i.e. these macro but not the internal behaviors of a particle P by human senses or via instrumental, includes the size, magnitudes or eigenvalues of states, ..., etc.

Certainly, the out-observation is the fundamental for quantitative research on matters of human beings. Usually, a dynamic equation (1.2) on a particle P is established by the principle of stationary action $\delta S = 0$ with

$$S = \int_{t_1}^{t_2} dt\, L(q(t), \dot{q}(t)) \tag{2.1}$$

in classical mechanics, where $q(t), \dot{q}(t)$ are respectively the generalized coordinates, the velocities and $L(q(t), \dot{q}(t))$ the *Lagrange function* on the particle, and

$$S = \int_{\tau_2}^{\tau_1} d^4x \mathcal{L}(\phi, \partial_\mu \psi) \tag{2.2}$$

in field theory, where ψ is the state function and $\mathcal{L}$ the *Lagrangian density* with τ_1, τ_2 the limiting surfaces of integration by viewed P an independent system of dynamics or a field. The principle of stationary action $\delta S = 0$ respectively induced the *Euler-Lagrange equations*

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0 \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} = 0 \tag{2.3}$$

in classical mechanics and field theory, which enables one to find the dynamic equations of particles by proper choice of L or $\mathcal{L}$. For examples, let

$$\mathcal{L}_S = \frac{i\hbar}{2}\left(\frac{\partial \psi}{\partial t}\overline{\psi} - \frac{\partial \overline{\psi}}{\partial t}\psi\right) - \frac{1}{2}\left(\frac{\hbar^2}{2m}|\nabla\psi|^2 + V|\psi|^2\right),$$
$$\mathcal{L}_D = \overline{\psi}\left(i\gamma^\mu \partial_\mu - \frac{mc}{\hbar}\right)\psi,$$
$$\mathcal{L}_{KG} = \frac{1}{2}\left(\partial_\mu \psi \partial^\mu \psi - \left(\frac{mc}{\hbar}\right)^2 \psi^2\right).$$

Then we respectively get the Schrödinger equation (1.3) or the Dirac equation

$$\left(i\gamma^\mu \partial_\mu - \frac{mc}{\hbar}\right)\psi(t, x) = 0 \tag{2.4}$$

for a free fermion $\psi(t, x)$ and the Klein-Gordon equation

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\psi(x, t) + \left(\frac{mc}{\hbar}\right)^2 \psi(x, t) = 0 \tag{2.5}$$

for a free boson $\psi(t, x)$ hold in relativistic forms by (2.3), where $\hbar = 6.582 \times 10^{-22}$MeV s is the Planck constant, c is the speed of light,

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right), \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$
$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right),$$
$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial x_1}, -\frac{\partial}{\partial x_2}, -\frac{\partial}{\partial x_3}\right)$$

and $\gamma^\mu = \left(\gamma^0, \gamma^1, \gamma^2, \gamma^3\right)$ with

$$\gamma^0 = \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

with the usual Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Furthermore, let $\mathcal{L} = \sqrt{-g}R$, where $R = g^{\mu\nu}R_{\mu\nu}$, the Ricci scalar curvature on the gravitational field. The equation (2.3) then induces the vacuum Einstein gravitational field equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0. \tag{2.6}$$

Usually, the equation established on the out-observations only characterizes those of coherent behaviors of all parts in a particle P. For example, a water molecule H_2O obeys the Schrödinger equation (1.3), we assume its Hydrogen atom H and oxygen atom O also obey the Schrödinger equation (1.3) as a matter of course. However, the divisibility of matter initiates human beings to search elementary constituting cells of matter, i.e. elementary particles such as those of quarks, leptons with interaction quanta including photons and other particles of mediated interactions, also with those of their antiparticles at present ([14]), and unmatters between a matter and its antimatter which is partially consisted of matter but others antimatter ([8-19]). For example, a baryon is predominantly formed from three quarks, and a meson is mainly composed of a quark and an antiquark in the models of Sakata, or Gell-Mann and Ne'eman on hadron and meson, such as those shown in Fig. 2, where, $q_i \in \{\mathbf{u}, \mathbf{d}, \mathbf{c}, \mathbf{s}, \mathbf{t}, \mathbf{b}\}$ denotes a quark for $i = 1, 2, 3$ and $\overline{q}_2 \in \left\{\overline{\mathbf{u}}, \overline{\mathbf{d}}, \overline{\mathbf{c}}, \overline{\mathbf{s}}, \overline{\mathbf{t}}, \overline{\mathbf{b}}\right\}$, an antiquark. But a free quark was never found in experiments. We can not even conclude the Schrödinger equation (1.3) is the right equation (1.2) for quarks because it is established on an independent particle, can not be divided again in mathematics.

