A Comparative Analysis of QADA-KF with JPDAF for Multitarget Tracking in Clutter

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Abstract—This paper presents a comparative analysis of performances of two types of multi-target tracking algorithms: 1) the Joint Probabilistic Data Association Filter (JPDAF), and 2) classical Kalman Filter based algorithms for multi-target tracking improved with Quality Assessment of Data Association (QADA) method using optimal data association. The evaluation is based on Monte Carlo simulations for difficult maneuvering multiple-target tracking (MTT) problems in clutter.

Keywords: Data association, JPDAF, Belief Functions, QADA, PCR6 rule, Multitarget Tracking.

I. INTRODUCTION

Multiple-target tracking (MTT) is a principle component of surveillance systems. The main objective of MTT is to estimate jointly, at each observation time moment, the number of targets continuously moving in a given region and their trajectories from the noisy sensor data. In a single-sensor case, the multitarget tracker receives a random number of measurements due to the uncertainty which results in low detection and false alarms, arising independently of the targets of interest. Because of the fact that detection probability is not perfect, some targets may go undetected at some sampling intervals. Additional complications appear, apart from the process and measurement noises, associated with a measurement origin uncertainty, missed detection, cancelling (death) of targets, etc.

Data association (DA) is a primary task of modern MTT systems [1]–[3]. It entails selecting the most trustable associations between uncertain sensor's measurements and existing targets at a given time. In the presence of a dense MTT environment, with false alarms and sensor detection probabilities less than unity, the problem of DA becomes more complex, because it should contend with many possibilities of pairings, some of which are in practice very imprecise, unreliable, and could lead to critical association mistakes in the overall tracking process.

In order to deal with these complex associations the most recent method to evaluate the Quality Assessment of Data Association (QADA) encountered in multiple target tracking applications in a mono-criterion context was proposed by Dezert and Benameur [4], and extended in [5] for the multi-criteria context. It is based on belief functions (BF) for achieving the quality of pairings belonging to the optimal data assignment solution based on its consistency with respect to all the second best solutions, provided by a chosen algorithm. Most recently, in [6] the authors did discuss and propose the

way in which Kalman filter (KF) could be enhanced in order to reflect the knowledge obtained based on the QADA method, called QADA-KF method.

Taking into account that QADA assumes the reward matrix is known, regardless of the manner in which it is obtained by the user, in this paper we propose and test the performance of two possible versions of QADA-KF. The first one utilizes the assignment matrix, provided by the Global Nearest Neighbour (GNN) method, called QADA-GNN KF approach. The second one utilizes the assignment matrix, provided by the Probabilistic Data Association (PDA) method, called QADA-PDA KF method.

These two QADA-KF methods are compared with the Joint Probabilistic Data Association Filter (JPDAF) [7]–[9] which is an extension of the Probabilistic Data Association Filter (PDAF) [1] to a fixed and known number of targets. JPDAF uses joint association events and joint association probabilities in order to avoid conflicting measurement-to-track assignments by making a soft (probabilistic) assignment of all validated measurements to multiple targets.

The main objective of this paper is to compare the performances of: (i) classical MTT algorithms based on the GNN approach for data association, utilizing Kinematic only Data (KDA) and Converted Measurement Kalman Filter (CMKF); (ii) QADA-GNN KF based MTT; (iii) QADA-PDA KF based MTT; (iiii) JPDAF based MTT. The evaluation is based on a Monte Carlo simulation for particular difficult maneuvering MTT problems in clutter.

This paper is organised as follows. In Section II the JPDAF is described and discussed. Section III is devoted to QADA based KF. Data association methods, providing an assignment matrix for QADA are discussed in Section IV. Two particular simulation MTT scenarios and results are presented for the KDA, QADA-GNN KF, QADA-PDA KF, and JPDAF in Section V. Conclusions are made in Section VI.

II. JOINT PROBABILISTIC DATA ASSOCIATION FILTER

The Joint Probabilistic Data Association Filter (JPDAF) is an extension of the Probabilistic Data Association Filter (PDAF) for tracking multiple targets in clutter [1], [2], [10]–[12]. This Bayesian tracking filter uses the probabilistic assignment of all validated measurements belonging to the target gate to update its estimate. The preliminary version of JPDAF

was proposed by Bar-Shalom in 1974 [13], then updated and finalized in [7]–[9]. The assumptions of JPDAF are the following:

- the number N_T of established targets in clutter is known;
- all the information available from the measurements $\mathbf{Z}^k$ up to time k is summarized by the sufficient statistic $\hat{\mathbf{x}}^t(k|k)$ (the approximate conditional mean), and covariance $\mathbf{P}^t(k|k)$ for each target t;
- the real state $\mathbf{x}^t(k)$ of a target t at time k is modeled by a Gaussian pdf $\mathcal{N}(\mathbf{x}^t(k); \hat{\mathbf{x}}^t(k|k), \mathbf{P}^t(k|k))$;
- each target t follows its own dynamic model;
- each target generates at most one measurement at each observation time and there are no merged measurements;
- each target is detected with some known detection probability P_d^t;
- the false alarms (FA) are uniformly distributed in surveillance area and their number follows a Poisson pmf with FA density λ_{FA}.

In JPDAF, the measurement to target association probabilities are computed jointly across the targets and only for the latest set of measurements. This appealing theoretical (0-scan-back) approach however can give rise to very high combinatorics complexity if there are several persistent interferences, typically when several targets are crossing or if they move closely during several consecutive scans. Moreover some track coalescence effects may also appear which degrades substantially the JPDAF performances as it will shown in section V. These limitations of JPDAF have already been reported in [14]. Here we briefly recall the basics of JPDAF. For more details, please refer to [1], [2], [10]–[12], [15].

