International Journal of Applied Operational Research
Vol. 9, No. 1, pp. 1-9, Winter 2019

Journal homepage: ijorlu.liau.ac.ir

A new approach for solving neutrosophic integer programming problems

A. H. Nafei*, S. H. Nasseri

Received: 25 August 2018**;** **Accepted:** 9 December 2018

Abstract Linear programming is one of the most important usages of operation research methods in real life, that includes of one objective function and one or several constraints which can be in the form of equality and inequality. Most of the problems in the real world are include of inconsistent and astute uncertainty, because of this reason we can't obtain the optimal solution easily. In this paper, we introduce a new model for Neutrosophic Integer Programming Problems where the coefficient of problems are neutrosophic numbers and by using a new score function will propose a method for solving them.

Keyword: Integer Programming, Neutrosophic, Neutrosophic Integer Programming, Neutrosophic Number.

1 Introduction

The Fuzzy Sets (FSs) theory which is proposed by Zadeh [11] is an applied approach to overcoming uncertainty, so that is assigned a membership function to any non-deterministic event. In fuzzy sets, the membership degree of an element in [0, 1] expresses the degree of belongingness of an element to a fuzzy set. Sometimes because of uncertainty determining the degree of membership isn't possible, for this reason, in 1975 Zadeh [12] proposed Interval Fuzzy Sets (IFSs) to express the uncertainty in the membership function. An interval-valued fuzzy set is a fuzzy set in which the membership degree is assumed to belong to an interval. Attanasov[2,3] by adding the degree of non-membership introduced another extension of fuzzy sets namely intuitionistic fuzzy sets which the degree of elements belong to an intuitionistic fuzzy set, respectively are presented by a membership degree and a non-membership degree in [0,1]. In 1995, Smarandache [7, 8] introduced neutrosophy which is the study of neutralities as an extension of dialectics. In [5] Mai Mohamed et al. by introducing a new score function proposed a novel method for neutrosophic integer programming problems.

Neutrosophic is the derivative of neutrosophy, and it includes neutrosophic set, neutrosophic probability, neutrosophic statistics, and neutrosophic logic. Neutrosophic theory means neutrosophy applied in many fields of sciences, to solve problems related to indeterminacy. Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle both incomplete and indeterminate

* **Corresponding Author.** (✉)
E-mail: Amir.nafei@e.gzhu.cn (A.H.Nafei)
A.H.Nafei
Department of Mathematics and Information Science, Guangzhou University, Guangzhou, China, 510006
S.H.Nasseri
Department of Mathematics and Big Data, Foshan University, Foshan, China
Department of mathematics, University of Mazandaran, Babolsar, Iran

information.[8] Neutrosophic sets characterized by three independent degrees namely truth-membership degree (*T*), indeterminacy-membership degree(*I*), and falsity-membership degree (*F*), where *T, I, F* are standard or non-standard subsets of $]0^-,1^+[$. The decision makers in neutrosophic set want to increase the degree of truth-membership and decrease the degree of indeterminacy and falsity membership. In this paper, we present a new method for solving integer programming problems under a neutrosophic environment with triangular neutrosophic numbers, and with using of a score function we convert neutrosophic integer problems into crisp problems. Integer programming problems can be defined as linear programming problems with integer restrictions on decision variables. When some, but not all decision variables are restricted to be an integer, this problem called a mixed integer problem, and when all decision variables are integers, it's a pure integer program. Integer programming plays an important role in supporting managerial decisions. In integer programming problems the decision maker may not be able to specify the objective function and constraints function precisely.

The structure of this paper is marshaled as follows: in section 2 briefly review of neutrosophic sets, single value neutrosophic sets, Complement, union and intersection between neutrosophic sets and describes the formulation of integer programming problems. In section 3 the proposed method for solving integer neutrosophic problems is presented. In section 4 we use an example to illustrate the proposed method. Conclusions are discussed in section 5.

2 preliminaries

2.1.Definitions

In this section, we briefly review of some preliminaries about neutrosophic sets, single-valued neutrosophic sets and some other details about them from ([1],[10],[4],[6]).

Definition 1. Let X be a space of objectives and $x \in X$. A neutrosophic set N in X is characterized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$ and a falsity-membership function $F_N(x)$, where $T_N(x), I_N(x)$ and $F_N(x)$ are real standard or real non-standard subsets of $\left]0^-,1^+\right[$, That is $T_N(x): X \to \left]0^-,1^+\right[$, $I_N(x): X \to \left]0^-,1^+\right[$ and $F_N(x): X \to \left]0^-,1^+\right[$. There is no a restriction on the sum of $T_N(x), I_N(x), F_N(x)$, So $\bar{0} \le T_N(x) + I_N(x) + F_N(x) \le 3^+$.

