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A LINEAR FRACTIONAL PROGRAMMING PROBLEM IN NEUTROSOPHIC ENVIRONMENT

S.N. Mohamed Assarudeen, N. Suresh Kumar and S. Suresh

PG & Research Department of Mathematics
Islamiah College(Autonomous), Vaniyambadi - 635 752
Tirupattur Dt, Tamilnadu, India.

Abstract

In this paper a fractional linear programming problem is solved in Neutrosophic Environment. The proposed method for solving is based on modified simplex procedure for fractional function. This method is explained details with the help of Numerical illustration.

Keywords: Fractional Linear Programming Problem, Neutrosophic Environment, Modified Simplex Procedure, Fractional Function.

1. Introduction

Fractional programming problem is that in which the objective function is the ratio of numerator and denominator. These types of problems have attracted considerable research and interest. Since these are useful in production planning, financial and corporate planning, health care and hospital planning etc.

Algorithms for solving linear fractional programming problems are well known by many. Charnes and Copper [1] replaces a linear fractional program by one equivalent linear fractional program, in which one extra constraint and one extra variable has been added. The usual simplex algorithm computes the optimum solution. Chadha-Caldile [2] solves a system of linear of inequalities in which the objective function is expressed as one of the constraint along with the given set of linear constraints of the problem. Resently Tantawy [7] has suggested a feasible direction approach and a duality approach to solve a linear fractional programming problem. Here our aim is to find the solution of fractional programming problems in Neutrosophic environment (i.e. objective function is the ratio of numerator and denominator of linear functions). For it, we use modified simplex method. These methods are very easy to understand and apply.

Preliminaries are given in the next section. The steps of the proposed Modified Simplex Algorithm in Neutrosophic Environment are presented. Numerical illustration have been worked out in the paper and finally we present the references.

2. Preliminaries

Definition 2.1 (Single Valued Neutrosophic Number). *Let $w_a, u_a, y_a \in [0,1]$ be any real numbers. A single valued neutrosophic number $\tilde{a} = \{< a_1, b_1, c_1 >; w_a, u_a, y_a\}$ is defined as a special neutrosophic set on the real number set R, Whose truth-membership, indeterminacy-membership, and a flasty-membership are given as follows:-*

$$\mu_a(x) = \begin{cases} \dfrac{(x-a)w_a}{b_1 - a_1} & (a_1 \leqslant x < b_1) \\ w_a & (x = b_1) \\ \dfrac{(c_1 - x)w_a}{c_1 - d_1} & (b_1 \leqslant x < c_1) \\ 0 & Otherwise \end{cases}$$

$$\mu_a(x) = \begin{cases} \dfrac{(b_1 - x + u_a(x-a))}{b_1 - a_1} & (a_1 \leqslant x < b_1) \\ u_a & (x = b_1) \\ \dfrac{(x - b_1 + u_a(c_1 - x))}{c_1 - d_1} & (b_1 \leqslant x < c_1) \\ 1 & Otherwise \end{cases}$$

$$\mu_a(x) = \begin{cases} \dfrac{(b_1 - x + y_a(x-a))}{b_1 - a_1} & (a_1 \leqslant x < b_1) \\ y_a & (x = b_1) \\ \dfrac{(x - b_1 y_a(c_1 - x))}{c_1 - d_1} & (b_1 \leqslant x < c_1) \\ 1 & Otherwise \end{cases}$$

respectively.

If $a_1 \geqslant 0$ and at least $c_1 > 0$, then $\tilde{a} =< (a_1, b_1, c_1); w_a, u_a, y_a >$ is called a positive single valued triangular neutrosophic number, denoted by $\tilde{a} > 0$.

Similarly, If $c_1 \leqslant 0$ and at least $a_1 < 0$, then $\tilde{a} =< (a_1, b_1, c_1); w_a, u_a, y_a >$ is called a negative single valued triangular neutrosophic number, denoted by $\tilde{a} < 0$.

An ill-known quantity of the range, which is approximately equal to b_1 may be represented by a single valued triangular neutrosophic number $\tilde{a} = \{\langle a_1, b_1, c_1 \rangle ; w_a, u_a, y_a\}$.

