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A new hybrid time series forecasting model based on the neutrosophic set and quantum optimization algorithm

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ABSTRACT

This article acquaints a new method to forecast the time series dataset based on neutrosophic-quantum optimization approach. This study uses neutrosophic set (NS) theory to represent the inherited uncertainty of time series dataset with three different memberships as truth, indeterminacy and false. We refer such representations of time series dataset as neutrosophic time series (NTS). This NTS is further utilized for modeling and forecasting time series dataset. Study showed that the performance of NTS modeling approach is highly dependent on the optimal selection of the universe of discourse and its corresponding intervals. To resolve this issue, this study selects quantum optimization algorithm (QOA) and ensembles with the NTS modeling approach. QOA improves the performance of the NTS modeling approach by selecting the globally optimal universe of discourse and its corresponding intervals from the list of local optimal solutions. The proposed hybrid model (i.e., NTS-QOA model) is verified and validated with datasets of university enrollment of Alabama (USA), Taiwan futures exchange (TAIFEX) index and Taiwan Stock Exchange Corporation (TSEC) weighted index. Various experimental results signify the efficiency of the proposed NTS-QOA model over existing benchmark models in terms of average forecasting error rates (AFERs) of 0.44%, 0.066% and 1.27% for the university enrollment, TAIFEX index and TSEC weighted index, respectively.

1. Introduction

Zadeh [1] made his first endeavor to represent the uncertainty in 1965, which is found at various events. This theory is called as a fuzzy set theory. It is based on the non-probabilistic representation of uncertainty inherited in the set of events, where the degree of membership of each event must lie between 0 and 1. This concept of uncertainty representation is appropriate for the events, where the information about their occurrence is completely available. Using fuzzy set theory, Song and Chissom [2] initially introduced a model, which was entitled as the “fuzzy time series (FTS)”. In this modeling approach, each historical time series datum is characterized by the fuzzified linguistic variable. Then, forecasting results are derived by establishing the fuzzy logic relationships (i.e., decision rules) between the fuzzified linguistic variables [3]. Later, various forecasting models have been introduced by the researchers using this approach of Song and Chissom [2]. For example, researchers have illustrated the applications of FTS modeling approach in forecasting the university enrollment dataset of Alabama [4,5]. However, a few researchers have additionally focused on forecasting the daily temperature [6–8]. In the recent years, many researchers have been getting attracted toward the application of the FTS modeling approach in financial time series forecasting. In literature, numerous FTS forecasting models are available for forecasting the financial time series, such as the TAIFEX [9–14] and the TIFEX [13,15–17].

Many researchers focused on resolving various domain specific problems in the FTS modeling approach. Cheng et al. [18] integrated the FCM clustering algorithm with the FTS to partition the time series dataset effectively. Wong et al. [19] proposed an adaptive time-variant FTS forecasting model to automatically select the window size during the forecasting. Qiu et al. [20] suggested a weighted method for fuzzy relationships in the FTS modeling approach, so that forecasting accuracy can be improved. Liu [21] introduced an improved FTS model by considering trapezoidal fuzzy number for the forecasted value. Singh and Borah [22] incorporated new data discretization approach with the FTS modeling approach for resolving the problem of determination of effective interval

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lengths. Similarly, Gupta et al. [23] proposed a novel hybrid model by integrating an automatic clustering approach with the FTS modeling approach. Bas et al. [24] suggested the application of artificial neural network in the FTS modeling approach for the determination of fuzzy relationships.

Study shows that forecasting accuracy of the FTS modeling approach mainly depends on two factors [3], viz., determination of the effective lengths of intervals and establishment of fuzzy logic relationships (i.e., called the decision rules). However, determination of effective decision rules mostly relies upon the appropriate selection of the intervals. In FTS modeling approach, researchers utilize various optimization algorithms to resolve the issue of appropriate selection of the intervals. In the FTS modeling approach, researchers mainly use Genetic Algorithm (GA) [25], Particle Swarm Optimization (PSO) [16,17], Simulated Annealing (SA) [26], Geese Movement Based Optimization Algorithm (GMBOA) [27], Harmony Search Algorithm [28] and Quantum Optimization Algorithm (QOA) [29], for the appropriate selection of intervals from the universe of discourse.

Time series dataset often contains inadequate or incomplete information. In case of fuzzy based modeling approach (i.e., FTS modeling approach), uncertainties inherited in time series dataset is characterized only by using truth membership. To overcome this problem, intuitionistic fuzzy sets (IFS) [30] theory has been introduced. This IFS is an extension way to manage uncertainty in situations where available information is not adequate for defining the uncertainty. In IFS, this loss of information is represented by using both membership and non-membership degree functions. This concept can be useful to represent time series dataset in terms of both truth membership and false membership.

Real-time problems often contain more inadequate or incomplete information, whose characterization need to pay more granular attention. In this case, NS theory can be a better option for the researchers [31]. This theory was proposed by Smarandache [32], which can characterize the uncertainties in terms of three degrees of memberships, viz., truth membership (T_f), indeterminacy membership (I_f) and false membership (F_f). Recent application of NS theory is found in classification and visualization of stock index dataset in terms of the T_f, I_f and F_f [33]. Except this work [33], no other work is reported in the literature that could present a forecasting model for time series dataset exclusively based on NS theory. This motivates us to propagate our study toward the direction of designing time series forecasting model using NS theory and QOA.

While propagating our study, it has been observed that the modeling of time series and its forecasting inherits the following major problems as

- To distinguish an approach that can provide more effective representation for time series dataset using NS theory.
- To identify a way to deal with three degrees of memberships, viz., T_f, I_f and F_f together.
- To identify a way to generate optimal forecasting results from the provided inputs.

In the following, we have discussed these problems more elaborately along with their proposed solutions, which can be regarded as the contributions of the study.

1. *Discussion on Problem 1:* For this problem, a mathematical approach is required that should have the capability for converting time series dataset into NS. Time series dataset is mostly in crisp form and the particular approach should be able to convert them into NS using three degrees of memberships of NS, i.e., T_f, I_f and F_f. This study adopts the mathematical formulas as introduced in [33] for finding degrees of memberships of time series dataset. Then, time series dataset is represented using these degrees of memberships. A complete process of representing time series dataset is termed as *neutrosophication*,[1] and that time series dataset is called as *neutrosophic time series (NTS)*. The main advantageous of NTS over FTS is that former approach can represent the uncertainties in a more granular way by considering not only T_f (as in the case of FTS), but also I_f and F_f. This makes more robust to the NTS modeling approach over FTS modeling approach.
2. *Discussion on Problem 2:* In the NTS modeling approach, the three degrees of memberships associated with time series dataset are utilized for further processing. However, for any expert system, it would be very difficult to deal with these three degrees of memberships together. One solution of this problem is to recognize an approach to integrate these three memberships together, so that it can represent inheritance of complete uncertainty available in time series dataset. Our study find entropy [34] as a suitable approach to combine these three disintegrated memberships into one parameter of uncertainty measurement. In [33], researchers have embraced entropy for integrating degrees of memberships available in neutrosophic information system. By adopting the similar approach as discussed in [33], this study also represents the degrees of memberships of the NTS using corresponding entropy values. These entropy values are additionally used to express the NTS in the form of decision rules. We refer to such rules as *neutrosophic entropy relationship (NER)*.
3. *Discussion on Problem 3:* To obtain forecasting results from the NTS, an approach is required that ought to have the capability of converting NS into the crisp set. This study refers this operation as *deneutrosophication*.[2] Due to non-stationary nature of time series dataset, this operation is unfit to express NS into original crisp information. It can be regarded as the limitation of this operation. Such sort of issue has also faced by many researchers in the domain of FTS modeling approach during the defuzzification operation [35]. This problem arises due to ill-defined boundary for the time series dataset, that is called the universe of discourse. Appropriate representation of uncertainty in time series dataset depends on the universe of discourse. And, the intervals corresponding to the universe of discourse also play a significant role, because they actively participate in the neutrosophication. Hence, overall accuracy in time series forecasting can be improved during the deneutrosophication operation by selecting appropriate universe of discourse and its corresponding intervals. As discussed above, many researchers used different optimization algorithms to resolve this issue. Recently, Singh et al. [29] presented QOA, which mimics the quantum behavior of the particles. This study adopts this algorithm to resolve the problem of optimal selection of the universe of discourse and its corresponding intervals.

By concluding the above literature review and discussion, we propose the uses of QOA with NS theory for designing time series forecasting model. The proposed model is trained with the university enrollment dataset of Alabama [2]. Finally, the proposed model is validated with the TAIFEX and TSEC datasets. In case of forecasting all these datasets, it is observed that the proposed model has higher forecasting accuracy as compared to various FTS

[1] The term "neutrosophication" is identical to "fuzzification" as in the case of fuzzy set theory.

[2] The term "deneutrosophication" is identical to "defuzzification" as in the case of fuzzy set theory.

and non-FTS based models.

The remainder of this article is organized as follows. Section 2 provides the basics of NS followed by overview of QOA. The proposed NTS-QOA model is discussed in Section 3. In Section 4, empirical analysis for forecasting the time series dataset is discussed. Finally, conclusion is discussed in Section 5.

2. Background for the study

In this section, we provide the basics of NS theory followed by an overview of QOA and its application in finding the optimal solution of a generalized objective function.

2.1. Neutrosophic set

Various theories of NS are presented as follows [33,36]:

Definition 1. NS [36]Assume that U is a universe of discourse. A NS $\mathbb{N}$ for all $u_i \in U$ can be represented by a truth membership T_f, an indeterminacy membership I_f and a false membership F_f, where T_f, I_f, $F_f : U \rightarrow [0, 1]$ and $\forall u_i \in U, u_i \equiv (T_f(u_i), I_f(u_i), F_f(u_i)) \in \mathbb{N}$.

Neutrosophic membership functions are used to determine the values of T_f, I_f and F_f for the elements of the universe of discourse U. It is defined next.

Definition 2. Neutrosophic membership function [33]A *neutrosophic membership function* for the element $u_i \in U$ can be defined in terms of truth membership function T_f, an indeterminacy membership function I_f and a false membership function F_f as follows(1)$T_f(u_i) = \frac{u_i - \min(U)}{\max(U) - \min(U)}(2)F_f(u_i) = 1 - T_f(u_i)(3)I_f(u_i) = \sqrt{T_f(u_i)^2 + F_f(u_i)^2}$In Eq. (1), *min* and *max* represent the minimum and maximum functions, which return the minimum and maximum values from the U, respectively.

$$T_f(u_i) = \frac{u_i - \min(U)}{\max(U) - \min(U)} \tag{1}$$

$$F_f(u_i) = 1 - T_f(u_i) \tag{2}$$

$$I_f(u_i) = \sqrt{T_f(u_i)^2 + F_f(u_i)^2} \tag{3}$$

Definition 3. Neutrosophication [33]The operation of *neutrosophication* transforms a crisp set into a NS. Thus, a *neutrosifier* $\mathfrak{N}$ is applied to a crisp subset i of the universe of discourse U that yields a neutrosophic subset $\mathfrak{N}(i : N)$ as expressed below:

$$\mathfrak{N}(i : N) = \int_U (T_f(u_i), I_f(u_i), F_f(u_i))N(u_i) \tag{4}$$

Here, $(T_f(u_i), I_f(u_i), F_f(u_i))N(u_i)$ represents the product of a scalar $T_f(u_i), I_f(u_i), F_f(u_i)$ and the NS $N(u_i)$; and $\int$ is the union of the family of NS $T_f(u_i), I_f(u_i), F_f(u_i)N(u_i)$, $u_i \in U$.

Definition 4. Deneutrosophication [33]The operation of *deneutrosophication* transforms a NS into a crisp set.

Next, we will introduce the notion of NTS. Before comprehending what a NTS is, an example is presented here.

Example 1. Let us consider the stock index prices of an organization, which consist of records of a particular month from the first day to the last day of the month. To describe the trading behavior of the stock index prices, we use different semantic variables, which are commonly used in our daily life. These semantic variables may be chosen as low, very low, moderate, good, very good, high, very high and so on. At every moment, there are changes observed in the prices with respect to time t. If one describes the stock index prices at time $t - 1$ as low, then their description can be changed as high after a certain interval of time $t + 1$. That is, their interpretation for the price at time $t - 1$ becomes true to false after time $t + 1$. In most of the cases, interpretation of the prices is inexpressive due to its uncertain nature. This inexpressiveness nature of human cognitive process is called indeterministic, i.e., a situation of the human cognitive process which is unable to distinguish the occurrence of an event between truthfulness and falseness. This kind of indeterministic situation is always observed in our daily life. Due to non-stationary and uncertain nature, we often use three different kinds of semantic variables to express our views regarding the prices as (a) first variable that reflects the positive meaning or truthfulness of occurrence of an event, (b) second variable that reflects the negative meaning or falseness of occurrence of an event, and (c) finally third variable that reflects a situation in between truthfulness and falseness of occurrence of an event, i.e., indeterministic. By combining these three different natures of variables, NS theory has been proposed to solve the problems. If one records the observations of daily stock index in terms of semantic variables that express the meaning of truth, indeterministic and false simultaneously, we will obtain a new time series whose representation comprises of these three semantic variables together. That is, each record in this new time series represents a neutrosophic circumstance, where a set of records "R" can be expressed as "truth-R", "indeterministic-R" and "false-R". Such a transformation of time series in terms of three semantic variables together is called the NTS. With due course of time, meaning of semantic variables may change due to changes in time series, but their method of articulation remains the same in the NTS, i.e., three semantic variables together for a single observation or record.