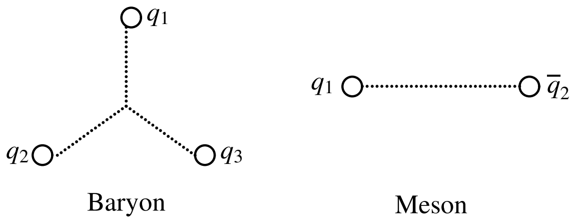

Fig. 2

Then, *why is it believed without a shadow of doubt that the dynamical equations of elementary particles such as those of quarks, leptons with interaction quanta are (*1.3*) in physics?* It is because that all our observations come from a macro viewpoint, the human beings, not the particle itself, which rationally leads to H. Everett's multiverse interpretation on the superposition by letting parallel equations for the wave functions $\psi(t, x)$ on positions of a particle in 1957 ([2]). We only hold coherent behaviors of elementary particles, such as those of quarks, leptons with interaction quanta and their antiparticles by (1.3), not the individual, and it is only an equation on those of particles viewed abstractly to be a geometrical point or an independent field from a macroscopic point, which leads physicists to assume the internal structures mechanically for hold the behaviors of particles such as those shown in Fig. 2 on hadrons. However, such an assumption is a little ambiguous in logic, i.e. we can not even conclude which is the point or the independent field, the hadron or its subparticle, the quark.

In fact, a point is non-divisible in geometry. Even so, the assumption on the internal structure of particles by physicists was mathematically verified by extending Banach spaces to extended Banach spaces on topological graphs $\overrightarrow{G}$ in [12]:

Let $(\mathscr{V}; +, \cdot)$ be a Banach space over a field $\mathscr{F}$ and $\overrightarrow{G}$ a strong-connected topological graph with vertex set V and arc set X. A vector labeling $\overrightarrow{G}^L$ on $\overrightarrow{G}$ is a $1-1$ mapping $L: \overrightarrow{G} \to \mathscr{V}$ such that $L: (u, v) \to L(u, v) \in \mathscr{V}$ for $\forall (u, v) \in X\left(\overrightarrow{G}\right)$ and it is a $\overrightarrow{G}$-flow if it holds with

$$L(u, v) = -L(v, u) \quad \text{and} \quad \sum_{u \in N_G(v)} L(v^u) = \mathbf{0}$$

for $\forall (u, v) \in X\left(\overrightarrow{G}\right)$, $\forall v \in V(\overrightarrow{G})$, where $\mathbf{0}$ is the zero-vector in $\mathscr{V}$.

For $\overrightarrow{G}$-flows $\overrightarrow{G}^L$, $\overrightarrow{G}^{L_1}$, $\overrightarrow{G}^{L_2}$ on a topological graph $\overrightarrow{G}$ and $\xi \in \mathscr{F}$ a scalar, it is clear that $\overrightarrow{G}^{L_1} + \overrightarrow{G}^{L_2}$ and $\xi \cdot \overrightarrow{G}^L$ are also $\overrightarrow{G}$-flows, which implies that all $\overrightarrow{G}$-flows on $\overrightarrow{G}$ form a linear space over $\mathscr{F}$ with unit $\mathbf{O}$ under operations $+$ and $\cdot$, denoted by $\overrightarrow{G}^{\mathscr{V}}$, where $\mathbf{O}$ is such a $\overrightarrow{G}$-flow with vector $\mathbf{0}$ on (u, v) for $\forall (u, v) \in X\left(\overrightarrow{G}\right)$. Then, it was shown that $\overrightarrow{G}^{\mathscr{V}}$ is a Banach space, and furthermore a Hilbert space if introduce

$$\left\|\overrightarrow{G}^L\right\| = \sum_{(u,v) \in X\left(\overrightarrow{G}\right)} \|L(u, v)\|,$$

$$\left\langle \overrightarrow{G}^{L_1}, \overrightarrow{G}^{L_2} \right\rangle = \sum_{(u,v) \in X\left(\overrightarrow{G}\right)} \langle L_1(u, v), L_2(u, v) \rangle$$

