On the Refined AH-Isometry and Its Applications in Refined Neutrosophic Surfaces

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Abstract

The aim of this paper is to generalize the neutrosophic AH-isometry into the system of refined neutrosophic numbers, where it presents an isometer between the refined neutrosophic space with one/two neutrosophic dimensions and the cartesian product of classical Euclidean spaces.Also, many refined neutrosophic geometrical surfaces such as refined circles and lines will be handled according to the isometry.

Keywords: Refined Neutrosophic Space; refined neutrosophic geometry; semi homomorphism; refined AH-isometry

1.Introduction

Neutrosophic logic is a generalization of intuitionistic fuzzy logic by adding an indeterminacy I with property $I=I^2$.

On the other hand, neutrosophic sets played an interesting role in pure mathematics such as topology and analysis [3], spaces [1], and algebraic structures [2-12].

Neutrosophic spaces theory began with Agboola et.al [8,9], where they studied neutrosophic vector spaces and their properties.

In [26] the concept of the neutrosophic plane with N neutrosophic dimensions is obtained. In addition, Euclidean geometric concepts are extended neutrosophically such as neutrosophic distance, neutrosophic midpoint, neutrosophic vectors, neutrosophic circles, and lines.

This work finds an isometry between the real refined neutrosophic space and the Cartesian product of classical Euclidean spaces, which means that the foundations and laws of refined neutrosophic geometry will be established.

2. Preliminaries

Definition 2.1: The refined neutrosophic real number has the form $a_0 + a_1I_1 + a_2I_2$ where $a_0, a_1, a_2 \in R$ are real numbers. (We write the refined neutrosophic numbers by the previous form instead of the equivalent form (a_0, a_1I_1, a_2I_2)

Remark 2.2 : $I_1.I_1 = I_1, I_1.I_2 = I_2.I_1 = I_1$

Definition 2.3:

Let $R(I_1, I_2) = \{a_0 + a_1I_1 + a_2I_2;\ a_0, a_1, a_2 \in R\}$ be the refined neutrosophic field, we say

$a_0 + a_1I_1 + a_2I_2 \leq b_0 + b_1I_1 + b_2I_2$ if and only if $a_0 \leq b_0$, $a_0 + a_1 + a_2 \leq b_0 + b_1 + b_2$ and

$a_0 + a_2 \leq b_0 + b_2$.

Doi : https://doi.org/10.54216/GJMSA.020103
Received: March 16, 2022 Accepted: July 09, 2022

Theorem2.4:

The previous relation is a partial order relation (reflexive, anti-symmetric, and transitive).

3. Main concepts and results

In the beginning, we will define some basic concepts in refined neutrosophic real numbers, hence we will study its relation with classical real numbers.

Remark 3.1:

According to Theorem 2.4, we can define positive refined neutrosophic real numbers as follows.

$a_0 + a_1I_1 + a_2I_2 \geq 0 = 0 + 0I_1 + 0I_2$ implies that $a_0 \geq 0$, $a_0 + a_1 + a_2 \geq 0$,$a_0 + a_2 \geq 0$.

Absolute value on $R(I_1, I_2)$ can be defined as follows:

$|a_0 + a_1I_1 + a_2I_2| = |a_0| + [|a_0 + a_1 + a_2| - |a_0 + a_2|]I_1 + [|a_0 + a_2| - |a_0|]I_2$, we can see that

$|a_0 + a_1I_1 + a_2I_2| \geq 0$. (Under the defined partial order relation).

We can compute the square root of a refined neutrosophic positive real number as follows:

$\sqrt{a_0 + a_1I_1 + a_2I_2} = \sqrt{a_0} + [\sqrt{a_0 + a_1 + a_2} - \sqrt{a_0 + a_2}]I_1 + [\sqrt{a_0 + a_2} - \sqrt{a_0}]I_2$.

It is clear that $\left(\sqrt{a_0} + [\sqrt{a_0 + a_1 + a_2} - \sqrt{a_0 + a_2}]I_1 + [\sqrt{a_0 + a_2} - \sqrt{a_0}]I_2\right)^2 = a_0 + a_1I_1 + a_2I_2$ and $\sqrt{a_0 + a_1I_1 + a_2I_2} \geq 0$.

