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A novel perspective for Q-neutrosophic soft relations and their application in decision making

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Abstract

Although neutrosophic soft sets are quite successful in expressing neutral uncertain information, this mathematical approach is insufficient if the dimension of uncertain information increases. Therefore, in order to present a better approach to the uncertainty problems encountered, Q-neutrosophic soft sets (Abu Qamar in Entropy 20:172, 2018), which are a generalization of neutrosophic soft sets, are proposed for processing two-dimensional uncertain information. Moreover, relations are one of the methods preferred by researchers to explain the correspondences between objects in uncertain environments. The purpose of this paper is to explain the correspondence between objects in a better way. For this, the Q-neutrosophic soft set theory defined in Abu Qamar and Hassan (Entropy 20:172,2018) has been modified by taking inspiration from the definition of neutrosophic soft sets proposed by Deli and Broumi (J Intell Fuzzy Syst 28:2233–2241, 2015). In this way, it has been ensured that the two-dimensional information can be used more effectively in uncertainty problems encountered in many real-life problems. In this paper, Q-neutrosophic soft relations are defined by referring to the modified Q-neutrosophic soft set theory and analyzed in detail. Later, concepts related to Q-neutrosophic soft relations such as composition, inverse, functions, equivalence classes, and partitions are given. These concepts are especially exemplified by selecting uncertainty problems that can be encountered in real life and some related properties and theorems are presented. Finally, in an uncertainty problem, a decision-making algorithm is proposed by using the correspondence between objects and a comparison is given.

Keywords Q-neutrosophic soft set · Q-neutrosophic soft relation · Q-neutrosophic soft function · Decision making

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1 Introduction

There are many uncertainty problems that can be encountered in many areas of our daily life, such as engineering and physics. For this reason, expressing uncertainty problems in the most accurate way and thus obtaining the most accurate result in solving the problem are the two most important factors to manage the best decision-making process. Researchers have brought many different mathematical models to the literature that can bring this process to the most accurate result. Although fuzzy sets proposed by Zadeh (1965) is the first mathematical approach put forward in this area, it is a very successful theory. Even today, many mathematical approaches such as interval-valued intuitionistic FS (Atanassov 1999), neutrosophic set (briefly N-set) (Smarandache 2005), single valued NS (Wang et al. 2010), picture fuzzy set (Cuong and Kreinovich 2014), pythagorean fuzzy set (Yager 2013), fermatean fuzzy sets (Senapati and Yager 2020), hypersoft set (Smarandache 2018, 2019), which are different extensions of this theory, are put forward.

One of the theories that come to the fore with the study of fuzzy sets to overcome uncertainty is the N-sets proposed by Smarandache (2005). The fact that this theory is also a generalization of intuitionistic fuzzy set (Atanassov 1986) strengthens the reasons for this theory to be studied. It is known that the theory used to express uncertainty consists of a logic set with three components, which are independent from each other, consisting of the truth, indeterminacy and falsity memberships (Smarandache and Neutrosophy 1998). Therefore, the most important advantage of this mathematical approach is its ability to process indeterminate data that are not processed by fuzzy set and intuitionistic fuzzy set.

Although fuzzy sets and their various extensions are very useful mathematical approaches to overcome uncertainty, it is not practical and useful to use these theories on the uncertainty problem. Molodtsov (1999), who thinks that the reason for this situation is due to the lack of a parameterization tool, proposed the soft set (briefly S-set) theory in the literature. Thanks to this important feature, which is not found in any of the other mathematical approaches developed for uncertainty, interest in soft sets continues to increase day by day. For this reason, since the introduction of soft sets, many different types of sets such as soft multi set theory (Alkhazaleh et al. 2011), soft expert set (Alkhazaleh and Salleh 2011), vague S-set (Xu et al. 2010), interval-valued vague S-set (Alhazaymeh and Hassan 2012, 2013a, b), virtual fuzzy parametrized S-set (Dalkılıç and Demirtas 2020), virtual fuzzy parametrized fuzzy S-set (Dalkılıç 2020), neutrosophic S-set (briefly NS-set) (Maji 2013; Deli and Broumi 2015; Deli 2017; Alkhazaleh 2016), neutrosophic parameterized NS-set (Deli 2015), Q-NS-set (Abu Qamar and Hassan 2018) have been brought to the literature. Ongoing research on soft sets shows us that this mathematical approach is a very successful theory in expressing uncertainty (De and Dalk 2019; D 2020; Dalkılıç and Demirtaş 2020; Demirtaş and Dalkılıç 2020; Demirtaş et al. 2020; Dalkılıç 2020; Dalkılıç and Demirtas 2020; Da 2021).

N-set and S-set theories, which are the most popular theories used in uncertainty environments today, were combined by Maji (2013) and the NS-set theory was proposed as a new hybrid set type in this field. In this way, soft sets can be used more practically to express uncertainty problems. The NS-set definition of Maji (2013) was modified by Deli and Broumi (2015), allowing it to be used more practically in uncertainty problems and a comparison between definitions was also made. In addition, basic set operations have been redefined based on this definition. Many researchers have studied this proposed mathematical approach to uncertainty problems: For example; Mukherjee and Sarkar (2014) solved a medical diagnosis decision-making problem using NSSs and discussed about the theory.

Moreover, şahin and Küçük (2014) studied some algebraic properties by introducing a novel style of NS-set theory. Apart from that, Hussain and Shabir (2015) investigated on algebraic operations of theory. Sumathi and Arockiarani (2016) also studied the NS-sets. In addition to these, a lot of research has been done on the NS-set (Maji 2014; Mukherjee and Sadhan 2015; Cuong et al. 2016; Chatterjee et al. 2016; Marei 2018). NS-sets are sufficient to express neutral uncertain information, but not all data sets may consist of neutral information. For this reason, Q-NS-sets that allow the processing of two-dimensional information were proposed by Abu Qamar and Hassan (2018). In this way, a more useful hybrid cluster model was built in modeling real-life problems where uncertainty occurred to a great extent. Researchers who find this situation quite remarkable have recently preferred to use this mathematical approach (Hassan et al. 2018; Abu Qamar and Hassan 2018a, b; Khan et al. 2019; Qamar and Hassan 2019a, b).

Relations are one of the most important mathematical methods used to explain the correspondence between objects in uncertainty environments. Different types of relations have been introduced into the literature by making use of the mathematical approaches put forward to eliminate the uncertainty mentioned above. If we briefly refer to these relationships, firstly, fuzzy relations are only useful in modeling an uncertainty that expresses the degree of relations of two objects. For this reason, intuitionistic fuzzy relations (Bustince and Burillo 1996) and intuitionistic fuzzy soft relations (Dinda and Samanta 2010) were suggested, but they were insufficient to express the correspondence between objects in the most accurate way because they do not handle indeterminacy degrees of membership in these relations. For this reason, Deli and Broumi (2015) suggested NS-relations by making use of the NS-sets defined by Maji (2013). Later, the NS-relations given by Deli and Broumi (2015) were modified by Da (2021) in order to express the correspondence between objects in a more practical way. This has been an important achievement in explaining the correspondence between objects. In addition, Q-NS-relations have been brought to the literature by Abu Qamar and Hassan (2018) in expressing the correspondence between objects in two-dimensional uncertainty environments. Especially in order to express uncertainty problems in the best way, it is necessary to determine the correspondence between objects in the most accurate way. Therefore, advances in this area are very important and many researchers have paid attention to this situation in their recent studies (Kanwal and Shabir 2018, 2019; Zheng and Deng 2018; Mordeson et al. 2018; Kanwal et al. 2020). The importance of this article on the literature is as follows:

- Better detection of correspondence is given by examining the relationships between objects in uncertain environments based on Q-NS-sets, which is a more comprehensive mathematical model.
- The determination of the relations between the correspondences has been examined in a different way than the traditional way, and their superior sides have been emphasized. (For the only similar modern approach available in the literature, see (Da 2021))
- In particular, the importance of correspondence between objects is emphasized in order to get better results in decision-making problems.

In this paper, the Q-NS-set definition given by Abu Qamar and Hassan (2018) was modified by inspiring from Deli and Broumi (2015)'s definition of NS-set, and thus we provide the opportunity to express uncertainty problems in a more practical way. Moreover, Q-NS-relations studied by Abu Qamar and Hassan (2018) have been analyzed using the definition we have suggested. To facilitate analysis, we organized our paper as follows: In section 2, we recall some basic notions in N-set, S-set, NS-set and (multi) Q-N-set. Then, the Q-NS-sets proposed by Abu Qamar and Hassan (2018) have been modified. In section 3, we characterize the idea of Q-NS-relation and present the composition and inverse operations of these relations with some basic properties. In section 4, various types of Q-NS-relations are defined. In addition, the equivalence classes and partitions of Q-NS-sets are analyzed. In section 5, the concept of Q-NS-function is defined and some special types of this concept are presented together with related theorems. In section 6, we present an application of Q-NSrelations in a decision-making problem. Also a comparison has been added. Finally, we conclude the paper in section 7.

2 Preliminaries

In this section, we recall some basic notions in N-set, S-set, NS-set and (multi) Q-N-set. Then, the Q-NS-sets proposed by Abu Qamar and Hassan (2018) have been modified.

Throughout this paper, let P be a set of parameters and K, L, M be non-empty subsets of P. Also, let U be an initial universe, and 2^U denotes the power set of U.

Definition 2.1 (Smarandache 2005) A N-set X on the universe of discourse U is defined as:

$$X = \{\langle u, T_X(u), I_X(u), F_X(u)\rangle : u \in U\} \tag{2.1}$$

where ${}^-0 \leq T_X(u) + I_X(u) + F_X(u) \leq 3^+$ and $T, I, F : U \to]^-0, 1^+[$.

Note that the set of all N-sets over U will be denoted by $2^{N(U)}$.

Definition 2.2 (Molodtsov 1999) A pair (F, P) is called a S-set over U, where F is a mapping given by $F : P \to 2^U$. For $p \in P$, $F(p)$ may be considered as the set of p-approximate elements of the S-set (F, P), i.e.,

$$(F, P) = \{(p, F(p)) : p \in P\}. \tag{2.2}$$

Firstly, NS-set defined by Maji (2013) and later this concept has been modified by Deli and Broumi (2015) as given below:

Definition 2.3 (Deli and Broumi 2015) A NS-set $(\widehat{F}, P)$ over U is a set defined by a set valued function $\widehat{F}$ representing a mapping $\widehat{F} : P \to 2^{N(U)}$ where $\widehat{F}$ is called approximate function of the NS-set $(\widehat{F}, P)$, i.e.,

$$(\widehat{F}, P) = \left\{ \left(p, \langle u, T_{\widehat{F}(p)}(u), I_{\widehat{F}(p)}(u), F_{\widehat{F}(p)}(u)\rangle : u \in U\right) : p \in P\right\} \tag{2.3}$$

where $0 \leq T_{\widehat{F}(p)}(u) + I_{\widehat{F}(p)}(u) + F_{\widehat{F}(p)}(u) \leq 3$ and $T_{\widehat{F}(p)}(u), I_{\widehat{F}(p)}(u), F_{\widehat{F}(p)}(u) \in [0, 1]$ called the true membership, indeterminacy-membership, false membership functions of $\widehat{F}(p)$, respectively.

Definition 2.4 (Abu Qamar and Hassan 2018) Let Q and U be two nonempty set. A Q-N-set Γ_Q in U and Q is an object of the form

$$\Gamma_Q = \left\{ \langle (u, q), T_{\Gamma_Q}(u, q), I_{\Gamma_Q}(u, q), F_{\Gamma_Q}(u, q)\rangle : u \in U, q \in Q\right\} \tag{2.4}$$

where $^{-}0 \leq T_{\Gamma_Q}(u,q) + I_{\Gamma_Q}(u,q) + F_{\Gamma_Q}(u,q) \leq 3^{+}$ and $T_{\Gamma_Q}, I_{\Gamma_Q}, F_{\Gamma_Q} : U \times Q \to]^{-}0, 1^{+}[$ are the truth membership, indeterminacy membership and falsity membership functions, respectively.

Note that the set of all Q-N-sets over U will be denoted by $2^{N_Q(U)}$.

Now, we modify the Q-NS-set definition given by Abu Qamar and Hassan (2018), inspired by the definition of NS-set proposed by Deli and Broumi (2015), as follows:

Definition 2.5 Let Q be a nonempty set. A pair $(\widehat{\Gamma}_Q, P)$ is called a Q-NS-set over U, where $\widehat{\Gamma}_Q : P \to 2^{N_Q^1(U)}$, i.e.,

$$(\widehat{\Gamma}_Q, P) = \left\{ \left(p, \langle (u,q), T_{\widehat{\Gamma}_{Q(p)}}(u,q), I_{\widehat{\Gamma}_{Q(p)}}(u,q), F_{\widehat{\Gamma}_{Q(p)}}(u,q) \rangle : u \in U, q \in Q \right) : p \in P \right\} \tag{2.5}$$

such that

$$\widehat{\Gamma}_Q(p) = \left\{ \langle (u,q), T_{\widehat{\Gamma}_{Q(p)}}(u,q), I_{\widehat{\Gamma}_{Q(p)}}(u,q), F_{\widehat{\Gamma}_{Q(p)}}(u,q) \rangle : u \in U, q \in Q \right\} \tag{2.6}$$

where

$$0 \leq T_{\widehat{\Gamma}_{Q(p)}}(u,q) + I_{\widehat{\Gamma}_{Q(p)}}(u,q) + F_{\widehat{\Gamma}_{Q(p)}}(u,q) \leq 3$$

and $T_{\widehat{\Gamma}_{Q(p)}}(u,q), I_{\widehat{\Gamma}_{Q(p)}}(u,q), F_{\widehat{\Gamma}_{Q(p)}}(u,q) \in [0,1]$ called the truth membership, indeterminacy membership, falsity membership functions of $\widehat{\Gamma}_Q(p)$, respectively.

