A Chi-Square Distance-Based Similarity Measure Of Single-Valued Neutrosophic Under Set And Applications

T. Deepika[1], K. Mohana[2]

[1]PG Student & [2]Assistant Professor,

Nirmala College For Women, Coimbatore.

E-mail: deepikathangavel23@gmail.com[1],riyaraju1116@gmail.com[2]

Abstract:

The aim of this paper is to propose a new similarity measure of single valued neutrosophic under sets (SVNUSs). Numerical examples are provided to show the superiority of the proposed similarity measure comparing with the existing similarity measures of SVNUSs. A weighted similarity is also put forward based on the proposed similarity. Some examples are given to show the effectiveness and practicality of the proposed similarity in pattern recognition, medical diagnosis and multi-attribute decision making problems under single-valued neutrosophic environment.

Keywords: *Chi-square distance measure, similarity measure, multi-attribute decision making, single-valued neutrosophic under set.*

1 Introduction:

Since fuzzy set was first proposed by Zadeh and has achieved a great success in various fields due to its capability of handling uncertainty [3, 5, 8, 14, 21, 30]. Over the last decades, some extended fuzzy sets have been introduced by researchers, such as intuitionistic fuzzy set [1], vague set [10], interval-valued intuitionistic fuzzy set [2] and hesitant fuzzy set [29]. In recent years, intuitionistic fuzzy set has received a lot attention and been applied to many fields, such as management decision, pattern recognition and medical diagnosis [4, 6, 18, 20, 28]. In practice, indeterminate and inconsistent information may occur, then intuitionistic fuzzy set cannot deal with these situations well because it only contains the true membership degree and the false membership degree (non-membership degree).

For example, when an authority wants to choose the best candidate, ten experts are invited to take part in the decision. For one candidate he gained 10 votes from the experts. There are 3 votes yes", 2 votes "no", 2 "gave up"" and 3 "undecided". In this case, intuitionistic fuzzy set cannot describe it well. To overcome this shortcoming, Smarandache [27] introduced a concept of neutrosophic set, which is an extension of intuitionistic fuzzy set from philosophical point of view. Neutrosophic set is defined as a set containing the degree of truth, indeterminacy, and falsity. For afore-mentioned example, the vote result can be expressed by a neutrosophic set. However, the original neutrosophic set is difficult to apply in practical problems. To apply it easily in science and engineering fields, Wang et al. [31] introduced the concept of single-valued neutrosophic set (SVNS), which is a subclass of Smarandache's neutrosophic set. Because SVNS is easy to express, it has been a useful mathematical tool for handling various practical problems involving imprecise, indeterminacy, and inconsistent data [11, 15, 19, 23, 25].

In recent years, the study and applications of information measures of fuzzy sets have received a lot attention. Similarity measure is one of the most important measurement tools for comparing the degree of similarity between two objects. Since Li and Chen [17] introduced the definition of the similarity measure between two intuitionistic fuzzy sets. Since then intuitionistic fuzzy similarity measures have received great attention. From a different point of view, many similarity measures are proposed and applied to solve various practical problems of Multi-Attribute Decision Making (MADM), pattern recognition and medical diagnosis, etc. [12, 13, 16, 22, 32, 34].

As an extension of intuitionistic fuzzy set, some similarity measures of neutrosophic set is developed from those of intuitionistic fuzzy sets, and some new similarity measures are also proposed, but the references are still rare ([35]). Based on vector similarity functions, some similarity measures between simplified neutrosophic sets are put forwards, such as similarity measures based on Jaccard, Dice, and cosine functions [33-35].

Neutrosophic Under Set and Logic [9] were defined for the first time by Florentin Smarandache in 1995 and presented to various international and national conferences and seminars between 1995-2016 and first time published in 2007. They are totally different from other sets/logics/probabilities/statistics.

They extended the neutrosophic set respectively to Neutrosophic Overset {when some neutrosophic component is > 1}, Neutrosophic Under set {when some neutrosophic component is < 0}, and to Neutrosophic Offset {when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and other neutrosophic component < 0}.