It is obvious from the above example that NS is very useful to represent the non-stationary nature of time series. This study referred this approach as NTS. Mathematically, it can be defined as follows.

Definition 5. NTSAssume that $X(t)(t = \ldots, 0, a, b, \ldots)$ is a function of time t with sequence of observations $\{u_1, u_2, \ldots, u_n\}$. Assume U is the universe of discourse for these observations such that each $u_i \in U$. Let NS $N(t)$ be a collection of $<T_f(u_i), I_f(u_i), F_f(u_i)>$, which are defined on the U. Then, $N(t)$ is called a NTS on $X(t)$.

By considering the above Example 1, in Definition 5, $N(t)$ can be regarded as a NTS and $<T_f(u_i), I_f(u_i), F_f(u_i)>$ (i=1,2, ...) as its possible semantic variables in terms of three degrees of memberships, viz., $<T_f(u_i), I_f(u_i), F_f(u_i)>$. For different time intervals, the values of $N(t)$ can be changed; therefore, $N(t)$ is represented here as a function of time t. As the universe of discourse U can vary with time t, we utilize $X(t)$ to represent the sequence of observations $\{u_1, u_2, \ldots, u_n\}$.

The main difference between the FTS and NTS is that for the same set of observations, the former uses only one semantic variable, while the later uses three semantic variables simultaneously.

Entropy provides the measure of uncertainty of any observation [37]. Majumdar and Samanta [38] illustrated that uncertainty in NS is influenced by two main factors, viz., T_f and F_f. In their measure of uncertainty, they have given less importance to the third factor, which is I_f. Due to the non-linear nature of time series dataset, it always inherits much uncertainties, which need to pay much attention during their representation and modeling. By considering these three degrees of memberships, Singh and Rabadiya [33] introduced a formula to compute the entropy for NS. Experimental results also demonstrated the effectiveness of their approach in information classification and visualization. Therefore, in case of NTS modeling approach, the limitation of Majumdar and Samanta [38] approach of computing entropy is overcome by representing each time series value u_i in terms of three degrees of memberships (i.e., $T_f(u_i)$, $I_f(u_i)$ and $F_f(u_i)$), where each of them has given equal importance. The formula of computing entropy for the NTS dataset is given as follows:

Definition 6. Entropy of NTSFor any u_i in $X(t)$, there exists a collection of <$T_f(u_i)$, $I_f(u_i)$, $F_f(u_i)$> in $N(t)$, whose entropy is denoted as a function $E_N(u_i)$, where $E_N(u_i): N(t) \to [0, 1]$, and can be defined as follows(5)$E_N(u_i) = 1 - \frac{1}{n}\sum_{u_i \in U}(T_f(u_i) + I_f(u_i) + F_f(u_i)) \times E_1 E_2 E_3$Here, $E_1 = |T_f(u_i) - T_f^c(u_i)|$, $E_2 = |I_f(u_i) - {}_I^{fc}(u_i)|$, and $E_3 = |F_f(u_i) - F_f^c(u_i)|$.

Definition 7. Neutrosophic entropy relationship (NER)Assume that $N(t-1) = E_{\mathbb{N}}(u_i)$ and $N(t) = E_{\mathbb{N}}(u_j)$. The relationship between $N(t-1)$ and $N(t)$ is referred as a NER, which can be represented as(6)$E_{\mathbb{N}}(u_i) \to E_{\mathbb{N}}(u_j)$,where $E_{\mathbb{N}}(u_i)$ and $E_{\mathbb{N}}(u_j)$ are referred as *previous state* and *current state* of the NER, respectively.

Definition 8. Neutrosophic entropy relationship group (NERG) Assume the following NERs as
$E_{\mathbb{N}}(u_i) \to E_{\mathbb{N}}(u_{k1})$,
$E_{\mathbb{N}}(u_i) \to E_{\mathbb{N}}(u_{k2})$,
...
$E_{\mathbb{N}}(u_i) \to E_{\mathbb{N}}(u_{km})$

The NERs having the same previous state can be grouped into a same neutrosophic entropy relationship group (NERG), which can be represented as

$$E_{\mathbb{N}}(u_i) \to E_{\mathbb{N}}(u_{k1}), E_{\mathbb{N}}(u_{k2}), \ldots, E_{\mathbb{N}}(u_{km}) \quad (7)$$

2.2. Quantum optimization algorithm (QOA)

In quantum computing, the entanglement of quantum particles is necessary for quantum mechanics. This unique property of quantum has no analog in classical mechanics. Therefore, two particles can be considered as entangled when the energy between them has been large at any time in the past [39]. However, none of these particles are involved in the emission of other particles. Entanglement is commonly used in quantum communication, quantum cryptography, and quantum computing [39,40]. Schrödinger introduced the concept of “entanglement” in quantum mechanics [41]. Based on this quantum mechanics, Singh et al. [29] proposed the quantum optimization algorithm (QOA). In the following, we briefly provide the mechanism of the QOA [29], as follows.

For simplicity, initially a search agent q in two-dimensional search space is considered, and try to solve the optimization problem using Schrödinger equation as follows:

$$q = |\phi| \times Q_j + (1 - |\phi|) \times Q_h, \quad 0 < |\phi| < 1 \quad (8)$$

Here, Eq. (8) consists of two wave functions Q_{ij} and Q_{hi} as solutions, and ϕ is any complex number, which is equal to $a + bi$, i.e., $\phi = a + bi$. Here, a and b are two real numbers, and i is an imaginary number, which can be defined as $i = \sqrt{-1}$. The real value of $\phi = a + bi$ can be obtained by taking modulus of ϕ as $|\phi| = \sqrt{a^2 + b^2}$.

Now, for the negative exponential function, with density function $P(x)$, the cumulative distribution function is given as

$$P(x) = \frac{1}{L}e^{-2/L} \quad (9)$$

$$\int_{-\infty}^{x} P(x)dx = D(x) = 1 - e^{-2/L} \quad (10)$$

In Eq. (10), L is the search scope of search agent.

By taking natural log (ln) on both sides of Eq. (10), we get:

$$\int_{-\infty}^{x} P(x)dx = -(L/2) \quad \ln \quad (1 - D(x)) \quad (11)$$

If $r = D(x)$ is uniformly distributed random decimal fraction between 0 to 1, then the exponential variable associated with r is given by:

$$\begin{aligned} \int_{-\infty}^{x} P(x)dx &= -(L/2) \quad \ln \quad (1 - r) \\ \int_{-\infty}^{x} P(x)dx &= -(L/2) \quad \ln \quad r \end{aligned} \quad (12)$$

As it is exponential random generator, $1 - r$ can be replaced by r. Now, obtain the position of search agent using Monte Carlo method [42] as

$$y = |q \pm \frac{L}{2}\ln(1/v)| \quad v \sim U(0, 1) \quad (13)$$

where, v is a random number, whose value lies between [0, 1]. Now, compute the search scope of search agent q as

$$L = 2 \cdot |q - y| \quad (14)$$

Now, the final position y_f for the search agent q can be given as

$$y_f = q \pm \alpha \cdot |q - y| \cdot \ln(1/v) \quad (15)$$

where, α is the adjustment parameter for the algorithm. Therefore, the quantum mechanism for update the position of search agent is defined as

$$\vec{P}(x+1) = \vec{P}(x) + y_f \quad (16)$$

Here, $\vec{P}(x+1)$ and $\vec{P}(x)$ represent the new and old positions of the search agent q in problem space, respectively.

2.3. Generalized problem formulation and its solution using QOA

A single objective optimization problem (SOOP) includes at least one objective, whose optimal solution is required to obtain [43]. An example of formulating generalized SOOP using one objective is given next.

A generalized optimization problem can be formulated as

$$\text{Optimize (Max. or Min.)}\ \Theta(x) = \sum_{m=1}^{M} \lambda_m \Theta_j(x), \quad (17)$$

subject to the linear constraints

$$\begin{aligned} \lambda_m \geq 0; &\quad m = 1, 2, \ldots, M; \\ \Theta_j(x) \geq 0; &\quad j = 1, 2, \ldots, J; \\ x_i^{(L)} \leq x_i \leq x_i^{(U)}; &\quad i = 1, 2, \ldots, n \end{aligned}$$

Solution: A solution for the SOOP is λ, which consists of n decision variables, where $\lambda = \{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_m\}$. Here, last condition set denotes variable bound, which restricts the value of each x_i. Each x_i should fall within lower bound $x_i^{(L)}$ and upper $x_i^{(U)}$ bound. Further, all other steps are explained as

- Choose initial parameters as $\lambda = \{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_m\}$, $x_i^{(L)}$, and $x_i^{(U)}$. To obtain the solution for the SOOP, maximum number of iterations is defined.
- Initially, assume that the initial search agents q_i are the best search agents, and explore it in the given search space.
- Initialize the search agents q_i using Schrödinger equation in the search space as

$$q_i = |\phi| \times Q_j + (1 - |\phi|) \times Q_h, \quad 0 < |\phi| < 1 \tag{18}$$

Here, Eq. (8) consists of two wave functions Q_{ij} and Q_{hi} as solutions, and ϕ is any complex number, which is equal to $a + bi$, i.e., $\phi = a + bi$. Here, a and b are two real numbers, and i is an imaginary number, which can be defined as $i = \sqrt{-1}$. The real value of $\phi = a + bi$ can be obtained by taking modulus of ϕ as $|\phi| = \sqrt{a^2 + b^2}$.
- Obtain the current position of each search agent q_i in terms of x_i as $\vec{P}(x_i)$, where $\vec{P}(x_i)$ can be defined as

$$\vec{P}(x_i) = \frac{1}{L_i} e^{-2/L_i} \tag{19}$$

- Compute the displacement of each search agent q_i within range 0 to 1.

$$D(q_i) = \int_0^1 P(x_i)dx = e^{-2/L_i} \tag{20}$$

- Generate the random position y_i for each search agent q_i, based on Eq. (13).

$$y_i = |q_i \pm \frac{L_i}{2} \ln(1/v)|, \ v \in [0, 1] \tag{21}$$

where, v is a random number, whose value lies between [0, 1].
- Compute the search scope of each search agent with respect to search elements ω_m and random position y_i, based on Eq. (14) as

$$L_i = 2 \cdot |\omega_m - y_i| \in \vec{P}(x_i) \tag{22}$$

where, L_i represents the search scope of each search agent q_i.
- Calculate the final position y_{fi} for each search agent q_i, using Eq. (15).

$$y_{fi} = q_i \pm \alpha \cdot |\omega_m - y_i| \cdot \ln(1/v) \tag{23}$$

- For each search agent q_i, fitness value is obtained, and the $\vec{P}(x_i)$ is updated if a better solution exists as compared to previous one.
- To update the fitness value of each search agent q_i, update the position $\vec{P}(x_i)$ as

$$\vec{P}(x_i + 1) = \vec{P}(x_i) + y_{fi} \tag{24}$$

Here, $\vec{P}(x_i + 1)$ and $\vec{P}(x_i)$ represent the new and old positions of search agent in problem space, respectively.
- Stop the algorithm, if stopping criterion is satisfied; else return to Step 7.
- After stopping criteria is satisfied, select the best optimal solution.

3. The proposed NTS-QOA model

In this section, the proposed NTS-QOA model is introduced for time series forecasting. An algorithmic representation of the proposed model is presented as Algorithm 1. Initially, the experiment is performed by employing the university enrollment dataset of Alabama. This dataset is shown in Table 1. Each phase of the proposed model is explained next.