As can be understood from Table 1, the definition of Q-NS-set suggested in this paper is more general. It is aimed to generalize the definition of Q-NS-set in order to explain the correspondences between objects in a better way. Because a parameter that does not belong to the parameter set may have a relationship between other parameters. Not ignoring this situation can help us to express the uncertainty environments in the best way possible.

Note that the set of all multi Q-NS-sets over U and Q will be denoted by $2^{NS_Q(U)}$.

Definition 2.6 Let $(\widehat{\Gamma}_Q, P), (\widehat{\Lambda}_Q, P) \in 2^{NS_Q(U)}$. Then,

(i) $(\widehat{\Gamma}_Q, P)$ is a Q-NS-subset of $(\widehat{\Lambda}_Q, P)$, denoted by $(\widehat{\Gamma}_Q, P) \widehat{\subseteq} (\widehat{\Lambda}_Q, P)$, if $\widehat{\Gamma}_Q(p) \subseteq \widehat{\Lambda}_Q(p)$; $\forall p \in P$, that is $T_{\widehat{\Gamma}_{Q(p)}}(u,q) \leq T_{\widehat{\Lambda}_{Q(p)}}(u,q)$, $I_{\widehat{\Gamma}_{Q(p)}}(u,q) \geq I_{\widehat{\Lambda}_{Q(p)}}(u,q)$ and $F_{\widehat{\Gamma}_{Q(p)}}(u,q) \geq F_{\widehat{\Lambda}_{Q(p)}}(u,q)$; $\forall (u,q) \in U \times Q$.

Table 1 Comparison for definitions of Q-NS-set

Definition 2.5	In Abu Qamar and Hassan (2018)
$(\widehat{\Gamma}_Q, P) = \left\{ \left(p, \widehat{\Gamma}_Q(p) \right) : p \in P \right\}$ where $\widehat{\Gamma}_Q : P \to 2^{N_Q^1(U)}$.	$(\widehat{\Gamma}_Q, K) = \left\{ \left(p, \widehat{\Gamma}_Q(p) \right) : p \in K \right\}$ where $\widehat{\Gamma}_Q : K \to 2^{N_Q^1(U)}$ such that $\widehat{\Gamma}_Q(p) = \emptyset$ if $p \notin K$.

(*ii*) $(\widehat{\Gamma}_Q, P)$ and $(\widehat{\Lambda}_Q, P)$ are equal, denoted by $(\widehat{\Gamma}_Q, P) = (\widehat{\Lambda}_Q, P)$, if and only if $(\widehat{\Gamma}_Q, P)\widehat{\subseteq}(\widehat{\Lambda}_Q, P)$ and $(\widehat{\Lambda}_Q, P)\widehat{\subseteq}(\widehat{\Gamma}_Q, P)$; $\forall u \in U$.

Example 2.7 Let's examine the popularity of the computer a person wants to use. Let $U = \{u_1, u_2, u_3\}$ be the set of available computers, $Q = \{q_1, q_2\}$ be the set of companies from which computers can be purchased and $\{p_1, p_2\}$ be the set of parameters expressing the properties of computers. Suppose Q-NS-set $(\widehat{\Gamma}_Q, P)$ express the effect of parameters on the popularity of a computer in a particular company as follows:

$$(\widehat{\Gamma}_Q, P) = \left\{ \begin{array}{l} \left(p_1, \begin{array}{c} \langle (u_1, q_1), 0.55, 0.76, 0.25\rangle, \langle (u_1, q_2), 0.47, 0.35, 0.72\rangle, \\ \langle (u_2, q_1), 0.67, 0.44, 0.15\rangle, \langle (u_2, q_2), 0.78, 0.24, 0.67\rangle, \\ \langle (u_3, q_1), 0.85, 0.62, 0.45\rangle, \langle (u_3, q_2), 0.54, 0.74, 0.47\rangle \end{array} \right), \\ \left(p_2, \begin{array}{c} \langle (u_1, q_1), 0.43, 0.46, 0.14\rangle, \langle (u_1, q_2), 0.48, 0.65, 0.67\rangle, \\ \langle (u_2, q_1), 0.23, 0.56, 0.75\rangle, \langle (u_2, q_2), 0.35, 0.74, 0.85\rangle, \\ \langle (u_3, q_1), 0.62, 0.25, 0.57\rangle, \langle (u_3, q_2), 0.89, 0.46, 0.32\rangle \end{array} \right) \end{array} \right\}$$

Here, the $T_{\widehat{\Gamma}_{Q_i}}$, $I_{\widehat{\Gamma}_{Q_i}}$ and $F_{\widehat{\Gamma}_{Q_i}}$ represent the degree of true popularity, the degree of indeterminacy popularity and the the degree of falsity popularity of a computer in a particular company, respectively.

3 Relations on Q-NS-sets

In this section, we define the concept cartesian product (briefly CP) for two Q-NS-sets considering the modified Q-NS-sets, and thus we characterize the idea of Q-NS-relation. Then, we give the composition and inverse operations for these relationships with some basic properties.

Definition 3.1 Let $(\widehat{\Gamma}_Q, P), (\widehat{\Lambda}_Q, P) \in 2^{NS_Q(U)}$. Then, $(\widehat{\Gamma}_Q, P) \times (\widehat{\Lambda}_Q, P) = (\widehat{\Omega}_Q, P \times P)$ is the CP of $(\widehat{\Gamma}_Q, P)$ and $(\widehat{\Lambda}_Q, P)$, where $(a, b) \in P \times P$, $\widehat{\Omega}_Q : P \times P \to 2^{NS_Q(U)}$ and $\widehat{\Omega}_Q(a, b) = \Gamma(a)_Q \times \Lambda(b)_Q$, i.e.,

$$(\widehat{\Omega}_Q, P \times P) = \left\{ \left((a, b), \widehat{\Omega}_Q(a, b) \right) : (a, b) \in P \times P \right\} \tag{3.1}$$

such that

$$\widehat{\Omega}_Q(a, b) = \left\{ \langle (u, q), T_{\widehat{\Omega}_Q(a,b)}(u, q), I_{\widehat{\Omega}_Q(a,b)}(u, q), F_{\widehat{\Omega}_Q(a,b)}(u, q)\rangle : (u, q) \in U \times Q \right\} \tag{3.2}$$

where $T_{\widehat{\Omega}_Q(a,b)}(u, q), I_{\widehat{\Omega}_Q(a,b)}(u, q), F_{\widehat{\Omega}_Q(a,b)}(u, q)$ are the truth, indeterminacy and falsity membership functions of $(\widehat{\Omega}_Q, P \times P)$ such that $T_{\widehat{\Omega}_Q(a,b)}, I_{\widehat{\Omega}_Q(a,b)}, F_{\widehat{\Omega}_Q(a,b)}(u) : U \to [0, 1]$ and for all $(u, q) \in U \times Q$ and $(a, b) \in P \times P$ we have:

$$T_{\widehat{\Omega}_Q(a,b)}(u, q) = min\left\{ T_{\widehat{\Gamma}_Q(a)}(u, q), T_{\widehat{\Lambda}_Q(b)}(u, q) \right\},$$

$$I_{\widehat{\Omega}_{Q(a,b)}}(u,q) = max\Big\{ I_{\widehat{\Gamma}_{Q(a)}}(u,q), I_{\widehat{\Lambda}_{Q(b)}}(u,q) \Big\}$$

and

$$F_{\widehat{\Omega}_{Q(a,b)}}(u,q) = max\Big\{ F_{\widehat{\Gamma}_{Q(a)}}(u,q), F_{\widehat{\Lambda}_{Q(b)}}(u,q) \Big\}.$$

Example 3.2 Let $U = \{u_1, u_2, u_3\}$, $Q = \{q_1, q_2\}$, $P = \{p_1, p_2\}$, $(\widehat{\Gamma}_Q, P)$ be given as in Example 2.7 and Q-NS-set $(\widehat{\Lambda}_Q, P)$ over U be given as follows:

$$(\widehat{\Lambda}_Q, P) = \left\{ \begin{array}{l} \left(p_1, \begin{array}{c} \langle (u_1,q_1), 0.76, 0.56, 0.85\rangle, \langle (u_1,q_2), 0.84, 0.65, 0.24\rangle, \\ \langle (u_2,q_1), 0.18, 0.56, 0.43\rangle, \langle (u_2,q_2), 0.72, 0.56, 0.62\rangle, \\ \langle (u_3,q_1), 0.45, 0.24, 0.37\rangle, \langle (u_3,q_2), 0.32, 0.54, 0.41\rangle \end{array} \right), \\ \left(p_2, \begin{array}{c} \langle (u_1,q_1), 0.36, 0.52, 0.44\rangle, \langle (u_1,q_2), 0.82, 0.54, 0.78\rangle, \\ \langle (u_2,q_1), 0.28, 0.68, 0.55\rangle, \langle (u_2,q_2), 0.66, 0.48, 0.59\rangle, \\ \langle (u_3,q_1), 0.25, 0.56, 0.74\rangle, \langle (u_3,q_2), 0.92, 0.64, 0.25\rangle \end{array} \right) \end{array} \right\}.$$

Then, $(\widehat{\Omega}_Q, P \times P) = (\widehat{\Gamma}_Q, P) \times (\widehat{\Lambda}_Q, P)$ is

$$(\widehat{\Omega}, P \times P) = \Big\{ \widehat{\Gamma}_Q(p_1) \times \widehat{\Lambda}_Q(p_1), \widehat{\Gamma}_Q(p_1) \times \widehat{\Lambda}_Q(p_2), \widehat{\Gamma}_Q(p_2) \times \widehat{\Lambda}_Q(p_1), \widehat{\Gamma}_Q(p_2) \times \widehat{\Lambda}_Q(p_2) \Big\}$$

where

$$\widehat{\Gamma}_Q(p_1) \times \widehat{\Lambda}_Q(p_1) = \left\{ \begin{array}{c} \langle (u_1,q_1), 0.55, 0.76, 0.85\rangle, \langle (u_1,q_2), 0.47, 0.65, 0.72\rangle, \\ \langle (u_2,q_1), 0.18, 0.56, 0.43\rangle, \langle (u_2,q_2), 0.72, 0.56, 0.67\rangle, \\ \langle (u_3,q_1), 0.45, 0.62, 0.45\rangle, \langle (u_3,q_2), 0.32, 0.74, 0.47\rangle \end{array} \right\}$$

and so on.

Definition 3.3 Let $(\widehat{\Gamma}_Q, P), (\widehat{\Lambda}_Q, P) \in 2^{NS_Q(U)}$. Then a Q-NS-relation from $(\widehat{\Gamma}_Q, P)$ to $(\widehat{\Lambda}_Q, P)$ is a Q-NS-subset of $(\widehat{\Gamma}_Q, P) \times (\widehat{\Lambda}_Q, P)$, and is of the form $(\widehat{\mathfrak{R}}_Q, K \times L)$, where $K \times L \subseteq P \times P$ and $\widehat{\mathfrak{R}}_Q(a,b) \subseteq (\widehat{\Gamma}_Q, P) \times (\widehat{\Lambda}_Q, P)$, $\forall (a,b) \in K \times L$, i.e.,

$$(\widehat{\mathfrak{R}}_Q, K \times L) = \Big\{ \Big((a,b), \widehat{\mathfrak{R}}_Q(a,b) \Big) : (a,b) \in K \times L \subseteq P \times P \Big\} \tag{3.3}$$

such that

$$\widehat{\mathfrak{R}}_Q(a,b) = \Big\{ \langle (u,q), T_{\widehat{\mathfrak{R}}_{Q(a,b)}}(u,q), I_{\widehat{\mathfrak{R}}_{Q(a,b)}}(u,q), F_{\widehat{\mathfrak{R}}_{Q(a,b)}}(u,q) \rangle : (u,q) \in U \times Q \Big\} \tag{3.4}$$

where

$$T_{\widehat{\mathfrak{R}}_{Q(a,b)}}(u,q) = min\Big\{ T_{\widehat{\Gamma}_{Q(a)}}(u,q), T_{\widehat{\Lambda}_{Q(b)}}(u,q) \Big\},$$

$$I_{\widehat{\mathfrak{R}}_{Q(a,b)}}(u,q) = max\Big\{ I_{\widehat{\Gamma}_{Q(a)}}(u,q), I_{\widehat{\Lambda}_{Q(b)}}(u,q) \Big\}$$

and

$$F_{\widehat{\mathfrak{R}}_{Q(a,b)}}(u,q) = max\left\{F_{\widehat{\Gamma}_{Q(a)}}(u,q), F_{\widehat{\Lambda}_{Q(b)}}(u,q)\right\}.$$

If $(\widehat{\mathfrak{R}}_Q, K \times L)$ is a Q-NS-relation from $(\widehat{\Gamma}_Q, P)$ to $(\widehat{\Gamma}_Q, P)$, then is called a Q-NS-relation on $(\widehat{\Gamma}_Q, P)$.