A. JPDAF principle

Let's consider a cluster[1] of $T \geq 2$ targets $t = 1, \ldots, T$. The set of m_k measurements available at scan k is denoted $\mathbf{Z}(k) = \{\mathbf{z}_i(k), i = 1, \ldots, m_k\}$. Each measurement $\mathbf{z}_i(k)$ of $\mathbf{Z}(k)$ either originates from a target or from a FA. Denote $\hat{\mathbf{z}}^t(k|k-1)$ as the predicted measurement for target t, and all the possible innovations that could be used in the Kalman Filter to update the target state estimate are denoted $\tilde{\mathbf{z}}_i^t(k) \triangleq \mathbf{z}_i(k) - \hat{\mathbf{z}}^t(k|k-1), i = 1, \ldots, m_k$. In JPDAF, instead of using a particular innovation $\tilde{\mathbf{z}}_i^t(k)$, it uses the weighted innovation $\tilde{\mathbf{z}}^t(k) = \sum_{i=1}^{m_k} \beta_i^t(k) \tilde{\mathbf{z}}_i^t(k)$, where $\beta_i^t(k)$ is the probability that the measurement $\mathbf{z}_i(k)$ originates from target t. $\beta_0^t(k)$ is the probability that none measurements originate from the target t. The core of JPDAF is the computation of the a posteriori association probabilities $\beta_i^t(k), i = 0, 1, \ldots, m_k$ based on all possible joint association events $\Theta(k) = \bigcap_{i=1}^{m_k} \Theta_i^{t_i}(k)$, where $\Theta_i^{t_i}(k)$ is the event that mesurement $\mathbf{z}_i(k)$ originates from target[2] t_i, $0 \leq t_i \leq N_T$. More precisely, one has to compute for $i = 1, \ldots, m_k$, $\beta_i^t(k) = \sum_{\Theta(k)} P\{\Theta(k)|\mathbf{Z}^k\} \hat{\omega}_{it}(\Theta(k))$ and $\beta_0^t(k) = 1 - \sum_{i=1}^{m_k} \beta_i^t(k)$, where $\mathbf{Z}^k$ is the set of all

[1]A cluster is a group of targets which have some measurements in common in their validation gates (i.e. non-empty intersections).
[2]By convention and notation convenience, $t_i = 0$ means that the origin of measurement $\mathbf{z}_i$ is a FA.

measurements available up to time k, and $\hat{\omega}_{it}(\Theta(k))$ are the corresponding components of the association matrix characterizing the possible joint association $\Theta(k)$.

B. Feasible joint association events

Validation gates are used for finding the feasible joint events but not in the evaluation of their probabilities [12] (p. 388–389). To describe the observation situation, it uses the validation matrix $\Omega = [\omega_{it}]$, $i = 1, \ldots, m_k$ and $t = 0, \ldots, N_T$ with elements $\omega_{it} \in \{0, 1\}$ to indicate whether or not the measurement $\mathbf{z}_i$ lies in the validation gate of target t. Because each measurement can potentially originate from a FA, all elements of the first column of Ω corresponding to index $t = 0$ (meaning FA, or none of the targets) are equal to one. From this validation matrix, all possible feasible joint association events $\hat{\Omega}(\Theta(k)) = [\hat{\omega}_{it}(\Theta(k))]$ where $\omega_{it}(\Theta(k)) = 1$ if $\Theta_i^t(k) \in \Theta(k)$, and zero otherwise, are realized satisfying the following *feasibility conditions*:

- a measurement can have only one origin, that is for all i

$$\sum_{t=0}^{N_T} \hat{\omega}_{it}(\Theta(k)) = 1 \tag{1}$$

- at most one measurement can originate from a target

$$\sum_{i=1}^{m_k} \hat{\omega}_{it}(\Theta(k)) \leq 1, \quad \text{for } t = 1, \ldots, N_T \tag{2}$$

The generation of all possible feasible joint association events is computationally expensive for complicated MTT scenarios, which is a serious limitation of JPDAF for real-world scenarios. A simple Matlab™ algorithm for the generation of matrices $\hat{\Omega}(\Theta(k))$ is given in [15] (pp. 56–57), which is based on DFS (Depth First Search) detailed by Zhou in [16], [17], previously coded in FORTRAN in [18].

C. Feasible joint association probabilities

Thanks to Bayes formula, the computation of the a posteriori joint association probabilities $P\{\Theta(k)|\mathbf{Z}^k\}$ involved in the derivation of $\beta_i^t(k)$ can be expressed as (see [1], [2], [10]–[12], [15] for full derivations)

$$
\begin{aligned}
P\{\Theta(k)|\mathbf{Z}^k\} &= \frac{1}{c} \cdot p[\mathbf{Z}(k)|\Theta(k), m_k, \mathbf{Z}^{k-1}] P\{\Theta(k)|m_k\} \\
&= \frac{1}{c} \cdot \frac{\phi(\Theta(k))!}{m_k!} \mu_F(\phi(\Theta(k))) V^{-\phi(\Theta(k))} \\
&\quad \times \prod_{i=1}^{m_k} \left[f_{t_i}(\mathbf{z}_i(k)) \right]^{\tau_i(\Theta(k))} \\
&\quad \times \prod_{t=1}^{T} (P_d^t)^{\delta_t(\Theta(k))} (1 - P_d^t)^{1 - \delta_t(\Theta(k))}
\end{aligned}
\tag{3}
$$

where c is a normalization constant, V is the volume of the surveillance region, and the indicators $\delta_t(\Theta(k))$ (target detec-

tion indicator), $\tau_i(\Theta(k))$ (measurement association indicator), $\phi(\Theta(k))$ (FA indicator) are defined by

$$\delta_t(\Theta(k)) \triangleq \sum_{i=1}^{m_k} \hat{\omega}_{it}(\Theta(k)) \leq 1 \qquad t = 1, \ldots, N_T \quad (4)$$

$$\tau_i(\Theta(k)) \triangleq \sum_{t=1}^{T} \hat{\omega}_{it}(\Theta(k)) \quad (5)$$

$$\phi(\Theta(k)) \triangleq \sum_{i=1}^{m_k} [1 - \tau_i(\Theta(k))] \quad (6)$$

$\mu_F(\phi(\Theta(k)))$ is the prior pmf of the number of false measurements (the clutter model) and

$$f_{t_i}(\mathbf{z}_i(k)) \triangleq \mathcal{N}[\mathbf{z}_i(k); \hat{\mathbf{z}}^{t_i}(k|k-1), \mathbf{S}^{t_i}(k)] \quad (7)$$

where $\mathbf{S}^{t_i}(k)$ is the predicted covariance matrix of innovation $\mathbf{z}_i(k) - \hat{\mathbf{z}}^{t_i}(k|k-1)$.