Algorithm 1. The proposed NTS-QOA model for time series forecasting.

```
procedure NTS-QOA model()
  Input: t_i (i = 1, 2, ..., n), where each t_i represents time series value.
  Output: forecasted t_i
  for ∀t_i
    Define the universe discourse for the time series dataset.
    Partition the universe of discourse into n-equal lengths of intervals.
    Formulate the optimization problem.
    Apply QOA to minimize the AFER (refer to Eq. (26)).
    Update the intervals and define the new universe of discourse.
    Apply the neutrosophication process to represent time series dataset
into the NTS dataset.
    Compute the entropy for the NTS dataset.
    Establish the NERs among the entropy values.
    Create the NERGs among the NERs.
    Perform the deneutrosophication operation to obtain the forecasted
time series values.
    Evaluate the performance of the proposed model.
    Select the universe of discourse and its corresponding intervals that
generate optimal error.
  end for
  return forecasted t_i.
```

- *Define the universe discourse for the time series dataset.* Assume that E_{min} and E_{max} be the minimum and maximum values of the time series dataset. Based on E_{min} and E_{max}, the universe of discourse U_0 can be defined as $U_0 = [E_{min} - A_N, E_{max} + A_P]$, where A_N and A_P be the two adjustment factors.
[Explanation:] From Table 1, we have $E_{min} = 13{,}055$ and $E_{max} = 19{,}337$. Initially, we assume that $A_N = 3555$ and $A_P = 10{,}663$. Now, the universe of discourse U_0 is defined as $U_0 = [9500, 30{,}000]$, where $min(U_0) = 9500$ and $max(U_0) = 30{,}000$.
- *Partition the universe of discourse into n-equal lengths of intervals.* For partitioning the universe of discourse U_0 into n-equal lengths of intervals, following Eq. (25) is used as

$$a_i = [\min(U_0) + (i-1)\frac{\max(U_0) - \min(U_0)}{j}, \min(U_0) + i\frac{\max(U_0) - \min(U_0)}{j}] \tag{25}$$

for $i = 1, 2, \ldots, n$, and j is the number of intervals.
Using Eq. (25), the set of intervals evolved from the universe of discourse U_0 can be defined as $a_1 = [L_{B1}, U_{B1}]$, $a_2 = [L_{B2}, U_{B2}]$, $a_3 = [L_{B3}, U_{B3}]$, $\ldots$, $a_n = [L_{Bn}, U_{Bn}]$, where $L_{Bi}, U_{Bi} \in a_i$ and $a_i \in U$.

Table 1
The university enrollment dataset of Alabama [2].

Year	Actual enrollment	Year	Actual enrollment	Year	Actual enrollment	Year	Actual enrollment	Year	Actual enrollment
1971	13,055	1976	15,311	1981	16,388	1986	15,984	1991	19,337
1972	13,563	1977	15,603	1982	15,433	1987	16,859	1992	18,876
1973	13,867	1978	15,861	1983	15,497	1988	18,150	–	–
1974	14,696	1979	16,807	1984	15,145	1989	18,970	–	–
1975	15,460	1980	16,919	1985	15,163	1990	19,328	–	–

[Explanation:] By assuming $j = 5$ in Eq. (25), the universe of discourse $U_0 = [9500, 30{,}000]$ is partitioned into 5-equal lengths of intervals as $a_1 = [9500, 13{,}600]$, $a_2 = [13{,}600, 17{,}700]$, ..., $a_5 = [25{,}900, 30{,}000]$. In this set of intervals, we can assume that $U_{B1} = 13{,}600 \in a_1$, $U_{B2} = 17{,}700 \in a_2$, ..., $U_{B5} = 30{,}000 \in a_5$.

- *Formulate the optimization problem.* An optimization problem can be formulated based on the objective function. In this study, the main objective is to improve the forecasting accuracy by selecting the optimal intervals from the universe of discourse U_0. For this purpose, performance of the model is required to evaluate by adopting suitable evaluation parameter. This parameter can also be used as an objective function. In literature, the average forecasting error rate (AFER) is chosen by most of the researchers [44]. It is obvious that if the proposed model selects the optimal intervals, then it will definitely minimize the AFER. Now, based on the AFER, we can formulate the objective function as

$$Minimize\ AFER = \frac{\sum_{i=1}^{N} |A_i - F_i|/A_i}{N} \tag{26}$$

subject to the constraints:

$$\begin{array}{r} U_{B1} \le \max(U_0) \\ U_{B2} \le \max(U_0) \\ \vdots \\ U_{Bn} \le \max(U_0) \\ U_{B1} < U_{B2} < U_{B3} < \cdots < U_{Bn} \end{array} \tag{27}$$

In Eq. (26), each F_i and A_i represent the forecasted and actual value of a particular year/day i, respectively; and N is the total number of years/days to be forecasted.

[Explanation:] We have the set of upper bounds $U_{B1} = 13{,}600$, $U_{B2} = 17{,}700$, ..., $U_{B7} = 30{,}000$ (refer to above example). In terms of these set of upper bounds, following constraints are defined to minimize the AFER (as defined in Eq. (26)) as

$$\begin{array}{r} 13{,}600 \le 30{,}000 \\ 17{,}700 \le 30{,}000 \\ \vdots \\ 30{,}000 \le 30{,}000 \\ 13{,}600 < 17{,}700 < 21{,}800 < \cdots < 30{,}000 \end{array}$$

- *Apply QOA to minimize the AFER.* Various sub-steps include in this process are explained next.
  - *Arrange n-number of intervals into two-dimensional search space.* For the set of intervals $a_1 = [L_{B1}, U_{B1}]$, $a_2 = [L_{B2}, U_{B2}]$, ..., $a_n = [L_{Bn}, U_{Bn}]$, their upper bounds $U_{B1}, U_{B2}, \ldots, U_{Bn}$ are arranged in ascending order as $U_{B1} < U_{B2} < \cdots < U_{Bn}$. Before initiating the iteration process, it is assumed that this set of upper bounds is the personal best set of upper bounds (i.e., the set of upper bounds that gives the minimum AFER value).
    [Explanation:] For the universe of discourse $U_0 = [9500, 30{,}000]$, the set of intervals are defined as $a_1 = [9500, 13{,}600]$, $a_2 = [13{,}600, 17{,}700]$, ..., $a_5 = [25{,}900, 30{,}000]$. This set of intervals consists of upper bounds 13,600, 17,700, ..., 30,000. Now, these upper bounds are arranged in ascending order as 13,600 < 17,700 < ... < 30,000. Initially, it is assumed that this set of upper bounds generate optimal result in terms of the AFER.
  - *Initialize the search agents $q_i (i = 1, 2, 3, \ldots, n)$ for each of the upper bounds U_{Bi} as*

$$\begin{aligned} q_1 &= |\phi| \times Q_{1j} + (1 - |\phi|) \times Q_{h1} \in U_{B1} \\ q_2 &= |\phi| \times Q_{2j} + (1 - |\phi|) \times Q_{h2} \in U_{B2} \\ &\vdots \\ q_n &= |\phi| \times Q_{nj} + (1 - |\phi|) \times Q_{hn} \in U_{Bn} \end{aligned} \tag{28}$$

[Explanation:] For the set of upper bounds 13,600, 17,700, ..., 30,000, search agents are initialized as $q_1, q_2, \ldots, q_5$, respectively. In Eq. (28), values of wave functions Q_{ij} and Q_{hi} are assumed between random number range [0, 1]. The real value of $\phi = a + bi$ can be obtained as $|\phi| = \sqrt{a^2 + b^2}$, where it is assumed that $a \in [0, 1]$ and $b \in [0, 1]$. Now, based on these assumptions, the search agents $q_1, q_2, \ldots, q_5$ can be defined in terms of Eq. (28) as

$$\begin{aligned} q_1 &= |\phi| \times Q_{1j} + (1 - |\phi|) \times Q_{h1} \in 13{,}600 \\ q_2 &= |\phi| \times Q_{2j} + (1 - |\phi|) \times Q_{h2} \in 17{,}700 \\ &\vdots \\ q_5 &= |\phi| \times Q_{5j} + (1 - |\phi|) \times Q_{h5} \in 30{,}000 \end{aligned} \tag{29}$$

For Eq. (29), search agents and their corresponding computation values are shown in Table 2.

- *Obtain the current position of each search agent q_i in terms of position $\vec{P}(x_i)$ as*

$$\vec{P}(x_1) = \frac{1}{L_1} e^{-2/L_1} \in q_1 \vec{P}(x_2) = \frac{1}{L_2} e^{-2/L_2} \in q_2 \vdots \vec{P}(x_n) = \frac{1}{L_n} e^{-2/L_n} \in q_n \tag{30}$$

Here, each $\vec{P}(x_i)$ and L_i represent the current position and search scope of each search agent q_i, respectively, where $i = 1, 2, 3, \ldots, n$.

[Explanation:] Based on Eq. (30), current positions of the search agents $q_1, q_2, \ldots, q_5$ can be computed as

$$\begin{aligned} \vec{P}(x_1) &= \frac{1}{L_1} e^{-2/L_1} = \frac{1}{13{,}600} e^{-2/13{,}600} = 74 \times 10^{-6} \\ &\in q_1 \vec{P}(x_2) = \frac{1}{L_2} e^{-2/L_2} = \frac{1}{17{,}700} e^{-2/17{,}700} \\ &= 56 \times 10^{-6} \in q_2 \vdots \vec{P}(x_5) = \frac{1}{L_5} e^{-2/L_5} \\ &= \frac{1}{30{,}000} e^{-2/30{,}000} = 33 \times 10^{-6} \in q_5 \end{aligned} \tag{31}$$

In Eq. (31), each search scope L_i represents the current position of the search agent q_i. Hence, for the search scopes $L_1, L_2, \ldots, L_5$, we have considered the initial assumed values of the upper bounds $U_{B1}, U_{B2}, \ldots, U_{B5}$, respectively, in Eq. (31).

- *Compute the displacement of each search agent q_i between range 0 to 1 as*

$$\begin{aligned} D(q_1) &= \int_0^1 |\vec{P}(x_1)| dx = e^{-2/L_1} D(q_2) = \int_0^1 |\vec{P}(x_2)| dx \\ &= e^{-2/L_2} \vdots D(q_n) = \int_0^1 |\vec{P}(x_n)| dx \\ &= e^{-2/L_n} \end{aligned} \tag{32}$$

[Explanation:] The displacement of each search agent $q_1, q_2, \ldots, q_5$ can be computed using Eq. (32) as

$$\begin{aligned} D(q_1) &= \int_0^1 |\vec{P}(x_1)| dx = e^{-2/L_1} = e^{-2/13{,}600} = 0.99 D(q_2) \\ &= \int_0^1 |\vec{P}(x_2)| dx = e^{-2/L_2} = e^{-2/17{,}700} \\ &= 0.99 \vdots D(q_5) = \int_0^1 |\vec{P}(x_5)| dx = e^{-2/L_5} \\ &= e^{-2/30{,}000} = 0.99 \end{aligned}$$

Table 2
Search agents and their corresponding computation values.

Search agent	a	b	$\lvert\phi\rvert = \sqrt{a^2+b^2}$	$Q_{ij}(i = 1, 2, 3, \ldots, 5)$	$Q_{hi}(i = 1, 2, 3, \ldots, 5)$	Final value
q_1	0.34	0.90	0.96	0.37	0.11	0.36
q_2	0.78	0.39	0.87	0.24	0.40	0.26
q_3	0.10	0.13	0.16	0.94	0.96	0.96
q_4	0.58	0.06	0.58	0.23	0.35	0.28
q_5	0.82	0.02	0.82	0.04	0.17	0.06

- *Generate the random position y_i for each search agent q_i as*

$$\begin{aligned} y_1 &= \left|q_1 \pm \frac{L_1}{2}\ln(1/v)\right|,\ v \in [0,1] \\ y_2 &= \left|q_2 \pm \frac{L_2}{2}\ln(1/v)\right|,\ v \in [0,1] \\ &\vdots \\ y_n &= \left|q_n \pm \frac{L_n}{2}\ln(1/v)\right|,\ v \in [0,1] \end{aligned} \tag{33}$$

where, v is a random number, whose value lies between [0, 1].
[Explanation:] Random position for each search agent q_1, q_2, ..., q_5 is generated using Eq. (33) as

$$\begin{aligned} y_1 &= q_1 - \frac{L_1}{2}\ln(1/v) = 0.36 - \frac{13,600}{2}\ln(1/0.65) = 1272.5 \\ y_2 &= q_2 - \frac{L_2}{2}\ln(1/v) = 0.26 - \frac{17,700}{2}\ln(1/0.73) = 1209 \\ &\vdots \\ y_5 &= q_5 - \frac{L_5}{2}\ln(1/v) = 0.06 - \frac{30,000}{2}\ln(1/0.55) = 3894.6 \end{aligned}$$

Here, "$-$" operator is used in the place of "$\pm$" (refer to Eq. (33)). The value of search agent q_1, q_2, ..., q_5 can be obtained from Table 2.