Definition 2. A Single-Valued Neutrosophic (SVN) set N through X taking the form $N = \{x, T_N(x), \mathrm{F}_N(x), \mathrm{I}_N(x); x \in X\}$, where X be a universe of discourse, $T_N(x): X \to [0,1]$, $F_N(x): X \to [0,1]$ and $I_N(x): X \to [0,1]$ with $0 \le T_N(x) + F_N(x) + I_N(x) \le 3$ for all $x \in X$. $T_N(x), F_N(x)$ and $I_N(x)$, respectively represent truth membership, falsity membership and indeterminacy membership degree of x to N.

Definition 3. if N be a single valued neutrosophic set, then the complement of N is calculated as follows:

$$T'_N(x) = F_N(x), \ \ \mathrm{I}'_N(x) = 1 - I_N(x), \ \ F'_N(x) = T_N(x), \ \forall x \in X.$$

Definition 4. The union of two single-valued neutrosophic sets N and M is a single-valued neutrosophic set R, whose truth membership function, indeterminacy-membership function and falsity membership function is related to those of A and B by

$T_R(x) = \max(T_N(x), T_M(x)),\ I_R(x) = \max(I_N(x), I_M(x)),\ \ F_R(x) = \min(F_N(x), F_M(x)),\ \ \forall x \in X.$

Definition 5. The intersection of two single-valued neutrosophic sets N and M is a single-valued neutrosophic set S, whose truth membership function, indeterminacy-membership function and falsity membership function is related to those of A and B by

$T_S(x) = \min(T_N(x), T_M(x)),\ I_S(x) = \min(I_N(x), I_M(x)),\ \ F_S(x) = \max(F_N(x), F_M(x)),\ \ \forall x \in X.$

Definition 6. Let $N = [(a^l, a^m, a^u); \mu_N, \nu_N, \gamma_N]$ and $M = [(\mathrm{b}^l, \mathrm{b}^m, \mathrm{b}^u); \mu_M, \nu_M, \gamma_M]$ be two triangular neutrosophic numbers, the mathematical operations between N and M are as follows:

$$N + M = [(a^l + b^l, a^m + b^m, a^u + b^u); \mu_N \wedge \mu_M, \nu_N \vee \nu_M, \gamma_N \vee \gamma_M], \tag{1}$$

$$N - M = [(a^l - b^l, a^m - b^m, a^u - b^u); \mu_N \wedge \mu_M, \nu_N \vee \nu_M, \gamma_N \vee \gamma_M], \tag{2}$$

$$kN = \begin{cases} [(ka^l, ka^m, ka^u); \mu_N, \nu_N, \gamma_N], & k > 0, \\ [(ka^u, ka^m, ka^l); \mu_N, \nu_N, \gamma_N], & k < 0, \end{cases} \tag{3}$$

Where $"\wedge"$ and $"\vee"$ are respectively mean that minimum and maximum operators.

Remark 1.We show the set of all triangular neutrosophic numbers by $N(\mathbb{R})$.

In the next definition, we will define one of the important concepts which have a key role to preparing the solving methods of mathematical programming where the parameters of the models are considered based on a kind of uncertainty such as fuzziness, intuitionistic, and neutrosophic.

Here we give the general definition for ordering on the set of $N(\mathbb{R})$.

Definition 7. A ranking function R on $N(\mathbb{R})$ is a map from $N(\mathbb{R})$ to the real numbers line, where the natural ordering there exists. Consider two triangular neutrosophic numbers as: $N = [(a^l, a^m, a^u); \mu_N, \nu_N, \gamma_N]$ And $M = [(\mathrm{b}^l, \mathrm{b}^m, \mathrm{b}^u); \mu_M, \nu_M, \gamma_M]$. Then, the ranking function is defined as follows:

i. If $R(N) > R(M)$ then $N > M$.
ii. If $R(N) < R(M)$ then $N < M$.
iii. If $R(N) = R(M)$ then $N = M$.

2.2 Neutrosophic Integer Programming Problems

An integer programming problem with neutrosophic factors is presented as bellows:

$$\begin{aligned} &Max\ \mathrm{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j \\ &s.t.\ \sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i \quad i = 1,2,\ldots,m, \\ &\qquad x_j \ge 0, \qquad j = 1,2,\ldots,n. \end{aligned} \tag{4}$$

where x_j is an integer variable and $\tilde{c}_j, \tilde{a}_{ij}\ \tilde{b}_i$ represented the neutrosophic numbers.