Definition 3.4 Let $\widehat{\mathfrak{R}}_Q$ be a Q-NS-relation on $(\widehat{\Gamma}_Q, P)$. Then,

(*i*) the domain of $\widehat{\mathfrak{R}}_Q$ $\left(\widehat{DM}_{\widehat{\mathfrak{R}}_Q}\right)$ is defined as the Q-NS-set $\left(\widehat{DM}_{\widehat{\mathfrak{R}}_Q}, K_1\right)$, where $K_1 = \{a \in K : \widehat{\mathfrak{R}}_Q(a,b) \in \widehat{\mathfrak{R}}_Q, \exists b \in L\}$ and $\widehat{DM}_{\widehat{\mathfrak{R}}_Q}(a_1) = \widehat{\Gamma}_Q(a_1), \forall a_1 \in K_1$.

(*ii*) the range of $\widehat{\mathfrak{R}}_Q$ $\left(\widehat{RN}_{\widehat{\mathfrak{R}}_Q}\right)$ is defined as the Q-NS-set $(\widehat{RN}_{\widehat{\mathfrak{R}}_Q}, L_1)$ where $L_1 = \{b \in L : \widehat{\mathfrak{R}}_Q(a,b) \in \widehat{\mathfrak{R}}_Q, \exists a \in K\}$ and $\widehat{RN}_{\widehat{\mathfrak{R}}_Q}(b_1) = \widehat{\Lambda}_Q(b_1), \forall b_1 \in L_1$.

Example 3.5 Consider Example 3.2. Let

$$\widehat{\mathfrak{R}}_Q = \left\{\widehat{\Gamma}_Q(p_1) \times \widehat{\Lambda}_Q(p_1), \widehat{\Gamma}_Q(p_1) \times \widehat{\Lambda}_Q(p_2)\right\} \subseteq (\widehat{\Gamma}, P) \times (\widehat{\Lambda}, P).$$

be a Q-NS-relation. Then $\widehat{DM}_{\widehat{\mathfrak{R}}_Q} = \left(\widehat{DM}_{\widehat{\mathfrak{R}}_Q}, K_1\right)$ where $\widehat{DM}_{\widehat{\mathfrak{R}}_Q}(p_1) = \widehat{\Gamma}_Q(p_1)$ for $K_1 = \{p_1\}$ and $\widehat{RN}_{\widehat{\mathfrak{R}}} = \left(\widehat{RN}_{\widehat{\mathfrak{R}}}, L_1\right)$ where $\widehat{RN}_{\widehat{\mathfrak{R}}}(p_1) = \widehat{\Lambda}(p_1)$, $\widehat{RN}_{\widehat{\mathfrak{R}}}(p_2) = \widehat{\Lambda}(p_2)$ for $L_1 = \{p_1, p_2\}$.

Definition 3.6 The identity Q-NS-relation $\widehat{I}_{(\widehat{\Gamma}_Q, P)}$ on a Q-NS-set $(\widehat{\Gamma}_Q, P)$ is defined as $\widehat{\Gamma}_Q(a)\widehat{I}_{(\widehat{\Gamma}_Q, P)}\widehat{\Gamma}_Q(b)$ if and only if $a = b$.

Example 3.7 In Example 3.2, the

$$(\widehat{\mathfrak{R}}_Q, K \times L) = \left\{ \begin{array}{l} \left((p_1,p_1), \begin{array}{c} \langle(u_1,q_1), 0.55, 0.76, 0.85\rangle, \langle(u_1,q_2), 0.47, 0.65, 0.72\rangle, \\ \langle(u_2,q_1), 0.18, 0.56, 0.43\rangle, \langle(u_2,q_2), 0.72, 0.56, 0.67\rangle, \\ \langle(u_3,q_1), 0.45, 0.62, 0.45\rangle, \langle(u_3,q_2), 0.32, 0.74, 0.47\rangle \end{array} \right), \\ \left((p_2,p_2), \begin{array}{c} \langle(u_1,q_1), 0.36, 0.52, 0.44\rangle, \langle(u_1,q_2), 0.48, 0.65, 0.78\rangle, \\ \langle(u_2,q_1), 0.23, 0.68, 0.75\rangle, \langle(u_2,q_2), 0.35, 0.74, 0.85\rangle, \\ \langle(u_3,q_1), 0.25, 0.56, 0.74\rangle, \langle(u_3,q_2), 0.89, 0.64, 0.32\rangle \end{array} \right) \end{array} \right\}$$

is an identity Q-NS-relation.

Definition 3.8 Let $(\widehat{\Gamma}_Q, P), (\widehat{\Lambda}_Q, P) \in 2^{NS_Q(U)}$ and $(\widehat{\mathfrak{R}}_Q, K \times L)$ be a Q-NS-relation from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$. Then the inverse of $(\widehat{\mathfrak{R}}_Q, K \times L)$, $\left(\widehat{\mathfrak{R}}_Q^{-1}, L \times K\right)$ is a Q-NS-relation and is defined as:

$$\widehat{\mathfrak{R}}_Q^{-1}(b,a) = \widehat{\mathfrak{R}}_Q(a,b), \quad \forall (a,b) \in K \times L \subseteq P \times P.$$

Example 3.9 Consider Example 3.5. Then we have,

$$\widehat{\mathfrak{R}}_Q^{-1} = \left\{\widehat{\Lambda}_Q(p_1) \times \widehat{\Gamma}_Q(p_1), \widehat{\Lambda}_Q(p_2) \times \widehat{\Gamma}_Q(p_1)\right\} \subseteq (\widehat{\Lambda}, P) \times (\widehat{\Gamma}, P).$$

Theorem 3.10 *Let* $(\widehat{\Gamma}_Q, P), (\widehat{\Lambda}_Q, P) \in 2^{NS_Q(U)}$ *and* $\left(\widehat{\mathfrak{R}}_Q^1, K \times L\right)$, $\left(\widehat{\mathfrak{R}}_Q^2, K \times L\right)$ *are Q-NS-relations from* $(\widehat{\Gamma}_Q, P)$ *to* $(\widehat{\Lambda}_Q, P)$*. Then,*

$$(i)\left(\left(\widehat{\mathfrak{R}}_Q^1, K\times L\right)^{-1}\right)^{-1} = \left(\widehat{\mathfrak{R}}_Q^1, K\times L\right).$$

$$(ii)\left(\widehat{\mathfrak{R}}_Q^1, K\times L\right)\widehat{\subseteq}\left(\widehat{\mathfrak{R}}_Q^2, K\times L\right) \Rightarrow \left(\widehat{\mathfrak{R}}_Q^1, K\times L\right)^{-1}\widehat{\subseteq}\left(\widehat{\mathfrak{R}}_Q^2, K\times L\right)^{-1}.$$

Proof (i) Since $\left(\left(\widehat{\mathfrak{R}}_Q^1\right)^{-1}\right)^{-1} = \widehat{\mathfrak{R}}_Q^1(a,b)$, then $\left(\left(\widehat{\mathfrak{R}}_Q^1, K\times L\right)^{-1}\right)^{-1} = \left(\widehat{\mathfrak{R}}_Q^1, K\times L\right)$; $\forall(a,b)\in K\times L\subseteq P\times P$.

(ii) $\widehat{\mathfrak{R}}_Q^1(a,b)\subseteq \widehat{\mathfrak{R}}_Q^2(a,b) \Rightarrow \left(\widehat{\mathfrak{R}}_Q^1\right)^{-1}(b,a)\subseteq\left(\widehat{\mathfrak{R}}_Q^2\right)^{-1}(b,a)$ and thus

$$\left(\widehat{\mathfrak{R}}_Q^1, K\times L\right)^{-1}\widehat{\subseteq}\left(\widehat{\mathfrak{R}}_Q^2, K\times L\right)^{-1};\ \forall(a,b)\in K\times L\subseteq P\times P.$$

□

Definition 3.11 Let $(\widehat{\Gamma}_Q, P), (\widehat{\Lambda}_Q, P), (\widehat{\Omega}_Q, P)\in 2^{NS_Q(U)}$ and $(\widehat{\mathfrak{R}}_Q^1, K\times L)$, $(\widehat{\mathfrak{R}}_Q^2, L\times M)$ are Q-NS-relations from $(\widehat{\Gamma}_Q, P)$ to $(\widehat{\Lambda}_Q, P)$ and from $(\widehat{\Lambda}_Q, P)$ to $(\widehat{\Omega}_Q, P)$ respectively, where $(a,b)\in K\times L\subseteq P\times P$ and $(b,c)\in L\times M\subseteq P\times P$, then the composition of the Q-NS-relations $(\widehat{\mathfrak{R}}_Q^1, K\times L)$ and $(\widehat{\mathfrak{R}}_Q^2, L\times M)$ denoted by $\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1$ from $(\widehat{\Gamma}_Q, P)$ to $(\widehat{\Omega}_Q, P)$ is defined as:

$$\left(\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1, K\times M\right) = \left\{\left((a,c), \left(\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1\right)(a,c)\right) : (a,c)\in K\times M\subseteq P\times P\right\} \quad (3.5)$$

such that

$$\left(\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1\right)(a,c) = \left\{\left\langle (u,q), \begin{matrix} T_{\left(\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1\right)(a,c)}(u,q), \\ I_{\left(\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1\right)(a,c)}(u,q), \\ F_{\left(\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1\right)(a,c)}(u,q) \end{matrix} \right\rangle : (u,q)\in U\times Q\right\} \quad (3.6)$$

where

$$\begin{aligned}
T_{\left(\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1\right)(a,c)}(u,q) &= max\left\{T_{\widehat{\mathfrak{R}}_Q^1(a,b)}(u,q), T_{\widehat{\mathfrak{R}}_Q^2(b,c)}(u,q)\right\}\\
&= max\left\{min\left\{T_{\widehat{\Gamma}_Q(a)}(u,q), T_{\widehat{\Lambda}_Q(b)}(u,q)\right\}, min\left\{T_{\widehat{\Lambda}_Q(b)}(u,q), T_{\widehat{\Omega}_Q(c)}(u,q)\right\}\right\},\\
I_{\left(\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1\right)(a,c)}(u,q) &= min\left\{I_{\widehat{\mathfrak{R}}_Q^1(a,b)}(u,q), I_{\widehat{\mathfrak{R}}_Q^2(b,c)}(u,q)\right\}\\
&= min\left\{max\left\{I_{\widehat{\Gamma}_Q(a)}(u,q), I_{\widehat{\Lambda}_Q(b)}(u,q)\right\}, max\left\{I_{\widehat{\Lambda}_Q(b)}(u,q), I_{\widehat{\Omega}_Q(c)}(u,q)\right\}\right\},\\
F_{\left(\widehat{\mathfrak{R}}_Q^2\circ\widehat{\mathfrak{R}}_Q^1\right)(a,c)}(u,q) &= min\left\{F_{\widehat{\mathfrak{R}}_Q^1(a,b)}(u,q), F_{\widehat{\mathfrak{R}}_Q^2(b,c)}(u,q)\right\}\\
&= min\left\{max\left\{F_{\widehat{\Gamma}_Q(a)}(u,q), F_{\widehat{\Lambda}_Q(b)}(u,q)\right\},\right.\\
&\quad\left. max\left\{F_{\widehat{\Lambda}_Q(b)}(u,q), F_{\widehat{\Omega}_Q(c)}(u,q)\right\}\right\}.
\end{aligned}$$

Example 3.12 Let $U=\{u_1,u_2\}$ be a set of televisions, $Q=\{q_1 : philips,\ q_2 : sony\}$ be a set of brands, $P=\{p_1,p_2,p_3\}$ be the set of parameters that want to be examined for different brands of televisions. For $i = 1, 2, 3, 4$, the parameters p_i stand for "fun to watch", "expensive", "modern" and "easy to use", respectively.