Example 3.2:

- $x = 2 - I_1 + 3I_2$ is a refined neutrosophic positive real number, since.

$2 \geq 0$, $(2 - 1 + 3) = 4 \geq 0$, $2 + 3 = 5 \geq 0$.

- $2 + I_1 + 2I_2 \geq 1 - I_2$, that is because $2 \geq 1$, $(2 + 1 + 2) = 5 \geq (1 - 1) = 0$, and

$(2 + 2) = 4 \geq (1 - 1) = 0$.

- $|-1 - 3I_1 + 3I_2| = |-1| + [|-1 - 3 + 3| - |-1 + 3|]I_1 + [|-1 + 3| - |-1|]I_2 = 1 - I_1 + I_2$
- $\sqrt{2 + I_1 + I_2} = \sqrt{2} + [\sqrt{4} - \sqrt{3}]I_1 + [\sqrt{3} - \sqrt{2}]I_2 = \sqrt{2} + [2 - \sqrt{3}]I_1 + [\sqrt{3} - \sqrt{2}]I_2$

Definition 3.3:

We define the refined neutrosophic plane with N neutrosophic dimensions (N-dimensions) as follows:

$R(I_1, I_2) \times R(I_1, I_2) \times R(I_1, I_2) \times \underbrace{\dots\dots}_{N-times} \times R(I_1, I_2)$

Doi : https://doi.org/10.54216/GJMSA.020103
Received: March 16, 2022 Accepted: July 09, 2022

Example 3.4:

$R(I_1, I_2) = \{a_0 + a_1 I_1 + a_2 I_2;\ a_0, a_1, a_2 \in R\}$ is a refined neutrosophic plane with one N-dimension.

$R(I_1, I_2)^2 = \{(a_0 + a_1 I_1 + a_2 I_2, b_0 + b_1 I_1 + b_2 I_2);\ a_0, a_1, a_2, b_0, b_1, b_2 \in R\}$ is a refined neutrosophic plane with two N-dimension.

In the following, we will focus on the case of two N-dimensional refined neutrosophic plane.

Definition 3.5:

Let $A(a_0 + a_1 I_1 + a_2 I_2, b_0 + b_1 I_1 + b_2 I_2)$,$B(c_0 + cI_1 + c_2 I_2, d_0 + d_1 I_1 + d_2 I_2)$ be two refined neutrosophic points form $R(I_1, I_2)^2$, we define:

$\overrightarrow{AB} = ([a_0 + a_1 I_1 + a_2 I_2] - [c_0 + cI_1 + c_2 I_2], [b_0 + b_1 I_1 + b_2 I_2] - [d_0 + d_1 I_1 + d_2 I_2])$, is called a refined neutrosophic vector with two N-dimensions.

Definition 3.6: Let $\vec{u} = (a_0 + a_1 I_1 + a_2 I_2, b_0 + b_1 I_1 + b_2 I_2)$ be a refined neutrosophic vector, we define its norm as follows:

$||\vec{u}|| = \sqrt{(a_0 + a_1 I_1 + a_2 I_2)^2 + (b_0 + b_1 I_1 + b_2 I_2)^2}$.

It easy to see that $||\vec{u}|| \geq 0$, according to remark 3.3.

Definition 3.7: Let $M = R(I_1, I_2)^2 \times R(I_1, I_2)^2$, $V = R^3 \times R^3$ be the refined neutrosophic plane with two N-dimensions and Cartesian product of the classical Euclidean space R^3 with itself respectively, we define the refined AH-isometry map as follows:

$f: M \to V; f(a_0 + a_1 I_1 + a_2 I_2, b_0 + b_1 I_1 + b_2 I_2) = \big((a_0, a_0 + a_1 + a_2, a_0 + a_2), (b_0, b_0 + b_1 + b_2, b_0 + b_2)\big)$

We can define the refined one-dimensional AH-isometry between $R(I_1, I_2)$ and the space $R \times R \times R$ as follows:

$$g: R(I_1, I_2) \to R^3; g(a_0 + a_1 I_1 + a_2 I_2) = (a_0, a_0 + a_1 + a_2, a_0 + a_2)$$

Remark 3.8: The refined one-dimensional AH-isometry is an algebraic isomorphism between$R(I_1, I_2)$ and $R \times R \times R$.