Definition 2.2. *Let $\tilde{a}^n = \{\langle a_1, b_1, c_1 \rangle ; w_a, u_a, y_a\}$ and $\tilde{b}^n = \{\langle a_2, b_2, c_2 \rangle ; w_b, u_b, y_b\}$ be two single valued triangular neutrosophic numbers and $\gamma \neq 0$. Then*

1. $\tilde{a}^n + \tilde{b}^n = \{\langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}$

2. $\tilde{a}^n - \tilde{b}^n = \{\langle a_1 - c_2, b_1 - b_2, c_1 - a_2\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}$

3. $$\tilde{a}^n.\tilde{b}^n = \begin{cases} \{\langle a_1a_2, b_1b_2, c_1c_2\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}, (c_1 > 0, c_2 > 0) \\ \{\langle a_1c_2, b_1b_2, c_1a_2\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}, (c_1 < 0, c_2 > 0) \\ \{\langle c_1c_2, b_1b_2, a_1a_2\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}, (c_1 < 0, c_2 < 0) \end{cases}$$

4. $$\frac{\tilde{a}^n}{\tilde{b}^n} = \begin{cases} \left\{\left\langle \frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}\right\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\right\}, (c_1 > 0, c_2 > 0) \\ \left\{\left\langle \frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\right\}, (c_1 < 0, c_2 > 0) \\ \left\{\left\langle \frac{c_1}{c_2}, \frac{b_1}{b_2}, \frac{a_1}{a_2}\right\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\right\}, (c_1 < 0, c_2 < 0) \end{cases}$$

5. $$\gamma\tilde{a}^n = \begin{cases} \{(\gamma a_1^l, \gamma b_1^m, \gamma c_1^u); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}, (\gamma > 0) \\ \{(\gamma c_1^u, \gamma b_1^m, \gamma a_1^l); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}, (\gamma < 0). \end{cases}$$

2.1. Neutrosophic Linear Fractional Programming Problem

A maximization linear fractional programming problem may be stated as:

$$Max.\tilde{z}^n = \frac{c\tilde{x}^n + \alpha}{d\tilde{x}^n + \beta} \tag{2.1}$$

$$\begin{aligned} s.t \qquad A\tilde{x}^n &\leqslant \tilde{B}^n \\ \tilde{x}^n &\geqslant \{\langle 0,0,0\rangle ; 1,0,0\} \end{aligned}$$

Where $\tilde{x}^n, \tilde{z}^n, \tilde{B}^n$ are Neutrosophic Single Valued Triangular Number.

3. Modified Simplex Algorithm in Neutrosophic Environment

Step: 1. *Converts the inequality constraints to equations by introducing the non-negative slack or surplus variables. The coefficients of slack or surplus variables are always taken zero in the objective function.*

Step: 2. *Constructs the simplex table by using the following notations. Let $\tilde{x}_A^n$ be the initial basic feasible solution of the given problem such that*

$$A\tilde{x}_B^n = \tilde{B}^n$$

$$\begin{aligned} \tilde{x}_B^n &= \tilde{B}^n A^{-1} \\ \text{Where } \tilde{B}^n &=< (a,b,c); w_a, u_a, y_a >, \\ Z_1 &= c_B \tilde{x}_B + \alpha \\ Z_2 &= d_B \tilde{x}_B + \beta \end{aligned}$$

where c_B and d_B are the vectors having their components as the coefficients associated with the basic variables in the numerator and denominator of the objective function respectively.

Step: 3. *First we compute the values for $\tilde{z}_1, \tilde{z}_2$ and z. Where*

$$\begin{aligned} \tilde{z}_1 &= \sum \{(co - efficientofc_j) \times \tilde{B}^n\} + \alpha \\ \tilde{z}_2 &= \sum \{(co - efficientofd_j) \times \tilde{B}^n\} + \beta \\ z &= \frac{\tilde{z}_1}{z_2} \end{aligned}$$

Step: 4. *To calculate Δ_j^1 row is $c_j - \sum c_B \tilde{x}_B^n$ for each variable x_j. In this way we calculate the values of Δ_j^2 row is $d_j - \sum d_B \tilde{x}_B^n$ for each variable x_j.*

Step: 5. *Now, compute the evaluation Δ_j for each variable x_j (column vector x_j) by the formula*

$$\Delta_j = \tilde{Z}_2 \left(c_j - \sum c_B \tilde{x}_B^n \right) - \tilde{Z}_1 \left(d_j - \sum d_B \tilde{x}_B^n \right)$$

Step: 6. *To find the Entering variable, in the most positive value of Δ_j*

Step: 7. *Next we find the Minimum ratio column $\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$.*

Step: 8. *Then we find the Leaving variable, to select the least positive value of the Minimum ratio $\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$ column.*

Step: 9. *If all Δ_j less than (or) equal to $\{\langle 0,0,0\rangle ; 1,0,0\}$, the optimal solution is obtained. Then we get the required solution. Finally stop the iteration.*