- *Compute the search scope of each search agent q_i with respect to upper bound $U_{Bi} \in U$ and corresponding random position y_i as*

$$L_1 = 2\cdot|U_{B1} - y_1| \in \vec{P}(x_1) L_2 = 2\cdot|U_{B2} - y_2| \in \vec{P}(x_2) \vdots L_n = 2\cdot|U_{Bn} - y_n| \in \vec{P}(x_n) \tag{34}$$

[Explanation:] For each search agent q_1, q_2, ..., q_5, their corresponding search scope L_1, L_2, ..., L_5, respectively, can be computed using Eq. (34) as

$$L_1 = 2\cdot|U_{B1} - y_1| = 2\cdot|13,600 - 1272.5| = 24,655 \in \vec{P}(x_1) L_2 = 2\cdot|U_{B2} - y_2| = 2\cdot|17,700 - 1209| = 32,982 \in \vec{P}(x_2) \vdots L_5 = 2\cdot|U_{B5} - y_5| \in \vec{P}(x_5) = 2\cdot|30,000 - 3894.6| = 52,210.8 \in \vec{P}(x_5)$$

- *Obtain the final position for each search agent q_i as*

$$\begin{aligned} y_{f1} &= q_1 \pm \alpha\cdot|U_{B1} - y_1|\cdot\ln(1/v) \\ y_{f2} &= q_2 \pm \alpha\cdot|U_{B2} - y_2|\cdot\ln(1/v) \\ &\vdots \\ y_{fn} &= q_n \pm \alpha\cdot|U_{Bn} - y_n|\cdot\ln(1/v) \end{aligned} \tag{35}$$

Here, each y_{fi} represents the corresponding final position for the search agent q_i and α represents the adjustment parameter for the algorithm.
[Explanation:] Final position for each search agent q_1, q_2, ..., q_5 is computed using Eq. (35) as

$$\begin{aligned} y_{f1} &= q_1 - \alpha\cdot|U_{B1} - y_1|\cdot\ln(1/v) = 0.36 - 0.3\cdot|13,600 - 1272.5|\cdot\ln(1/0.65) = -691.53 \\ y_{f2} &= q_2 - \alpha\cdot|U_{B2} - y_2|\cdot\ln(1/v) = 0.26 - 0.3\cdot|17,700 - 1272.5|\cdot\ln(1/0.73) = -673.32 \\ &\vdots \\ y_{f5} &= q_5 - \alpha\cdot|U_{B5} - y_5|\cdot\ln(1/v) = 0.06 - 0.3\cdot|30,000 - 3894.6|\cdot\ln(1/0.55) = -2237.9 \end{aligned}$$

Here, "$-$" operator is used in the place of "$\pm$" (refer to Eq. (35)). Here, the value of α is assumed as 0.3.

- *Update the position of each search agent q_i with respect to upper bound $U_{Bi} \in U$ and corresponding old position $\vec{P}(x_i)$ as*

$$\vec{P}(x_1 + 1) = \vec{P}(x_1) + y_{f1} \in x_1 \vec{P}(x_2 + 1) = \vec{P}(x_2) + y_{f2} \in x_2 \vdots \vec{P}(x_n + 1) = \vec{P}(x_n) + y_{fn} \in x_n \tag{36}$$

Here, $\vec{P}(x_i + 1)$ and $\vec{P}(x_i)$ represent the new and old positions of each search agent q_i in the search space, respectively.
[Explanation:] For each search agent q_1, q_2, ..., q_5, position is updated based on Eq. (36) as

$$\vec{P}(x_1 + 1) = \vec{P}(x_1) + y_{f1} = 13,600 - 691.53 = 12,908.47 \in x_1 \vec{P}(x_2 + 1) = \vec{P}(x_2) + y_{f2} = 17,700 - 673.32 = 17,026.68 \in x_2 \vdots \vec{P}(x_5 + 1) = \vec{P}(x_5) + y_{f5} = 30,000 - 2237.9 = 27,762.1 \in x_5$$

- *Update the intervals and define the new universe of discourse.* The new set of intervals can be defined as $a_1(new) = [L_{B1}, \vec{P}(x_1 + 1)]$, $a_2(new) = [\vec{P}(x_1 + 1), \vec{P}(x_2 + 1)], \ldots, a_n(new) = [\vec{P}(x_{n-1} + 1), \vec{P}(x_n + 1)]$; where the new universe of discourse U_{1st} can be defined as $U_{1st} = [L_{B1}, \vec{P}(x_n + 1)]$. Here, L_{B1} and $\vec{P}(x_n + 1)$ represent the lower and upper bounds of the new universe of discourse U_{1st}, respectively. Note that universe of discourse obtained after ith iteration is representation as U_{ith}, where i = 1, 2, ..., n.
[Explanation:] By following sub-step 4.8, the new set of intervals can be defined as $a_1(new)$ = [9500, 12,908.47], $a_2(new)$ = [12,908.47, 17,026.68], ..., $a_5(new)$ = [23,338.1, 27,762.10]. For this new set of intervals, their universe of discourse can be defined as U_{1st} = [9500, 27,762.10].
- *Apply the neutrosophication process to represent time series dataset into the NTS dataset.* Using NS theory, time series dataset can be represented into the NTS dataset as

$$\mathbb{N}_{t_i} = \frac{t_i}{< T_f(t_i), I_f(t_i), F_f(t_i) >} \tag{37}$$

In Eq. (37), T_f, I_f and F_f represent the truth membership, indeterminacy membership and false membership functions, respectively. Here, T_f, I_f, F_f: $U_{1st} \rightarrow [0, 1]$ and $\forall t_i \in U_{1st}, t_i \equiv t_i(T_f(t_i), I_f(t_i), F_f(t_i)) \in \mathbb{N}_{t_i}$. Here, the values of $T_f(t_i)$, $I_f(t_i)$ and $F_f(t_i)$ can be computed in terms of the new universe of discourse $U_{1st} = [L_{B1}, \vec{P}(x_n + 1)]$ using Eqs. (38–40) as

$$T_f(t_i) = \frac{t_i - L_{B1}}{\vec{P}}(x_n + 1) - L_{B1} \tag{38}$$

$$F_f(t_i) = 1 - T_f(t_i) \tag{39}$$

$$I_f(t_i) = \sqrt{T_f(t_i)^2 + F_f(t_i)^2} \tag{40}$$

In this way, the time series dataset can be represented into the NTS dataset.

Table 3
The NTS representation of the university enrollment dataset of Alabama.

Year	Actual enrollment	NTS	Entropy	Year	Actual enrollment	NTS	Entropy
1971	13,055	$\frac{13,055}{<0.19,0.81,0.83>}$	0.84	1982	15,603	$\frac{15,603}{<0.33,0.67,0.74>}$	0.96
1972	13,563	$\frac{13,563}{<0.22,0.78,0.81>}$	0.88	1983	15,861	$\frac{15,861}{<0.34,0.66,0.74>}$	0.97
1973	13,867	$\frac{13,867}{<0.23,0.77,0.80>}$	0.89	1984	16,807	$\frac{16,807}{<0.40,0.60,0.72>}$	0.98
1974	14,696	$\frac{14,696}{<0.28,0.72,0.77>}$	0.93	1985	16,919	$\frac{16,919}{<0.40,0.60,0.72>}$	0.98
1975	15,460	$\frac{15,460}{<0.32,0.68,0.75>}$	0.96	1986	15,497	$\frac{15,497}{<0.32,0.68,0.75>}$	0.96
1976	15,311	$\frac{15,311}{<0.31,0.69,0.75>}$	0.95	1987	18,970	$\frac{18,970}{<0.51,0.49,0.70>}$	0.99
1977	16,388	$\frac{16,388}{<0.37,0.63,0.73>}$	0.98	1988	19,328	$\frac{19,328}{<0.53,0.47,0.70>}$	0.99
1978	15,433	$\frac{15,433}{<0.32,0.68,0.75>}$	0.96	1989	19,337	$\frac{19,337}{<0.53,0.47,0.70>}$	0.99
1979	15,984	$\frac{15,984}{<0.35,0.65,0.73>}$	0.97	1990	15,145	$\frac{15,145}{<0.30,0.70,0.76>}$	0.95
1980	16,859	$\frac{16,859}{<0.40,0.60,0.72>}$	0.98	1991	15,163	$\frac{15,163}{<0.31,0.69,0.75>}$	0.95
1981	18,150	$\frac{18,150}{<0.47,0.53,0.70>}$	0.99	1992	18,876	$\frac{18,876}{<0.51,0.49,0.70>}$	0.99

Table 4
The NERs for the university enrollment dataset of Alabama.

NERs	NERs	NERs	NERs	NERs
0.84 → 0.88	0.88 → 0.89	0.89 → 0.93	0.93 → 0.96	0.96 → 0.95
0.95 → 0.98	0.98 → 0.96	0.96 → 0.97	0.97 → 0.98	0.98 → 0.99
0.99 → 0.96	0.96 → 0.97	0.97 → 0.98	0.98 → 0.98	0.98 → 0.96
0.96 → 0.99	0.99 → 0.99	0.99 → 0.99	0.99 → 0.95	0.95 → 0.95
0.95 → 0.99				

[Explanation:] Here, an example is presented to demonstrate the application of neutrosophication process that applied on a time series value. Consider an enrollment value for the year “1971”, which is “13,055”. Now, its neutrosophication can be done using Eqs. (38–40) as

$$T_f(13,055) = \frac{t_i - L_B}{U_B(new) - L_B} = \frac{13,055 - 9500}{27,762.44 - 9500} = 0.19 \tag{41}$$

$$F_f(13,055) = 1 - T_f(t_i) = 1 - 0.07 = 0.81 \tag{42}$$

$$I_f(13,055) = \sqrt{T_f(t_i)^2 + F_f(t_i)^2} = \sqrt{0.19^2 + 0.81^2} = 0.83 \tag{43}$$

Now, based on $T_f(13,055)$, $F_f(13,055)$ and $I_f(13,055)$, the enrollment value “13,055” is represented into the NTS as

$$\mathbb{N}_{13,055} = \frac{t_i}{< T_f(t_i), I_f(t_i), F_f(t_i) >} = \frac{13,055}{< 0.19, 0.81, 0.83 >} \tag{44}$$

A complete representation of the university enrollment dataset of Alabama into the NTS dataset is shown in Table 3.

- *Compute the entropy for the NTS dataset.* The entropy of the NTS dataset can be computed (refer to Eq. (5)) as

$$E_{\mathbb{N}}(t_i) = 1 - \frac{1}{n} \sum_{t_i \in U_{1st}} (T_f(t_i) + I_f(t_i) + F_f(t_i)) \times E_1 E_2 E_3 \tag{45}$$

Here, $E_1 = |T_f(t_i) - T_f^c(t_i)|$, $E_2 = |I_f(t_i) - I_f^c(t_i)|$, and $E_3 = |F_f(t_i) - F_f^c(t_i)|$. Here, $U_{1st} = [L_{B1}, \vec{P}(x_n + 1)]$ is the new universe of discourse as obtained in Step 5.

[Explanation:] Here, an example is presented to compute entropy of the NTS dataset. Consider the NTS representation for the enrollment value “13,055” (refer to Eq. (44)). Now, its corresponding entropy value can be obtained using Eq. (45) as

$$E_{\mathbb{N}}(13,055) = 1 - \frac{1}{3} \sum_{t_i \in a_i(new)} (0.19 + 0.81 + 0.83) \times 0.86 \times 0.86 \times 0.86 = 0.59$$

Here, $E_1 = |0.19 - 0.81| = 0.62$, $E_2 = |0.81 - 0.19| = 0.62$, and $E_3 = |0.83 - 0.17| = 0.66$.

In this way, entropy for the NTS dataset can be computed in terms of the new universe of discourse $U_{1st} = [L_{B1}, \vec{P}(x_n + 1)]$. The entropy values of the university enrollment dataset of Alabama are shown in Table 3.

- *Establish the NERs among the entropy values.* The NER can be established between two consecutive entropy values of the NTS dataset. It is mentioned in Definition 7 that a NER exhibits relationship between two or more sequential entropy values. For example, $E_{\mathbb{N}}(t_i)$ and $E_{\mathbb{N}}(t_{i+1})$ be two sequential entropy values in the NTS dataset. By following Definition 7, the NER can be set-up between them as $E_{\mathbb{N}}(t_i) \rightarrow E_{\mathbb{N}}(t_{i+1})$.

[Explanation:] In Table 3, entropy values for Years 1973 and 1974 are 0.89 and 0.93, respectively. So, we can set-up a NER between 0.89 and 0.93 as 0.89 → 0.93. In this way, we have obtained the NERs for the NTS dataset of the university enrollment. All these NERs are shown in Table 4.

- *Create the NERGs among the NERs.* The NERGs can be created between two or more NERs. In the set of NERs, the relationship which have same previous state can be grouped together, which is called the NERGs.

[Explanation:] Here, an example is presented to create the NERG from the list of NERs available in Table 4. In this table, there are three NERs having the same previous state as 0.95 → 0.95, 0.95 → 0.96, and 0.95 → 0.97. By following Definition 8, the NERG can be formed among these NERs as

Table 5
The NERGs of the university enrollment dataset of Alabama.