Proof.

Let $w_1 = a_0 + a_1 I_1 + a_2 I_2, w_2 = b_0 + b_1 I_1 + b_2 I_2$ be two refined neutrosophic real numbers, then.

$g(w_1 + w_2) = g([a_0 + b_0] + [a_1 + b_1]I_1 + [a_2 + b_2]I_2)$

$= (a_0 + b_0, a_0 + a_1 + a_2 + b_0 + b_1 + b_2, a_0 + a_2 + b_0 + b_2)$

$= (a_0, a_0 + a_1 + a_2, a_0 + a_2) + (b_0, b_0 + b_1 + b_2, b_0 + b_2)$

$= g(a_0 + a_1 I_1 + a_2 I_2) + g(b_0 + b_1 I_1 + b_2 I_2) = g(w_1) + g(w_2)$.

$g(w_1.w_2) = g\big((a_0 + a_1 I_1 + a_2 I_2).(b_0 + b_1 I_1 + b_2 I_2)\big)$

$= g(a_0 b_0 + [a_1 b_1 + a_1 b_0 + a_1 b_1 + a_1 b_2 + a_2 b_1]I_1 + [a_0 b_2 + a_2 b_0 + a_2 b_2]I_2)$

$= (a_0 b_0, a_0 b_0 + a_1 b_1 + a_1 b_0 + a_1 b_1 + a_1 b_2 + a_2 b_1 + a_0 b_2 + a_2 b_0 + a_2 b_2, a_0 b_0 + a_0 b_2 + a_2 b_0 + a_2 b_2)$

$= (a_0, a_0 + a_1 + a_2, a_0 + a_2).(b_0, b_0 + b_1 + b_2, b_0 + b_2)$

$= g(a_0 + a_1 I_1 + a_2 I_2).g(b_0 + b_1 I_1 + b_2 I_2) = g(w_1).g(w_2)$.

g is a correspondence one-to-one, that is because $ker(g) = \{0\}$, and for every

$(a_0, a_1, a_2) \in R \times R \times R$,there exists $x = a_0 + ([a_1 - a_2])I_1 + (a_2 - a_0)I_2 \in R(I_1, I_2)$such that $g(x) = (a_0, a_1, a_2)$.

Thus, g is isomorphism.

Doi : https://doi.org/10.54216/GJMSA.020103

Received: March 16, 2022 Accepted: July 09, 2022

Theorem 3.9:

Let $f: M \to V; f(a_0 + a_1I_1 + a_2I_2, b_0 + b_1I_1 + b_2I_2) = \big((a_0, a_0 + a_1 + a_2, a_0 + a_2), (b_0, b_0 + b_1 + b_2, b_0 + b_2)\big)$ be the refined AH-isometry defined abobe, we have:

(a). f preserves addition operation between vectors.

(b). f preserves distances between points.

(c). f is a bijection one-to-one between M and V.

(d). Multiplying a refined neutrosophic vector by a refined neutrosophic real number is preserved up to refined AH-isometry, i.e.

The direct image of a refined neutrosophic vector multiplied by a refined neutrosophic real number is exactly equal to its refined AH- isometric image multiplied by the one-dimensional refined AH- isometric image of the corresponding refined neutrosophic real number.

Proof.

(a). Let $\vec{u} = (a_0 + a_1I_1 + a_2I_2, b_0 + b_1I_1 + b_2I_2), \vec{v} = (c_0 + cI_1 + c_2I_2, d_0 + d_1I_1 + d_2I_2)$ be two refined neutrosophic vectors, we have.

$f(\vec{u} + \vec{v}) = f([a_0 + c_0] + [a_1 + c_1]I_1 + [a_2 + c_2]I_2, [b_0 + d_0] + [b_1 + d_1]I_1 + [b_2 + d_2]I_2)$

$= \big((a_0 + c_0, a_0 + a_1 + a_2 + c_0 + c_1 + c_2, a_0 + a_2 + c_0 + c_2), (b_0 + d_0, b_0 + b_1 + b_2 + d_0 + d_1 + d_2, b_0 + b_2 + d_0 + d_2)\big)$

$= \big((a_0, a_0 + a_1 + a_2, a_0 + a_2), (b_0, b_0 + b_1 + b_2, b_0 + b_2)\big) + \big((c_0, c_0 + c_1 + c_2, c_0 + c_2), (d_0, d_0 + d_1 + d_2, d_0 + d_2)\big) = f(\vec{u}) + f(\vec{v})$.