Suppose three users' views on the televisions are given in the form of Q-NS-sets $(\widehat{\Gamma}_Q, P)$, $(\widehat{\Lambda}_Q, P)$ and $(\widehat{\Omega}_Q, P)$ as follows:

$$(\widehat{\Gamma}_Q, P) = \left\{ \begin{array}{l} \left(p_1, \begin{array}{l} \langle (u_1, q_1), 0.23, 0.44, 0.36\rangle, \langle (u_1, q_2), 0.44, 0.55, 0.75\rangle, \\ \langle (u_2, q_1), 0.25, 0.47, 0.38\rangle, \langle (u_2, q_2), 0.77, 0.52, 0.84\rangle \end{array}\right), \\ \left(p_2, \begin{array}{l} \langle (u_1, q_1), 0.15, 0.64, 0.53\rangle, \langle (u_1, q_2), 0.48, 0.62, 0.88\rangle, \\ \langle (u_2, q_1), 0.74, 0.34, 0.25\rangle, \langle (u_2, q_2), 0.12, 0.46, 0.77\rangle \end{array}\right), \\ \left(p_3, \begin{array}{l} \langle (u_1, q_1), 0.65, 0.84, 0.14\rangle, \langle (u_1, q_2), 0.35, 0.57, 0.79\rangle, \\ \langle (u_2, q_1), 0.49, 0.57, 0.89\rangle, \langle (u_2, q_2), 0.91, 0.26, 0.72\rangle \end{array}\right), \\ \left(p_4, \begin{array}{l} \langle (u_1, q_1), 0.53, 0.3, 0.25\rangle, \langle (u_1, q_2), 0.77, 0.87, 0.93\rangle, \\ \langle (u_2, q_1), 0.22, 0.32, 0.19\rangle, \langle (u_2, q_2), 0.39, 0.86, 0.24\rangle \end{array}\right) \end{array} \right\},$$

$$(\widehat{\Lambda}_Q, P) = \left\{ \begin{array}{l} \left(p_1, \begin{array}{l} \langle (u_1, q_1), 0.53, 0.31, 0.91\rangle, \langle (u_1, q_2), 0.16, 0.72, 0.83\rangle, \\ \langle (u_2, q_1), 0.73, 0.88, 0.15\rangle, \langle (u_2, q_2), 0.61, 0.24, 0.43\rangle \end{array}\right), \\ \left(p_2, \begin{array}{l} \langle (u_1, q_1), 0.33, 0.54, 0.74\rangle, \langle (u_1, q_2), 0.56, 0.79, 0.98\rangle, \\ \langle (u_2, q_1), 0.42, 0.18, 0.55\rangle, \langle (u_2, q_2), 0.67, 0.44, 0.74\rangle \end{array}\right), \\ \left(p_3, \begin{array}{l} \langle (u_1, q_1), 0.66, 0.24, 0.61\rangle, \langle (u_1, q_2), 0.85, 0.77, 0.34\rangle, \\ \langle (u_2, q_1), 0.22, 0.37, 0.19\rangle, \langle (u_2, q_2), 0.81, 0.66, 0.26\rangle \end{array}\right), \\ \left(p_4, \begin{array}{l} \langle (u_1, q_1), 0.62, 0.45, 0.44\rangle, \langle (u_1, q_2), 0.49, 0.55, 0.22\rangle, \\ \langle (u_2, q_1), 0.33, 0.73, 0.92\rangle, \langle (u_2, q_2), 0.15, 0.66, 0.44\rangle \end{array}\right) \end{array} \right\},$$

$$(\widehat{\Omega}_Q, P) = \left\{ \begin{array}{l} \left(p_1, \begin{array}{l} \langle (u_1, q_1), 0.73, 0.45, 0.66\rangle, \langle (u_1, q_2), 0.77, 0.65, 0.84\rangle, \\ \langle (u_2, q_1), 0.78, 0.55, 0.84\rangle, \langle (u_2, q_2), 0.44, 0.22, 0.76\rangle \end{array}\right), \\ \left(p_2, \begin{array}{l} \langle (u_1, q_1), 0.55, 0.42, 0.45\rangle, \langle (u_1, q_2), 0.88, 0.65, 0.22\rangle, \\ \langle (u_2, q_1), 0.32, 0.66, 0.18\rangle, \langle (u_2, q_2), 0.36, 0.84, 0.11\rangle \end{array}\right), \\ \left(p_3, \begin{array}{l} \langle (u_1, q_1), 0.43, 0.44, 0.23\rangle, \langle (u_1, q_2), 0.33, 0.87, 0.92\rangle, \\ \langle (u_2, q_1), 0.59, 0.33, 0.38\rangle, \langle (u_2, q_2), 0.22, 0.64, 0.83\rangle \end{array}\right), \\ \left(p_4, \begin{array}{l} \langle (u_1, q_1), 0.44, 0.62, 0.43\rangle, \langle (u_1, q_2), 0.11, 0.77, 0.42\rangle, \\ \langle (u_2, q_1), 0.22, 0.36, 0.69\rangle, \langle (u_2, q_2), 0.56, 0.64, 0.22\rangle \end{array}\right) \end{array} \right\}.$$

In this case, the relationships and effects between the desired parameters for televisions of various brands can be expressed. For example; if $(\widehat{\mathfrak{R}}^1_Q, K \times L)$, $(\widehat{\mathfrak{R}}^2_Q, L \times M)$ are Q-NS-relations from $(\widehat{\Gamma}_Q, P)$ to $(\widehat{\Lambda}_Q, P)$ and from $(\widehat{\Lambda}_Q, P)$ to $(\widehat{\Omega}_Q, P)$ respectively, where $K \times L \subseteq P \times P$, $L \times M \subseteq P \times P$ for $K = \{p_1, p_3\}$, $L = \{p_4\}$, $M = \{p_2\}$, then these Q-NS-relations are defined as:

$$(\widehat{\mathfrak{R}}^1_Q, K \times L) = \left\{ \begin{array}{l} \left((p_1, p_4), \begin{array}{l} \langle (u_1, q_1), 0.23, 0.45, 0.44\rangle, \langle (u_1, q_2), 0.44, 0.55, 0.75\rangle, \\ \langle (u_2, q_1), 0.25, 0.73, 0.92\rangle, \langle (u_2, q_2), 0.15, 0.66, 0.84\rangle \end{array}\right), \\ \left((p_3, p_4), \begin{array}{l} \langle (u_1, q_1), 0.62, 0.84, 0.44\rangle, \langle (u_1, q_2), 0.35, 0.57, 0.79\rangle, \\ \langle (u_2, q_1), 0.33, 0.73, 0.92\rangle, \langle (u_2, q_2), 0.15, 0.66, 0.72\rangle \end{array}\right) \end{array} \right\},$$

$$(\widehat{\mathfrak{R}}^2_Q, L \times M) = \left\{ \left((p_4, p_2), \begin{array}{l} \langle (u_1, q_1), 0.55, 0.45, 0.45\rangle, \langle (u_1, q_2), 0.49, 0.65, 0.22\rangle, \\ \langle (u_2, q_1), 0.32, 0.73, 0.92\rangle, \langle (u_2, q_2), 0.15, 0.84, 0.44\rangle \end{array}\right) \right\}.$$

The Q-NS-relation $(\widehat{\mathfrak{R}}^1_Q, K \times L)$ describes the effect of parameters in K on being "easy to use" where it measures the true, indeterminacy and falsity degrees for a "fun to watch" or "modern" television from various brands to be "easy to use". Similarly, the Q-NS-relation $(\widehat{\mathfrak{R}}^2_Q, L \times M)$ expresses the effect of "easy to use" on being "expensive", where it measures the true, indeterminacy and falsity degrees for a “easy to use” television from various brands to be “expensive”.

Now, the combination of these two Q-NS-relations described can help us understand the impact that a “fun to watch” or “modern” television from various brands can have on being “expensive”. Then, the composition between the Q-NS-relations $(\widehat{\mathfrak{R}}^1_Q, K \times L)$ and $(\widehat{\mathfrak{R}}^2_Q, L \times M)$ is

$$(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q, K \times M) = \left\{ \begin{array}{l} \left((p_1, p_2), \begin{array}{l} \langle (u_1, q_1), 0.55, 0.45, 0.44 \rangle, \langle (u_1, q_2), 0.49, 0.55, 0.22 \rangle, \\ \langle (u_2, q_1), 0.32, 0.73, 0.92 \rangle, \langle (u_2, q_2), 0.15, 0.66, 0.44 \rangle \end{array} \right), \\ \left((p_3, p_2), \begin{array}{l} \langle (u_1, q_1), 0.62, 0.45, 0.44 \rangle, \langle (u_1, q_2), 0.49, 0.57, 0.22 \rangle, \\ \langle (u_2, q_1), 0.33, 0.73, 0.92 \rangle, \langle (u_2, q_2), 0.15, 0.66, 0.44 \rangle \end{array} \right) \end{array} \right\}.$$

$T_{\left(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q\right)(a,c)}$, $I_{\left(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q\right)(a,c)}$ and $F_{\left(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q\right)(a,c)}$ represent respectively, the degrees of true, indeterminacy and falsity for a “fun to watch” or “modern” television from various brands to be “expensive”. For example, the term $\langle (u_2, q_1), 0.33, 0.73, 0.92 \rangle$ for (p_3, p_2) means that the “modern” of (u_2, q_1) has a 0.33 truth degree of in being “expensive”, 0.73 indeterminacy degree of in being “expensive”, 0.92 falsity degree of in being “expensive”.

Theorem 3.13 *Let* $(\widehat{\Gamma}_Q, P), (\widehat{\Lambda}_Q, P), (\widehat{\Omega}_Q, P) \in 2^{NS_Q(U)}$ *and* $(\widehat{\mathfrak{R}}^1_Q, K \times L)$, $(\widehat{\mathfrak{R}}^2_Q, L \times M)$ *be Q-NS-relations from* $(\widehat{\Gamma}_Q, P)$ *to* $(\widehat{\Lambda}_Q, P)$ *and from* $(\widehat{\Lambda}_Q, P)$ *to* $(\widehat{\Omega}_Q, P)$ *respectively, where* $K \times L \subseteq P \times P, L \times M \subseteq P \times P$. *Then*

$$\left(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q, K \times M\right)^{-1} = \left(\left(\widehat{\mathfrak{R}}^1_Q\right)^{-1} \circ \left(\widehat{\mathfrak{R}}^2_Q\right)^{-1}, M \times K\right).$$

Proof Since $(\widehat{\mathfrak{R}}^1_Q, K \times L)$, $(\widehat{\mathfrak{R}}^2_Q, L \times M)$ are Q-NS-relations from $(\widehat{\Gamma}_Q, P)$ to $(\widehat{\Lambda}_Q, P)$ and $(\widehat{\Lambda}_Q, P)$ to $(\widehat{\Omega}_Q, P)$ respectively, then $(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q, K \times M) \widehat{\subseteq} (\widehat{\Gamma}_Q, P) \times (\widehat{\Omega}_Q, P)$. Thus, for $(a, c) \in K \times M, (u, q) \in U \times Q$,

$$\begin{aligned} T_{\left(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q\right)^{-1}(c,a)}(u,q) =& T_{\left(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q\right)(a,c)}(u,q) \\ =& max\left\{ T_{\widehat{\mathfrak{R}}^1_Q(a,b)}(u), T_{\widehat{\mathfrak{R}}^2_Q(b,c)}(u) \right\} \\ =& max\left\{ T_{\widehat{\mathfrak{R}}^2_Q(b,c)}(u,q), T_{\widehat{\mathfrak{R}}^1_Q(a,b)}(u,q) \right\} \\ =& max\left\{ min\left\{ T_{\widehat{\Lambda}_Q(b)}(u,q), T_{\widehat{\Omega}_Q(c)}(u,q) \right\}, min\left\{ T_{\widehat{\Gamma}_Q(a)}(u,q), T_{\widehat{\Lambda}_Q(b)}(u,q) \right\} \right\} \\ =& max\left\{ min\left\{ T_{\widehat{\Omega}_Q(c)}(u,q), T_{\widehat{\Lambda}_Q(b)}(u,q) \right\}, min\left\{ T_{\widehat{\Lambda}_Q(b)}(u,q), T_{\widehat{\Gamma}_Q(a)}(u,q) \right\} \right\} \\ =& max\left\{ T_{\left(\widehat{\mathfrak{R}}^2_Q\right)^{-1}(c,b)}(u,q), T_{\left(\widehat{\mathfrak{R}}^1_Q\right)^{-1}(b,a)}(u,q) \right\} \\ =& T_{\left(\left(\widehat{\mathfrak{R}}^1_Q\right)^{-1} \circ \left(\widehat{\mathfrak{R}}^2_Q\right)^{-1}\right)(c,a)}(u,q) \end{aligned}$$

Similar results can be shown for $I_{\left(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q\right)^{-1}(c,a)}(u,q)$ and $F_{\left(\widehat{\mathfrak{R}}^2_Q \circ \widehat{\mathfrak{R}}^1_Q\right)^{-1}(c,a)}(u,q)$. This completes the proof. □

4 Partitions on Q-NS-sets

This section has been created to bring the various types of Q-NS-relations defined in the previous section to the literature. Moreover, the equivalence classes and partitions of Q-NS-sets are analyzed by giving them together with relevant important theorems.

Definition 4.1 Let $\widehat{\mathfrak{R}}_Q$ be a Q-NS-relation on $(\widehat{\Gamma}_Q, P)$, then

(*i*) $(\widehat{\mathfrak{R}}_Q, K \times L)$ is reflexive Q-NS-relation (briefly R-Q-NSR) if $\widehat{\mathfrak{R}}_Q(a,a) \in (\widehat{\mathfrak{R}}_Q, K \times L)$, $\forall a \in P, (a,a) \in K \times L \in P \times P$.

(*ii*) $(\widehat{\mathfrak{R}}_Q, K \times L)$ is symmetric Q-NS-relation (briefly S-Q-NSR) if $\widehat{\mathfrak{R}}_Q(a,b) \in (\widehat{\mathfrak{R}}_Q, K \times L) \Longrightarrow \widehat{\mathfrak{R}}_Q(b,a) \in (\widehat{\mathfrak{R}}_Q, K \times L), \forall a,b \in P\ (a,b) \in K \times L \subseteq P \times P$.