This is no surprise with respect to the classical fuzzy set/ logic, intuitionistic fuzzy set/ logic, or classical/ imprecise probability, where the values are not allowed outside the interval [0, 1], since our real-world has numerous examples and applications of over-/under-/off-neutrosophic components.

We find that the existing similarity measures have shortcomings, and the detail analysis can be found in Example 1. Then the aim of this paper is to develop a new similarity measure of SVUNSs based on Chi-square distance measure, which is an important measure in statistical theory. We will show the advantage of the proposed similarity measure with existing similarity measures of SVNUSs through comparison with some numerical examples. Three examples are provided to demonstrate the effectiveness and practicality of the proposed similarity in the application of pattern recognition, medical diagnosis and multi-attribute decision making.

2. Preliminaries:

In this section, some basic concepts and properties of SVNSs and similarity measure are presented. Smarandache [27] originally introduced a concept of neutrosophic set from philosophical point of view.

Definition 2.1([26]):

Let X be a universal set. A set is called a neutrosophic set, if it is characterized by three parameters: truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$ .That is A has the following form:

$$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$$

Here $T_A(x), I_A(x), F_A(x) : X \rightarrow]^-0, 1+[,]^-0, 1^+[$ is non-standard interval, and they satisfy

$$^-0 \leq \sup TA(x) + \sup IA(x) + \sup FA(x) \leq 3^+$$

A neutrosophic is defined as a set containing the degree of truth, indeterminacy, and falsity. However, the original neutrosophic set is difficult to apply in practical problems. To apply it easily in science and engineering fields, Wang et al. [30] introduced single-valued neutrosophic set (SVNS), which is a subclass of Smarandache's neutrosophic set.

Definition 2.2([26]):

Let X be a universal set. A set A is called a SVNS, if it is characterized by three parameters: truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. That is, A has the following form:

$$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$$

Here $T_A(x), I_A(x), F_A(x) : X \rightarrow [0, 1]$, and they satisfy $0 \leq T_A(x)+ I_A(x)+ F_A(x) \leq 3$. For convenience, when X = {x}, we briefly denote the element $< x, T_A(x), I_A(x), F_A(x) >$ of A by $< T_A, I_A, F_A >$. Element $< T_A, I_A, F_A >$ is often named as a single-valued neutrosophic value (SVNV).

Definition 2.3 [31]:

Let X be a universal set, and $A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$ and $B = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$ are two SVNSs in X, then

(i) The complement of a SVNS A is

$$A^C = \{< x, F_A(x), 1 - I_A(x), T_A(x) > | x \in X\}$$

(ii)$A \subseteq B$ if and only if

$$T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \leq F_B(x),$$

for all x in X.

(iii) A = B if and only if $A \subseteq B$ and $B \subseteq A$.

In the following discussion, we always use SV NSs(X) to denote the set of all SVNSs in X.

Definition 4 will introduce the definition of a similarity measure between two SVNSs A and B.

Definition 2.4 [35]:

Let A and B be two SVNSs, and S is a mapping S : SVNSs(X) × SVNSs(X)→ [0, 1]. We call S (A,B) the similarity measure between A and B if it satisfies the following properties:

(i) $0 \leq S(A,B) \leq 1$,

(ii)S (A,B) = 1 if and only if A = B,

(iii)S (A,B) = S(B,A),

(iv) If $A \subseteq B \subseteq C$, then $S(A,C) \leq \min \{S(A,B), S(B,C)\}$.

Definition 2.5 ([9]):

Let X be a universe of discourse and the neutrosophic set $A \subset X$. Let T(x), I(x), F(x) be the functions that describe the degrees of membership, Intermediate membership and non-membership respectively, of a generic element $x \in X$ with respect to the set A.

$$T(x), I(x), F(x): X \rightarrow [\psi, 1]$$

Where $\psi < 0 < 1$ and ψ is called under limit

$$T(x), I(x), F(x) \in [\psi, 1]$$

Under set: T, F, I any one <0

A Single-Valued Neutrosophic Under set A is defined as:

$$A = \{(x, \langle T(x), I(x), F(x) \rangle), x \in X\},$$

such that there exists at least one element in A that has at least one neutrosophic component that is < 0, and no element has neutrosophic components that are > 1.