NERGs
0.84 → 0.88
0.89 → 0.93
0.88 → 0.89
0.93 → 0.96
0.95 → 0.95, 0.99, 0.98
0.96 → 0.95, 0.97, 0.97, 0.99
0.97 → 0.98, 0.98
0.98 → 0.98, 0.96, 0.99, 0.96
0.99 → 0.99, 0.99, 0.95, 0.96

0.95 → 0.95, 0.96, 0.97. In this way, we have created the NERGs for the university enrollment dataset of Alabama, which are presented in Table 5.

- *Perform the deneutrosophication operation to obtain the forecasted time series values.* Deneutrosophication process is applied to obtain the forecasted values from the NTS dataset. Phases involved in deneutrosophication operation is presented as follows.
  - For a particular day/year t_i, obtain the corresponding entropy value as $E_{\mathbb{N}}(t_i)$.
  - Find the NERG for the corresponding $E_{\mathbb{N}}(t_i)$, which can be represented in the following form as

$$E_{\mathbb{N}}(t_i) \rightarrow E_{\mathbb{N}}(t_{k1}), E_{\mathbb{N}}(t_{k2}), E_{\mathbb{N}}(t_{k3}), \ldots, E_{\mathbb{N}}(t_{kn}) \quad (46)$$

  Here, $E_{\mathbb{N}}(t_i)$ is called the previous state of the neutrosophic time series value for the day/year t_i; whereas $E_{\mathbb{N}}(t_{k1}), E_{\mathbb{N}}(t_{k2}), E_{\mathbb{N}}(t_{k3}), \ldots, E_{\mathbb{N}}(t_{kn})$ are called the current state of the NTS values for the days/years t_{k1}, t_{k2}, t_{k3}, ..., t_{kn}, respectively.
  - Obtain the desired entropy available in the current state of the NERG as

$$E_{\mathbb{N}}(desired) = \frac{1}{n}\sum_{i=1}^{n} E_{\mathbb{N}}(t_{kn}) \quad (47)$$

  - Obtain the actual time series values associated with the previous state's of the NERG as t_i
  - Apply the following deneutrosophication formula to calculate the forecasted value for the day/year t_{i+1} as

$$Forecast(t_{i+1}) = \frac{E_{\mathbb{N}}(desired) \times t_i}{E_{\mathbb{N}}(t_i)} \quad (48)$$

  Here, the value of $E_{\mathbb{N}}(desire)$ can be obtained from Eq.(47). **[Explanation:]** Suppose, we want to forecast the enrollment for the year “1976”. For this year, the corresponding entropy value can be obtained from Table 3, which is 0.95. Then, obtain the NERG for the entropy value 0.95 from Table 5 as 0.95 → 0.95, 0.99, 0.98, where 0.95 is the called the previous state, and 0.95, 0.99, 0.98 are called the current state of the NERG. Now, by using Eq. (47), compute the desired entropy available in current state of the NERG as

$$E_{\mathbb{N}}(desired) = \frac{0.95 + 0.99 + 0.98}{3} = 0.97 \quad (49)$$

  Now, based on Eq. (48), obtain the forecasted enrollment for the year 1976 as

$$Forecast(1976) = \frac{0.97 \times 15,311}{0.95} = 15,633.33 \quad (50)$$

  In this way, we have obtained the forecasted values for the university enrollment dataset of Alabama based on the proposed model.
- *Evaluate the performance of the proposed model.* **[Explanation:]** The performance of the proposed model is evaluated using the AFER (refer to Eq. (26)). If the AFER value indicates that there is a further scope in improving the performance of the proposed model, then iterate the process again from sub-step 4.6.
- *Select the universe of discourse and its corresponding intervals that generate optimal error.* **[Explanation:]** After certain number of iterations, computation is stopped if satisfactory result is obtained. In this study, the proposed NTS-QOA model is iterated for 100 times to get the optimal solution. The universe of discourse and its corresponding intervals that generate optimal AFER can be considered as the global best solution.

In the following discussion, a complete process of application of QOA in the proposed NTS-QOA model has been briefly presented.

[Discussion:] Before the application of QOA in the proposed NTS-QOA model, an initial assumption is made for the universe of discourse as U_0 = [9500, 30,000] for the university enrollment dataset of Alabama. In Step 2 of the proposed model, we have partitioned the U_0 = [9500, 30,000] into 5-equal lengths of intervals as a_1 = [9500, 13,600], a_2 = [13,600, 17,700], a_3 = [17,700, 21,800], a_4 = [21,800, 25,900] and a_5 = [25,900, 30,000]. Initially, 5 search agents $q_i(i = 1, 2, \ldots, 5)$ are declared for these 5-equal lengths of intervals. In the searching of optimal universe of discourse and its corresponding intervals, upper bounds that are represented as $U_{B1} = 13,600 \in a_1$, $U_{B2} = 17,700 \in a_2$, $U_{B3} = 21,800 \in a_3$, $U_{B4} = 25,900 \in a_4$ and $U_{B5} = 30,000 \in a_5$ in Step 2 are considered. Now, each upper bound is assigned to each search agent as $13,600 \in q_1$, $17,700 \in q_2$, $21,800 \in q_3$, $25,900 \in q_4$ and $30,000 \in q_5$. The initial random position (i.e., RM) of each search agent (i.e., $q_i(i = 1, 2, \ldots, 5)$) for the universe of discourse U_0 = [9500, 30,000] is shown in Fig. 1 in the form of an array. Due to ease of computation, upper bound assigned to each search agent in this array is arranged in such a way that $U_{B1} < U_{B2} < U_{B3} < U_{B4} < U_{B5}$.

For finding the optimal solution, these 5 search agents (refer to Fig. 1) are allowed to move other positions, and their movement is recorded and considered as the 1st iteration. During the movement of each search agent from one position to another, the upper bounds that belong to new array always need to be adjusted in such a way that it must always follow the ascending order of sequence as $U_{B1} < U_{B2} < U_{B3} < U_{B4} < U_{B5}$. During the movement, various

q_1	q_2	q_3	q_4	q_5	Universe of discourse	Iteration	Position
13600	17700	21800	25900	30000	$U_0 = [9500, 30000]$	Not initiate	RM
12908.47	17026.68	20648.80	23338.10	27762.10	$U_{1st} = [9500, 27762.10]$	1st	LB
13599.97	16682.80	18341.70	25233.42	25579.60	$U_{99th} = [9500, 25579.60]$	99th	LB
13303.33	15054.50	21633.69	23019.50	24866.90	$U_{100th} = [9500, 24866.90]$	100th	GB

Fig. 1. Position of each search agent before and after the iterations. Here, abbreviations RM, LB and GB in the column-Position stand for random, local best and global best, respectively.

Table 6
Selected values of parameters associated with 5 search agents (for the 1st, 99th and 100th iterations).

Iteration	Search agent	a	b	$\|\phi\|$	Q_{ij}	Q_{hi}	q_n	$\vec{P}(x_i)$	$D(q_i)$	v	y_i	L_i	α	y_{fi}	$\vec{P}(x_i+1)$
1st	q_1	0.34	0.9	0.96	0.37	0.11	0.36	74×10^{-6}	0.99	0.65	1272.5	24,655	0.30	−691.53	12,908.47
	q_2	0.78	0.39	0.87	0.24	0.40	0.26	56×10^{-6}	0.99	0.73	1209.0	32,980	0.30	−673.32	17,026.68
	q_3	0.10	0.13	0.16	0.94	0.96	0.96	46×10^{-6}	0.99	0.65	2040.2	39,520	0.30	−1151.2	20,648.80
	q_4	0.58	0.06	0.58	0.23	0.35	0.28	39×10^{-6}	0.99	0.45	4491.2	42,818	0.30	−2561.9	23,338.10
	q_5	0.82	0.02	0.82	0.04	0.17	0.06	33×10^{-6}	0.99	0.55	3894.6	52,211	0.30	−2237.9	27,762.10
99th	q_1	0.62	0.47	0.78	0.35	0.83	0.46	74×10^{-6}	0.99	0.01	13,600	0.92	0.53	−0.0276	13,599.97
	q_2	0.59	0.55	0.81	0.92	0.29	0.80	56×10^{-6}	0.99	0.34	4147.2	27,106	0.53	−1017.2	16,682.80
	q_3	0.76	0.75	1.07	0.38	0.57	0.37	46×10^{-6}	0.99	0.16	8675.5	26,249	0.53	−3458.3	18,341.70
	q_4	0.08	0.05	0.09	0.53	0.78	0.76	39×10^{-6}	0.99	0.79	1326.5	49,147	0.53	−666.58	25,233.42
	q_5	0.93	0.13	0.94	0.57	0.47	0.56	33×10^{-6}	0.99	0.31	7630.1	44,740	0.53	−4420.4	25,579.60
100th	q_1	0.66	0.16	0.68	0.12	0.5	0.24	74×10^{-6}	0.99	0.81	622.54	25,955	0.25	−296.67	13,303.33
	q_2	0.96	0.34	1.02	0.59	0.22	0.6	56×10^{-6}	0.99	0.24	5485.7	24,429	0.25	−2645.5	15,054.50
	q_3	0.75	0.26	0.79	0.51	0.7	0.55	46×10^{-6}	0.99	0.93	344.09	42,912	0.25	−166.31	21,633.69
	q_4	0.89	0.96	1.31	0.55	0.14	0.68	39×10^{-6}	0.99	0.35	5905	39,990	0.25	−2880.5	23,019.50
	q_5	0.15	0.26	0.3	0.84	0.25	0.43	33×10^{-6}	0.99	0.2	10,485	39,030	0.25	−5133.1	24,866.90

Table 7
List of intervals and corresponding universe of discourse associated with 5 search agents (for the 1st, 99th and 100th iterations).

Iteration	$a_1(new)$	$a_2(new)$	$a_3(new)$	$a_4(new)$	$a_5(new)$	Universe of discourse
1st	[9500, 12,908.47]	[12,908.47, 17,026.68]	[17,026.68, 20,648.80]	[20,648.80, 23,338.10]	[23,338.10, 27,762.10]	[9500, 27,762.10] $\in U_{1st}$
99th	[9500, 13,599.97]	[13,599.97, 16,682.80]	[16,682.80, 18,341.70]	[18,341.70, 25,233.42]	[25,233.42, 25,579.60]	[9500, 25,579.60] $\in U_{99th}$
100th	[9500, 13,303.33]	[13,303.33, 15,054.50]	[15,054.50, 21,633.69]	[21,633.69, 23,019.50]	[23,019.50, 24,866.90]	[9500, 24,866.90] $\in U_{100th}$

parameters associated with QOA are either need to be assumed or computed. For the 1st iteration, values of parameters associated with the 5 search agents are listed in Table 6. Assumption and various computation process for these parameters are discussed in sub-step 4.1 to sub-step 4.8 (refer to Section 3). In the following, we discuss about the searching process of the optimal universe of discourse and its intervals from the randomly selected universe of discourse U_0 = [9500, 30,000], so that the AFER can be minimized.

- For the 1st iteration, position of each search agent is updated based on the universe of discourse U_0 = [9500, 30,000]. The new position of each search agent is considered as their local best position (i.e., LB). These local best positions for the 1st iteration are shown in Fig. 1. Using the local best positions of the search agents, we define new set of intervals along with the new universe of discourse. The new universe of discourse obtained from the 1st iteration is represented as U_{1st} = [9500, 27,762.10] (refer to Fig. 1). Intervals associated with this universe of discourse are listed in Table 7. Based on the U_{1st}, the NERGs are created from the NERs. Finally, the 1st iteration is ended up by performing deneutrosophication operation (refer to Step 10 in Section 3). The forecasted university enrollment values obtained after the 1st iteration are depicted in Table 8. After the 1st iteration, the proposed NTA-QOA model attains the AFER 1.83% (refer to Table 8).
- For the 99th iteration, updated position of each search agent is shown in Fig. 1. These local best positions for the 99th iteration are shown in Fig. 1. For the 99th iteration, values of parameters associated with the 5 search agents are listed in Table 6. Using the local best positions of the search agents, we define new set of intervals along with the new universe of discourse. The new universe of discourse obtained from the 99th iteration is represented as U_{99th} = [9500, 25,579.60] (refer to Fig. 1). Intervals associated with this universe of discourse are listed in Table 7. Based on the U_{99th}, the forecasted university enrollment values are depicted in Table 8. In this case, the proposed NTA-QOA model attains the AFER 0.49%.
- Finally, after attaining 100 iterations, computation process of the proposed NTS-QOA model is stopped. For the 100th iteration, updated position of each search agent is shown in Fig. 1. For the 100th iteration, values of parameters associated with the 5 search agents are listed in Table 6. The new universe of discourse obtained from the 100th iteration is represented as U_{100th} = [9500, 24,866.90] (refer to Fig. 1). Intervals associated with this universe of discourse are listed in Table 7. The forecasted university enrollment values after the 100th iteration are shown in Table 8. After the 100th iteration, the proposed NTA-QOA model attains the AFER 0.44%.