(*iii*) $(\widehat{\mathfrak{R}}_Q, K \times L)$ is transitive Q-NS-relation (briefly T-Q-NSR) if $\widehat{\mathfrak{R}}_Q(a,b) \in (\widehat{\mathfrak{R}}_Q, K \times L)$ and $\widehat{\mathfrak{R}}_Q(b,c) \in (\widehat{\mathfrak{R}}_Q, K \times L) \Longrightarrow \widehat{\mathfrak{R}}_Q(a,c) \in (\widehat{\mathfrak{R}}_Q, K \times L)$, $\forall a,b,c \in P\ (a,b),(a,c),(b,c) \in K \times L \subseteq P \times P$.

(*iv*) $(\widehat{\mathfrak{R}}_Q, K \times L)$ is a Q-NS-equivalence relation (briefly Q-NSER) if it is R-Q-NSR, S-Q-NSR and T-Q-NSR.

Example 4.2 Suppose a Q-NS-relation $(\widehat{\mathfrak{R}}_Q, K \times L)$ for $(\widehat{\Gamma}_Q, P)$ given in Example 2.7 is defined by:

$$(\widehat{\mathfrak{R}}_Q, K \times L) = \left\{ \widehat{\Gamma}_Q(p_1) \times \widehat{\Gamma}_Q(p_1), \widehat{\Gamma}_Q(p_1) \times \widehat{\Gamma}_Q(p_2), \widehat{\Gamma}_Q(p_2) \times \widehat{\Gamma}_Q(p_1), \widehat{\Gamma}_Q(p_2) \times \widehat{\Gamma}_Q(p_2) \right\}.$$

This Q-NS-relation is a Q-NSER.

Definition 4.3 Let $(\widehat{\Gamma}_Q, P) \in 2^{NS_Q(U)}$. Then the Q-NS-equivalence class of $\widehat{\Gamma}_Q(a)$ is defined as

$$[\widehat{\Gamma}_Q(a)] = \left\{ \widehat{\Gamma}_Q(b) : \widehat{\Gamma}_Q(b) \widehat{\mathfrak{R}}_Q \widehat{\Gamma}_Q(a), \forall a,b \in P \right\}. \tag{4.1}$$

Example 4.4 Consider Example 4.2. Then have $[\widehat{\Gamma}_Q(p_1)] = \{\widehat{\Gamma}_Q(p_1), \widehat{\Gamma}_Q(p_2)\} = [\widehat{\Gamma}_Q(p_2)]$.

Lemma 4.5 *Let* $(\widehat{\Gamma}_Q, P) \in 2^{NS_Q(U)}$ *and* $(\widehat{\mathfrak{R}}_Q, K \times L)$ *be a Q-NSER on* $(\widehat{\Gamma}_Q, P)$. *Then,*

$$[\widehat{\Gamma}_Q(a)] = [\widehat{\Gamma}_Q(b)] \Leftrightarrow \widehat{\Gamma}_Q(a) \widehat{\mathfrak{R}}_Q \widehat{\Gamma}_Q(b);\ \exists \widehat{\Gamma}_Q(a), \widehat{\Gamma}_Q(b) \in (\widehat{\Gamma}_Q, P).$$

Proof $(\Rightarrow)$ If $\widehat{\Gamma}_Q(a_1) \in [\widehat{\Gamma}_Q(a)]$, then $\widehat{\Gamma}_Q(a_1) \widehat{\mathfrak{R}}_Q \widehat{\Gamma}_Q(a)$. Moreover, since $(\widehat{\mathfrak{R}}_Q, K \times L)$ is a Q-NSER (i.e., using the transitive property), then $\widehat{\Gamma}_Q(a_1) \in [\widehat{\Gamma}_Q(b)]$. Thus $[\widehat{\Gamma}_Q(a)] \subseteq [\widehat{\Gamma}_Q(b)]$. Similarly $[\widehat{\Gamma}_Q(b)] \subseteq [\widehat{\Gamma}_Q(a)]$. Hence, $[\widehat{\Gamma}_Q(a)] = [\widehat{\Gamma}_Q(b)]$.

$(\Leftarrow)$ Since $(\widehat{\mathfrak{R}}_Q, K \times L)$ is a Q-NSER (i.e., using the reflexive property), then $\widehat{\Gamma}_Q(b) \widehat{\mathfrak{R}}_Q \widehat{\Gamma}_Q(b)$. Hence $\widehat{\Gamma}_Q(b) \in [\widehat{\Gamma}_Q(b)] = [\widehat{\Gamma}_Q(a)]$ which gives $\widehat{\Gamma}_Q(a) \widehat{\mathfrak{R}}_Q \widehat{\Gamma}_Q(b)$. □

Definition 4.6 Let $(\widehat{\Gamma}_Q, P) \in 2^{NS_Q(U)}$. Then, a collection of nonempty Q-NS-subsets $\rho = \left\{ \left(\left[\widehat{\Gamma}_Q \right]_i, P_i \right) : i \in I \right\}$ of $(\widehat{\Gamma}_Q, P)$ is called a Q-NS-set partition (briefly Q-NS-PRTN) of $(\widehat{\Gamma}_Q, P)$ such that $(\widehat{\Gamma}_Q, P) = \widehat{\bigcup}_i \left(\left[\widehat{\Gamma}_Q \right]_i, P_i \right)$ and $P_i \cap P_j = \emptyset$, whenever $i \neq j$.

Example 4.7 Let $U = \{u_1, u_2, u_3\}$, $Q = \{q_1, q_2\}$, $P = \{p_1, p_2, p_3\}$ and $(\widehat{\Gamma}_Q, P)$ is a Q-NS-set over U defined as:

$$(\widehat{\Gamma}_Q, P) = \left\{ \begin{array}{l} \widehat{\Gamma}_Q(p_1) = \left\{ \begin{array}{l} \langle (u_1, q_1), 0.92, 0.23, 0.33 \rangle, \langle (u_1, q_2), 0.54, 0.74, 0.22 \rangle, \\ \langle (u_2, q_1), 0.18, 0.38, 0.26 \rangle, \langle (u_2, q_2), 0.66, 0.45, 0.34 \rangle, \\ \langle (u_3, q_1), 0.22, 0.87, 0.11 \rangle, \langle (u_3, q_2), 0.45, 0.85, 0.55 \rangle \end{array} \right\}, \\ \\ \widehat{\Gamma}_Q(p_2) = \left\{ \begin{array}{l} \langle (u_1, q_1), 0.35, 0.68, 0.49 \rangle, \langle (u_1, q_2), 0.77, 0.95, 0.22 \rangle, \\ \langle (u_2, q_1), 0.51, 0.27, 0.19 \rangle, \langle (u_2, q_2), 0.76, 0.48, 0.59 \rangle, \\ \langle (u_3, q_1), 0.22, 0.59, 0.42 \rangle, \langle (u_3, q_2), 0.29, 0.64, 0.27 \rangle \end{array} \right\}, \\ \\ \widehat{\Gamma}_Q(p_3) = \left\{ \begin{array}{l} \langle (u_1, q_1), 0.16, 0.65, 0.84 \rangle, \langle (u_1, q_2), 0.88, 0.54, 0.26 \rangle, \\ \langle (u_2, q_1), 0.33, 0.56, 0.52 \rangle, \langle (u_2, q_2), 0.15, 0.58, 0.44 \rangle, \\ \langle (u_3, q_1), 0.23, 0.86, 0.17 \rangle, \langle (u_3, q_2), 0.39, 0.65, 0.52 \rangle \end{array} \right\} \end{array} \right\}$$

Suppose $P_1 = \{p_1, p_3\}$, $P_2 = \{p_2\}$, where $\left(\left[\widehat{\Gamma}_Q\right]_1, P_1\right) = \left\{\left[\widehat{\Gamma}_Q\right]_1(p_1), \left[\widehat{\Gamma}_Q\right]_1(p_3)\right\}$ and $\left(\left[\widehat{\Gamma}_Q\right]_2, P_2\right) = \left\{\left[\widehat{\Gamma}_Q\right]_2(p_2)\right\}$ are Q-NS-subsets of $(\widehat{\Gamma}_Q, P)$ such that $\left[\widehat{\Gamma}_Q\right]_i = \widehat{\Gamma}_Q$ for $i = 1, 2$. Since $P_1 \cap P_2 = \emptyset$ and $\left(\left[\widehat{\Gamma}_Q\right]_1, P_1\right) \widehat{\cup} \left(\left[\widehat{\Gamma}_Q\right]_2, P_2\right) = (\widehat{\Gamma}_Q, P)$, then

$$\left\{ \left(\left[\widehat{\Gamma}_Q\right]_1, P_1\right), \left(\left[\widehat{\Gamma}_Q\right]_2, P_2\right) \right\}$$

is a Q-NS-PRTN of $(\widehat{\Gamma}_Q, P)$.

Definition 4.8 Let $(\widehat{\Gamma}_Q, P) \in 2^{NS_Q(U)}$. Then, the elements of Q-NS-PRTN are called a Q-NS-set-block of $(\widehat{\Gamma}_Q, P)$.

Theorem 4.9 *Let $(\widehat{\Gamma}_Q, P) \in 2^{NS_Q(U)}$ and $\left\{ \left(\left[\widehat{\Gamma}_Q\right]_i, P_i\right) : i \in I \right\}$ be a Q-NS-PRTN of $(\widehat{\Gamma}_Q, P)$. The Q-NS-relation defined on $(\widehat{\Gamma}_Q, P)$ as $\widehat{\Gamma}_Q(a) \widehat{\mathfrak{R}}_Q \widehat{\Gamma}_Q(b)$ is a Q-NSER if and only if $\widehat{\Gamma}_Q(a)$ and $\widehat{\Gamma}_Q(b)$ are elements of the same block.*

Proof (i) It is clear that any element of $(\widehat{\Gamma}_Q, P)$ is in the same block itself. Thus $\widehat{\mathfrak{R}}_Q(a, a) \in (\widehat{\mathfrak{R}}_Q, K \times L)$ and $(\widehat{\mathfrak{R}}_Q, K \times L)$ is R-Q-NSR.

(ii) If $\widehat{\mathfrak{R}}_Q(a, b) \in (\widehat{\mathfrak{R}}_Q, K \times L)$,then $\widehat{\Gamma}_Q(a)$ and $\widehat{\Gamma}_Q(b)$ are in the same block, i.e., $\widehat{\mathfrak{R}}_Q(b, a) \in (\widehat{\mathfrak{R}}_Q, K \times L)$ and so it is S-Q-NSR.

(iii) If $\widehat{\mathfrak{R}}_Q(a, b) \in (\widehat{\mathfrak{R}}_Q, K \times L)$ and $\widehat{\mathfrak{R}}_Q(b, c) \in (\widehat{\mathfrak{R}}_Q, K \times L)$, then $\widehat{\Gamma}_Q(a)$, $\widehat{\Gamma}_Q(b)$ and $\widehat{\Gamma}_Q(c)$ are in the same block, i.e., $\widehat{\mathfrak{R}}_Q(a, c) \in (\widehat{\mathfrak{R}}_Q, K \times L)$ and so it is T-Q-NSR. □

Remark 4.10 The Q-NS-relation given in Theorem 4.9 is called a Q-NSER determined by the Q-NS-PRTN ρ. For example, the Q-NSER determined by $\rho = \left\{ \left(\left[\widehat{\Gamma}_Q\right]_1, P_1\right), \left(\left[\widehat{\Gamma}_Q\right]_2, P_2\right) \right\}$ in Example 4.7 is as follows:

$$\widehat{\mathfrak{R}} = \left\{ \begin{array}{c} \widehat{\Gamma}_Q(p_1) \times \widehat{\Gamma}_Q(p_1), \widehat{\Gamma}_Q(p_2) \times \widehat{\Gamma}_Q(p_2), \widehat{\Gamma}_Q(p_3) \times \widehat{\Gamma}_Q(p_3), \\ \widehat{\Gamma}_Q(p_1) \times \widehat{\Gamma}_Q(p_3), \widehat{\Gamma}_Q(p_3) \times \widehat{\Gamma}_Q(p_1) \end{array} \right\}$$

Theorem 4.11 *Let $(\widehat{\Gamma}_Q, P) \in 2^{NS_Q(U)}$. Corresponding to every Q-NSER defined on $(\widehat{\Gamma}_Q, P)$ there exists a Q-NS-PRTN on $(\widehat{\Gamma}_Q, P)$ and this Q-NS-PRTN precisely consists of the Q-NS-equivalence classes of $(\widehat{\mathfrak{R}}_Q, K \times L)$.*

Proof Let $[\widehat{\Gamma}_Q(a)]$ be Q-NS-equivalence class with respect to a Q-NS-relation $(\widehat{\mathfrak{R}}_Q, K \times L)$ on $(\widehat{\Gamma}_Q, P)$. Here, we can denote $[\widehat{\Gamma}_Q(a)]$ as $(\widehat{\Gamma}_Q, P_a)$ for $P_a = \left\{ b \in P : \widehat{\mathfrak{R}}_Q(b, a) \in (\widehat{\mathfrak{R}}_Q, K \times L) \right\}$. Thus, we have to show that the collection $\left\{ (\widehat{\Gamma}_Q, P_a) : a \in P \right\}$ is a Q-NS-PRTN ρ of $(\widehat{\Gamma}_Q, P)$. For this we must show from Definition 4.6 that the following two conditions are satisfied:

(i) $(\widehat{\Gamma}_Q, P) = \widehat{\cup}_{a \in P}(\widehat{\Gamma}_Q, P_a)$,

(ii) $P_a \cap P_b = \emptyset$ for $a \neq b$.