Remark 1. We also can consider the integer neutrosophic programming problems in which integer neutrosophic numbers represent the values of variables, but because of in real calculate we always prefer to obtain the crisp value as an optimal solution, so in this research, we always consider the values of x_j as a real integer number.

The truth, indeterminacy and falsity membership functions of each triangular neutrosophic number same as N are defined as follows, respectively:

$$T_N(x)=\begin{cases}\dfrac{x-a^l}{a^m-a^l}, & a^l \le x \le a^m,\\ \dfrac{a^u-x}{a^u-a^m}, & a^m \le x \le a^u,\\ 0, & \text{O.W},\end{cases} \tag{5}$$

$$I_N(x)=\begin{cases}\dfrac{a^m-x}{a^m-a^l}, & a^l \le x \le a^m,\\ \dfrac{x-a^m}{a^u-a^m}, & a^m \le x \le a^u,\\ 0, & \text{O.W},\end{cases} \tag{6}$$

$$F_N(x)=\begin{cases}\dfrac{a^m-x}{a^m-a^l}, & a^l \le x \le a^m,\\ \dfrac{x-a^m}{a^u-a^m}, & a^m \le x \le a^u,\\ 1, & \text{O.W},\end{cases} \tag{7}$$

The membership functions of the above triangular neutrosophic number are presented in Fig. 1.

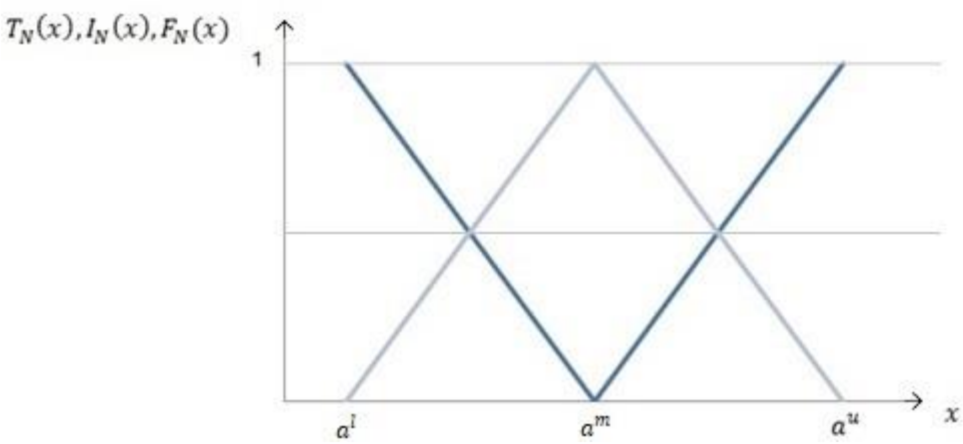

Fig. 1. Truth, falsity and indeterminacy membership functions of a triangular neutrosophic number

So the maximum value of the objective function for truth membership and the minimum values of the objective function for indeterminacy and falsity memberships can obtain as follows:
$f_{\max} = \max\{f(x_i^*)\}$ and $f_{\min} = \min\{f(x_i^*)\}$ for $(i = 1,\ldots,n)$ also $f_{\max}^F = f_{\max}^T - P(f_{\max}^T - f_{\min}^T)$, $f_{\min}^F = f_{\min}^T$ and $f_{\max}^I = f_{\max}^I$, $f_{\min}^I = f_{\min}^I - K(f_{\max}^T - f_{\min}^T)$ where P and K are real numbers in (0,1).
Therefore the truth membership of objective function will be shown as:

$$T(f(x)) = \begin{cases} 1 & f \le f_{\min} \\ \dfrac{f_{\max} - f(x)}{f_{\max} - f_{\min}} & f_{\min} < f(x) \le f_{\max} \\ 0 & f(x) > f_{\max} \end{cases} \tag{8}$$

Also the indeterminacy membership of objective function will be shown as:

$$I(f(x)) = \begin{cases} 0 & f \le f_{\min} \\ \dfrac{f(x) - f_{\max}}{f_{\max} - f_{\min}} & f_{\min} < f(x) \le f_{\max} \\ 0 & f(x) > f_{\max} \end{cases} \tag{9}$$

And finally the falsity membership of objective function will be shown as follow:

$$F(f(x)) = \begin{cases} 0 & f \le f_{\min} \\ \dfrac{f(x) - f_{\min}}{f_{\max} - f_{\min}} & f_{\min} < f(x) \le f_{\max} \\ 1 & f(x) > f_{\max} \end{cases} \tag{10}$$

The neutrosophic set for the j^{th} variable x_j will be shown as follows:

$$T(x_j) = \begin{cases} 1 & x_j \le 0 \\ \dfrac{m_j - x_j}{m_j} & 0 < x_j \le m_j \\ 0 & x_j > m_j \end{cases} \tag{11}$$

$$I(x_j) = \begin{cases} 0 & x_j \le 0 \\ \dfrac{x_j - m_j}{m_j + n_j} & 0 < x_j \le m_j \\ 0 & x_j > m_j \end{cases} \tag{12}$$

$$F(x_j) = \begin{cases} 0 & x_j \le 0 \\ \dfrac{x_j}{m_j + n_j} & 0 < x_j \le m_j \\ 1 & x_j > m_j \end{cases} \tag{13}$$

Where, m_j and n_j are integer numbers.