Two versions of JPDAF have been proposed [1], [7]–[9]:

- **Parametric JPDAF**: Knowing the spatial density λ_{FA} of the false measurements, and using a Poisson pmf $\mu_F(\phi(\Theta(k))) = \frac{(\lambda_{\text{FA}} V)^{\phi(k)}}{\phi(k)!} e^{-\lambda_{\text{FA}} V}$, results in

$$P\{\Theta(k)|\mathbf{Z}^k\} = \frac{1}{c_1} \cdot \prod_{i=1}^{m_k} [\lambda_{\text{FA}}^{-1} \cdot f_{t_i}(\mathbf{z}_i(k))]^{\tau_i(\Theta(k))}$$
$$\times \prod_{t=1}^{N_T} [P_d^t]^{\delta_t(\Theta(k))} [1 - P_d^t]^{1-\delta_t(\Theta(k))} \quad (8)$$

where c_1 is a normalization constant.

- **Non parametric JPDAF**: Using a diffuse prior pmf of number of FA $\mu_F(\phi(k)) = \epsilon, \forall \phi(k)$, results in

$$P\{\Theta(k)|\mathbf{Z}^k\} = \frac{\phi(k)!}{c_2} \cdot \prod_{i=1}^{m_k} [V f_{t_i}(\mathbf{z}_i(k))]^{\tau_i(\Theta)}$$
$$\times \prod_{t=1}^{N_T} [P_d^t]^{\delta_t(\Theta(k))} [1 - P_d^t]^{1-\delta_t(\Theta(k))} \quad (9)$$

where c_2 is a new normalization constant.

D. JPDAF state estimation

Once all feasible joint association events $\Theta(k)$ have been generated and their a posteriori probabilities $P\{\Theta(k)|\mathbf{Z}^k\}$ determined, all the marginal association probabilities $\beta_i^t(k) = \sum_{\Theta(k)} P\{\Theta(k)|\mathbf{Z}^k\} \hat{\omega}_{it}(\Theta(k))$ and $\beta_0^t(k) = 1 - \sum_{i=1}^{m_k} \beta_i^t(k)$ are computed. The state update and prediction are done with PDAF equations[3] given by

$$\hat{\mathbf{x}}^t(k|k) = \sum_{i=0}^{m_k} \beta_i^t(k) \hat{\mathbf{x}}_i^t(k|k) \quad (10)$$

with $\hat{\mathbf{x}}_i^t(k|k)$ given by

$$\hat{\mathbf{x}}_{i>0}^t(k|k) = \hat{\mathbf{x}}^t(k|k-1) + \mathbf{K}^t(k) \tilde{\mathbf{z}}_i^t(k) \quad (11)$$
$$\hat{\mathbf{x}}_{i=0}^t(k|k) = \hat{\mathbf{x}}^t(k|k-1) \quad (12)$$

Using (11) and (12) in (10), then

$$\hat{\mathbf{x}}^t(k|k) = \hat{\mathbf{x}}^t(k|k-1) + \mathbf{K}^t(k) \sum_{i=1}^{m_k} \beta_i^t(k) \tilde{\mathbf{z}}_i^t(k) \quad (13)$$

$$\mathbf{P}^t(k|k) = \beta_0^t(k) \mathbf{P}^t(k|k-1) + (1 - \beta_0^t(k)) \mathbf{P}_c^t(k) + \tilde{\mathbf{P}}^t(k) \quad (14)$$

with

$$\mathbf{P}_c^t(k) = [\mathbf{I} - \mathbf{K}^t(k) \mathbf{H}(k)] \mathbf{P}^t(k|k-1) \quad (15)$$

$$\tilde{\mathbf{P}}^t(k) = \mathbf{K}^t(k) \left[\sum_{i=1}^{m_k} \beta_i^t(k) \tilde{\mathbf{z}}_i^t(k) \tilde{\mathbf{z}}_i^{t'}(k) - \tilde{\mathbf{z}}(k) \tilde{\mathbf{z}}'(k) \right] \mathbf{K}'(k) \quad (16)$$

and

$$\mathbf{K}^t(k) \triangleq \mathbf{P}^t(k|k-1) \mathbf{H}'(k) \mathbf{S}^t(k)^{-1} \quad (17)$$
$$\tilde{\mathbf{z}}_i^t(k) \triangleq \mathbf{z}_i(k) - \hat{\mathbf{z}}^t(k|k-1) \quad (18)$$
$$\tilde{\mathbf{z}}^t(k) \triangleq \sum_{i=1}^{m_k} \beta_i^t(k) \tilde{\mathbf{z}}_i^t(k) \quad (19)$$

It has been proved in [1] that $\tilde{\mathbf{P}}(k)$ is always a semi-positive matrix. The target state prediction $\hat{\mathbf{x}}^t(k+1|k)$ and $\mathbf{P}^t(k+1|k)$ are obtained by the classical Kalman Filter (KF) equations [1] (assuming linear kinematic models), or by Extended KF equations. They will not be repeated here [2], [10].

In summary, JPDAF is well theoretically founded and it does not require high memory (0-scan-back). It provides pretty good results on simple MTT scenarios (with non persisting interferences) with moderate FA densities. However the number of feasible joint association matrices increases exponentially with problem dimensions (m_k and N_T) which makes the JPDAF intractable for complex dense MTT scenarios.

III. QADA BASED KALMAN FILTER

The aim of this paper is to compare the performance of the JPDAF based MTT algorithm with the classical[4] MTT algorithm, using the CMKF based on kinematics measurements, but improved by the QADA method.

The main idea behind the QADA method, proposed recently by Dezert and Benameur [4] is to compare the values $a_1(i,j)$ in the first optimal DA solution $\mathbf{A}_1$ with the corresponding values $a_2(i,j)$ in second assignment solution $\mathbf{A}_2$, and to identify if there is a change of the optimal pairing (i,j). In the MTT context (i,j) means an association between measurement $\mathbf{z}_j$ and target T_i. QADA establishes a quality indicator associated with this pairing, depending on the stability of the pairing and also, on its relative impact in the global reward. The proposed method works also when the 1st and 2nd optimal assignments $\mathbf{A}_1$ and $\mathbf{A}_2$ are not unique, i.e., there are multiplicities available. In such a situation, the establishment of quality indicators could help in selecting one particular optimal assignment solution among multiple possible choices.