Since $\widehat{\mathfrak{R}}_Q$ is a Q-NSER (i.e., using the reflexise property), $\widehat{\mathfrak{R}}_Q(a, a) \in (\widehat{\mathfrak{R}}_Q, K \times L)$, $\forall a \in P$, i.e., (i) is implemented.

Let $p \in P_a \cap P_b$ for (ii). Since $\widehat{\Gamma}_Q(p) \in (\widehat{\Gamma}_Q, P_a)$ and $\widehat{\Gamma}_Q(p) \in (\widehat{\Gamma}_Q, P_b)$, then $\widehat{\mathfrak{R}}_Q(p, a) \in (\widehat{\mathfrak{R}}_Q, K \times L)$ and $\widehat{\mathfrak{R}}_Q(p, b) \in (\widehat{\mathfrak{R}}_Q, K \times L)$. Then, using the transitive property of $\widehat{\mathfrak{R}}_Q$, we have $\widehat{\mathfrak{R}}_Q(a, b) \in (\widehat{\mathfrak{R}}_Q, K \times L)$. Moreover, using Lemma 4.5 we have $[\widehat{\Gamma}_Q(a)] = [\widehat{\Gamma}_Q(b)]$. However, since $P_a = P_b$ is obtained from here, then this is a contradiction, i.e. $P_a \neq P_b$, hence $P_a \cap P_b = \emptyset$. □

Remark 4.12 The Q-NS-PRTN in Theorem 4.11 where we construct all the Q-NS-equivalence classes of $(\widehat{\mathfrak{R}}_Q, K \times L)$ is called the quotient Q-NS-set of $(\widehat{\Gamma}, P)$ and is denoted by $(\widehat{\Gamma}_Q, P) \setminus (\widehat{\mathfrak{R}}_Q, K \times L)$.

5 Functions on Q-NS-sets

In this section, the concept of Q-NS-function is defined by using the Q-NS-relations given in this paper. In addition, some special types of this concept are analyzed together with the related theorems.

Definition 5.1 Let $(\widehat{\Gamma}_Q, P)$ and $(\widehat{\Lambda}_Q, P)$ be two nonempty Q-NS-set. Then a Q-NS-relation $\mathfrak{f}_Q$ from $(\widehat{\Gamma}_Q, P)$ and $(\widehat{\Lambda}_Q, P)$ is called a Q-NS-function if every element in the domain has a unique element in the range. We write $\mathfrak{f}_Q : (\widehat{\Gamma}_Q, P) \to (\widehat{\Lambda}_Q, P)$. If $\widehat{\Gamma}_Q(a)\mathfrak{f}_Q\widehat{\Lambda}_Q(b)$ then $\mathfrak{f}_Q(\widehat{\Gamma}_Q(a)) = \widehat{\Lambda}_Q(b)$; $\forall a, b \in P$.

Example 5.2 Reconsider Example 3.2. Then, the Q-NS-relation $\mathfrak{f}_Q$ forms a Q-NS-function from $(\widehat{\Gamma}_Q, P)$ to $(\widehat{\Lambda}_Q, P)$ as follows:

$$\mathfrak{f}_Q = \left\{ \begin{array}{l} \left((p_1, p_2), \begin{array}{c} \langle (u_1, q_1), 0.36, 0.76, 0.44 \rangle, \langle (u_1, q_2), 0.47, 0.54, 0.78 \rangle, \\ \langle (u_2, q_1), 0.28, 0.68, 0.55 \rangle, \langle (u_2, q_2), 0.66, 0.48, 0.67 \rangle, \\ \langle (u_3, q_1), 0.25, 0.62, 0.74 \rangle, \langle (u_3, q_2), 0.54, 0.74, 0.47 \rangle \end{array} \right), \\ \left((p_2, p_2), \begin{array}{c} \langle (u_1, q_1), 0.36, 0.52, 0.44 \rangle, \langle (u_1, q_2), 0.48, 0.65, 0.78 \rangle, \\ \langle (u_2, q_1), 0.23, 0.68, 0.75 \rangle, \langle (u_2, q_2), 0.35, 0.74, 0.85 \rangle, \\ \langle (u_3, q_1), 0.25, 0.56, 0.74 \rangle, \langle (u_3, q_2), 0.89, 0.64, 0.32 \rangle \end{array} \right) \end{array} \right\}.$$

Definition 5.3 Let $\mathfrak{f}_Q : (\widehat{\Gamma}_Q, P) \to (\widehat{\Lambda}_Q, P)$ be a Q-NS-function. Then,

(i) $\mathfrak{f}_Q$ is injective Q-NS-function (briefly I-Q-NSF) if $\widehat{\Gamma}_Q(a_1) \neq \widehat{\Gamma}_Q(a_2)$ implying $\mathfrak{f}_Q(\widehat{\Gamma}_Q(a_1)) \neq \mathfrak{f}_Q(\widehat{\Gamma}_Q(a_2))$; $\widehat{\Gamma}_Q(a_1), \widehat{\Gamma}_Q(a_2) \in (\widehat{\Gamma}_Q, P)$.

(ii) $\mathfrak{f}_Q$ is surjective Q-NS-function (briefly S-Q-NSF) if $\mathfrak{f}((\widehat{\Gamma}_Q, P)) = (\widehat{\Lambda}_Q, P)$.

(iii) $\mathfrak{f}_Q$ is bijective Q-NS-fuction (briefly B-Q-NSF) if it is both I-Q-NSF and S-Q-NSF.

(iv) $\mathfrak{f}_Q$ is a constant Q-NS-fuction (briefly C-Q-NSF) if all elements in $\widehat{DM}_{\mathfrak{f}_Q}$ have the same image.

(v) $\mathfrak{f}_Q$ is an identity Q-NS-function (briefly ID-Q-NSF) if the Q-NS-function $\widehat{I}_{\mathfrak{f}_Q}$ on a Q-NS-set $(\widehat{\Gamma}_Q, P)$ is defined by $\widehat{I}_{\mathfrak{f}_Q} : (\widehat{\Gamma}_Q, P) \to (\widehat{\Gamma}_Q, P)$ as $\widehat{I}_{\mathfrak{f}_Q}(\widehat{\Gamma}_Q(a)) = \widehat{\Gamma}_Q(a)$; $\forall \widehat{\Gamma}_Q(a) \in (\widehat{\Gamma}_Q, P)$.

Definition 5.4 Let $\mathfrak{f}_Q : (\widehat{\Gamma}_Q, P) \to (\widehat{\Lambda}_Q, P)$, $\mathfrak{g}_Q : (\widehat{\Lambda}_Q, P) \to (\widehat{\Omega}_Q, P)$ be two Q-NS-functions. Then $\mathfrak{g}_Q \circ \mathfrak{f}_Q : (\widehat{\Gamma}_Q, P) \to (\widehat{\Lambda}_Q, P)$ is also a Q-NS-function defined by $(\mathfrak{g}_Q \circ \mathfrak{f}_Q)(\widehat{\Gamma}_Q(a)) = \mathfrak{g}_Q(\mathfrak{f}_Q(\widehat{\Gamma}_Q(a)))$.

Definition 5.5 Let $\mathfrak{f}_Q : (\widehat{\Gamma}_Q, P) \to (\widehat{\Lambda}_Q, P)$ be a B-Q-NSF. Then the inverse Q-NS-relation $\mathfrak{f}_Q^{-1} : (\widehat{\Lambda}_Q, P) \to (\widehat{\Gamma}_Q, P)$ is called the inverse Q-NS-function.

Theorem 5.6 *If* $\mathfrak{f}_Q : (\widehat{\Gamma}_Q, P) \to (\widehat{\Lambda}_Q, P)$ *is B-Q-NSF then* $\mathfrak{f}_Q^{-1} : (\widehat{\Lambda}_Q, P) \to (\widehat{\Gamma}_Q, P)$ *is also a B-Q-NSF.*

Proof Let $\widehat{\Lambda}_Q(b_1) \neq \widehat{\Lambda}_Q(b_2)$; $\widehat{\Lambda}_Q(b_1), \widehat{\Lambda}_Q(b_2) \in (\widehat{\Lambda}_Q, P)$. So, this we should prove $\mathfrak{f}_Q^{-1}(\widehat{\Lambda}_Q(b_1)) \neq \mathfrak{f}_Q^{-1}\widehat{\Lambda}_Q(b_2)$. Let $\mathfrak{f}_Q^{-1}(\widehat{\Lambda}_Q(b_1)) = \widehat{\Gamma}_Q(a_1)$, $\mathfrak{f}_Q^{-1}(\widehat{\Lambda}_Q(b_2)) = \widehat{\Gamma}_Q(a_2)$: i.e., $\widehat{\Lambda}_Q(b_1) = \mathfrak{f}_Q(\widehat{\Gamma}_Q(a_1))$, $\widehat{\Lambda}_Q(b_2) = \mathfrak{f}_Q(\widehat{\Gamma}_Q(a_2))$. Since $\mathfrak{f}_Q$ is B-Q-NSF, $\mathfrak{f}_Q(\widehat{\Gamma}_Q(a_1)) \neq \mathfrak{f}_Q(\widehat{\Gamma}_Q(a_2))$ implies $\widehat{\Gamma}_Q(a_1) \neq \widehat{\Gamma}_Q(a_2)$. Thus $\mathfrak{f}_Q^{-1}$ is B-Q-NSF.

Since $\mathfrak{f}_Q$ is S-Q-NSF, then $\exists \widehat{\Lambda}_Q(b) \in (\widehat{\Lambda}_Q, P)$ such that $\mathfrak{f}_Q(\widehat{\Gamma}_Q(a)) = \widehat{\Lambda}_Q(b)$ implies $\widehat{\Gamma}_Q(a) = \mathfrak{f}_Q^{-1}(\widehat{\Lambda}_Q(b))$; $\exists \widehat{\Gamma}_Q(a) \in (\widehat{\Gamma}_Q, P)$. Thus $\mathfrak{f}_Q^{-1}$ is S-Q-NSF. Hence, $\mathfrak{f}_Q^{-1}$ is B-Q-NSF. □

Theorem 5.7 *Let* $\mathfrak{f}_Q : (\widehat{\Gamma}_Q, P) \to (\widehat{\Lambda}_Q, P)$, $\mathfrak{g}_Q : (\widehat{\Lambda}_Q, P) \to (\widehat{\Omega}_Q, P)$ *be two B-Q-NSFs. Then,*

(i) $\mathfrak{g}_Q \circ \mathfrak{f}_Q : (\widehat{\Gamma}_Q, P) \to (\widehat{\Lambda}_Q, P)$ *is a B-Q-NSF.*

(ii) $\left(\mathfrak{g}_Q \circ \mathfrak{f}_Q\right)^{-1} = \mathfrak{f}_Q^{-1} \circ \mathfrak{g}_Q^{-1}$.

Proof (i) Let $\widehat{\Gamma}_Q(a_1) \neq \widehat{\Gamma}_Q(a_2)$; $\widehat{\Gamma}_Q(a_1), \widehat{\Gamma}_Q(a_2) \in (\widehat{\Gamma}_Q, P)$. So, this we should prove $(\mathfrak{g}_Q \circ \mathfrak{f}_Q)(\widehat{\Gamma}_Q(a_1)) \neq (\mathfrak{g}_Q \circ \mathfrak{f}_Q)(\widehat{\Gamma}_Q(a_2))$. Since $\mathfrak{f}_Q$, $\mathfrak{g}_Q$ are B-Q-NSFs; then $\mathfrak{f}_Q(\widehat{\Gamma}_Q(a_1)) \neq \mathfrak{f}_Q(\widehat{\Gamma}_Q(a_2))$, $\mathfrak{g}_Q\left(\mathfrak{f}_Q(\widehat{\Gamma}_Q(a_1))\right) \neq \mathfrak{g}_Q\left(\mathfrak{f}_Q(\widehat{\Gamma}_Q(a_2))\right)$. Thus $(\mathfrak{g}_Q \circ \mathfrak{f}_Q$ is B-Q-NSF. Since $\mathfrak{g}_Q$ is S-Q-NSF, then $\exists \widehat{\Lambda}_Q(b) \in (\widehat{\Lambda}_Q, P)$ such that $\mathfrak{g}_Q(\widehat{\Lambda}_Q(b)) = \widehat{\Omega}_Q(c)$; $\exists \widehat{\Omega}_Q(c) \in (\widehat{\Omega}_Q, P)$. Also; since $\mathfrak{f}_Q$ is S-Q-NSF, then $\mathfrak{f}_Q(\widehat{\Gamma}_Q(a)) = \widehat{\Lambda}_Q(b)$; $\exists \widehat{\Gamma}_Q(a) \in (\widehat{\Gamma}_Q, P)$. Moreover; since $(\mathfrak{g}_Q \circ \mathfrak{f}_Q)(\widehat{\Gamma}_Q(a)) = \widehat{\Omega}_Q(c)$, $\forall \widehat{\Omega}_Q(c) \in (\widehat{\Omega}_Q, P)$; then $\mathfrak{g}_Q \circ \mathfrak{f}_Q$ is S-Q-NSF. Thus, $\mathfrak{g}_Q \circ \mathfrak{f}_Q$ is B-Q-NSF.