.(b). We must prove that the norm of the classical vector $\overrightarrow{f(u)}$, is exactly equal to the refined one- dimensional isometric image of the norm of the neutrosophic vector $\vec{u}$.

$\|f(\vec{u})\|^2 = \big(a_0{}^2 + b_0{}^2, (a_0 + a_1 + a_2)^2 + (b_0 + b_1 + b_2)^2, (a_0 + a_2)^2 + (b_0 + b_2)^2\big)$

Now,

$\|\vec{u}\|^2 = (a_0 + a_1I_1 + a_2I_2)^2 + (b_0 + b_1I_1 + b_2I_2)^2 = \big(a_0{}^2 + b_0{}^2 + \big(a_1{}^2 + b_1{}^2 + 2a_0a_1 + 2b_0b_1 + 2a_1a_2 + 2b_1b_2\big)I_1 + \big(a_2{}^2 + b_2{}^2 + 2a_0a_2 + 2b_0b_2\big)I_2\big)$.

$g(\|\vec{u}\|^2) = \big(a_0{}^2 + b_0{}^2, a_0{}^2 + b_0{}^2 + a_1{}^2 + b_1{}^2 + 2a_0a_1 + 2b_0b_1 + 2a_1a_2 + 2b_1b_2 + a_2{}^2 + b_2{}^2 + 2a_0a_2 + 2b_0b_2, a_0{}^2 + b_0{}^2 + a_2{}^2 + b_2{}^2 + 2a_0a_2 + 2b_0b_2\big) = \big(a_0{}^2 + b_0{}^2, (a_0 + a_1 + a_2)^2 + (b_0 + b_1 + b_2)^2, (a_0 + a_2)^2 + (b_0 + b_2)^2\big) = \|f(\vec{u})\|^2$.

(c).Let $w_1 = a_0 + a_1I_1 + a_2I_2, w_2 = b_0 + b_1I_1 + b_2I_2, w_3 = c_0 + cI_1 + c_2I_2, w_4 = d_0 + d_1I_1 + d_2I_2$

Suppose that $f(w_1, w_2) = f(w_3, w_4)$, hence

$\big((a_0, a_0 + a_1 + a_2, a_0 + a_2), (b_0, b_0 + b_1 + b_2, b_0 + b_2)\big) = \big((c_0, c_0 + c_1 + c_2, c_0 + c_2), (d_0, d_0 + d_1 + d_2, d_0 + d_2)\big)$, thus

$a_0 = c_0, a_0 + a_1 + a_2 = c_0 + c_1 + c_2, a_0 + a_2 = c_0 + c_2, b_0 = d_0, b_0 + b_1 + b_2 = d_0 + d_1 + d_2, b_0 + b_2 = d_0 + d_2$, so $a_0 = c_0, a_1 = c_1, a_2 = c_2, b_0 = d_0, b_1 = d_1, b_2 = d_2$, so f is surjective.

It is clear that f is injective, thus it is a bijection.

(d).Consider the following refined vector $\vec{u} = (a_0 + a_1I_1 + a_2I_2, b_0 + b_1I_1 + b_2I_2)$ with the following refined neutrosopjic real number $X = x_0 + x_1I_1 + x_2I_2$, we have.

$X\vec{u} = (x_0 + x_1I_1 + x_2I_2)(a_0 + a_1I_1 + a_2I_2, b_0 + b_1I_1 + b_2I_2)$

$X\vec{u} = \big((x_0 + x_1I_1 + x_2I_2)(a_0 + a_1I_1 + a_2I_2), (x_0 + x_1I_1 + x_2I_2)(b_0 + b_1I_1 + b_2I_2)\big)$

Doi : https://doi.org/10.54216/GJMSA.020103
Received: March 16, 2022 Accepted: July 09, 2022