Step: 10. *Otherwise, continued in Step-(3) to Step-(9).*

4. Numerical Example

Problem 4.1. *Find the solution of following linear fractional programming problem*

$$max.\tilde{Z} = \frac{\tilde{x}_1 + 2\tilde{x}_2}{2\tilde{x}_1 - \tilde{x}_2 + 2}$$

$$s.t \quad -\tilde{x}_1 + 2\tilde{x}_2 \leqslant \{\langle 1,2,3\rangle ; 0.6, 0.5, 0.3\}$$

$$\tilde{x}_1 + \tilde{x}_2 \leqslant \{\langle 2,4,6\rangle ; 0.2, 0.6, 0.5\}$$

$$\tilde{x}_1^n, \tilde{x}_2^n \geqslant \{\langle 0,0,0\rangle ; 1, 0, 0\}$$

Solution: The Objective function is,

$$max.Z = \frac{\tilde{x}_1 + 2\tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4 + 0}{2\tilde{x}_1 - \tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4 + 2}$$

After adding slack variables, $\tilde{x}_3$ and $\tilde{x}_4$, the constraints become

$$-\tilde{x}_1 + 2\tilde{x}_2 + \tilde{x}_3 = \{\langle 1,2,3\rangle ; 0.6, 0.5, 0.3\}$$

$$\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_4 = \{\langle 2,4,6\rangle ; 0.2, 0.6, 0.5\}$$

$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geqslant \{\langle 0,0,0\rangle ; 1, 0, 0\}$$

Table:1.1

	$\alpha = 0$		c_j	1	2	0	0	Min.Ratio
B.V	$\beta = 2$		d_j	2	-1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_3$	0	0	$\{< 1,2,3 >;$ $0.6, 0.5, 0.3\}$	-1	2	1	0	
$\tilde{x}_4$	0	0	{¡2,4,6¿; 0.2,0.6,0.5}	1	1	0	1	
$\tilde{z}_1$				$\Delta_{11}^{(1)}$	$\Delta_{12}^{(1)}$	$\Delta_{13}^{(1)}$	$\Delta_{14}^{(1)}$	$\Delta_j^{(1)}$
$\tilde{z}_2$				$\Delta_{21}^{(1)}$	$\Delta_{22}^{(1)}$	$\Delta_{23}^{(1)}$	$\Delta_{24}^{(1)}$	$\Delta_j^{(2)}$
$\tilde{z}$				Δ_1	Δ_2	Δ_3	Δ_4	Δ_j

In table 1.2 calculations are given below.

$$\begin{aligned}
\tilde{z}_1 &= \sum c_B \tilde{x}_B^n + \alpha = (0)(\{< 1,2,3 >; 0.6, 0.5, 0.3 >\}) \\
&\quad +(0)(\{< 2,4,6 >; 0.2, 0.6, 0.5\}) + 0 \\
&= \{< 0,0,0 >; 0.2, 0.6, 0.5\} \\
\tilde{z}_2 &= \sum d_B \tilde{x}_B^n + \beta = (0)(\{< 1,2,3 >; 0.6, 0.5, 0.3\}) \\
&\quad +(0)(\{< 2,4,6 >; 0.2, 0.6, 0.5\}) + 2 \\
&= \{< 2,2,2 >; 0.2, 0.6, 0.5\} \\
\tilde{z} &= \frac{\tilde{z}_1}{\tilde{z}_2} = \frac{\{< 0,0,0 >; 0.2, 0.6, 0.5\}}{\{< 2,2,2 >; 0.2, 0.6, 0.5\}} = \{< 0,0,0 >; 0.2, 0.6, 0.5\}
\end{aligned}$$