(ii) We have from Theorem 3.13 that $\left(\left[\widehat{\mathfrak{R}}_Q\right]_2 \circ \left[\widehat{\mathfrak{R}}_Q\right]_1\right)^{-1} = \left[\widehat{\mathfrak{R}}_Q\right]_1^{-1} \circ \left[\widehat{\mathfrak{R}}_Q\right]_2^{-1}$. Hence, since $\mathfrak{f}_Q$, $\mathfrak{g}_Q$, $\mathfrak{g}_Q \circ \mathfrak{f}_Q$ are B-Q-NSFs, then $(\mathfrak{g}_Q \circ \mathfrak{f}_Q)^{-1} = \mathfrak{f}_Q^{-1} \circ \mathfrak{g}_Q^{-1}$. □

6 Decision making in an uncertainty problem

In this section, we propose an application with the help of Q-NS-relations in order to determine the decision-making process of an uncertainty problem in the best way.

Suppose the current encountered uncertainty problem is given as follows:

Suppose a private school wants to choose the student that best suits the desired parameters for a science competition. For this, let $U = \{u_1, u_2, u_3, u_4\}$ be the set of four students who make it to the final, $Q = \{q_1, q_2\}$ be the set of nationality of students and $P = \{p_1, p_2, p_3\}$ be the set of parameters required by the school administration. For $i = 1, 2, 3$, the parameters p_i stand for “successful”, “determined” and “hardworking”, respectively.

Let's assume that two juries selected by the school administration express their evaluations for students with the help of the Q-NS-sets $(\widehat{\Gamma}_Q, P)$ and $(\widehat{\Lambda}_Q, P)$ as follows:

$$(\widehat{\Gamma}_Q, P) = \left\{ \begin{array}{l} \left(p_1, \begin{array}{l} \langle (u_1,q_1), 0.23, 0.56, 0.15 \rangle, \langle (u_1,q_2), 0.56, 0.67, 0.33 \rangle, \\ \langle (u_2,q_1), 0.33, 0.94, 0.50 \rangle, \langle (u_2,q_2), 0.24, 0.33, 0.12 \rangle, \\ \langle (u_3,q_1), 0.43, 0.55, 0.81 \rangle, \langle (u_3,q_2), 0.45, 0.84, 0.58 \rangle, \\ \langle (u_4,q_1), 0.42, 0.58, 0.75 \rangle, \langle (u_4,q_2), 0.38, 0.61, 0.29 \rangle \end{array} \right), \\ \left(p_2, \begin{array}{l} \langle (u_1,q_1), 0.43, 0.56, 0.27 \rangle, \langle (u_1,q_2), 0.47, 0.55, 0.33 \rangle, \\ \langle (u_2,q_1), 0.32, 0.66, 0.43 \rangle, \langle (u_2,q_2), 0.55, 0.74, 0.27 \rangle, \\ \langle (u_3,q_1), 0.22, 0.87, 0.43 \rangle, \langle (u_3,q_2), 0.42, 0.76, 0.45 \rangle, \\ \langle (u_4,q_1), 0.87, 0.27, 0.41 \rangle, \langle (u_4,q_2), 0.24, 0.17, 0.56 \rangle \end{array} \right), \\ \left(p_3, \begin{array}{l} \langle (u_1,q_1), 0.55, 0.44, 0.11 \rangle, \langle (u_1,q_2), 0.24, 0.67, 0.31 \rangle, \\ \langle (u_2,q_1), 0.22, 0.37, 0.65 \rangle, \langle (u_2,q_2), 0.11, 0.58, 0.63 \rangle, \\ \langle (u_3,q_1), 0.27, 0.38, 0.19 \rangle, \langle (u_3,q_2), 0.41, 0.52, 0.48 \rangle, \\ \langle (u_4,q_1), 0.66, 0.58, 0.66 \rangle, \langle (u_4,q_2), 0.69, 0.42, 0.86 \rangle \end{array} \right) \end{array} \right\}$$

and

$$(\widehat{\Lambda}_Q, P) = \left\{ \begin{array}{l} \left(p_1, \begin{array}{l} \langle (u_1,q_1), 0.28, 0.43, 0.19 \rangle, \langle (u_1,q_2), 0.84, 0.36, 0.28 \rangle, \\ \langle (u_2,q_1), 0.92, 0.44, 0.41 \rangle, \langle (u_2,q_2), 0.22, 0.61, 0.11 \rangle, \\ \langle (u_3,q_1), 0.38, 0.54, 0.85 \rangle, \langle (u_3,q_2), 0.84, 0.36, 0.54 \rangle, \\ \langle (u_4,q_1), 0.44, 0.58, 0.55 \rangle, \langle (u_4,q_2), 0.94, 0.44, 0.26 \rangle \end{array} \right), \\ \left(p_2, \begin{array}{l} \langle (u_1,q_1), 0.66, 0.52, 0.63 \rangle, \langle (u_1,q_2), 0.44, 0.56, 0.87 \rangle, \\ \langle (u_2,q_1), 0.76, 0.44, 0.37 \rangle, \langle (u_2,q_2), 0.57, 0.38, 0.74 \rangle, \\ \langle (u_3,q_1), 0.11, 0.65, 0.38 \rangle, \langle (u_3,q_2), 0.23, 0.76, 0.55 \rangle, \\ \langle (u_4,q_1), 0.64, 0.37, 0.64 \rangle, \langle (u_4,q_2), 0.25, 0.87, 0.56 \rangle \end{array} \right), \\ \left(p_3, \begin{array}{l} \langle (u_1,q_1), 0.55, 0.27, 0.18 \rangle, \langle (u_1,q_2), 0.44, 0.39, 0.44 \rangle, \\ \langle (u_2,q_1), 0.23, 0.92, 0.26 \rangle, \langle (u_2,q_2), 0.33, 0.25, 0.47 \rangle, \\ \langle (u_3,q_1), 0.17, 0.72, 0.42 \rangle, \langle (u_3,q_2), 0.41, 0.11, 0.42 \rangle, \\ \langle (u_4,q_1), 0.66, 0.47, 0.33 \rangle, \langle (u_4,q_2), 0.47, 0.59, 0.32 \rangle \end{array} \right) \end{array} \right\}.$$

Considering the evaluations made by the jury members, we propose the algorithm given below in order to determine the student with the best relationship between the parameters:

Step 1: Input the Q-NS-sets $(\widehat{\Gamma}_Q, P)$, $(\widehat{\Lambda}_Q, P) \in 2^{NS_Q(U)}$.

Step 2: Construct a Q-NS-relation $(\widehat{\mathfrak{R}}_Q, K \times L)$ from $(\widehat{\Gamma}_Q, P)$ to $(\widehat{\Lambda}_Q, P)$ as requested for $K \times L \subseteq P \times P$.

Step 3: Calculate the score values of the relation between the parameters in the determined relation $\widehat{\mathfrak{R}}_Q$ using the formula:

$$\Theta^{k,l}_{\widehat{\mathfrak{R}}_Q}(u_i, q_j) = T_{\widehat{\mathfrak{R}}_Q(p_k,p_l)}(u_i, q_j) + I_{\widehat{\mathfrak{R}}_Q(p_k,p_l)}(u_i, q_j) - F_{\widehat{\mathfrak{R}}_Q(p_k,p_l)}(u_i, q_j) \tag{6.1}$$

for all $(u_i, q_j) \in U \times Q$ and $(p_k, p_l) \in K \times L \subseteq P \times P$. **Step 4:** Input the all score values in matrix form:

$$\mathcal{M}_{\widehat{\mathfrak{R}}_Q} = \begin{pmatrix} \Delta^{1,1}_{\widehat{\mathfrak{R}}_Q}(u_1,q_1) & \cdots & \Delta^{1,l'}_{\widehat{\mathfrak{R}}_Q}(u_1,q_1) & \cdots & \Delta^{k',1}_{\widehat{\mathfrak{R}}_Q}(u_1,q_1) & \cdots & \Delta^{k',l'}_{\widehat{\mathfrak{R}}_Q}(u_1,q_1) \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \Delta^{1,1}_{\widehat{\mathfrak{R}}_Q}(u_1,q_m) & \cdots & \Delta^{1,l'}_{\widehat{\mathfrak{R}}_Q}(u_1,q_m) & \cdots & \Delta^{k',1}_{\widehat{\mathfrak{R}}_Q}(u_1,q_m) & \cdots & \Delta^{k',l'}_{\widehat{\mathfrak{R}}_Q}(u_1,q_m) \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \Delta^{1,1}_{\widehat{\mathfrak{R}}_Q}(u_n,q_1) & \cdots & \Delta^{1,l'}_{\widehat{\mathfrak{R}}_Q}(u_n,q_1) & \cdots & \Delta^{k',1}_{\widehat{\mathfrak{R}}_Q}(u_n,q_1) & \cdots & \Delta^{k',l'}_{\widehat{\mathfrak{R}}_Q}(u_n,q_1) \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \Delta^{1,1}_{\widehat{\mathfrak{R}}_Q}(u_n,q_m) & \cdots & \Delta^{1,l'}_{\widehat{\mathfrak{R}}_Q}(u_n,q_m) & \cdots & \Delta^{k',1}_{\widehat{\mathfrak{R}}_Q}(u_n,q_m) & \cdots & \Delta^{k',l'}_{\widehat{\mathfrak{R}}_Q}(u_n,q_m) \end{pmatrix}_{nm \times k'l'} \tag{6.2}$$

where

$$\Delta^{k,l}_{\widehat{\mathfrak{R}}_Q}(u_i,q_j) = \begin{cases} \Theta^{k,l}_{\widehat{\mathfrak{R}}_Q}(u_i,q_j), & (p_k,p_l) \in K \times L \subseteq P \times P \\ 0, & otherwise \end{cases}$$

is a mapping given by $\Delta^{k,l}_{\widehat{\mathfrak{R}}_Q} : U \times Q \to [-1, 2]$. Here $1 \le k \le k'$, $1 \le l \le l'$ and $1 \le i \le n$, $1 \le j \le m$.

Step 5: Calculate the total value $\biguplus_{\widehat{\mathfrak{R}}_Q}(u_i, q_j)$ of element (u_i, q_j):

$$\biguplus_{\widehat{\mathfrak{R}}_Q}(u_i,q_j) = \sum_{k=1}^{k'} \sum_{l=1}^{l'} \Delta^{k,l}_{\widehat{\mathfrak{R}}_Q}(u_i,q_j).$$

Step 6: Find r, s, for which $\biguplus_{\widehat{\mathfrak{R}}_Q}(u_r, q_s) = max\left\{ \biguplus_{\widehat{\mathfrak{R}}_Q}(u_i, q_j) \right\}$.

Remark 6.1 If there is more than one value (u_r, q_s), any of these objects can be selected.

Now, using the evaluations of the jury members, a Q-NS-relation $\widehat{\mathfrak{R}}_Q$ from $(\widehat{\Gamma}_Q, P)$ to $(\widehat{\Lambda}_Q, P)$ is given in line with the requests of the school administration as follows:

$$\widehat{\mathfrak{R}}_Q = \left\{ \begin{array}{l} \left((p_1,p_3), \begin{array}{c} \langle (u_1,q_1), 0.23, 0.56, 0.18 \rangle, \langle (u_1,q_2), 0.44, 0.67, 0.44 \rangle, \\ \langle (u_2,q_1), 0.23, 0.94, 0.50 \rangle, \langle (u_2,q_2), 0.24, 0.33, 0.47 \rangle, \\ \langle (u_3,q_1), 0.17, 0.72, 0.81 \rangle, \langle (u_3,q_2), 0.41, 0.84, 0.58 \rangle, \\ \langle (u_4,q_1), 0.42, 0.58, 0.75 \rangle, \langle (u_4,q_2), 0.38, 0.61, 0.32 \rangle \end{array} \right), \\ \left((p_2,p_1), \begin{array}{c} \langle (u_1,q_1), 0.28, 0.56, 0.27 \rangle, \langle (u_1,q_2), 0.47, 0.55, 0.33 \rangle, \\ \langle (u_2,q_1), 0.32, 0.66, 0.43 \rangle, \langle (u_2,q_2), 0.22, 0.74, 0.27 \rangle, \\ \langle (u_3,q_1), 0.22, 0.87, 0.85 \rangle, \langle (u_3,q_2), 0.42, 0.76, 0.54 \rangle, \\ \langle (u_4,q_1), 0.44, 0.58, 0.55 \rangle, \langle (u_4,q_2), 0.24, 0.44, 0.56 \rangle \end{array} \right), \\ \left((p_2,p_3), \begin{array}{c} \langle (u_1,q_1), 0.43, 0.56, 0.27 \rangle, \langle (u_1,q_2), 0.44, 0.55, 0.44 \rangle, \\ \langle (u_2,q_1), 0.23, 0.92, 0.43 \rangle, \langle (u_2,q_2), 0.33, 0.74, 0.47 \rangle, \\ \langle (u_3,q_1), 0.17, 0.87, 0.43 \rangle, \langle (u_3,q_2), 0.41, 0.76, 0.45 \rangle, \\ \langle (u_4,q_1), 0.66, 0.47, 0.41 \rangle, \langle (u_4,q_2), 0.24, 0.59, 0.56 \rangle \end{array} \right) \end{array} \right\}.$$