The construction of the quality indicator is based on belief functions (BF) and the Proportional Conflict Redistribution

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[3]for the decoupled version of JPDAF. For the coupled version of JPDAF, see [10], [12].

[4]Classical MTT algorithms are those based on hard assignment of a chosen measurement to a given target.

fusion rule no.6 (PCR6), defined within Dezert-Smarandache Theory (DSmT) [19]. It depends on the type of the pairing matching, and it is described in detail in [4].

In [6], the authors discuss and propose the way in which Kalman filter could be improved in order to reflect the knowledge obtained based on the QADA method.

Let's briefly recall what kind of information is obtained, having in hand the quality matrix, derived by QADA, in the MTT context. It gives knowledge about the confidence $q(i, j)$ in all pairings $(T_i, \mathbf{z}_j), i = 1, .., m; j = 1, .., n$, chosen in the first best assignment solution. The smaller quality (confidence) of hypothesis "$\mathbf{z}_j$ *belongs to* T_i" means, that the particular measurement error covariance $\mathbf{R}$ was increased and the filter should not trust fully in the actual (true) measurement $\mathbf{z}(k+1)$.

Having this conclusion in mind, the authors propose, such a behaviour of the measurement error covariance to be modelled by $\mathbf{R} = \frac{\mathbf{R}}{q(T_i, \mathbf{z}_j)}$, for every pairing, chosen in the first best assignment and based on the corresponding quality value obtained. Then, when the Kalman filter gain decreases the true measurement $\mathbf{z}_j(k+1)$ is trusted less in the updated state estimate $\hat{\mathbf{x}}(k+1|k+1)$.

IV. Building assignment matrix for QADA

Data Association (DA) is a central problem in the modern MTT systems [1], [2]. It consists in finding the global optimal assignments of targets $T_i, i = 1, ..., m$ to some measurements $\mathbf{z}_j, j = 1, ..., n$ at a given time k by maximizing the overall gain in such a way, that no more than one target is assigned to a measurement, and reciprocally. The $m \times n$ reward (gain/payoff) matrix $\mathbf{\Omega} = [\omega(i, j)]$ is defined by its elements $\omega(i, j) > 0$, representing the gain of the association of target T_i with the measurement $\mathbf{z}_j$.

These values are usually homogeneous to the likelihood ratios and could be established in different ways, described below. They provide the assignment matrix utilized by QADA in order to obtain the quality of pairings (interpreted as a confidence score) belonging to the optimal data assignment solution based on its consistency (stability) with respect to all the second best solutions, provided for a chosen algorithm.

QADA assumes the reward matrix is known, regardless of the manner in which it is obtained by the user. In this paper we propose two versions of QADA-KF. The first one utilizes the assignment matrix built from the single normalized distances, provided by the Global Nearest Neighbour method, called QADA-GNN KF method. The second one utilizes the assignment matrix, built from the posterior association probabilities, provided by the Probabilistic Data Association (PDA) method, called QADA-PDA KF method.

A. Assignment matrix based on GNN method

The GNN method finds and propagates the single most likely hypothesis during each scan to update KF. It is a hard (i.e., binary) decision approach, as compared to the JPDAF which is a soft (i.e., probabilistic) decision approach using all validated measurements with their probabilities of association. GNN method was applied in [6] and [20] to

obtain the assignment matrix, utilized in QADA. In this case the elements of assignment matrix $\omega(i, j), i = 1, .., m; j = 1, ..., n$ represent the normalized distances $d(i, j) \triangleq [(\mathbf{z}_j(k) - \hat{\mathbf{z}}_i(k|k-1))'\mathbf{S}^{-1}(k)(\mathbf{z}_j(k) - \hat{\mathbf{z}}_i(k|k-1))]^{1/2}$ between the validated measurement $\mathbf{z}_j$ and target T_i satisfying the condition $d^2(i, j) \leq \gamma$. The distance $d(i, j)$ is computed from the measurement $\mathbf{z}_j(k)$ and its prediction $\hat{\mathbf{z}}_i(k|k-1)$ (see [1] for details), and the inverse of the covariance matrix $\mathbf{S}(k)$ of the innovation, computed by the tracking filter. The threshold γ, for which the probability of given observation to fall in the gate is 0.99, could be defined from the table of the Chi-square distribution with M degrees of freedom and allowable probability of a valid observation falling outside the gate. In this case the DA problem consists in finding the best assignment, that minimizes the overall cost.

B. Assignment matrix based on PDA method

The Probabilistic Data Association (PDA) method [1] calculates the association probabilities for validated measurements at a current time moment to the target of interest. PDA assumes the following hypotheses according to each validated measurement:

- $H_i(k)$: $\mathbf{z}_j(k)$ is a measurement, originated from the target of interest, $i = 1, ..., m$
- $H_0(k)$: no one of the validated measurement originated from the target of interest

If N observations fall within the gate of track i, $N + 1$ hypotheses will be formed.

The probability of H_0 is proportional to $p_{i0} = \lambda_{\text{FA}}^N(1 - P_g P_d)$, and the probability of H_j $(j = 1, 2, .., N)$ is proportional to

$$p_{ij} = \frac{\lambda_{\text{FA}}^{N-1} P_g P_d \cdot e^{-\frac{d_{ij}^2}{2}}}{(2\pi)^{M/2} \cdot \sqrt{|\mathbf{S}_{ij}|}} \tag{20}$$

where P_g is the a priori probability that the correct measurement is in the validation gate[5] [1]; P_d is the target detection probability; λ_{FA} is the spatial density of FA. The probabilities p_{ij} can be rewritten as [1]

$$p_{ij} = \begin{cases} \frac{b}{b + \sum_{l=1}^{N} \alpha_{il}} & \text{for } j = 0 \text{ (no valid observ.)} \\ \\ \frac{\alpha_{ij}}{b + \sum_{l=1}^{N} \alpha_{il}} & \text{for } 1 \leq j \leq N \end{cases} \tag{21}$$

where

$$b \triangleq (1 - P_g P_d)\lambda_{\text{FA}}(2\pi)^{M/2}\sqrt{|\mathbf{S}_{ij}|} \tag{22}$$

and

$$\alpha_{ij} \triangleq P_d \cdot e^{-\frac{d_{ij}^2}{2}} \tag{23}$$

The assignment matrix used in QADA method is established from all p_{ij} given by (21) related with all association hypotheses. This matrix will have m rows (where m is the number of all targets of interest), and $N + 1$ columns for the hypotheses generated. The $(N + 1)$th column will include the values p_{i0} associated with $H_0(k)$.