3. A new Chi-square distance-based similarity:

This section contains two subsections. The first subsection will propose a new similarity measure between two SVNUSs based on Chi-square distance measure. The second subsection will compare the proposed similarity measure with existing similarity measures of SVNUSs.

Definition: 3.1.1

Let A and B be two SVNUSs, and S is a mapping S : SVNUSs(X) × SVNUSs(X)→ [ψ, 1]. We call S (A,B) the similarity measure between A and B if it satisfies the following properties:

(i) $\psi \le S(A,B) \le 1$,

(ii) $S(A,B) = 1$ if and only if $A = B$,

(iii) $S(A,B) = S(B,A)$,

(iv) If $A \subseteq B \subseteq C$, then $S(A,C) \le \min\{S(A,B), S(B,C)\}$.

3.1 A NEW PROPOSED SIMILARITY BASED ON CHI-SQUARE DISTANCE:

This section will propose a new similarity measure between two SVNUSs based on Chi-square distance measure. The name of the Chi-square distance measure is derived from Pearson's Chi -squared test statistic $\chi^2(x,y)=\sum_{i=1}^{n}\frac{(x_i-y_i)^2}{x_i}$ which is used to compare two discrete probability distributions. However, as a distance measure, the function d(x, y) should be symmetric for two objects x and y (i.e.,) d (x, y) = d (y, x).Then Chi-square distance measure of two real vectors x = (x_1, x_2, ... x_n) and y = (y_1, y_2, ... y_n) is proposed as the following formula [25]:

$$d(x,y)=\sum_{i=1}^{n}\frac{(x_i-y_i)^2}{x_i+y_i} \text{--------(1)}$$

Chi-square distance measure is one of most important distance measure used in face recognition [25]. To avoid the fact that the denominator is zero, the numerical value is meaningless. For later use, we proposed a revised version of Chi-square distance measure formula as follows:

$$d(x,y)=\sum_{i=1}^{n}\frac{(x_i-y_i)^2}{2+x_i+y_i} \text{----------(2)}$$

Let X = {x_1, x_2, ... x_n}be a universe set. Then for two given SVNUSs A = {< x_i, T_A (x_i), I_A (x_i), F_A (x_i) >|$x_i \in$ X} and B = {< x_i, T_B (x_i), I_B (x_i), F_B (x_i) >|$x_i \in$ X}, the new neutrosophic fuzzy information measure based on Chi-square distance S $\overset{\Delta}{=}$ S (A,B) is constructed as follows:

$$S=1-\frac{1}{2n}\sum_{i=1}^{n}\left[\frac{\left(T_A(x_i)-T_B(x_i)\right)^2}{2+T_A(x_i)+T_B(x_i)}+\frac{\left(I_A(x_i)-I_B(x_i)\right)^2}{2+I_A(x_i)+I_B(x_i)}+\frac{\left(F_A(x_i)-F_B(x_i)\right)^2}{2+F_A(x_i)+F_B(x_i)}+\left|m_A(x_i)-m_B(x_i)\right|\right] \text{-------(3)}$$

Where $m_j(x_i)=\frac{1+T_j(x_i)-F_j(x_i)}{2}$, j = 1,..., n .

To prove the information measure (3) is a valid similarity measure, we need the following lemma which can be easily proved by straightforward calculation.

Lemma 1: Let a, b, c be three real numbers, and $\psi \le a \le b \le c$. Then

(i) $$\frac{(a-c)^2}{2+a+c} \ge \frac{(a-b)^2}{2+a+b}$$

(ii) $$\frac{(a-c)^2}{2+a+c} \ge \frac{(b-c)^2}{2+b+c}$$

Theorem 1: Let $X = \{x_1, x_2, ..., x_n\}$ be a universe set. $A = \{< x_i, T_A(x_i), I_A(x_i), F_A(x_i) > | x_i \in X\}$ and $B = \{< x_i, T_B(x_i), I_B(x_i), F_B(x_i) > | x_i \in X\}$, are two SVNUSs. Then information measure S(A,B) given by (2) is a valid similarity measure between SVNUSs A and B. That is, S (A, B) satisfies the properties (i)-(iv) of Definition 3.1.1.