$X\vec{u} = \big(a_0x_0 + (a_0x_0 + a_0x_1 + a_1x_1 + a_1x_2 + a_2x_2)I_1 + (a_0x_2 + a_2x_0 + a_2x_2)I_2, (b_0x_0 + (b_0x_0 + b_0x_1 + b_1x_1 + b_1x_2 + b_2x_2)I_1 + (b_0x_2 + b_2x_0 + b_2x_2)I_2)\big).$

$f(X\vec{u}) = \big((a_0x_0, a_0x_0 + a_0x_0 + a_0x_1 + a_1x_1 + a_1x_2 + a_2x_2 + a_0x_2 + a_2x_0 + a_2x_2, a_0x_0 + a_0x_2 + a_2x_0 + a_2x_2), (b_0x_0, b_0x_0 + b_0x_0 + b_0x_1 + b_1x_1 + b_1x_2 + b_2x_2 + b_0x_2 + b_2x_0 + b_2x_2, b_0x_0 + b_0x_2 + b_2x_0 + b_2x_2)\big).$

$f(X\vec{u}) = (x_0, x_0 + x_1 + x_2, x_0 + x_2)\big((a_0, a_0 + a_1 + a_2, a_0 + a_2), (b_0, b_0 + b_1 + b_2, b_0 + b_2)\big)$

$f(X\vec{u}) = g(x_0 + x_1I_1 + x_2I_2).f(a_0 + a_1I_1 + a_2I_2, b_0 + b_1I_1 + b_2I_2)$

$f(X\vec{u}) = g(X).f(\vec{u}).$

Definition 3.10: (refined neutrosophic circle)

Let $M(a_0 + a_1I_1 + a_2I_2, b_0 + b_1I_1 + b_2I_2)$ be a fixed refined neutrosophic point, we define the refined neutrosophic circle with center M and radius $R = r_0 + r_1I_1 + r_2I_2 \geq 0$ to be the set of all two N-dimensional points $N(X, Y) = N(x_0 + x_1I_1 + x_2I_2, y_0 + y_1I_1 + y_2I_2)$; $\text{dist}(M, N) = R$.

Theorem 3.11:

Let $M(a_0 + a_1I_1 + a_2I_2, b_0 + b_1I_1 + b_2I_2)$ be a fixed refined neutrosophic point, $R = r_0 + r_1I_1 + r_2I_2$ be a refined neutrosophic real positive number, we have:

(a). The equation of the refined circle with center M and radius R is:

$C: ([x_0 + x_1I_1 + x_2I_2] - [a_0 + a_1I_1 + a_2I_2])^2 + ([y_0 + y_1I_1 + y_2I_2] - [b_0 + b_1I_1 + b_2I_2])^2 = R^2.$

(b). The previous refined neutrosophic circle is equivalent to the following direct product of three classical circles.

$C_1: (x_0 - a_0)^2 + (y_0 - b_0)^2 = {r_0}^2.$

$C_2: ([x_0 + x_1 + x_2] - [a_0 + a_1 + a_2])^2 + ([y_0 + y_1 + y_2] - [b_0 + b_1 + b_2])^2 = (r_0 + r_1 + r_2)^2.$

$C_3: ([x_0 + x_2] - [a_0 + a_2])^2 + ([y_0 + y_2] - [b_0 + b_2])^2 = (r_0 + r_2)^2.$

Proof.

(a). By using the neutrosophic distance form defined above, we get:

$([x_0 + x_1I_1 + x_2I_2] - [a_0 + a_1I_1 + a_2I_2])^2 + ([y_0 + y_1I_1 + y_2I_2] - [b_0 + b_1I_1 + b_2I_2])^2 = R^2$

(b). To obtain the classical equivalent geometrical systems of the refined neutrosophic circle, it is sufficient to take its refined AH-isometric image as follows:

$f(([x_0 + x_1I_1 + x_2I_2] - [a_0 + a_1I_1 + a_2I_2])^2 + ([y_0 + y_1I_1 + y_2I_2] - [b_0 + b_1I_1 + b_2I_2])^2) = f(R^2)$ hence.