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[5]In our simulations, we use $P_g = 0.99$ and $P_d = 0.99$.

V. SIMULATION SCENARIOS AND RESULTS

The Converted Measurement KF is used in our MTT algorithm. We assume constant velocity target model. The process noise covariance matrix is: $Q = \sigma_v^2 Q_T$, where T is the sampling period, σ_v is the standard deviation of the process noise and Q_T is as given in [3]. Here are the results of KDA KF, QADA-GNN KF, QADA-PDA KF, and JPDAF for two interesting MTT scenarios.

A. Groups of targets simulation scenario

The noise-free groups of targets simulation scenario (Fig.1) consists of five air targets moving from North-West to South-East. For the clear explanation of the results, targets are numbered starting at the beginning with 1st target that has the greater y-coordinate and continuing to 5th target with the smallest y-coordinate. The three targets 2nd, 3rd, and 4th move together between them[6]. The stationary sensor is located at the origin with range 20000 m. The sampling period is $T_{scan} = 5$ sec and the measurement standard deviations are 0.2 deg and 35 m for azimuth and range respectively. The targets move with constant velocity $V = 100m/sec$. The group of three targets in the middle i.e. 2nd to 4th move without maneuvering keeping azimuth 135 deg from North. It is the main direction of the group's movement. The first target starts with azimuth 165 deg and moves towards the middle group of rectilinearly moving targets. When it approaches the group, it starts a turn to the left with -30 deg. Its initial azimuth of 165 deg is decreased by the angle of turn and becomes 135 deg, the main direction. The fifth target makes similar maneuver but in opposite direction - to the right. Its initial azimuth of 105 deg is increased by the turn of 30 deg and becomes 135 deg, and also coincides with the main direction. From 21th scan to 48th scan all the targets move rectilinearly in parallel. The distance between them is 150 m. From 48th scan, the first target makes a left turn to azimuth of 105 deg, that means -30 degrees with respect to the main direction and starts to go away from the middle group. The fifth target makes right turn to azimuth of 165 deg that means $+30$ deg from the main direction and also starts to go away. All maneuvers are with one and the same value of the angle (angle= 30 deg by absolute value), the same time duration and linear velocity. The absolute value of the corresponding transversal acceleration for all maneuvers is $1.163m/s^2$. The total number of scans for the simulations is 65. Fig. 2 shows the noised scenario for $\lambda_{FA} = 16 \cdot 10^{-10} m^{-2}$ yielding to 0.2 FA per gate on average.

Our results are based on Monte Carlo (MC) simulations with 200 independent runs in applying KDA based KF, QADA-GNN KF, QADA-PDA KF, and JPDAF[7]. We compare the performance of these methods with different criteria, and we use an idealized track initiation in order to prevent uncontrolled impact of this stage on the statistical parameters of the tracking process during MC simulations. The true targets positions (known in our simulations) for the first two scans are

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[6]Note that three targets move together in the center.

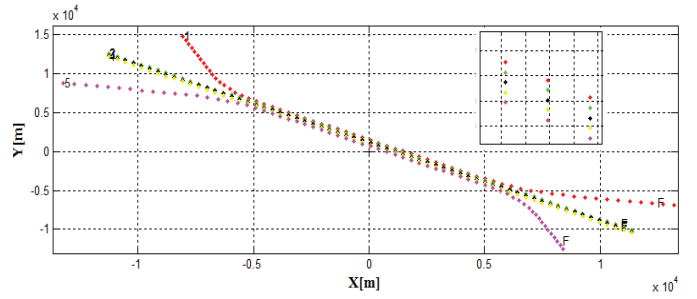[7]We have used the non parametric version of JPDAF in our simulations.

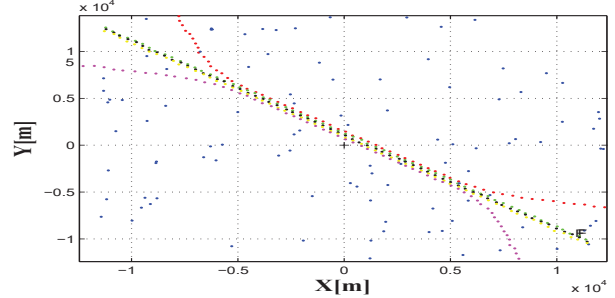Figure 1. Noise-free groups of targets Scenario.

Figure 2. Noised groups of targets Scenario with $\lambda_{FA} = 16 \cdot 10^{-10} m^{-2}$.

used for track initiation. The evaluation of MTT performance is based on the criteria of Track Purity (TP), Track Life (TL), and percentage of miscorrelation (pMC):

1) TP criteria examines the ratio between the number of particular performed (jth observation - ith track) associations (in case of detected target) over the total number of all possible associations during the tracking scenario, but TP cannot be used with JPDAF because JPDAF is a soft assignment method. Instead of TP, we define the Probabilistic Purity Index (PPI). It considers the measurement that has the highest association probability computed by the JPDAF and check, (and count) if this measurement originated from the target or not. PPI measures the ability of JPDAF to commit the highest probability to the correct target measurement in the soft assignment of all validated measurements.

2) TL is evaluated as an average number of scans before track's deletion. In our simulations, a track is cancelled and deleted from the list of tracked tracks, when during 3 consecutive scans it cannot be updated with some measurement because there is no validated measurement in the validation gate. When using JPDAF, the track is cancelled and deleted from the list of tracked tracks, when during 3 consecutive scans its own measurement does not fall in its gate. We call this, the "cancelling/deletion condition". The status of the tracked tracks is denoted "alive".

3) pMC examines the relative number of incorrect observation-to-track associations during the scans.

The MTT performance results for KDA only KF, QADA-GNN KF, QADA-PDA KF, and JPDAF for a low-noise case (0.2 FA per gate on average) are given in Table 1.