$$
\begin{aligned}
\Delta_j^{(1)} &= c_j - \sum c_B \tilde{x}_B \\
\Delta_{11}^{(1)} &= 1 - [(0)(-1) + (0)(1)] = 1 \\
\Delta_{12}^{(1)} &= 2 - [(0)(2) + (0)(1)] = 2 \\
\Delta_{13}^{(1)} &= 0 - [(0)(1) + (0)(0)] = 0 \\
\Delta_{14}^{(1)} &= 0 - [(0)(0) + (0)(1)] = 0 \\
\Delta_j^{(2)} &= d_j - \sum d_B \tilde{x}_B \\
\Delta_{21}^{(2)} &= 2 - [(0)(-1) + (0)(1)] = 2 \\
\Delta_{22}^{(2)} &= -1 - [(0)(2) + (0)(1)] = -1 \\
\Delta_{23}^{(2)} &= 0 - [(0)(1) + (0)(0)] = 0 \\
\Delta_{24}^{(2)} &= 0 - [(0)(0) + (0)(1)] = 0 \\
\Delta_j &= \tilde{z}_2 \times \Delta_j^{(1)} - \tilde{z}_1 \times \Delta_j^{(2)} \\
\Delta_1 &= (\{< 2,2,2 >; 0.2, 0.6, 0.5\})(1) - (\{< 0,0,0 >; 0.2, 0.6, 0.5\})(2) \\
&= \{< 2,2,2 >; 0.2, 0.6, 0.5\} \\
\Delta_2 &= (\{< 2,2,2 >; 0.2, 0.6, 0.5\})(2) - (\{< 0,0,0 >; 0.2, 0.6, 0.5\})(-1) \\
&= \{< 4,4,4 >; 0.2, 0.6, 0.5\} \\
\Delta_3 &= (\{< 2,2,2 >; 0.2, 0.6, 0.5\})(0) - (\{< 0,0,0 >; 0.2, 0.6, 0.5\})(0) \\
&= \{< 0,0,0 >; 0.2, 0.6, 0.5\} \\
\Delta_4 &= (\{< 2,2,2 >; 0.2, 0.6, 0.5\})(0) - (\{< 0,0,0 >; 0.2, 0.6, 0.5\})(0) \\
&= \{< 0,0,0 >; 0.2, 0.6, 0.5\}
\end{aligned}
$$

Table:1.2

	$\alpha = 0$		c_j	1	2	0	0	Min.Ratio
B.V	$\beta = 2$		d_j	2	-1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2 \downarrow$	$\tilde{x}_3$	$\tilde{x}_4$	
$\leftarrow \tilde{x}_3$	0	0	$\{\langle 1,2,3 \rangle; 0.6, 0.5, 0.3\}$	-1	2	1	0	$\{< \frac{1}{2}, 1, \frac{3}{2} >; 0.6, 0.5, 0.3\}$
$\tilde{x}_4$	0	0	$\{< 2,4,6 >; 0.2, 0.6, 0.5\}$	1	1	0	1	$\{\langle 2,4,6 \rangle; 0.2, 0.6, 0.5\}$
$\tilde{z}_1 = \{\langle 0,0,0 \rangle; 0.2, 0.6, 0.5\}$				1	2	0	0	$\Delta_j^{(1)}$
$\tilde{z}_2 = \{\langle 2,2,2 \rangle; 0.2, 0.6, 0.5\}$				2	-1	0	0	$\Delta_j^{(2)}$
$\tilde{z} = \{\langle 0,0,0 \rangle; 0.2, 0.6, 0.5\}$				$\{\langle 2,2,2 \rangle; 0.2, 0.6, 0.5\}$	$\{\langle 4,4,4 \rangle; 0.2, 0.6, 0.5\}$	$\{\langle 0,0,0 \rangle; 0.2, 0.6, 0.5\}$	$\{\langle 0,0,0 \rangle; 0.2, 0.6, 0.5\}$	Δ_j

Since $\tilde{z}_1 = \{\langle 0,0,0 \rangle; 0.2, 0.6, 0.5\}$,

$\tilde{z}_2 = \{\langle 2,2,2 \rangle; 0.2, 0.6, 0.5\}$

and $\tilde{z} = \{\langle 0,0,0 \rangle; 0.2, 0.6, 0.5\}$.

Therefore $\Delta_j = (\{\langle 2,2,2 \rangle; 0.2, 0.6, 0.5\}) \times \Delta_j^{(1)}$ as shown in table 1.2. Entering variable is $\tilde{x}_2$ and Leaving variable is $\tilde{x}_3$. in table 1.2 Go to next table.

In table 1.3 calculations are given below.