The matrix form of the score values expressed in the algorithm for the current uncertainty problem is as follows:

$$\mathcal{M}_{\widehat{\Re}_Q} = \begin{pmatrix} 0 & 0 & 0.61 & 0.57 & 0 & 0.72 & 0 & 0 & 0 \\ 0 & 0 & 0.67 & 0.69 & 0 & 0.55 & 0 & 0 & 0 \\ 0 & 0 & 0.67 & 0.55 & 0 & 0.72 & 0 & 0 & 0 \\ 0 & 0 & 0.10 & 0.69 & 0 & 0.60 & 0 & 0 & 0 \\ 0 & 0 & 0.08 & 0.24 & 0 & 0.61 & 0 & 0 & 0 \\ 0 & 0 & 0.67 & 0.64 & 0 & 0.72 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0.47 & 0 & 0.72 & 0 & 0 & 0 \\ 0 & 0 & 0.67 & 0.12 & 0 & 0.27 & 0 & 0 & 0 \end{pmatrix}_{8\times 9}$$

The total values of (u_i, q_j), which is the sum of the rows of the matrix $\mathcal{M}_{\widehat{\Re}_Q}$, are

$$\biguplus_{\widehat{\Re}_Q}(u_1, q_1) = 1.9, \quad \biguplus_{\widehat{\Re}_Q}(u_1, q_2) = 1.91, \quad \biguplus_{\widehat{\Re}_Q}(u_2, q_1) = 1.94, \quad \biguplus_{\widehat{\Re}_Q}(u_2, q_2) = 1.39,$$
$$\biguplus_{\widehat{\Re}_Q}(u_3, q_1) = 0.93, \quad \biguplus_{\widehat{\Re}_Q}(u_3, q_2) = 2.03, \quad \biguplus_{\widehat{\Re}_Q}(u_4, q_1) = 1.44, \quad \biguplus_{\widehat{\Re}_Q}(u_4, q_2) = 1.06.$$

Finally, for $\biguplus_{\widehat{\Re}_Q}(u_3, q_2) = max\left\{\biguplus_{\widehat{\Re}_Q}(u_i, q_j)\right\}$, we determine that the student (u_3, q_2) gets the highest value. In other words, through the evaluations made by the jury members, it was determined that the relationship between the desired parameters was stronger in student (u_3, q_2) than other students. Therefore, it is recommended that the student (u_3, q_2), whose relationship between parameters is more compatible, participate in the science competition.

A comparison Here, since the Q-NS-relations were first proposed by Abu Qamar and Hassan (2018) in 2018, the algorithm given in this paper and the decision-making approach given in Abu Qamar and Hassan (2018) are compared. For this, both decision-making approaches have been applied to the solution of the uncertainty problem given in this section, and a comparison table has been created as follows:

It is clear from Table 2 that differences were found in the results obtained between the proposed mathematical approaches. Especially when we apply the decision-making algorithm given in Abu Qamar and Hassan (2018) to the uncertainty problem given in this section, it is seen that it cannot determine the best student. Moreover, the fact that there are some "="s in the ranking among students is another disadvantage of the algorithm given in Abu Qamar and Hassan (2018). Therefore, we recommend using the algorithm given in this paper in order to express the decision-making process in uncertainty problems in a better way.

Table 2 Q-NS-relations approaches to decision making

Paper	Result of algorithm in the paper	The ranking order of objects
In Abu Qamar and Hassan (2018)	$\{(u_2, q_1), (u_3, q_2)\}$	$(u_2, q_1) = (u_3, q_2) \geq (u_1, q_2) \geq (u_1, q_1) = (u_4, q_1) \geq (u_2, q_2) \geq (u_4, q_2) \geq (u_3, q_1)$
This paper	$\{(u_3, q_2)\}$	$(u_3, q_2) \geq (u_2, q_1) \geq (u_1, q_2) \geq (u_1, q_1) \geq (u_4, q_1) \geq (u_2, q_2) \geq (u_4, q_2) \geq (u_3, q_1)$

7 Conclusions

In this paper, Q-neutrosophic soft sets, which are a generalization of neutrosophic soft sets, have been modified to better explain the correspondence between objects in the uncertain environment and the concept of the Q-neutrosophic soft relationship has been analyzed. The concept of Q-neutrosophic soft relationship is exemplified in the study that it can be very useful in solving uncertainty problems since it expresses two-dimensional information with a truth membership function, an indeterminacy membership function and a falsity membership function. In particular, the presence of a parameterization tool in the defined hybrid set is very useful in expressing the uncertainty problems encountered in real life. In addition, the concepts of inverse and composition of Q-neutrosophic soft functions and relations are studied in detail, together with related theorems and properties. Moreover, equivalence classes and partitions of Q-neutrosophic soft relations are defined and some properties are given by exemplifying related real-life uncertainty problems. In the last part of the paper, a very useful algorithm in solving uncertainty problems is proposed by using Q-neutrosophic soft relations and a comparison is given for the proposed algorithm.

Q-neutrosophic soft relations presented in our paper can be re-evaluated by generalizing neutrosophic soft sets on many mathematical models such as Q-neutrosophic soft expert set (Hassan et al. 2018), generalized Q-neutrosophic soft expert set (Abu Qamar and Hassan 2018a), multi Q-single valued neutrosophic soft expert set (Khan et al. 2019). Q-neutrosophic soft relationships allow two-dimensional information to define relations between inconsistent and uncertain data. In this way, Q-neutrosophic soft relationships can be used to manage the decision-making processes in uncertain environments that can be encountered in many fields such as medical diagnosis, physics, engineering.

Declarations

Conflict of interest The authors declare that they have no confict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study, including their legal guardians.

References

Abu Qamar M, Hassan N (2018) Q-neutrosophic soft relation and its application in decision making. Entropy 20(3):172

Abu Qamar M, Hassan N (2018) Generalized Q-neutrosophic soft expert set for decision under uncertainty. Symmetry 10(11):621

Abu Qamar M, Hassan N (2018) Entropy, measures of distance and similarity of Q-neutrosophic soft sets and some applications. Entropy 20(9):672

Alhazaymeh K, Hassan N (2013) Generalized interval-valued vague soft set. Appl Math Sci 7:6983–6988

Alhazaymeh K, Hassan N (2013) Possibility interval-valued vague soft set. Appl Math Sci 7:6989–6994

Alhazaymeh K, Hassan N (2012) Interval-valued vague soft sets and its application. Adv Fuzzy Syst 208489

Alkhazaleh S (2016) Time-neutrosophic soft set and its applications. J Intell Fuzzy Syst 30:1087–1098

Alkhazaleh S, Salleh AR, Hassan N (2011) Soft multi sets theory. Appl Math Sci 5:3561–3573

Alkhazaleh S, Salleh AR (2011) Soft expert sets. Adv Decis Sci 757868
Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96
Atanassov KT (1999) Interval valued intuitionistic fuzzy sets, In Intuitionistic Fuzzy Sets. Physica, Heidelberg, pp 139–177
Bustince H, Burillo P (1996) Structures on intuitionistic fuzzy relations. Fuzzy Sets Syst 78:293–303
Chatterjee R, Majumdar P, Samanta SK (2016) Interval-valued possibility quadripartitioned single valued neutrosophic soft sets and some uncertainty based measures on them. Neutrosophic Sets Syst 14:35–43
Cuong BC, Kreinovich V (2014) Picture fuzzy sets. J Comput Sci Cybernet 30(4):409–420
Cuong BC, Phong PH, Smarandache F (2016) Standard neutrosophic soft theory: some first results. Neutrosophic Sets Syst 12(2):80–91
Dalkılıç OO (2020) An application of VFPFSS's in decision making problems. J Polytech. https://doi.org/10.2339/politeknik.758474
Dalkılıç OO, Demirtas N (2020) VFP-soft sets and its application on decision making problems. J Polytech. https://doi.org/10.2339/politeknik.685634
Dalkılıç O, Demirtaş N (2020) Bipolar soft filter. J Univ Mathem 3(1):21–27
Dalkılıç OO (2020) A novel approach to soft set theory in decision-making under uncertainty. Int J Compt Mathem 1-11
Dalkılıç O (2021) Relations on neutrosophic soft set and their application in decision making. J Appl Mathem Comput 1-17
Deli I (2015) npn-soft sets theory and applications. Ann Fuzzy Math Inform 10(6):847–862
Deli I (2017) Interval-valued neutrosophic soft sets and its decision making. Int J Mach Learn Cybern 8(2):665–676
Deli I, Broumi S (2015) Neutrosophic soft matrices and NSM-decision making. J Intell Fuzzy Syst 28(5):2233–2241
Deli I, Broumi S (2015) Neutrosophic soft relations and some properties. Ann Fuzzy Math Inform 9:169–182
Demirtaş N, Dalkılıç O (2020) Decompositions of soft α-continuity and soft A-continuity. J New Theory 31:86–94
Demirtaş N, Hussaın S, Dalkılıç O (2020) New approaches of inverse soft rough sets and their applications in a decision making problem. J Appl Mathem Inform 38(3–4):335–349
Demirtaş N, Dalkılıç O (2019) An application in the diagnosis of prostate cancer with the help of bipolar soft rough sets. On Mathematics and Mathematics Education (ICMME 2019), KONYA, p 283
Dinda B, Samanta TK (2010) Relations on intuitionistic fuzzy soft sets. General Mathem Notes 1:74–83
Hassan N, Uluçay V, şahin M (2018) Q-neutrosophic soft expert set and its application in decision making. Int J Fuzzy Syst Appl 7(4):37–61
Hussain A, Shabir M (2015) Algebraic structures of neutrosophic soft sets. Neutrosophic Sets Syst 7:53–61
Kanwal RS, Qurashi SM, Shabir M (2020) Generalized approximation of substructures in quantales by soft relations. Comput Appl Math 39(1):1–22
Kanwal RS, Shabir M (2018) Approximation of ideals in semigroups by soft relations. J Intell Fuzzy Syst 35(3):3895–3908
Kanwal RS, Shabir M (2019) Rough approximation of a fuzzy set in semigroups based on soft relations. Comput Appl Math 38(2):89
Khan Q, Mahmood T, Hassan N (2019)Multi Q-single valued neutrosophic soft expert set and its application in decision making. Infinite Study
Maji PK (2013) Neutrosophic soft set. Ann Fuzzy Math Inform 5(1):157–168
Maji PK (2014) Weighted neutrosophic soft sets. Neutrosophic Sets Syst 6:6–11
Marei E (2018) Single valued neutrosophic soft approach to rough sets, theory and application. Neutrosophic Sets Syst 20:76–85
Molodtsov D (1999) Soft set theory first results. Comput Math Appl 37:19–31
Mordeson JN, Mathew S, Malik DS (2018) Generalized fuzzy relations. New Mathem Nat Comput 14(02):187–202
Mukherjee A, Sadhan S (2015) A new method of measuring similarity between two neutrosophic soft sets and its application in pattern recognition problems. Neutrosophic Sets Syst 8(3):63–68
Mukherjee A, Sarkar S (2014) Several similarity measures of neutrosophic soft sets and its application in real life problems. Ann Pure Appl Mathem 7(1):1–6
Qamar MA, Hassan N (2019) Characterizations of group theory under Q-neutrosophic soft environment. Infinite Study
Qamar MA, Hassan N (2019) An approach to Q-neutrosophic soft rings. Infinite Study
şahin R, Küçük A (2014) On similarity and entropy of neutrosophic soft sets. J Intell Fuzzy Syst 27(5):2417–2430

Senapati T, Yager RR (2020) Fermatean fuzzy sets. J Ambient Intell Human Comput 11(2):663–674
Smarandache F, Neutrosophy (1998) Neutrosophic probability, set, and logic; American Research Press: Rehoboth. IL, USA
Smarandache F (2005) Neutrosophic set-a generalisation of the intuitionistic fuzzy sets. Int J Pure Appl Mathem 24(3):287–297
Smarandache F (2018) Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. Neutrosophic Sets Syst 22(1):168–170
Smarandache F (2019) Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. Octogon Mathem Mag 27(1):413–418
Sumathi IR, Arockiarani I (2016) Cosine similarity measures of neutrosophic soft set. Ann Fuzzy Mathem Inform, pp 1-10
Wang H, Smarandache F, Zhang YQ, Sunderraman R (2010) Single valued neutrosophic sets. Multispace Multistruct 4:410–413
Xu W, Ma J, Wang S, Hao G (2010) Vague soft sets and their properties. Comput Math Appl 59:787–794
Yager RR (2013) Pythagorean fuzzy subsets, In: Proceedings of the Joint IFSA World Congress and NAFIPS Annual Meeting. pp 57-61
Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353
Zheng H, Deng Y (2018) Evaluation method based on fuzzy relations between Dempster-Shafer belief structure. Int J Intell Syst 33(7):1343–1363