(in %)	KDA	JPDAF	QADA-GNN	QADA-PDA
Average TL	50.27	66.46	81.94	90.85
Average pMC	3.35	2.98	2.10	1.75
Average TP	45.61	PPI=29.14	79.32	87.61

Table 1

GROUPS OF TARGETS SCENARIO: COMPARISON BETWEEN MTT
PERFORMANCE RESULTS FOR 0.2 FA PER GATE.

Figure 3. RMSE on X for track 1 with the four tracking methods.

According to all criteria, the QADA-PDA KF method shows the best performance, followed by QADA-GNN KF, and JPDAF. The KDA based KF approach, as one could expect, shows the worst performance. Performance results for a more noisy scenario with 0.4 FA per gate on average are given in Table 2.

(in %)	KDA	QADA-GNN	JPDAF	QADA-PDA
Average TL	43.54	70.51	70.94	84.17
Average pMC	3.90	3.33	2.71	3.11
Average TP	38.22	66.43	PPI=25.65	78.51

Table II

GROUPS OF TARGETS SCENARIO: COMPARISON BETWEEN MTT
PERFORMANCE RESULTS FOR 0.4 FA PER GATE.

Figure 4. RMSE on Y for track 1 with the four tracking methods.

As we see, the results for 0.4 FA per gate scenario are degraded in comparison to the low-noise case. The average miscorrelation for QADA-PDA is slightly higher than for JPDAF, probably because QADA method is based on the 1st and 2nd best solutions only, and more information (i.e. the 3rd best assignment solution) should be used in such case to improve QADA performance, which is left for further research. According to TL and TP, still QADA-PDA KF based MTT shows stably better performance than JPDAF.

JPDAF based MTT outperforms QADA-GNN KF and KDA KF based MTT approaches according to the considered criteria. In order to make a fair comparison between QADA KF and JPDAF, we will discuss also the root mean square errors (RMSE), associated with the filtered X and Y values, presented in Figs. 3–7. Figs. 3 and 4 show the mean square X and Y error filtered, associated with target 1, and compared for KDA KF, QADA-GNN KF, QADA-PDA KF, and JPDAF. Figs. 5 and 6 consider the same errors for the middle of track 3. All the results are compared to the sensor's errors along X and Y axis. We see that the RMSE on X filtered associated with KDA KF, QADA-GNN KF, and QADA-PDA KF are a little bit above from the sensor's error in the region where target 1 makes maneuvers. For scans [20, 50], target 1 is moving in parallel to the group of other targets running rectilinearly and then these errors are less than respective sensors's ones. The RMSE on X filtered associated with JPDAF performance is three times bigger in the region between scans 20th and 30th where target 1 starts moving in parallel to the rest of rectilinearly moving targets. The RMSE on Y filtered is high during the whole region, where target 1 moves in parallel way.

The RMSE on Y error filtered by JPDAF are especially critical for the middle track 3 which shows its poor performance in state estimation on Y direction. The RMSEs are more than 5 times bigger (in the region between scans 20th and 50th, where all five targets move in parallel) than the respective
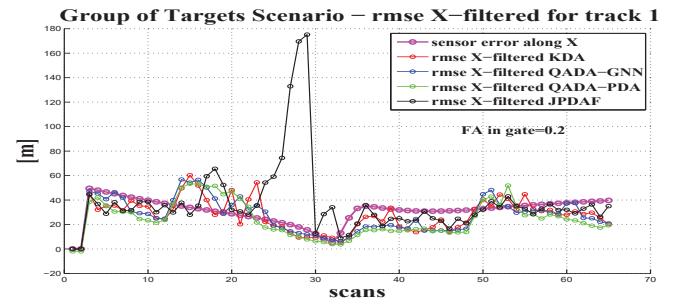
errors obtained by KDA KF, QADA-GNN KF, QADA-PDA KF, which are less than the sensor's error. The RMSE on X filtered obtained with JPDAF is under the sensor's error, beside KDA KF, QADA-GNN KF, QADA-PDA KF methods.
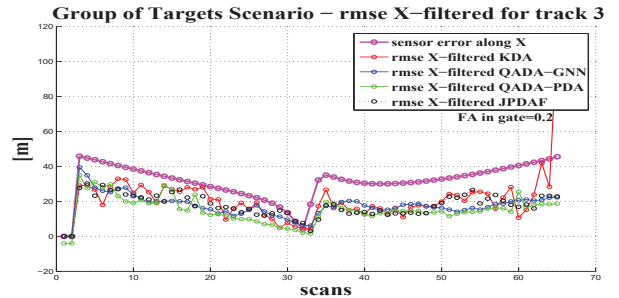

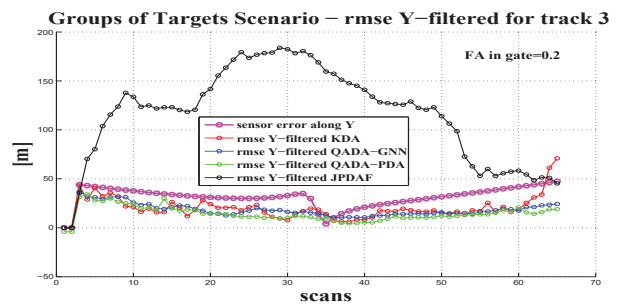Figure 5. RMSE on X for track 3 with the four tracking methods.

Figure 6. RMSE on Y for track 3 with the four tracking methods.

The large value of RMSE on Y using the JPDAF can be

explained by the specificity of the scenario because it has five targets moving closely during more than 30 consecutive scans with sensor's measurement errors, and false alarms density, which yields to spatial persisting interferences and track coalescence effects in JPDAF, as shown in Fig. 7, where the red and green plots are the tracks estimates. These effects degrades significantly the quality of JPDAF performance as already reported in [14].
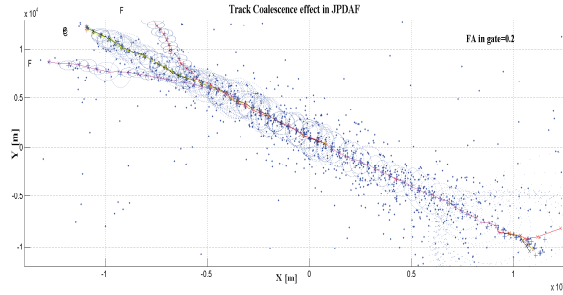

Figure 7. JPDAF track coalescence (one run) with $\lambda_{FA} = 16 \cdot 10^{-10} m^{-2}$.