Nevertheless the neutrosophic optimize integer programming problem can be written as follow:

$$\begin{aligned}
&\max T(x)\\
&\min I(x)\\
&\min F(x)\\
&s.t.\\
&\quad T(x) \geq F(x)\\
&\quad T(x) \geq I(x)\\
&\quad 0 \leq T(x)+I(x)+F(x) \leq 3\\
&\quad T(x), I(x), F(x) \geq 0\\
&\quad x \geq 0 \text{ is integer.}
\end{aligned} \tag{14}$$

The previous problem can be written to the equivalent form as follows:

$$\begin{aligned}
&\max \mu, \min \partial, \min \nu\\
&s.t.\\
&\quad \mu \leq T(x)\\
&\quad \nu \geq F(x)\\
&\quad \partial \geq I(x)\\
&\quad \mu \geq \nu\\
&\quad \mu \geq \partial\\
&\quad 0 \leq \mu+\nu+\partial \leq 3\\
&\quad x \geq 0, \ is \text{ integer,}
\end{aligned} \tag{15}$$

where μ represented the minimal degree of acceptation, ν represented the maximal rejection degree and ∂ represented the maximal degree of indeterminacy.

The previous model can be written to another type of neutrosophic optimization model where formulated as follow:

$$\begin{aligned}
&\max(\mu-\partial-\nu)\\
&s.t.\\
&\quad \mu \leq T(x)\\
&\quad \nu \geq F(x)\\
&\quad \partial \geq I(x)\\
&\quad \mu \geq \nu\\
&\quad \mu \geq \partial\\
&\quad 0 \leq \mu+\nu+\partial \leq 3\\
&\quad \mu, \nu, \partial \geq 0\\
&\quad x \geq 0 \text{ is integer}
\end{aligned} \tag{16}$$

where the above model is equivalent to:

$$
\begin{aligned}
& \min (1-\mu)+\nu+\partial \\
& s.t. \\
& \quad \mu \leq \mathrm{T}(x) \\
& \quad \nu \geq F(x) \\
& \quad \partial \geq \mathrm{I}(x) \\
& \quad \mu \geq \nu \\
& \quad \mu \geq \partial \\
& \quad 0 \leq \mu+\nu+\partial \leq 3 \\
& \quad x \geq 0, \text{ is integer.}
\end{aligned} \tag{17}
$$

3 The new method for solving Neutrosophic Integer Programming Problems

In this section, we introduced a new approach to finding the optimal solution for solving integer neutrosophic problems. At first, by using a score function, we convert neutrosophic integer programming problem into the crisp model and then by using of Branch and Bound Algorithm solve it same as the classic integer programming problem. The algorithm of the proposed method is presented as follows:

Step 1. Let $N=(a^l,a^m,a^u;\mu_N,\partial_N,\nu_N)$ be a triangular neutrosophic number, where a^l,a^m,a^u, represented the lower bound, median value and upper bound of triangular neutrosophic number, respectively. Also μ_N,∂_N,ν_N denotes the truth degree, indeterminacy degree and falsity degree of a triangular number N. The score function of this number is calculated as follow:

$$
S(N)=\frac{a^l+a^u+2a^m}{4}+|\,\mu_N-\partial_N-\nu_N\,|, \tag{18}
$$

By using of this score function, each single-valued triangular neutrosophic number will convert to a crisp number.

Step 2. Create the decision set which includes the highest degree of truth-membership and the least degree of falsity and indeterminacy memberships.

Step 3. Find the optimal solution to the linear programming model with the integer restrictions relaxed.

Step 4. At the previous node let the relaxed solution be the upper bound and the rounded-down integer solution be the lower bound.

Step 5. Select the variable with the greatest fractional part for branching. Create two new constraints for this variable reflecting the partitioned integer values. The result will be a new $\leq$ constraint and a new $\geq$ constraint.

Step 6. Create two new nodes, one for the $\geq$ constraint and one for the $\leq$ constraint.