for $\forall \overrightarrow{G}^L, \overrightarrow{G}^{L_1}, \overrightarrow{G}^{L_2} \in \overrightarrow{G}^{\mathscr{V}}$, where $\|L(u, v)\|$ is the norm of $L(u, v)$ and $\langle \cdot, \cdot \rangle$ the inner product in $\mathscr{V}$ if it is an inner space. The following result generalizes the representation theorem of Fréchet and Riesz on linear continuous functionals on $\overrightarrow{G}$-flow space $\overrightarrow{G}^{\mathscr{V}}$, which enables us to find $\overrightarrow{G}$-flow solutions on linear equations (1.2).

Theorem 2.1([12]) *Let* $\mathbf{T}: \overrightarrow{G}^{\mathscr{V}} \to \mathbb{C}$ *be a linear continuous functional. Then there is a unique* $\overrightarrow{G}^{\widehat{L}} \in \overrightarrow{G}^{\mathscr{V}}$ *such that*

$$\mathbf{T}\left(\overrightarrow{G}^L\right) = \left\langle \overrightarrow{G}^L, \overrightarrow{G}^{\widehat{L}} \right\rangle$$

for $\forall \overrightarrow{G}^L \in \overrightarrow{G}^{\mathscr{V}}$.

For non-linear equations (1.2), we can also get $\overrightarrow{G}$-flow solutions on them if $\overrightarrow{G}$ can be decomposed into circuits.

Theorem 2.2([12]) *If the topological graph* $\overrightarrow{G}$ *is strong-connected with circuit decomposition*

$$\overrightarrow{G} = \bigcup_{i=1}^{l} \overrightarrow{C}_i$$

such that $L(u^v) = L_i(\mathbf{x})$ *for* $\forall(u,v) \in X\left(\overrightarrow{C}_i\right)$, $1 \le i \le l$ *and the Cauchy problem*

$$\begin{cases} \mathscr{F}_i(\mathbf{x}, u, u_{x_1}, \cdots, u_{x_n}, u_{x_1x_2}, \cdots) = 0 \\ u|_{\mathbf{x}_0} = L_i(\mathbf{x}) \end{cases}$$

is solvable in a Hilbert space $\mathscr{V}$ *on domain* $\Delta \subset \mathbb{R}^n$ *for integers* $1 \le i \le l$, *then the Cauchy problem*

$$\begin{cases} \mathscr{F}_i(\mathbf{x}, X, X_{x_1}, \cdots, X_{x_n}, X_{x_1x_2}, \cdots) = 0 \\ X|_{\mathbf{x}_0} = \overrightarrow{G}^L \end{cases}$$

such that $L(u^v) = L_i(\mathbf{x})$ *for* $\forall(u,v) \in X\left(\overrightarrow{C}_i\right)$ *is solvable for* $X \in \overrightarrow{G}^{\mathscr{V}}$.

Theorems 2.1−2.2 conclude the existence of $\overrightarrow{G}$-flow solution on linear or non-linear differential equations for a topological graph $\overrightarrow{G}$, such as those of the Schrödinger equation (1.3), Dirac equation (2.4) and the Klein-Gordon equation (2.5), which all implies the rightness of physicists assuming the internal structures for hold the behaviors of particles because there are infinite many such graphs $\overrightarrow{G}$ satisfying conditions of Theorem 2.1 − 2.2, particularly, the bouquet $\overrightarrow{B}_N^{L_\psi}$, the dipoles $\overrightarrow{D}^{L_\psi}_{0,2N,0}$ for elementary particles in [13].

3 In-observations

An *in-observation* observes on the internal behaviors of a particle, particularly, a composed particle P. Let P be composed by particles $P_1, P_2, \cdots, P_m$. Different from out-observation from a macro viewing, in-observation requires the observer holding the respective behaviors of particles $P_1, P_2, \cdots, P_m$ in P, for instance an observer enters a water molecule H_2O receiving information on the Hydrogen or Oxygen atoms H, O.

For such an observation, there are 2 observing ways:

(1) there is an apparatus such that an observer can simultaneously observe behaviors of particles $P_1, P_2, \cdots, P_m$, i.e. $P_1, P_2, \cdots, P_m$ can be observed independently as particles at the same time for the observer;

(2) there are m observers $O_1, O_2, \cdots, O_m$ simultaneously observe particles $P_1, P_2, \cdots, P_m$, i.e. the observer O_i only observes the behavior of particle P_i for $1 \le i \le m$, called *parallel observing*, such as those shown in Fig. 3 for the water molecule H_2O with $m = 3$.