B. Crossing targets simulation scenario

The second considered (crossing targets) scenario (Fig. 8) consists of two maneuvering targets moving with constant velocity $38m/sec$. At the beginning, both targets move from West to East. The stationary sensor is located at the origin with range 1200 m. The sampling period is $Tscan = 1sec$ and the measurement standard deviations are 0.2 deg and 25 m for azimuth and range respectively.

The first target, having at the beginning greater y-coordinate, moves straightforward from West to East. Between the 8th and 12th scans it makes a 50 deg right turn, and then it moves straightforward during 8 scans. From the 20th scan to the 24th scan it makes a 50 deg left turn, and then it moves in East direction till the 41th scan. It makes a second 50 deg left turn between 41th and 45th scans, and then it moves straightforward during 8 scans. From 53th scan it makes a second 50 deg right turn till the 57th scan and then it moves in East direction. The trajectory of target 1 corresponds to the red plot of Fig. 8.

The second target makes a mirrored trajectory corresponding to the green plot of Fig. 8. From scan 1 to 8 it moves from West to East. During 8th to 12th scans it makes a 50 deg left turn. Then it moves straightforward during 8 scans. During 20th to 24th scans it makes a 50 deg right turn and then it moves in East direction till the 41th scan. It makes a second 50 deg right turn between the 41th and 45th scans, and then it moves straightforward during 8 scans. From the 53th scan it makes a second 50 deg left turn till the 57th scan and then it moves in East direction. The total number of scans for the simulations is 65. Fig. 9 shows the respective noised scenario for $\lambda_{FA} = 4 \cdot 10^{-7} m^{-2}$.

The MTT performance results obtained on the base of KDA only KF, QADA-GNN KF, QADA-PDA KF, and JPDAF for less noised case corresponding to 0.2 FA per gate are given in
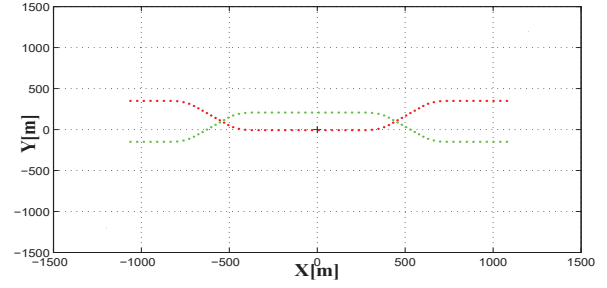

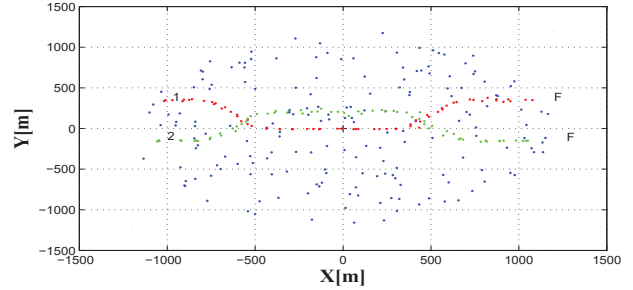Figure 8. Noise-free Crossing targets Scenario.

Figure 9. Noised Crossing targets Scenario with $\lambda_{FA} = 4 \cdot 10^{-7} m^{-2}$.

Table 3, and the performance results for a more noisy scenario with 0.4 FA per gate on average are given in Table 4.

(in %)	KDA	QADA-GNN	JPDAF	QADA-PDA
Average TL	77.06	88.93	91.25	93.47
Average pMC	2.40	2.24	2.08	2.11
Average TP	72.78	85.64	PPI=86.29	87.96

Table III
CROSSING TARGETS SCENARIO: COMPARISON BETWEEN MTT
PERFORMANCE RESULTS FOR FA IN GATE = 0.2.

(in %)	KDA	QADA-GNN	JPDAF	QADA-PDA
Average TL	58.80	77.20	82.87	83.18
Average pMC	3.61	3.63	2.94	3.40
Average TP	52.90	72.01	PPI=76.94	77.15

Table IV
CROSSING TARGETS SCENARIO: COMPARISON BETWEEN MTT
PERFORMANCE RESULTS FOR FA IN GATE = 0.4.

According to all criteria, the QADA-PDA KF shows again the best performance, but now JPDAF based MTT shows closed to QADA-PDA KF performance in comparison to the previous scenario, and exceeds the performance of QADA-GNN KF. Nevertheless the performances of all methods are deteriorated in more noised case, when one has 0.4 FA in gate on average, this tendency is still kept. JPDAF has better (than in the previous scenario) performance, but still QADA-PDA KF exceeds its performance.

Figures 10-13 show that the RMS errors associated with X and Y filtered are below the sensor's error. They confirm the better performance of JPDAF in this particular scenario with only two maneuvering targets, which is simpler than the groups of targets scenario.
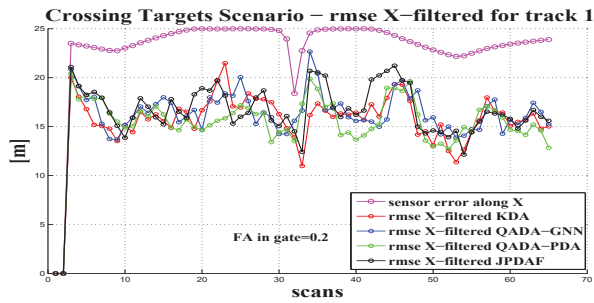
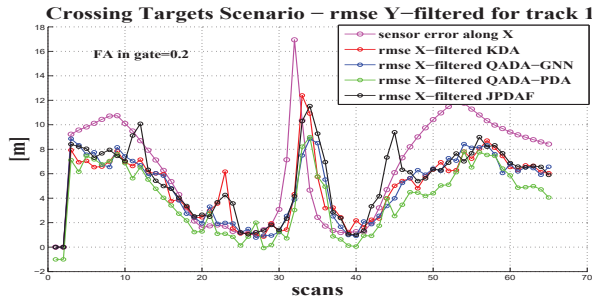Figure 10. RMSE on X for track 1 with the four tracking methods.