From Table 8, it is obvious that the proposed NTS-QOA model attained AFERs as 1.83%, 0.49% and 0.44% after the 1st, 99th and 100th iterations, respectively. Hence, the forecasted university enrollment values available after 100th iteration can be considered as the optimal one. And, the universe of discourse and its corresponding intervals through which these forecasted values are obtained, can be considered as the global best.

4. Empirical analysis

This section presents the forecasting results of the university enrollment dataset, TAIFEX index dataset and TSEC weighted index dataset. Forecasting results obtained by the proposed model is compared with various existing models. For comparison purpose, the AFER as defined in Eq. (26), is utilized.

4.1. Forecasting the university enrollment dataset

Initially, we present the empirical analysis of forecasting the university enrollment dataset. The performance of the proposed

Table 8
Forecasted university enrollment values obtained after the 1st, 99th and 100 iterations.

Iteration	Year	Actual enrollment	Entropy (using Eq. (5))	Desired entropy (using Eq. (47))	Forecasted enrollment (using Eq. (48))	AFER (%)
After 1st	1971	13,055	0.84	0.88	13,676.66	1.83
	1972	13,563	0.88	0.89	13,717.12	
	1973	13,867	0.89	0.93	14,490.23	
	1974	14,696	0.93	0.96	15,170.06	
	1975	15,460	0.96	0.97	15,621.04	
	1976	15,311	0.95	0.97	15,633.33	
	1977	16,388	0.98	0.97	16,220.77	
	1978	15,433	0.96	0.97	15,593.76	
	1979	15,984	0.97	0.98	16,148.78	
	1980	16,859	0.98	0.97	16,686.96	
	1981	18,150	0.99	0.97	17,783.33	
	1982	15,603	0.96	0.97	15,765.53	
	1983	15,861	0.97	0.98	16,024.51	
	1984	16,807	0.98	0.97	16,635.50	
	1985	16,919	0.98	0.97	16,746.35	
	1986	15,497	0.96	0.97	15,658.42	
	1987	18,970	0.99	0.97	18,586.76	
	1988	19,328	0.99	0.97	18,937.53	
	1989	19,337	0.99	0.97	18,946.35	
	1990	15,145	0.95	0.97	15,463.84	
	1991	15,163	0.95	0.97	15,482.22	
	1992	18,876	0.99	0.97	18,494.66	
After 99th	1971	13,055	0.88	0.91	13,500.05	0.49
	1972	13,563	0.91	0.93	13,861.08	
	1973	13,867	0.93	0.96	14,314.32	
	1974	14,696	0.96	0.98	15,002.16	
	1975	15,460	0.98	0.98	15,460.00	
	1976	15,311	0.97	0.97	15,311.00	
	1977	16,388	0.99	0.99	16,388.00	
	1978	15,433	0.97	0.97	15,433.00	
	1979	15,984	0.98	0.98	15,984.00	
	1980	16,859	0.99	0.99	16,859.00	
	1981	18,150	0.99	0.99	18,150.00	
	1982	15,603	0.98	0.98	15,603.00	
	1983	15,861	0.98	0.98	15,861.00	
	1984	16,807	0.99	0.99	16,807.00	
	1985	16,919	0.99	0.99	16,919.00	
	1986	15,497	0.98	0.98	15,497.00	
	1987	18,970	0.99	0.99	18,970.00	
	1988	19,328	0.98	0.98	19,328.00	
	1989	19,337	0.98	0.98	19,337.00	
	1990	15,145	0.97	0.97	15,145.00	
	1991	15,163	0.97	0.97	15,163.00	
	1992	18,876	0.99	0.99	18,876.00	
After 100th	1971	13,055	0.89	0.92	13,495.05	0.44
	1972	13,563	0.92	0.93	13,710.42	
	1973	13,867	0.93	0.96	14,314.32	
	1974	14,696	0.96	0.98	15,002.16	
	1975	15,460	0.98	0.98	15,460.00	
	1976	15,311	0.98	0.98	15,311.00	
	1977	16,388	0.99	0.99	16,388.00	
	1978	15,433	0.98	0.98	15,433.00	
	1979	15,984	0.99	0.99	15,984.00	
	1980	16,859	0.99	0.99	16,859.00	
	1981	18,150	0.99	0.99	18,150.00	
	1982	15,603	0.98	0.98	15,603.00	
	1983	15,861	0.99	0.99	15,861.00	
	1984	16,807	0.99	0.99	16,807.00	
	1985	16,919	0.99	0.99	16,919.00	
	1986	15,497	0.98	0.98	15,497.00	
	1987	18,970	0.98	0.98	18,970.00	
	1988	19,328	0.98	0.98	19,328.00	
	1989	19,337	0.97	0.97	19,337.00	
	1990	15,145	0.97	0.97	15,145.00	
	1991	15,163	0.97	0.97	15,163.00	
	1992	18,876	0.98	0.98	18,876.00	

Table 9
Comparison of the proposed NTS-QOA model with various existing models (for the university enrollment dataset).

Year	Actual enrollment	Model [2]	Model [4]	Model [5] (MEPA)	Model [5] (TFA)	Model [20]	Model [18]	Model [19]	Model [21]	Model [45]	Model [22]	Model [46]	Model [17]	Model [47]	Model [25]	Model [26]	Proposed model
1971	13,055	–	–	–	–	–	–	–	–	–	–	13,055	–	–	–	–	13,495.05
1972	13,563	14,000	14,025	15,430	14,230	14,195	14,242	13,563	13,500	13,563	13,563	14,250	13,480.38	14,000	–	13,706	13,710.42
1973	13,867	14,000	14,568	15,430	14,230	14,424	14,242	13,500	13,800	13,500	13,867	14,246	13,480.38	14,000	14,146	13,706	14,314.32
1974	14,696	14,000	14,568	15,430	14,230	14,593	14,242	14,500	14,700	14,500	14,696	14,246	14,780.95	14,000	14,878	14,749	15,002.16
1975	15,460	15,500	15,654	15,430	15,541	15,589	15,474	15,500	15,600	15,500	15,425	15,491	15,474.17	15,500	14,878	15,341	15,460
1976	15,311	16,000	15,654	15,430	15,541	15,645	15,474	15,466	15,400	15,500	15,420	15,491	15,523.89	16,000	15,609	15,346	15,311
1977	15,603	16,000	15,654	15,430	15,541	15,634	15,474	15,392	15,750	15,500	15,420	15,491	15,609.76	16,000	15,609	15,346	15,603
1978	15,861	16,000	15,654	15,430	16,196	16,100	15,474	15,549	15,400	15,500	15,923	16,345	15,562.98	16,000	16,214	15,923	15,861
1979	16,807	16,000	16,197	16,889	16,196	16,188	16,147	16,433	16,800	16,807	16,862	16,345	16,751.87	16,000	16,214	16,839	16,807
1980	16,919	16,813	17,283	16,871	16,196	17,077	16,988	16,656	17,100	16,919	17,192	15,850	16,837.83	16,833	16,818	17,046	16,919
1981	16,388	16,813	17,283	16,871	17,507	17,105	16,988	16,624	17,100	16,500	17,192	15,850	16,837.83	16,833	16,818	16,400	16,388
1982	15,433	16,789	16,197	15,447	16,196	16,369	16,147	15,556	15,300	15,500	15,425	15,850	15,901.87	16,833	15,609	15,455	15,433
1983	15,497	16,000	15,654	15,430	15,541	15,643	15,474	15,524	15,750	15,500	15,420	15,450	15,524.96	16,000	15,609	15,346	15,497
1984	15,145	16,000	15,654	15,430	15,541	15,648	15,474	15,497	15,400	15,500	15,420	15,450	15,527.33	16,000	14,146	15,346	15,145
1985	15,163	16,000	15,654	15,430	15,541	15,622	15,474	15,305	15,300	15,500	15,627	15,491	15,492.78	16,000	14,146	15,341	15,163
1986	15,984	16,000	15,654	15,430	15,541	15,623	15,474	15,308	15,750	15,984	15,627	15,491	15,570.69	16,000	16,818	15,923	15,984
1987	16,859	16,000	16,197	16,889	16,196	16,231	16,147	16,402	16,800	16,859	16,862	16,345	16,906.56	16,000	16,818	16,839	16,859
1988	18,150	16,813	17,283	16,871	17,507	17,090	16,988	18,500	17,100	18,500	17,192	17,950	16,837.83	16,833	17,992	18,007	18,150
1989	18,970	19,000	18,369	19,333	18,872	18,325	19,144	18,534	18,900	18,500	18,923	18,961	19,144.40	19,000	19,126	19,059	18,970
1990	19,328	19,000	19,454	19,333	18,872	19,000	19,144	19,345	19,200	19,337	19,333	18,961	19,144.40	19,000	19,126	19,197	19,328
1991	19,337	19,000	19,454	19,333	18,872	19,000	19,144	19,423	19,050	19,500	19,136	18,961	19,144.40	19,000	19,126	19,197	19,337
1992	18,876	–	–	19,333	18,872	19,000	19,144	18,752		18,704	19,136	18,961	19,144.40	19,000	19,126	19,059	18,876
AFER (%)	–	7.83	7.30	2.75	2.66	2.65	2.39	1.44	1.32	0.96	1.19	2.14	1.58	3.11	2.19	0.68	0.44

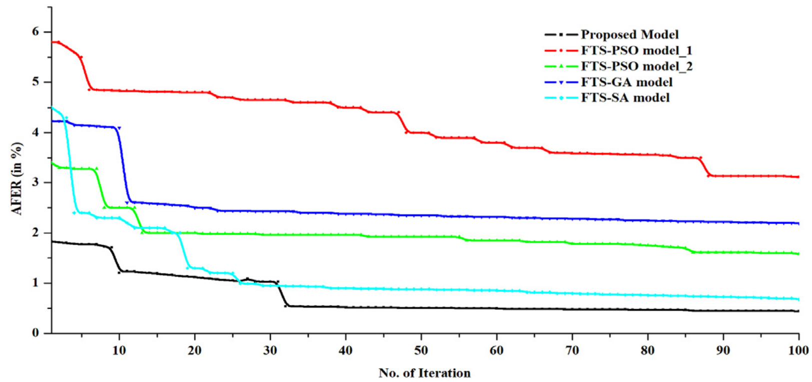

Fig. 2. A convergence curve showing the comparison between the proposed model and existing hybrid models.

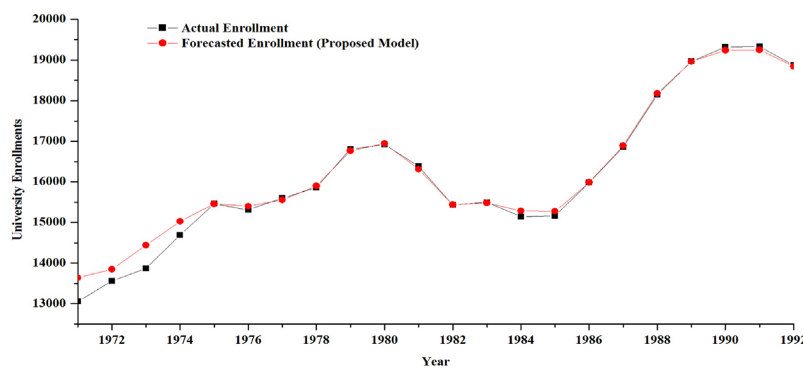

Fig. 3. A comparison of actual enrollment with forecasted enrollment based on the proposed model.

model is evaluated by comparing its forecasting accuracy with various existing benchmark models [2,4,5,18–22,45,46]. These forecasting results are presented in Table 9. From Table 9, it is observed that the proposed model exhibits a very minimum AFER, as compared to various existing benchmark models [2,4,5,18–22,45,46].

To validate the proposed hybrid model (i.e., NTS-QOA model), it has been compared with various well-known benchmark hybrid models of time series forecasting, such as FTS-GA model [25], FTS-PSO model_1 [47], FTS-PSO model_2 [17], and FTS-SA model [26]. The forecasted values of the enrollment dataset obtained through the proposed model and the considered benchmark hybrid models are also shown in Table 9 in terms of the AFER. From this table, it is obvious that the proposed hybrid model has the AFER 0.44%; whereas the existing hybrid models, such as the FTS-GA model [25], FTS-PSO model_1 [47], FTS-PSO model_2 [17], and FTS-SA model [26] have the AFERs 2.19%, 3.11%, 1.58%, and 0.68%, respectively. It indicates that the forecasting accuracy of the proposed model is also far better than the existing hybrid models [17,25,26,47]. The convergence analysis is also performed to compare the performance of the proposed model with the existing hybrid models [17,25,26,47]. From Fig. 2, it is obvious that the proposed model is fast convergent toward the optimal result in comparison to the existing hybrid models [17,25,26,47].