$$
\begin{aligned}
\tilde{z}_1 &= \sum c_B \tilde{x}_B + \alpha \\
&= (2)\left(\left\{< \frac{1}{2}, 1, \frac{3}{2} >; 0.6, 0.5, 0.3\right\}\right) + (0)\left(\left\{< \frac{-3}{2}, 3, \frac{9}{2} 9/2 >; 0.2, 0.6, 0.5\right\}\right) + 0 \\
&= \{< 1, 2, 3 >; 0.2, 0.6, 0.5\} \\
\tilde{z}_2 &= \sum d_B \tilde{x}_B + \beta(-1) \\
&= \left(\left\{< \frac{1}{2}, 1, \frac{3}{2} >; 0.6, 0.5, 0.3\right\}\right) + (0)\left(\left\{< \frac{-3}{2}, 3, \frac{9}{2} 9/2 >; 0.2, 0.6, 0.5\right\}\right) + 2 \\
&= \left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\} \\
\tilde{z} &= \frac{\tilde{z}_1}{\tilde{z}_2} = \frac{\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}}{\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}} = \left\{< \frac{2}{3}, 2, 6 >; 0.2, 0.6, 0.5\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\Delta_j^{(1)} = c_j - \sum c_B \tilde{x}_B \\
&\Delta_{11}^{(1)} = 1 - \left[(2)\left(\frac{-1}{2}\right) + (0)\left(\frac{3}{2}\right)\right] = 2 \\
&\Delta_{12}^{(1)} = 2 - [(2)(1) + (0)(0)] = 0 \\
&\Delta_{13}^{(1)} = 0 - \left[(2)(\frac{1}{2}) + (0)\left(\frac{-1}{2}\right)\right] = -1 \\
&\Delta_{14}^{(1)} = 0 - [(2)(0) + (0)(1)] = 0
\end{aligned}
$$

$$
\begin{aligned}
\Delta_j^{(2)} &= d_j - \sum d_B \tilde{x}_B \\
\Delta_{21}^{(2)} &= 2 - \left[(-1)\left(\frac{-1}{2}\right) + (0)\left(\frac{3}{2}\right)\right] = \frac{3}{2} \\
\Delta_{22}^{(2)} &= -1 - [(-1)(1) + (0)(0)] = -0 \\
\Delta_{23}^{(2)} &= 0 - \left[(-1)\left(\frac{1}{2}\right) + (0)\left(\frac{-1}{2}\right)\right] = \frac{1}{2} \\
\Delta_{24}^{(2)} &= 0 - [(-1)(0) + (0)(1)] = 0 \\
\Delta_j &= \tilde{z}_2 \times \Delta_j^{(1)} - \tilde{z}_1 \times \Delta_j^{(2)} \\
\Delta_1 &= \left(\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}\right) \times (2) - (\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}) \times \left(\frac{3}{2}\right) \\
&= \left\{< \frac{3}{2}, -1, \frac{-7}{2} >; 0.2, 0.6, 0.5\right\} \\
\Delta_2 &= \left(\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}\right) \times (0) - (\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}) \times (0) \\
&= \{< 0, 0, 0 >; 0.2, 0.6, 0.5\}
\end{aligned}
$$

$$
\begin{aligned}
\Delta_3 &= \left(\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}\right) \times (-1) - (\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}) \times \left(\frac{1}{2}\right) \\
&= \{< -2, -2, -2 >; 0.2, 0.6, 0.5\} \\
\Delta_4 &= \left(\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}\right) \times (0) - (\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}) \times (0) \\
&= \{< 0, 0, 0 >; 0.2, 0.6, 0.5\}
\end{aligned}
$$

Table:1.3

	$\alpha = 0$		c_j	1	2	0	0	Min.Ratio
B.V	$\beta = 2$		d_j	2	-1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}^n$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_2$	-1	2	$\{< \frac{1}{2}, 1, \frac{3}{2} >; 0.6, 0.5, 0.3\}$	$\frac{-1}{2}$	1	$\frac{1}{2}$	0	-
$\tilde{x}_4$	0	0	$\{\left\langle \frac{-3}{2}, 3, \frac{9}{2} \right\rangle; 0.2, 0.6, 0.5\}$	$\frac{3}{2}$	0	$\frac{-1}{2}$	1	-
$\tilde{z}_1 = \{\langle 1, 2, 3 \rangle; 0.2, 0.6, 0.5\}$				2	0	-1	0	$\Delta_j^{(1)}$
$\tilde{z}_2 = \{\left\langle \frac{3}{2}, 1, \frac{1}{2} \right\rangle; 0.2, 0.6, 0.5\}$				$\frac{3}{2}$	0	$\frac{1}{2}$	0	$\Delta_j^{(2)}$
$\tilde{z} = \{\left\langle \frac{2}{3}, 2, 6 \right\rangle; 0.2, 0.6, 0.5\}$				$\{\left\langle \frac{3}{2}, -1, \frac{-7}{2} \right\rangle; 0.2, 0.6, 0.5\}$	$\{\langle 0, 0, 0 \rangle; 0.2, 0.6, 0.5\}$	$\{\langle -2, -2, -2 \rangle; 0.2, 0.6, 0.5\}$	$\{\langle 0, 0, 0 \rangle; 0.2, 0.6, 0.5\}$	Δ_j

$$
\begin{aligned}
\text{Here} \quad \tilde{x}_1 &= \{\langle 0, 0, 0 \rangle; 1, 0, 0\}, \\
\tilde{x}_2 &= \left\{\left\langle \frac{1}{2}, 1, \frac{3}{2} \right\rangle; 0.6, 0.5, 0.3\right\} \\
\text{and} \quad \tilde{z} &= \frac{\tilde{z}_1}{\tilde{z}_2} = \left\{\left\langle \frac{2}{3}, 2, 6 \right\rangle; 0.2, 0.6, 0.5\right\}.
\end{aligned}
$$

Since all $\Delta_j \leqslant \{\langle 0, 0, 0 \rangle; 1, 0, 0\}$.