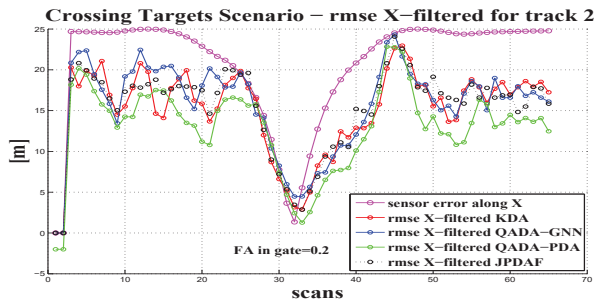Figure 11. RMSE on Y for track 1 with the four tracking methods.

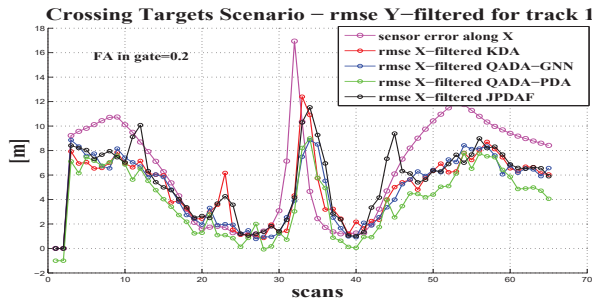Figure 12. RMSE on X for track 2 with the four tracking methods.

Figure 13. RMSE on Y for track 2 with the four tracking methods.

VI. CONCLUSIONS

This work evaluated with Monte Carlo simulations the efficiency of MTT performance in cluttered environment of four methods (i) classical MTT algorithm based on the GNN approach for data association, utilizing Kinematic only Data based Kalman Filter; (ii) QADA-GNN KF; (iii) QADA-PDA KF; and (iiii) JPDAF. The first scenario (groups of targets) shows the advantages of applying QADA-KF. According to all performance criteria, the QADA-PDA KF gives the best performance, followed by QADA-GNN KF, and JPDAF. The KDA KF approach shows the worst performance (as expected). This scenario is particularly difficult for JPDAF because of several closely spaced and rectilinearly moving targets in clutter during many consecutive scans, and it leads to track coalescence effects due to persisting interferences. As a result, the tracking performance of JPDAF is degraded. Because the complexity of the calculation for joint association probabilities grows exponentially with the number of targets, JPDAF requires almost 3 times more computational time in comparison to other methods in the first (complex) scenario. In the second (only two crossing targets) MTT scenario, JPDAF shows better tracking performances in comparison to QADA-GNN KF. It is able to track more precisely these only two targets, because of non persisting interferences. Overall, our analysis shows that QADA-PDA KF method is the best of the four approaches to track multiple targets in clutter with a tractable complexity.

REFERENCES

[1] Y. Bar-Shalom, T.E. Fortmann, *Tracking and Data Association*, Academic Press, 1988.
[2] Y. Bar-Shalom (Ed.), *Multitarget-Multisensor Tracking: Advanced Applications*, Artech House, Norwood, USA, 1990.
[3] S. Blackman, R. Popoli, Design and Analysis of Modern Tracking Systems, Artech House, 1999.
[4] J. Dezert, K. Benameur, *On the Quality of Optimal Assignment for Data Association*, Springer, L.N. in Compt. Sci., Vol.8764, pp. 374–382, 2014.
[5] J. Dezert et al., *On the Quality Estimation of Optimal Multiple Criteria Data Association Solutions*, Proc. of Fusion 2015, July 2015.
[6] J. Dezert, et al., Multitarget Tracking Performance based on the Quality Assessment of Data Association, Proc. of Fusion 2016, July 2016.
[7] Y. Bar-Shalom, T.E. Fortmann, M. Scheffe, *JPDA for Multiple Targets in Clutter*, Proc. Conf. on Inf. Sci. and Syst., Princeton, March 1980.
[8] T.E. Fortmann et al., *Multitarget Tracking using Joint Probabilistic Data Association*, in Proc. of IEEE CDC, Albuquerque, USA, Dec. 1980.
[9] T.E. Fortmann, et al., *Sonar Tracking of Multiple Targets Using JPDAF*, IEEE Journal of Oceanic Eng., Vol. 8, No.3, pp. 173–184, July 1983.
[10] Y. Bar-Shalom, X.-R. Li. *Multitarget-Multisensor Tracking: Principles and Techniques*, YBS Publishing, Storrs, CT, USA, 1995.
[11] Y. Bar-Shalom et al., *Estimation with Applications to Tracking and Navigation: Theory, Algorithms and Software*, John Wiley & Sons, 2001.
[12] Y. Bar-Shalom et al., *Tracking and Data Fusion: A Handbook of Algorithms*, YBS Publishing, Storrs, CT, USA, 2011.
[13] Y. Bar-Shalom, *Extension of the PDAF to Multitarget Environments*, Proc. 5th Symp. Nonlinear Estimation, San Diego, USA, Sept. 1974.
[14] R.J. Fitzgerald, *Track Biases and Coalescence with Probabilistic Data Association*, IEEE Trans. on AES, Vol. 21, No. 6, pp. 822–825, 1985.
[15] J. Dezert, *Introduction au pistage multi-cibles multi-senseurs*, ENSTA Course (in French), Sept. 2003.
[16] B. Zhou, *Multitarget Tracking in Clutter: Algorithms for Data Association and State Estimation*, Ph.D. Thesis, Penn. State Univ., 1992.
[17] B. Zhou, N.K. Bose, *Multitarget Tracking in Clutter: Fast Algorithms for Data Association*, IEEE Trans. on AES, Vol. 29, No. 2, April 1993.
[18] J. Dezert, *Poursuite Multi-Cibles Mono-Senseur*, ONERA Tech. Note 1988-10 (in French), France, 1988.
[19] F. Smarandache, J. Dezert (Editors), *Advances and Applications of DSmT for Information Fusion*, Volumes 1, 2, 3 & 4, ARP, 2004–2015. http://www.onera.fr/staff/jean-dezert?page=2
[20] J. Dezert et al., *The Impact of the Quality Assessment of Optimal Assignment for Data Association in Multitarget Tracking Context*, Cybernetics and Inf. Techn. J., Vol.15, No.7, pp. 88–98, 2015.