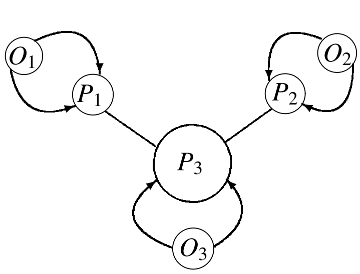

Fig. 3

Certainly, each of these observing views a particle in P to be an independent particle, which enables us to establish the dynamic equation (1.2) by Euler-Lagrange equation (2.3) for P_i, $1 \le i \le m$, respectively, and then we can apply the system of differential equations

$$\begin{cases} \dfrac{\partial L_1}{\partial \mathbf{q}} - \dfrac{d}{dt}\dfrac{\partial L_1}{\partial \dot{\mathbf{q}}} = 0 \\ \dfrac{\partial L_2}{\partial \mathbf{q}} - \dfrac{d}{dt}\dfrac{\partial L_2}{\partial \dot{\mathbf{q}}} = 0 \\ \cdots \\ \dfrac{\partial L_m}{\partial \mathbf{q}} - \dfrac{d}{dt}\dfrac{\partial L_m}{\partial \dot{\mathbf{q}}} = 0 \\ \mathbf{q}(t_0) = \mathbf{q}_0, \dot{\mathbf{q}}(t_0) = \dot{\mathbf{q}}_0 \end{cases} \tag{3.1}$$

for characterizing particle P in classical mechanics, or

$$\begin{cases} \dfrac{\partial \mathcal{L}_1}{\partial \psi} - \partial_\mu \dfrac{\partial \mathcal{L}_1}{\partial(\partial_\mu \psi)} = 0 \\ \dfrac{\partial \mathcal{L}_2}{\partial \psi} - \partial_\mu \dfrac{\partial \mathcal{L}_2}{\partial(\partial_\mu \psi)} = 0 \\ \cdots \\ \dfrac{\partial \mathcal{L}_m}{\partial \psi} - \partial_\mu \dfrac{\partial \mathcal{L}_m}{\partial(\partial_\mu \psi)} = 0 \\ \psi(t_0) = \psi_0 \end{cases} \tag{3.2}$$

for characterizing particle P in field theory, where the i^{th} equation is the dynamic equation of particle P_i with initial data $\mathbf{q}_0, \dot{\mathbf{q}}_0$ or ψ_0.

We discuss the solvability of systems (3.1) and (3.2). Let

$$S_{\mathbf{q}_i} = \left\{ (x_i, y_i, z_i)(\mathbf{q}_i, t) \in \mathbb{R}^3 \,\middle|\, \frac{\partial L_1}{\partial \mathbf{q}_i} - \frac{d}{dt}\frac{\partial L_1}{\partial \dot{\mathbf{q}}_i} = 0, \right. \\ \left. \mathbf{q}_i(t_0) = \mathbf{q}_0, \dot{\mathbf{q}}_i(t_0) = \dot{\mathbf{q}}_0 \right\}$$

for integers $1 \le i \le m$. Then, the system (3.1) of equations is solvable if and only if

$$\mathscr{D}(\mathbf{q}) = \bigcap_{i=1}^{m} S_{\mathbf{q}_i} \neq \emptyset. \tag{3.3}$$

Otherwise, the system (3.1) is non-solvable. For example, let particles P_1, P_2 of masses M, m be hanged on a fixed pulley, such as those shown in Fig. 4.

Then, the dynamic equations on P_1 and P_2 are respectively

$$P_1: \ \ddot{x} = g, \ x(t_0) = x_0 \text{ and } P_2: ddotx = -g, \ x(t_0) = x_0$$

but the system

$$\begin{cases} \ddot{x} = g \\ \ddot{x} = -g, \ x(t_0) = x_0 \end{cases}$$

is contradictory, i.e. non-solvable.