$\therefore$ The solution is the optimal basic feasible solution.

Problem 4.2. *Find the solution of following linear fractional programming problem:*

$$
\begin{aligned}
max.\tilde{Z} &= \frac{3\tilde{x}_1 + \tilde{x}_2}{3\tilde{x}_1 + \tilde{x}_2 + 6} \\
s.t \quad 5\tilde{x}_1 + 3\tilde{x}_2 &\leqslant \{\langle 5, 6, 7 \rangle; 0.2, 0.7, 0.5\} \\
7\tilde{x}_1 + \tilde{x}_2 &\leqslant \{\langle 3, 6, 9 \rangle; 0.4, 0.6, 0.3\} \\
\tilde{x}_1, \tilde{x}_2 &\geqslant \{\langle 0, 0, 0 \rangle; 1, 0, 0\}
\end{aligned}
$$

Solution: After adding slack variables, $\tilde{x}_3$ and $\tilde{x}_4$, the constraints become

$$
\begin{aligned}
5\tilde{x}_1 + 3\tilde{x}_2 + \tilde{x}_3 &= \{\langle 5, 6, 7 \rangle; 0.2, 0.7, 0.5\} \\
7\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_4 &= \{\langle 3, 6, 9 \rangle; 0.4, 0.6, 0.3\} \\
\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 &\geqslant \{\langle 0, 0, 0 \rangle; 1, 0, 0\}
\end{aligned}
$$

Table:2.1

	$\alpha = 0$		c_j	2	1	0	0	Min.Ratio
B.V	$\beta = 2$		d_j	3	1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_3$	0	0	$\{\langle 5,6,7\rangle;$ $0.2,0.7,0.5\}$	5	3	1	0	
$\tilde{x}_4$	0	0	$\{\langle 3,6,9\rangle;$ $0.4,0.6,0.3\}$	7	1	0	1	
$\tilde{z}_1$				$\Delta_{11}^{(1)}$	$\Delta_{12}^{(1)}$	$\Delta_{13}^{(1)}$	$\Delta_{14}^{(1)}$	$\Delta_j^{(1)}$
$\tilde{z}_2$				$\Delta_{21}^{(1)}$	$\Delta_{22}^{(1)}$	$\Delta_{23}^{(1)}$	$\Delta_{24}^{(1)}$	$\Delta_j^{(2)}$
$\tilde{z}$				Δ_1	Δ_2	Δ_3	Δ_4	Δ_j

Similarly we find the value of the Table:2.2 (In the Problem 4.1 calculation as well as).

Table:2.2

	$\alpha = 0$		c_j	2	1	0	0	Min.Ratio
B.V	$\beta = 2$		d_j	3	1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1 \downarrow$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_3$	0	0	$\{\langle 5,6,7\rangle;$ $0.2,0.7,0.5\}$	5	3	1	0	$\{\left\langle \frac{5}{5},\frac{6}{5},\frac{7}{5}\right\rangle;$ $0.2,0.7,0.5\}$
$\leftarrow$ $\tilde{x}_4$	0	0	$\{\langle 3,6,9\rangle;$ $0.4,0.6,0.3\}$	7	1	0	1	$\left\{\left\langle \frac{3}{7},\frac{6}{7},\frac{9}{7}\right\rangle;\right.$ $0.4,0.6,0.3\}$
$\tilde{z}_1 = \{\langle 0,0,0\rangle;$ $0.2,0.7,0.5\}$				2	1	0	0	$\Delta_j^{(1)}$
$\tilde{z}_2 = \{\langle 6,6,6\rangle;$ $0.2,0.7,0.5\}$				3	1	0	0	$\Delta_j^{(2)}$
$\tilde{z} = \{\langle 0,0,0\rangle;$ $0.2,0.7,0.5\}$				$\{\langle 12,12,12\rangle;$ $0.2,0.7,0.5\}$	$\{\langle 6,6,6\rangle;$ $0.2,0.7,0.5\}$	$\{\langle 0,0,0\rangle;$ $0.2,0.7,0.5\}$	$\{\langle 0,0,0\rangle;$ $0.2,0.7,0.5\}$	Δ_j

$$\text{Since } \tilde{z}_1 = \{\langle 0,0,0\rangle;0.2,0.7,0.5\},$$
$$\tilde{z}_2 = \{\langle 6,6,6\rangle;0.2,0.7,0.5\},$$
$$\text{and } \tilde{z} = \{\langle 0,0,0\rangle;0.2,0.7,0.5\}.$$