Proof:

(i)It is obvious that $\psi \le S(A, B) \le 1$.

(ii) When A = B, i.e. $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, $F_A(x_i) = F_B(x_i)$, for all x_i in X.

Then

$$m_A(x_i) = \frac{1+T_A(x_i)-F_A(x_i)}{2} = \frac{1+T_B(x_i)-F_B(x_i)}{2} = m_B(x_i)$$

Hence we have S (A, B) = 1.

(iii) The result is obvious.

(iv) If $A \subseteq B \subseteq C$, i.e.

$$T_A(x_i) \le T_B(x_i),\ I_A(x_i) \ge I_B(x_i),\ F_A(x_i) \ge F_B(x_i),$$

Then $\psi \le m_A(x_i) \le m_B(x_i) \le m_C(x_i)$,for all x_i in X.

Consequently, we can get

$$| m_A(x_i)- m_C(x_i)| \ge | m_A(x_i)- m_B(x_i)| ,\ | m_A(x_i)- m_C(x_i)| \ge | m_B(x_i)- m_C(x_i)|$$

Thus by (i) of Lemma 1, we have

$$\frac{(T_A(x_i)-T_C(x_i))^2}{2+T_A(x_i)+T_C(x_i)} \ge \frac{(T_A(x_i)-T_B(x_i))^2}{2+T_A(x_i)+T_B(x_i)},$$
$$\frac{(I_A(x_i)-I_C(x_i))^2}{2+I_A(x_i)+I_C(x_i)} \ge \frac{(I_A(x_i)-I_B(x_i))^2}{2+I_A(x_i)+I_B(x_i)},$$
$$\frac{(F_A(x_i)-F_C(x_i))^2}{2+F_A(x_i)+F_C(x_i)} \ge \frac{(F_A(x_i)-F_B(x_i))^2}{2+F_A(x_i)+F_B(x_i)}.$$

Then we can easily conclude that S(A, C) ≤ S (A, B). By (ii) of Lemma 2, we have

$$\frac{(I_A(x_i)-I_C(x_i))^2}{2+I_A(x_i)+I_C(x_i)} \ge \frac{(I_B(x_i)-I_C(x_i))^2}{2+I_B(x_i)+I_C(x_i)},$$
$$\frac{(F_A(x_i)-F_C(x_i))^2}{2+F_A(x_i)+F_C(x_i)} \ge \frac{(F_B(x_i)-F_C(x_i))^2}{2+F_B(x_i)+F_C(x_i)},$$
$$\frac{(T_A(x_i)-T_C(x_i))^2}{2+T_A(x_i)+T_C(x_i)} \ge \frac{(T_B(x_i)-T_C(x_i))^2}{2+T_B(x_i)+T_C(x_i)}.$$

Then we can easily conclude that S (A, C) ≤ S (B, C). Hence S (A, C) ≤ min{S(A,B), S(B,C)}. This completes the proof of Theorem 1. If we consider the important degree of $x_i \in X = \{x_1, x_2, ..., x_n\}$, then we can establish a weighted similarity measure $S_W \overset{\Delta}{=} S_W(A, B)$ between SVNUSs A and B as follows:

$$S_W = 1-\frac{1}{2}\sum_{i=1}^{n}\omega_i\left[\frac{(T_A(x_i)-T_B(x_i))^2}{2+T_A(x_i)+T_B(x_i)}+\frac{(I_A(x_i)-I_B(x_i))^2}{2+I_A(x_i)+I_B(x_i)} + \frac{(F_A(x_i)-F_B(x_i))^2}{2+F_A(x_i)+F_B(x_i)}+|m_A(x_i)-m_B(x_i)|\right] \quad \text{-------(4)}$$

where w_i (i = 1, 2,...,n) is the important degree of the element x_i, they satisfy $w_i \in [\psi, 1]$ and $\sum_{i=1}^{n}\omega_i = 1$. If we set w_i = (i = 1, 2, ..., n), then S_W (A, B)= S(A,B). Similar to the proof process of S_R(A, B) in Theorem 1, we can easily prove that the weighted similarity measure S_W (A, B) is also a valid similarity between two SVNUSs A and B . That is S_W (A, B) satisfies the properties (i)-(iv) of Definition 3.1.1.