$((x_0 - a_0)^2, ([x_0 + x_1 + x_2] - [a_0 + a_1 + a_2])^2, ([x_0 + x_2] - [a_0 + a_2])^2) + ((y_0 - b_0)^2, ([y_0 + y_1 + y_2] - [b_0 + b_1 + b_2])^2, ([y_0 + y_2] - [b_0 + b_2])^2) = ({r_0}^2, (r_0 + r_1 + r_2)^2, (r_0 + r_2)^2).$

Thus, we get:

$C_1: (x_0 - a_0)^2 + (y_0 - b_0)^2 = {r_0}^2.$

$C_2: ([x_0 + x_1 + x_2] - [a_0 + a_1 + a_2])^2 + ([y_0 + y_1 + y_2] - [b_0 + b_1 + b_2])^2 = (r_0 + r_1 + r_2)^2.$

$C_3: ([x_0 + x_2] - [a_0 + a_2])^2 + ([y_0 + y_2] - [b_0 + b_2])^2 = (r_0 + r_2)^2.$

Example 3.12:

Consider the following refined neutrosophic circle:

$C: (X - [1 - I_1 - I_2])^2 + (Y - [2 + 2I_1 - I_2])^2 = (3 - I_1 + I_2)^2$

It is equivalent to the direct product of the following three classical circles:

$C_1: (x_0 - 1)^2 + (y_0 - 2)^2 = 9$

Doi : https://doi.org/10.54216/GJMSA.020103

Received: March 16, 2022 Accepted: July 09, 2022

C_2: $([x_0 + x_1 + x_2] - 2)^2 + ([y_0 + y_1 + y_2] - 3)^2 = 9$

C_3: $([x_0 + x_2] - [-1])^2 + ([y_0 + y_2] - 1)^2 = 16$

Definition 3.13: (refined neutrosophic line)

We define the refined neutrosophic line by the set of all two N-dimensional points (X, Y) with the property

$AX + BY + C = 0; X = x_0 + x_1I_1 + x_2I_2, Y = y_0 + y_1I_1 + y_2I_2, A = a_0 + a_1I_1 + a_2I_2,\ B = b_0 + b_1I_1 + b_2I_2, C = c_0 + c_1I_1 + c_2I_2$.

Theorem 3.14:

Let $AX + BY + C = 0$ be an equation of a refined neutrosophic line d, this line is equivalent to the direct product of the following three classical lines.

d_1: $a_0x_0 + b_0y_0 + c_0 = 0$

d_2: $(a_0 + a_1 + a_2)(x_0 + x_1 + x_2) + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2) + (c_0 + c_1 + c_2) = 0$

d_3: $(a_0 + a_2)(x_0 + x_2) + (b_0 + b_2)(y_0 + y_2) + (c_0 + c_2) = 0$

Proof.

By taking the refined AH-isometric image to the equation $AX + BY + C = 0$, we get the proof.

Example 3.15:

Consider the following refined neutrosophic line:

d: $(-2 - I_1 - 2I_2)X + (1 + I_1 - 3I_2)Y + (2 + I_1 + I_2) = 0$

It is equivalent to the direct product of the following three classical lines:

d_1: $-2x_0 + y_0 + 2 = 0$.

d_2: $-5(x_0 + x_1 + x_2) - (y_0 + y_1 + y_2) + 4 = 0$.

d_3: $-4(x_0 + x_2) - 2(y_0 + y_2) + 3 = 0$.

Remark 3.16:

The inverse map of the two dimensional AH-isometry is:

$f^{-1}: R^3 \times R^3 \to R(I_1, I_2) \times R(I_1, I_2); f^{-1}\big((a, b, c), (t, m, n)\big) = (a + [b - c]I_1 + [c - a]I_2, t + [m - n]I_1 + [n - t]I_2)$.

The inverse of two N-dimensional AH-isometry can be used to turn any point from the classical Euclidean space $R^3 \times R^3$ into the refined neutrosophic equivalent point.

Conclusion

In this paper, we have defined the refined AH-isometry between a refined neutrosophic space with two dimensions and the Cartesian product of two classical spaces. Also, we have used this isometry to find the classical geometrical structure of refined neutrosophic circle and refined neutrosophic line, where we proved that a refined neutrosophic circle is equivalent to three classical circles, and the refined neutrosophic line is equivalent to three classical lines.

As a future research direction, we aim to find the isometry that describes the refined neutrosophic spaces with three dimensions.

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