In Fig. 3, a graph is depicted, which shows the comparison between actual enrollment and forecasted enrollment based on the proposed model. From this figure, it is obvious that the forecasted enrollment are very close to that of actual enrollment.

4.2. Forecasting the TAIFEX index dataset

The proposed NTS-QOA model is applied to forecast the TAIFEX index dataset for the period 8/3/1998–9/30/1998 (mm/dd/yyyy). Forecasted results are depicted in Table 10. In Table 10, forecasted results of the proposed model is compared with existing models [7,16,48,49,47]. On comparison, it has been observed that the proposed model outperforms selected existing models [7,16,48,49,47].

The proposed NTS-QOA model is validated by forecasting the TAIFEX index dataset for the period 1/19/2006–12/21/2006 (mm/dd/yyyy). In Table 11, a comparison of forecasted results is presented with existing models [7,16,48,49,47]. The proposed model exhibits AFER of 0.0432%, which is very small in comparison to existing models [7,16,48,49,47].

In Figs. 4 and 5, two different graphs are presented, which show the comparison between actual TAIFEX and forecasted TAIFEX based on the proposed model for the periods 8/3/1998–9/30/1998 and 1/19/2006–12/21/2006, respectively. From these figures, it is obvious that the forecasted TAIFEX values are very close to that of actual TAIFEX values.

4.3. Forecasting the TSEC weighted index dataset

The proposed model is verified by forecasting the TSEC weighted index from the period 1999–2002. Finally, it is validated by forecasting the TSEC weighted index from the period 2003–2004. Forecasted results of the proposed NTS-QOA model from the periods 1999–2002 and 2003–2004 are compared with various

Table 10
Comparison of the proposed model with various existing models (for the TAIFEX index dataset) from the period 8/3/1998–9/30/1998.

Date	Actual data	Model [48]	Model [48]	Model [49]	Model [7]	Model [47]	Model [16]	Proposed model
08/03/1998	7552.00	–	–	–	–	–	7559.00	
08/04/1998	7560.00	7450.00	7450.00	–	–	–	–	7563.00
08/05/1998	7487.00	7450.00	7450.00	–	–	–	–	7490.00
08/06/1998	7462.00	7450.00	7500.00	7450.00	–	–	7452.54	7465.00
08/07/1998	7515.00	7500.00	7500.00	7550.00	–	–	7518.11	7519.00
08/10/1998	7365.00	7450.00	7450.00	7350.00	–	–	7359.49	7375.00
08/11/1998	7360.00	7350.00	7300.00	7350.00	–	–	7359.49	7365.00
08/12/1998	7330.00	7300.00	7300.00	7350.00	7329.00	7320.77	7331.62	7335.00
08/13/1998	7291.00	7350.00	7300.00	7250.00	7289.50	7289.56	7285.63	7294.00
08/14/1998	7320.00	7100.00	7188.33	7350.00	7329.00	7320.77	7331.62	7322.00
08/15/1998	7300.00	7350.00	7300.00	7350.00	7289.50	7289.56	7291.67	7303.00
08/17/1998	7219.00	7300.00	7300.00	7250.00	7215.00	7222.19	7217.15	7225.00
08/18/1998	7220.00	7100.00	7100.00	7250.00	7215.00	7222.19	7217.15	7223.00
08/19/1998	7285.00	7300.00	7300.00	7250.00	7289.50	7289.56	7285.63	7290.00
08/20/1998	7274.00	7100.00	7188.33	7250.00	7289.50	7289.56	7279.59	7286.00
08/21/1998	7225.00	7100.00	7100.00	7250.00	7215.00	7222.19	7217.15	7227.00
08/24/1998	6955.00	7100.00	7100.00	6950.00	6949.50	6952.76	6950.85	6960.00
08/25/1998	6949.00	6850.00	6850.00	6950.00	6949.50	6952.76	6941.88	6952.00
08/26/1998	6790.00	6850.00	6850.00	6750.00	6796.00	6800.07	6784.34	6791.00
08/27/1998	6835.00	6650.00	6775.00	6850.00	6848.00	6850.12	6843.35	6840.00
08/28/1998	6695.00	6750.00	6750.00	6650.00	6698.50	6713.46	6700.46	6710.00
08/29/1998	6728.00	6750.00	6750.00	6750.00	6726.00	6713.46	6721.85	6733.00
08/31/1998	6566.00	6650.00	6650.00	6550.00	6569.50	6568.29	6562.63	6570.00
09/01/1998	6409.00	6450.00	6450.00	6450.00	6417.00	6416.74	6402.61	6415.00
09/02/1998	6430.00	6550.00	6550.00	6450.00	6417.00	6416.74	6417.14	6435.00
09/03/1998	6200.00	6350.00	6350.00	6250.00	6205.00	6195.00	6191.69	6202.00
09/04/1998	6403.20	6450.00	6450.00	6450.00	6417.00	6416.74	6402.61	6409.32
09/05/1998	6697.50	6550.00	6550.00	6650.00	6698.50	6713.46	6700.46	6700.95
09/07/1998	6722.30	6750.00	6750.00	6750.00	6726.00	6713.46	6721.85	6725.33
09/08/1998	6859.40	6850.00	6850.00	6850.00	6848.00	6850.12	6852.31	6860.98
09/09/1998	6769.60	6750.00	6750.00	6750.00	6763.00	6767.08	6770.64	6775.16
09/10/1998	6709.75	6650.00	6650.00	6750.00	6726.00	6713.46	6711.15	6721.74
09/11/1998	6726.50	6850.00	6775.00	6750.00	6726.00	6713.46	6721.85	6728.94
09/14/1998	6774.55	6850.00	6775.00	6817.00	6763.00	6767.08	6784.34	6780.35
09/15/1998	6762.00	6650.00	6775.00	6817.00	6763.00	6767.08	6770.64	6767.00
09/16/1998	6952.75	6850.00	6850.00	6817.00	6949.50	6952.76	6950.85	6955.46
09/17/1998	6906.00	6950.00	6850.00	6950.00	6904.50	6905.44	6903.03	6912.00
09/18/1998	6842.00	6850.00	6850.00	6850.00	6848.00	6850.12	6843.35	6845.00
09/19/1998	7039.00	6950.00	6850.00	7050.00	7064.00	7033.83	7042.60	7045.00
09/21/1998	6861.00	6850.00	6850.00	6850.00	6848.00	6850.12	6852.31	6865.00
09/22/1998	6926.00	6950.00	6850.00	6950.00	6904.50	6905.44	6932.90	6932.00
09/23/1998	6852.00	6850.00	6850.00	6850.00	6848.00	6850.12	6852.31	6855.00
09/24/1998	6890.00	6950.00	6850.00	6850.00	6904.50	6905.44	6891.10	6895.00
09/25/1998	6871.00	6850.00	6850.00	6850.00	6848.00	6850.12	6861.26	6873.00
09/28/1998	6840.00	6750.00	6750.00	6850.00	6848.00	6850.12	6843.35	6845.00
09/29/1998	6806.00	6750.00	6850.00	6850.00	6796.00	6800.07	6798.04	6815.00
09/30/1998	6787.00	6750.00	6750.00	6750.00	6796.00	6800.07	6784.34	6800.00
AFER (%)	–	1.02	0.88	0.43	0.11	0.12	0.067	0.066

Table 11
Comparison of the proposed model with various existing models (for the TAIFEX index dataset) from the period 1/19/2006–12/21/2006.

Date	Actual data	Model [48]	Model [48]	Model [49]	Model [7]	Model [47]	Model [16]	Proposed model
1/19/2006	7649	–	–	–	–	–	7650	7648
2/16/2006	7270	7279	7279	7279.5	7279.5	7271.5	7273.5	7273
3/16/2006	7002	7009	7009	7005	7005	7005	7005	7000
4/20/2006	6926	6910	6910	6916	6916	6916	6916	6915
5/18/2006	6750	6739	6739	6749	6741	6741	6741	6751
6/22/2006	6418	6428.5	6428.5	6428.5	6428.5	6418.5	6418.5	6418.5
7/20/2006	6413	6433.5	6423.5	6423.5	6421.5	6421.5	6421.5	6413.5
8/17/2006	6998	6988.1	6988.1	6988.1	6988.1	6988.1	6998.1	6998.1
9/21/2006	7064	7088	7088	7088	7088	7088	7088	7068
10/19/2006	6519	6538.5	6538.5	6538.5	6538.5	6538.5	6538.5	6518.5
11/16/2006	6656	6678	6678	6678	6668	6661	6661	6668
12/21/2006	6491	6471	6491	6491	6491	6491	6491	6491
AFER (%)	–	0.23	0.19	0.16	0.16	0.12	0.11	0.0432

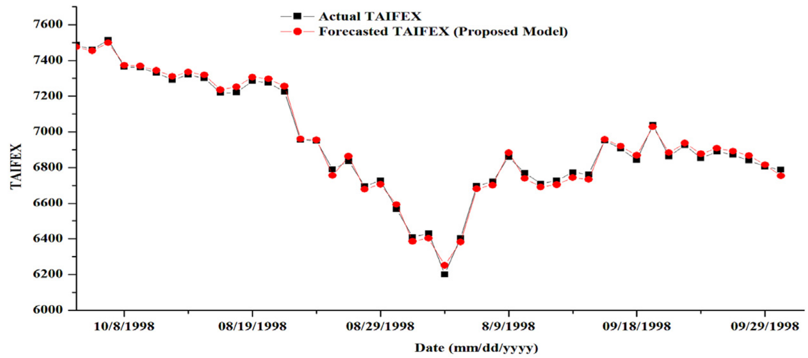

Fig. 4. A comparison of actual TAIFEX with forecasted TAIFEX based on the proposed model for the period 8/3/1998–9/30/1998.

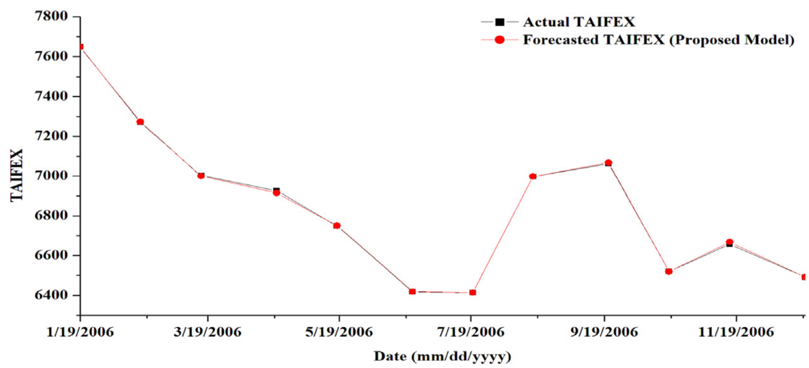

Fig. 5. A comparison of actual TAIFEX with forecasted TAIFEX based on the proposed model for the period 1/19/2006–12/21/2006.

Table 12
Comparison of the proposed model with various existing models (for the TSEC weighted index dataset) from the period 1999–2002.

Model	1999	2000	2001	2002	Average of AFER (%)
Huarng et al.'s model [50]	N/A	8.87	7.56	6.12	5.64
Yu and Huarng's model [9]	7.24	10.34	9.78	9.08	9.11
Univariate model [9]	10.56	14.45	8.12	6.8	9.98
AR-1 model [51]	7.02	8.09	6.73	6.26	7.03
AR-2 model [51]	7.13	7.58	7.23	6.02	6.99
Jiang et al.'s model [28]	1.45	7.12	6.83	5.57	5.24
Gupta et al.'s model [23]	1.10	2.28	3.82	2.14	2.34
Singh and Dhiman's model [27]	1.15	2.14	2.82	1.47	1.90
Proposed model	0.98	1.61	2.16	1.36	1.53

Table 13
Comparison of the proposed model with various existing models (for the TSEC weighted index dataset) from the period 2003–2004.