Similarly, let $\psi_i(x,t)$ be the state function of particle P_i, i.e. the solution of

$$\begin{cases} \frac{\partial \mathcal{L}_1}{\partial \psi_i} - \partial_\mu \frac{\partial \mathcal{L}_1}{\partial(\partial_\mu \psi_i)} = 0 \\ \psi(t_0) = \psi_0. \end{cases}$$

Then, the system (3.2) is solvable if and only if there is a state function $\psi(x,t)$ on P hold with each equation of system (3.2), i.e.

$$\psi(x,t) = \psi_1(x,t) = \cdots = \psi_m(x,t), \quad x \in \mathbb{R}^3,$$

which is impossible because if all state functions $\psi_i(x,t)$, $1 \le i \le m$ are the same, the particles $P_1, P_2, \cdots, P_m$ are nothing else but just one particle. Whence, the system (3.2) is non-solvable if $m \ge 2$, which implies we can not characterize the behavior of particle P by classical solutions of differential equations.

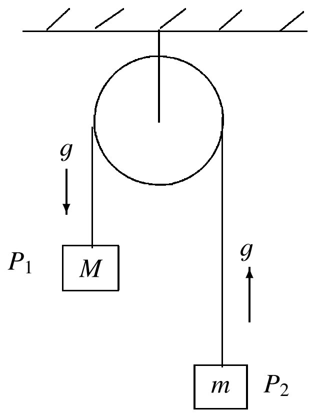

Fig. 4

For example, if the state function $\psi_O(x,t) = \psi_{H_1}(x,t) = \psi_{H_2}(x,t)$ in the water molecule H_2O for $x \in \mathbb{R}^3$ hold with

$$\begin{cases} -i\hbar \frac{\partial \psi_O}{\partial t} = \frac{\hbar^2}{2m_O}\nabla^2 \psi_O - V(x)\psi_O \\ -i\hbar \frac{\partial \psi_{H_1}}{\partial t} = \frac{\hbar^2}{2m_{H_1}}\nabla^2 \psi_{H_1} - V(x)\psi_{H_1} \\ -i\hbar \frac{\partial \psi_{H_2}}{\partial t} = \frac{\hbar^2}{2m_{H_2}}\nabla^2 \psi_{H_2} - V(x)\psi_{H_2} \end{cases}$$

Then $\psi_O(x,t) = \psi_{H_1}(x,t) = \psi_{H_2}(x,t)$ concludes that

$$A_O e^{-\frac{i}{\hbar}(E_O t - \mathbf{p}_O x)} = A_{H_1} e^{-\frac{i}{\hbar}(E_{H_1} t - \mathbf{p}_{H_1} x)} = A_{H_2} e^{-\frac{i}{\hbar}(E_{H_2} t - \mathbf{p}_{H_2} x)}$$

for $\forall x \in \mathbb{R}^3$ and $t \in \mathbb{R}$, which implies that

$$A_O = A_{H_1} = A_{H_2},\ E_O = E_{H_1} = E_{H_2} \text{ and } \mathbf{p}_O = \mathbf{p}_{H_1} = \mathbf{p}_{H_2},$$

a contradiction.

Notice that each equation in systems (3.1) and (3.2) is solvable but the system itself is non-solvable in general, and they are real in the nature. Even if the system (3.1) holds with condition (3.3), i.e. it is solvable, we can not apply the solution of (3.1) to characterize the behavior of particle P because such a solution only describes the coherent behavior of particles $P_1, P_2, \cdots, P_m$. Thus, we can not characterize the behavior of particle P by the solvability of systems (3.1) or (3.2). We should search new method to characterize systems (3.1) or (3.2).

Philosophically, the formula (1.1) is the understanding of particle P and all of these particles $P_1, P_2, \cdots, P_m$ are inherently related, not isolated, which implies that P naturally inherits a topological structure $G^L[P]$ in space of the nature, which is a vertex-edge labeled topological graph determined by:

$$V\left(G^L[P]\right) = \{P_1, P_2, \cdots, P_m\},$$
$$E\left(G^L[P]\right) = \{(P_i, P_j) | P_i \bigcap P_j \neq \emptyset,\ 1 \le i \neq j \le m\}$$

with labeling

$$L:\ P_i \to L(P_i) = P_i \quad \text{and}$$
$$L:\ (P_i, P_j) \to L(P_i, P_j) = P_i \bigcap P_j$$

for integers $1 \le i \neq j \le m$. For example, the topological graphs $G^L[P]$ of water molecule H_2O, meson and baryon in the quark model of Gell-Mann and Ne'eman are respectively shown in Fig. 5,

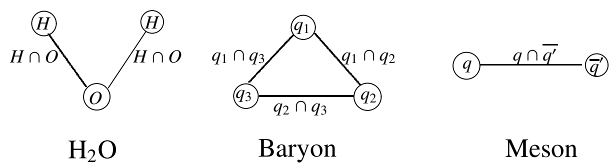

Fig. 5

where $O, H, q, \overline{q'}$ and $q_i, 1 \le i \le 3$ obey the Dirac equation but $O \cap H, q \cap \overline{q'}, q_k \cap q_l, 1 \le k, l \le 3$ comply with the Klein-Gordon equation.