3.2. Comparison Of Various Similarity Measures:

To demonstrate the validness and performance of the new proposed similarity measure, some numerical examples are used to compare it with existing similarity measures: Jaccard similarity $S_J(A,B)$, Dice similarity $S_D(A,B)$, Cosine Similarity $S_C(A,B)$, Improved cosine similarity $C_1(A,B)$and $C_2(A,B)$, Tangent function-based similarity $T_1(A,B)$, $T_2(A,B)$, and Cotangent function-based similarity $CoT_1(A,B)$, $CoT_2(A,B)$. These similarity measures are given as follows([33], [34], [35]):

$$S_J(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{S_{J1}}{S_{J2}}$$

where $S_{J1}=T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)$

and $S_{J2}=(T_A^{\,2}(x_i)+I_A^{\,2}(x_i)+F_A^{\,2}(x_i))+(T_B^{\,2}(x_i)+I_B^{\,2}(x_i)+F_B^{\,2}(x_i))-S_{J1}$

$$S_D(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{2(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{(T_A^{\,2}(x_i)+I_A^{\,2}(x_i)+F_A^{\,2}(x_i))+(T_B^{\,2}(x_i)+I_B^{\,2}(x_i)+F_B^{\,2}(x_i))},$$

$$S_C(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\sqrt{T_A^{\,2}(x_i)+I_A^{\,2}(x_i)+F_A^{\,2}(x_i)}\sqrt{T_B^{\,2}(x_i)+I_B^{\,2}(x_i)+F_B^{\,2}(x_i)}},$$

$$C_1(A,B)=\frac{1}{n}\sum_{i=1}^{n}\cos[\frac{\pi}{2}\max(|T_A(x_i)-T_B(x_i)|,|I_A(x_i)-I_B(x_i)|,|F_A(x_i)-F_B(x_i)|)],$$

$$C_2(A,B)=\frac{1}{n}\sum_{i=1}^{n}\cos[\frac{\pi}{6}(|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|)],$$

$$T_1(A,B)=1-\frac{1}{n}\sum_{i=1}^{n}\tan[\frac{\pi}{4}\max(|T_A(x_i)-T_B(x_i)|,|I_A(x_i)-I_B(x_i)|,|F_A(x_i)-F_B(x_i)|)],$$

$$T_2(A,B)=1-\frac{1}{n}\sum_{i=1}^{n}\tan[\frac{\pi}{12}(|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|)],$$

$$CoT_1(A,B)=\frac{1}{n}\sum_{i=1}^{n}\cot[\frac{\pi}{4}\max(|T_A(x_i)-T_B(x_i)|,|I_A(x_i)-I_B(x_i)|,|F_A(x_i)-F_B(x_i)|)],$$

$$CoT_2(A,B)=\frac{1}{n}\sum_{i=1}^{n}\cot[\frac{\pi}{4}+\frac{\pi}{12}(|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|)].$$

Example 1: Suppose that X = {x} , we consider pattern recognition problems with six pairs of SVNUSs shown in Table 1. The calculated numerical values of these 9 existing similarity measures and proposed similarity measure are shown in Table 1.