$\therefore\ \Delta_j = \left[\{\langle 6,6,6\rangle;0.2,0.7,0.5\} \times \Delta_j^{(1)}\right]$ as shown in table 2.2.
Entering variable is $\tilde{x}_1$ and Leaving variable is $\tilde{x}_4$. in table 2.2 Go to next table.
Similarly we find the value of the Table:2.3 (In the Problem 4.1 calculation as well as).

Table:2.3

	$\alpha = 0$		c_j	2	1	0	0	Min.Ratio
B.V	$\beta = 6$		d_j	3	1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2 \downarrow$	$\tilde{x}_3$	$\tilde{x}_4$	
$\leftarrow$ $\tilde{x}_3$	0	0	$\left\{\left\langle \frac{10}{7}, \frac{12}{7}, \frac{34}{7} \right\rangle; 0.2, 0.7, 0.5\right\}$	0	$\frac{16}{7}$	1	$\frac{-5}{7}$	$\left\{\left\langle \frac{-5}{8}, \frac{3}{4}, \frac{17}{8} \right\rangle; 0.2, 0.7, 0.5\right\}$
$\tilde{x}_1$	3	2	$\left\{\left\langle \frac{3}{7}, \frac{6}{7}, \frac{9}{7} \right\rangle; 0.4, 0.6, 0.3\right\}$	1	$\frac{1}{7}$	0	$\frac{1}{7}$	$\{\langle 3, 6, 9\rangle; 0.4, 0.6, 0.3\}$
$\tilde{z}_1 = \left\{\left\langle \frac{6}{7}, \frac{12}{7}, \frac{18}{7} \right\rangle; 0.2, 0.7, 0.5\right\}$				0	$\frac{5}{7}$	0	$\frac{-2}{7}$	$\Delta_j^{(1)}$
$\tilde{z}_2 = \left\{\left\langle \frac{51}{7}, \frac{60}{7}, \frac{69}{7} \right\rangle; 0.2, 0.7, 0.5\right\}$				0	$\frac{4}{7}$	0	$\frac{-3}{7}$	$\Delta_j^{(2)}$
$\tilde{z} = \left\{\left\langle \frac{6}{51}, \frac{1}{5}, \frac{18}{69} \right\rangle; 0.2, 0.7, 0.5\right\}$				$\{\langle 0, 0, 0\rangle; 0.2, 0.7, 0.5\}$	$\left\{\left\langle \frac{183}{49}, \frac{252}{49}, \frac{321}{49} \right\rangle; 0.2, 0.7, 0.5\right\}$	$\{\langle 0, 0, 0\rangle; 0.2, 0.7, 0.5\}$	$\left\{\left\langle \frac{-12}{7}, \frac{-12}{7}, \frac{-12}{7} \right\rangle; 0.2, 0.7, 0.5\right\}$	Δ_j

$$\text{Since } \tilde{z}_1 = \left\{\left\langle \frac{6}{7}, \frac{12}{7}, \frac{18}{7} \right\rangle; 0.2, 0.7, 0.5\right\},$$

$$\tilde{z}_2 = \left\{\left\langle \frac{51}{7}, \frac{60}{7}, \frac{69}{7} \right\rangle; 0.2, 0.7, 0.5\right\}$$

$$\text{and } \tilde{z} = \left\{\left\langle \frac{6}{51}, \frac{1}{5}, \frac{18}{69} \right\rangle; 0.2, 0.7, 0.5\right\}.$$

$$\therefore \Delta_j = \begin{bmatrix} \left(\left\{\left\langle \frac{51}{7}, \frac{60}{7}, \frac{69}{7} \right\rangle; 0.2, 0.7, 0.5\right\}\right) \times \Delta_j^{(1)} \\ -\left(\left\{\left\langle \frac{6}{7}, \frac{12}{7}, \frac{18}{7} \right\rangle; 0.2, 0.7, 0.5\right\}\right) \times \Delta_j^{(2)} \end{bmatrix} \text{as shown in table:2.3.}$$

Entering variable is $\tilde{x}_2$ and Leaving variable is $\tilde{x}_3$. in table:2.3 Go to next table. Similarly we find the value of the Table:2.4 (In the Problem 4.1 calculation as well as).