Model	2003	2004	Average of AFER (%)
Huarng et al.'s model [50]	5.41	6.87	6.14
Yu and Huarng's model [9]	8.67	8.97	8.82
Univariate model [9]	11.5	7.68	9.59
AR-1 model [51]	6.09	5.69	5.89
AR-2 model [51]	5.19	4.94	5.07
Jiang et al.'s model [28]	4.68	4.85	4.77
Gupta et al.'s model [23]	2.18	2.15	2.17
Singh and Dhiman's model [27]	1.17	2.15	1.66
Proposed model	0.68	0.84	0.76

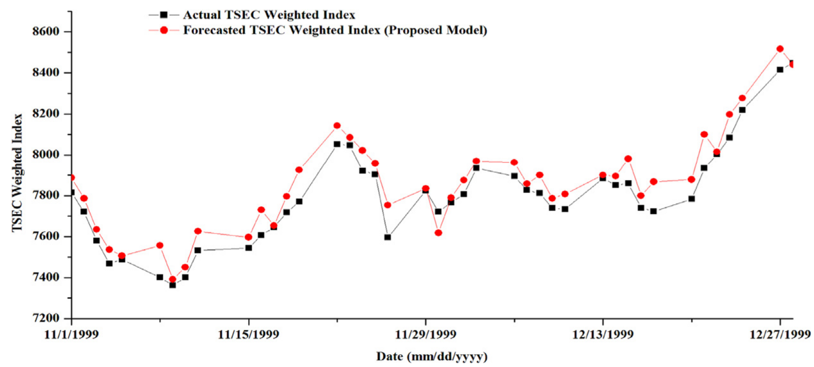

Fig. 6. A comparison of actual TSEC weighted index with forecasted TSEC weighted index for the year 1999 based on the proposed model.

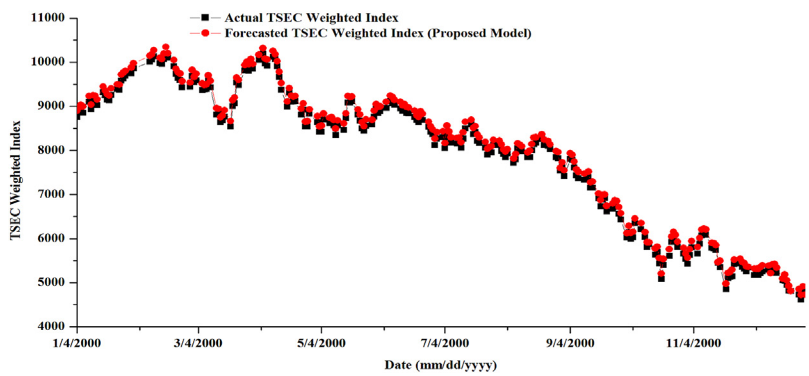

Fig. 7. A comparison of actual TSEC weighted index with forecasted TSEC weighted index for the year 2000 based on the proposed model.

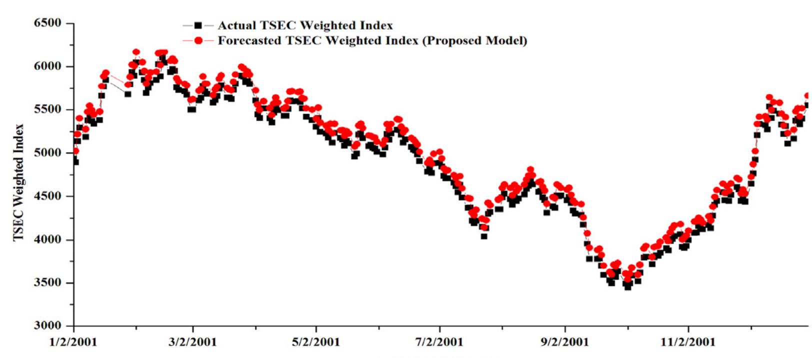

Fig. 8. A comparison of actual TSEC weighted index with forecasted TSEC weighted index for the year 2001 based on the proposed model.

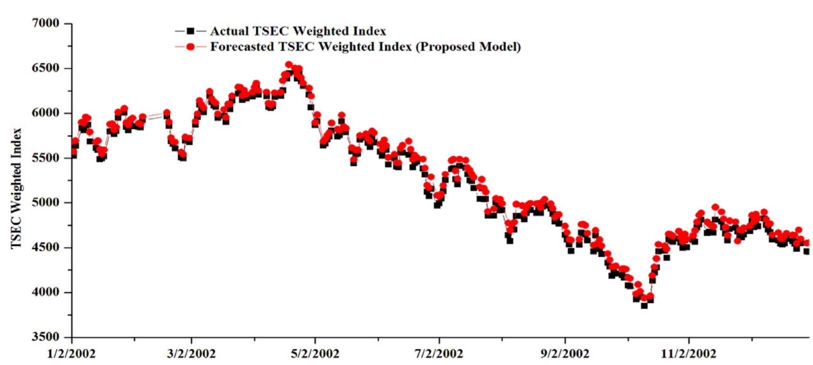

Fig. 9. A comparison of actual TSEC weighted index with forecasted TSEC weighted index for the year 2002 based on the proposed model.

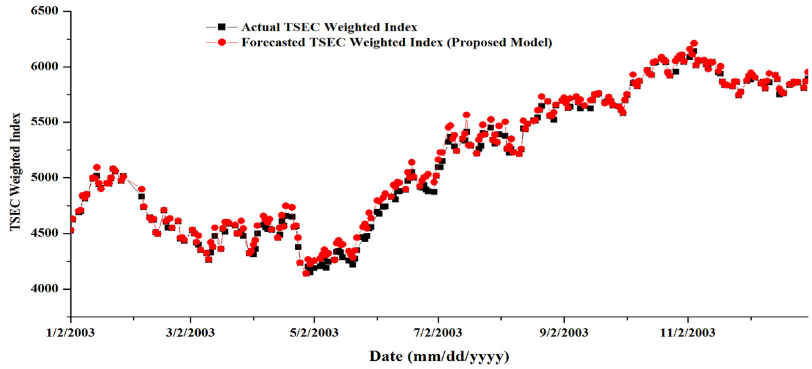

Fig. 10. A comparison of actual TSEC weighted index with forecasted TSEC weighted index for the year 2003 based on the proposed model.

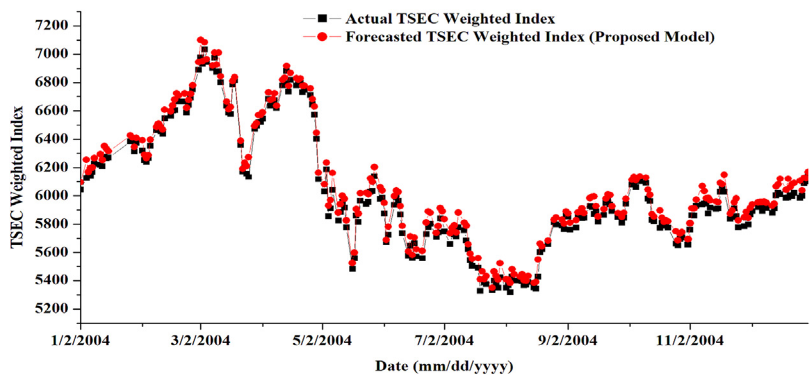

Fig. 11. A comparison of actual TSEC weighted index with forecasted TSEC weighted index for the year 2004 based on the proposed model.

existing models, as presented in articles [9,22,23,27,28,50–53]. Tables 12 and 13 demonstrate various comparison of the different models for forecasting the TSEC weighted index from the periods 1999–2002 and 2003–2004 by considering the average of AFER, respectively. It is obvious that the proposed model shows very smaller AFER than various existing models [50,9,51,22,52,53,28,23,27].

In Figs. 6–11, graphs are presented, which show the comparison between actual TSEC weighted index and forecasted TSEC weighted index for the period 1999–2004 based on the proposed model. From these figures, it is obvious that the forecasted TSEC weighted index values are very close to that of actual TSEC weighted index values.

4.4. Statistical analysis of the results

Performance of the proposed NTS-QOA has been evaluated using the statistical parameters, which include mean and standard deviations of the observed and forecasted values, correlation coefficient and Theil's U statistic. Definitions for all these parameters are given as follows [53]:

$$\text{Mean},\ \overline{M} = \frac{\sum_{i=1}^{n} Act_i}{n} \tag{51}$$

$$\text{Standarddeviation},\ S_d = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Act_i - \overline{M})^2} \tag{52}$$

Correlation coefficient,

$$CC = \frac{n\sum Act_i Fore_i - (\sum Act_i)(\sum Fore_i)}{\sqrt{n(\sum Act_i^2) - (\sum Act_i)^2}\sqrt{n(\sum Fore_i^2) - (\sum Fore_i)^2}} \tag{53}$$

$$\text{Theil's U statistic},\ U = \frac{\sqrt{\sum_{i=1}^{n}(Act_i - Fore_i)^2}}{\sqrt{\sum_{i=1}^{n} Act_i^2} + \sqrt{\sum_{i=1}^{n} Fore_i^2}} \tag{54}$$

Here, $Fore_i$ and Act_i represent the forecasted and actual values of particular day/year i, respectively, where n is the total number of days/years to be forecasted. In Eqs. (51 and 52), $\{Act_1, Act_2, \ldots, Act_n\}$ represent the actual time series values, and $\overline{M}$ represents the mean value of all these time series values. Similarly, using Eqs. (51 and 52), mean and standard deviation are obtained for the forecasted time series values. If the mean and standard deviation of the actual time series values are close to the mean and standard deviation of the forecasted time series values, then performance of the model can be considered as good in terms of forecasting accuracy. In Eq. (53), a correlation coefficient value CC must lie between the range $[-1, +1]$, i.e., $-1 \leq CC \leq +1$. A model can be considered as good if its CC value is greater than or equal to 0.8. In Eq. (54), the value of Theil's U statistic U must lie between the range [0, 1]. For good forecasting accuracy, the value of U must be close to 0.

Table 14
Statistical analysis of the forecasted results based on the proposed NTS-QOA model.

Data set (→)	University enrollment (see Table 9)		TAIFEX index (see Table 10)		TAIFEX index (see Table 11)		TSEC weighted index (see Table 12)		TSEC weighted index (see Table 13)	
Parameter (↓)	Actual value	Forecasted value	Actual value	Forecasted value	Actual value	Forecasted value	Actual value	Forecasted value	Actual value	Forecasted value
Mean	16,194.18	16,255.13	6964.25	6966.20	6846.33	6846.88	6445.84	6536.86	5597.84	5639.69
Standard deviation	1816.49	1734.55	331.58	335.07	378.38	377.82	796.49	796.61	515.16	507.11
Correlations	0.9978	–	0.9978	–	0.9999	–	0.9768	–	0.9898	–
Theil's U statistic	0.0002	–	0.0016	–	0.0062	–	0.0054	–	0.0055	–

All the statistics for forecasting the university enrollment, TAIFEX index and TSEC weighted index datasets based on the proposed NTS-QOA model are listed in Table 14. From Table 14, it can be observed that the means of actual and forecasted enrollments are 16,194.18 and 16,255.13, respectively, which are very close to each other. For the forecasted result presented in Table 10, the means of actual and forecasted TAIFEX are 6445.84 and 6536.86, respectively, which are also very close to each other. Similarly, the means of actual and forecasted TAIFEX are 5597.84 and 5639.69, respectively, which are also very close to each other in terms of forecasted result presented in Table 11. The means of actual and forecasted TSEC obtained through the proposed model (in terms of Tables 12 and 13) are very close to each other. Hence, in all the cases, the minimum differences between actual and forecasted means indicate good forecasting accuracy. The differences between actual and forecasted standard deviations in case of all the datasets are very low, which indicate the good predictive skill of the proposed NTS-QOA model. The correlation coefficients between actual and forecasted values in these datasets also indicate the efficiency of the proposed NTS-QOA model. The Theil's U statistics for all these datasets are close to 0, which indicate the effectiveness of the proposed NTS-QOA model. So, each statistic supports the NTS-QOA model, and it is strongly convinced with the outstanding performance in all cases of the university enrollment, TAIFEX index and TSEC weighted index datasets.

5. Conclusion

This study proposed a new hybrid model for time series forecasting based on the NS theory and QOA. NS theory was used to represent the time series dataset. This representation is termed as the NTS, and further used for modeling time series dataset. By employing the concept of entropy, this study established the NERs among the NTS representation of time series. These NERs were used in the deneutrosophication operation to obtain the forecasting results. The study showed that forecasting accuracy of the NTS modeling approach was mostly relied on the optimal selection of the universe of the discourse and its corresponding intervals. This significant research issue was resolved using the integration of QOA with the NTS modeling approach. The ensemble of QOA in the NTS modeling approach helped to search global optimal solution for the universe of discourse and its corresponding intervals from the list of local optimal solutions. The proposed NTS-QOA model was validated by the effective forecast of three different datasets, which include the university enrollment of Alabama, TAIFEX index and TSEC index. Forecasting results showed that the proposed model outperformed other existing models.

The limitation of the study is that the proposed NTS-QOA model is validated with univariate time series dataset. In the future, the proposed NTS-QOA model can be enhanced in such a way that it can be applied on forecasting the multivariate time series datasets.

Ethical approval

This article does not contain any studies with human participants performed by any of the authors.

Conflict of interest

The authors declare that they have no conflict of interest.

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