Step 7. Solve the relaxed linear programming model with the new constraint added at each of these nodes.

Step 8. The relaxed solution is the upper bound at each node, and the existing maximum integer solution is the lower bound.

Step 9. If the process produces a feasible integer solution with the greatest upper bound of these nodes. Integer solution (at any node) is the lower bound value of any ending node; the optimal integer solution has been reached. If feasible integer solution doses not emerge, a branch from the node with the greatest upper bound.

Step 10. Return to step 5.

For a minimization model, relaxed solutions are rounded up, and upper and lower bounds are reversed.

4 Numerical example

In this section, we present two examples to illustrating the efficiency of our proposed method. The order of each element for triangular neutrosophic numbers in all examples are considered as follows: lower bound, median bound and upper bound.

$$\begin{aligned} &max\tilde{Z} \approx \tilde{4}x_1 + \tilde{3}x_2 \\ &s.t. \quad \tilde{4}x_1 + \tilde{2}x_2 \,\tilde{\le}\, 12 \\ &\qquad\;\; \tilde{3}x_1 + \tilde{6}x_2 \,\tilde{\le}\, \tilde{5} \\ &\qquad\;\; x_1, x_2 \ge 0 \text{ integer} \end{aligned}$$

where x_1, x_2, x_3 are integer variables and

$$\begin{aligned} \tilde{4} &= (2,4,6), 0.8, 0.6, 0.4 \\ \tilde{3} &= (1,3,5), 0.75, 0.5, 0.3 \\ \tilde{4} &= (0,4,8), 1, 0.0, 0.5 \\ \tilde{2} &= (1,2,3), 1, 0.5, 0.5 \\ 12 &= (5,12,19), 1, 0.25, 0.25 \\ \tilde{3} &= (1,3,5), 0.75, 0.0, 0.25 \\ \tilde{6} &= (1,6,11), 1, 0, 0 \\ \tilde{5} &= (3,5,7), 0.8, 0, 6, 0.4 \end{aligned}$$

By using the score function proposed in Eq. (18) the previous problem will be converted to the crisp model as follows:

$$\begin{aligned} &max \;\; Z = 4.2x_1 + 3.05x_2 \\ &s.t. \qquad 4.5x_1 + 2x_2 \le 12.5 \\ &\qquad\quad\;\; 3.5x_1 + 6x_2 \le 5.4 \\ &\qquad\quad\;\; x_1, x_2 \ge 0 \end{aligned}$$

By following the steps that presented in the last section the optimal solution of the above problem is $x^* = (1,0)$ and $Z^* = 4.2$.

5 Conclusions

In this research, we concentered the integer neutrosophic programming problem and proposed a new method for solving these problems based on introducing a novel ordering approach. To increase the acceptance degree and reduce the degrees of indeterminacy and rejection we proposed a score function whereby using this function, we can convert every neutrosophic model into a crisp model. In particular, the illustrative example explored to solve the mentioned problem based on the established approach. The numerical example showed the efficiency of the proposed method.

References

1. Abdel-Baset, M., Hezam, I. M., Smarandache, F. (2016). Neutrosophic goal programming. Neutrosophic Sets & Systems, 11, 25-34.
2. Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87–96.
3. Atanassov, K, Gargov, G. (1989). Interval-valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 31(3), 343–349.
4. Hezam, I. M., Abdel-Baset, M., Smarandache, F. (2015). Taylor series approximation to solve neutrosophic multi-objective programming problem, 10, 39-45.
5. Mohamed, M., Abdel-Basset, M., Zaied, A. N. H., & Smarandache, F. (2017). Neutrosophic Integer Programming Problem, 15, 3-8.
6. Şahin, R., Yiğider, M. (2014). A Multi-criteria neutrosophic group decision-making method based TOPSIS for supplier selection. arXiv preprint arXiv:1412.5077.
7. Smarandache, F. (2004). A geometric interpretation of the neutrosophic set-A generalization of the intuitionistic fuzzy set. arXiv preprint math/0404520.
8. Smarandache, F. (2005). A generalization of the intuitionistic fuzzy set. International Journal of Pure and Applied Mathematics, 24, 287-297.
9. Smarandache, F. (1999). A Unifying Field in Logics: Neutrosophic Logic in Philosophy. American Research Press, 70, 1-141.
10. Wang, H., Smarandache, F., Zhang, Y. Q., Sunderraman, R., (2010). Single-valued neutrosophic sets, Multispace and Multistructure, 4, 410-413.
11. Zadeh, L. A. (1965). Fuzzy sets. Information and control, 8(3), 338-353.
12. Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. Information sciences, 8(3), 199-249.