Such a vertex-edge labeled topological graph $G^L[P]$ is called G^L-*solution* of systems (3.1)–(3.2). Clearly, the global behaviors of particle P are determined by particles $P_1, P_2, \cdots, P_m$. We can hold them on G^L-solution of systems (3.1) or (3.2). For example, let $u^{[v]}$ be the solution of equation at vertex $v \in V\left(G^L[P]\right)$ with initial value $u_0^{[v]}$ and $G^{L_0}[P]$ the initial G^L-solution, i.e. labeled with $u_0^{[v]}$ at vertex v. Then, a G^L-solution of systems (3.1) or (3.2) is *sum-stable* if for any number $\varepsilon > 0$ there exists $\delta_v > 0$, $v \in V(G^{L_0}[P])$ such that each $G^{L'}$-solution with

$$\left| u'^{[v]}_0 - u^{[v]}_0 \right| < \delta_v, \qquad \forall v \in V(G^{L_0}[P])$$

exists for all $t \ge 0$ and with the inequality

$$\left| \sum_{v \in V(G^{L'}[P])} u'^{[v]} - \sum_{v \in V(G^L[P])} u^{[v]} \right| < \varepsilon$$

holds, denoted by $G^L[P] \overset{\Sigma}{\sim} G^{L_0}[P]$. Furthermore, if there exists a number $\beta_v > 0$ for $\forall v \in V(G^{L_0}[P])$ such that every $G^{L'}[P]$-solution with

$$\left|u'^{[v]}_0 - u^{[v]}_0\right| < \beta_v, \qquad \forall v \in V(G^{L_0}[P])$$

satisfies

$$\lim_{t\to\infty}\left|\sum_{v\in V(G^{L'}[P])} u'^{[v]} - \sum_{v\in V(G^{L}[P])} u^{[v]}\right| = 0,$$

then the $G^L[P]$-solution is called asymptotically stable, denoted by $G^L[P] \overset{\Sigma}{\to} G^{L_0}[P]$. Similarly, the energy integral of G^L-solution is determined by

$$E(G^L[P]) = \sum_{G\le G^{L_0}[P]} (-1)^{|G|+1} \int_{\mathscr{O}_G} \left(\frac{\partial u^G}{\partial t}\right)^2 dx_1 dx_2 \cdots dx_{n-1},$$

where u^G is the $\mathbb{C}^2$ solution of system

$$\left.\begin{array}{l} \dfrac{\partial u}{\partial t} = H_v(t, x_1, \cdots, x_{n-1}, p_1, \cdots, p_{n-1}) \\ u|_{t=t_0} = u_0^{[v]}(x_1, x_2, \cdots, x_{n-1}) \end{array}\right\} v \in V(G)$$

and $\mathscr{O}_G = \bigcap\limits_{v\in V(G)} \mathscr{O}_v$ with $\mathscr{O}_v \subset \mathbb{R}^n$ determined by the vth equation

$$\left\{\begin{array}{l} \dfrac{\partial u}{\partial t} = H_v(t, x_1, \cdots, x_{n-1}, p_1, \cdots, p_{n-1}) \\ u|_{t=t_0} = u_0^{[v]}(x_1, x_2, \cdots, x_{n-1}). \end{array}\right.$$

All of these global properties were extensively discussed in [7–11], which provides us to hold behaviors of a composed particle P by its constitutions $P_1, P_2, \cdots, P_m$.