Table:2.4

	$\alpha = 0$		c_j	2	1	0	0	Min.Ratio
B.V	$\beta = 6$		d_j	3	1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_2$	1	1	$\left\{\left\langle \frac{-5}{8}, \frac{3}{4}, \frac{17}{8} \right\rangle; 0.2, 0.7, 0.5\right\}$	0	1	$\frac{7}{16}$	$\frac{-5}{16}$	
$\tilde{x}_1$	3	2	$\left\{\left\langle \frac{1}{8}, \frac{3}{4}, \frac{11}{8} \right\rangle; 0.2, 0.7, 0.5\right\}$	1	0	$\frac{-1}{6}$	$\frac{3}{16}$	
$\tilde{z}_1 = \left\{\left\langle \frac{-3}{8}, \frac{9}{4}, \frac{39}{8} \right\rangle; 0.2, 0.7, 0.5\right\}$				0	0	$\frac{-5}{16}$	$\frac{-1}{16}$	$\Delta_j^{(1)}$
$\tilde{z}_2 = \left\{\left\langle \frac{46}{8}, 9, \frac{98}{8} \right\rangle; 0.2, 0.7, 0.5\right\}$				0	0	$\frac{-1}{4}$	$\frac{-1}{4}$	$\Delta_j^{(2)}$
$\tilde{z} = \left\{\left\langle \frac{-3}{46}, \frac{1}{4}, \frac{39}{98} \right\rangle; 0.2, 0.7, 0.5\right\}$				$\{\langle 0, 0, 0\rangle; 0.2, 0.7, 0.5\}$	$\{\langle 0, 0, 0\rangle; 0.2, 0.7, 0.5\}$	$\left\{\left\langle \frac{-53}{32}, \frac{-9}{4}, \frac{-167}{64} \right\rangle; 0.2, 0.7, 0.5\right\}$	$\left\{\left\langle \frac{-29}{4}, 0, \frac{29}{4} \right\rangle; 0.2, 0.7, 0.5\right\}$	Δ_j

$$\text{Here } \tilde{x}_1 = \left\{\left\langle \frac{1}{8}, \frac{3}{4}, \frac{11}{8} \right\rangle; 0.2, 0.7, 0.5\right\},$$

$$\tilde{x}_2 = \left\{ \left\langle \frac{-5}{8}, \frac{3}{4}, \frac{17}{8} \right\rangle ; 0.2, 0.7, 0.5 \right\}$$
$$\text{and } \tilde{z} = \frac{\tilde{z}_1}{\tilde{z}_2} = \left\{ \left\langle \frac{-3}{46}, \frac{1}{4}, \frac{39}{98} \right\rangle ; 0.2, 0.7, 0.5 \right\}.$$

Since all $\Delta_j \leqslant \{\langle 0,0,0\rangle ; 1,0,0\}$.

$\therefore$ The current solution is the optimal basic feasible solution.

5. Conclusion

In this work, we have presented the simplex algorithm method for solving Neutrosophic linear fractional programming problem.

The solution of the given Problem (1) is

$$\tilde{x}_1 = \{\langle 0,0,0\rangle ; 1,0,0\},$$
$$\tilde{x}_2 = \left\{ \left\langle \frac{1}{2}, 1, \frac{3}{2} \right\rangle ; 0.6, 0.5, 0.3 \right\}$$
$$\text{and } \tilde{z} = \frac{\tilde{z}_1}{\tilde{z}_2} = \left\{ \left\langle \frac{2}{3}, 2, 6 \right\rangle ; 0.2, 0.6, 0.5 \right\}$$

The solution of the given Problem (2) is

$$\tilde{x}_1 = \left\{ \left\langle \frac{1}{8}, \frac{3}{4}, \frac{11}{8} \right\rangle ; 0.2, 0.7, 0.5 \right\},$$
$$\tilde{x}_2 = \left\{ \left\langle \frac{-5}{8}, \frac{3}{4}, \frac{17}{8} \right\rangle ; 0.2, 0.7, 0.5 \right\}$$
$$\text{and } \tilde{z} = \frac{\tilde{z}_1}{\tilde{z}_2} = \left\{ \left\langle \frac{-3}{46}, \frac{1}{4}, \frac{39}{98} \right\rangle ; 0.2, 0.7, 0.5 \right\}.$$

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