4 Reality

Generally, the reality is the state characters (1.1) of existed, existing or will exist things whether or not they are observable or comprehensible by human beings, and the observing objective is on the state of particles, which then enables us to find the reality of a particle. However, an observation is dependent on the perception of the observer by his organs or through by instruments at the observing time, which concludes that to hold the reality of a particle P can be only little by little, and determines local reality of P from a macro observation at a time t, no matter what P is, a macro or micro thing. Why is this happening because we always observe by one observer on one particle assumed to be a point in space, and then establish a solvable equation (1.2) on coherent, not individual behaviors of P. Otherwise, we get non-solvable equations on P contradicts to the law of contradiction, the foundation of classical mathematics which results in discussions following:

4.1 States of particles are multiverse

A particle P understood by formula (1.1) is in fact a multiverse consisting of known characters $\mu_1, \mu_2, \cdots, \mu_n$ and unknown characters $\nu_k, k \ge 1$, i.e. different characters characterize different states of particle P. This fact also implies that the multiverse exist everywhere if we understand a particle P with in-observation, not only those levels of $I - IV$ of Max Tegmark in [24]. In fact, the infinite divisibility of a matter M in philosophy alludes nothing else but a multiverse observed on M by its individual submatters. Thus, the nature of a particle P is multiple in front of human beings, with unity character appeared only in specified situations.

4.2 Reality only characterized by non-compatible system

Although the dynamical equations (1.2) established on unilateral characters are individually compatible but they must be globally contradictory with these individual features unless all characters are the same one. It can not be avoided by the nature of a particle P. Whence, the non-compatible system, particularly, non-solvable systems consisting of solvable differential equations are suitable tools for holding the reality of particles P in the world, which also partially explains a complaint of Einstein on mathematics, i.e. *as far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality* because the multiple nature of all things.

4.3 Reality really needs mathematics on graph

As we know, there always exists a universal connection between things in a family in philosophy. Thus, a family $\mathscr{F}$ of things naturally inherits a topological graph $G^L[\mathscr{F}]$ in space and we therefore conclude that

$$\mathscr{F} = G^L[\mathscr{F}] \tag{4.1}$$

in that space. Particularly, if all things in $\mathscr{F}$ are nothing else but manifolds $M_T(x_1, x_2, x_3; t)$ of particles P determined by equation

$$f_T(x_1, x_2, x_3; t) = 0, \qquad T \in \mathscr{F} \tag{4.2}$$

in $\mathbb{R}^3 \times \mathbb{R}$, we get a geometrical figure $\bigcup\limits_{T\in\mathscr{F}} M_T(x_1, x_2, x_3; t)$, a combinatorial field ([6]) for $\mathscr{F}$. Clearly, the graph $G^L[\mathscr{F}]$ characterizes the behavior of $\mathscr{F}$ no matter whether the system (4.2) is solvable or not. Calculation shows that the system (4.2) of equations is non-solvable or not dependent on

$$\bigcap_{T\in\mathscr{F}} M_T(x_1, x_2, x_3; t) = \emptyset \quad \text{or not.}$$

Particularly, if $\bigcap\limits_{T\in\mathscr{F}} M_T(x_1, x_2, x_3; t) = \emptyset$, the system (4.2) is non-solvable and we can not just characterize the behavior of $\mathscr{F}$ by the solvability of system (4.2). We must turn the contradictory system (4.2) to a compatible one, such as those shown in [10] and have to extend mathematical systems on graph $G^L[\mathscr{F}]$ ([12]) for holding the reality of $\mathscr{F}$.

Notice that there is a conjecture for developing mathematics in [4] called *CC conjecture* which claims that *any mathematical science can be reconstructed from or turned into combinatorization*. Such a conjecture is in fact a combinatorial notion for developing mathematics on topological graphs, i.e. finds the combinatorial structure to reconstruct or generalize classical mathematics, or combines different mathematical sciences and establishes a new enveloping theory on topological graphs for hold the reality of things $\mathscr{F}$.

5 Conclusion

Reality of a thing is hold on observation with level dependent on the observer standing out or in that thing, particularly, a particle classified to out- or in-observation, or parallel observing from a macro or micro view and characterized by solvable or non-solvable differential equations, consistent with the universality principle of contradiction in philosophy. For holding on the reality of things, the out-observation is basic but the in-observation is cardinal. Correspondingly, the solvable equation is individual but the non-solvable equations are universal. Accompanying with the establishment of compatible systems, we are also needed to characterize those of contradictory systems, particularly, non-solvable differential equations on particles and establish mathematics on topological graphs, i.e. *mathematical combinatorics*, and only which is the appropriate way for understanding the nature because all things are in contradiction.

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