Table 1: Values of the different similarity measures under different pairs of (A,B)

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**	**Case 6**
A	<0.3,-0.3,0.4>	<0.3,0.3,-0.4>	<-1,0,0>	<0.4,-0.2,0.6>	<0.4,0.4,-0.2>	<0.4,-0.4,0.2>
B	<-0.4,0.3,0.4>	<0.4,-0.3,0.3>	<0,1,-1>	<-0.2,0.2,0.3>	<0.5,-0.2,0.3>	<0.5,0.3,-0.2>
$S_J(A,B)$	-0.0625	-0.1169	0	0.0896	0.0882	0.0571
$S_D(A,B)$	-0.1334	-0.2647	0	0.1644	0.1622	0.1081
$S_C(A,B)$	-0.1339	-0.2647	0	0.1945	0.1622	0.1081
$C_1(A,B)$	**0.9998**	**0.9998**	0.9996	**0.9999**	**0.9999**	0.9998
$C_2(A,B)$	**0.9999**	**0.9999**	0.9996	**0.9999**	**0.9999**	0.9999
$T_1(A,B)$	0.9904	0.9918	0.9863	**0.9918**	**0.9918**	0.9904
$T_2(A,B)$	0.9941	0.9936	0.9863	0.9941	**0.9945**	**0.9945**
$CoT_1(A,B)$	0.5027	0.4404	0.6658	**0.4404**	**0.4404**	0.5027
$CoT_2(A,B)$	0.8445	0.8559	1.0039	0.8445	**0.8328**	**0.8328**
S(A,B)	0.6060	0.6292	-0.6667	0.7876	0.7570	0.8522

From Table 1, we can see that the similarity measures $C_1(A,B)$ and $C_2(A,B)$ cannot carry out the recognition between Case 1 and Case 2 & Case 4 and Case 5. An interesting counter-intuitive case occurs when three SVNUSs A =< 0.4, 0.4, -0.2 >, B =<0.5, -0.2, 0.3> and C =<-0.5, 0.3, 0.2>. They can be written in forms of intuitionistic fuzzy values as: A =< 0.4, -0.2 >, B =< 0.5, 0.3 > and C =< -0.5, 0.2 >, respectively.

4. Applications:

In the following discussion, we will give two examples in pattern recognition and medical diagnosis to demonstrate the effectiveness and practicability of the proposed similarity measure.

Example 4.1:

Assume that there are two patterns in $X = \{x_1, x_2\}$. The two patterns are expressed by SVNUSs, which are shown as follows:

$$A_1 = \{< x_1, 0.2, 0.0, -0.2 >, < x_2, -0.2, 0.0, 0.2 >, < x_3, 0.2, 0.0, -0.2 >\},$$

$$A_2 = \{< x_1, -0.4, 0.0, 0.4 >, < x_2, 0.4, 0.0, -0.4 >, < x_3, -0.4, 0.0, 0.4 >\}.$$

Assume that there is an object

$$B = \{< x_1, 0.3, 0.0, -0.3 >,< x_2, -0.3, 0.0, 0.3 >,< x_3, 0.2, 0.0, -0.3 >\}$$

Our task is to classify the object B in A_1 or A_2 . According to the recognition principle of maximum similarity measure between SVNUSs, the process of assigning the object B to A_1 or A_2 is described by

$$k = \arg\max_{1\le i\le 2}\{S_R(A_i, B)\} \text{-----(14)}$$

By Eq. (3), we can get the similarity measures between A_1, A_2 with B : $S(A_1,B) = 0.8610$, $S(A_2,B) =-0.7329$. Then the pattern B is classified in A_1 according to the recognition rule given. by Eq. (14). This result is consistent with our intuition.

Example 3:

We consider the following pattern recognition problem: There are three patterns A_1, A_2 and A_3 , which are represented by SVNUSs in universe set $X = \{x_1, x_2, x_3\}$, as follows:

$$A_1 = \{< x_1, 1.0, -0.2, 0.0 >,< x_2, -0.8, 0.3, 0.0 >,< x_3, 0.7, 0.1, -0.1 >\}$$

$$A_2 = \{< x_1, -0.8, 0.1, 0.1 >,< x_2, 1.0, 0.1, -0.2 >,< x_3, 0.9, -0.2, 0.1 >\}$$

$$A_3 = \{< x_1, 0.6, 0.3, -0.2 >,< x_2, 0.8, -0.2, 0.3 >,< x_3, -0.6, 0.3, 0.2 >\}$$

Given an unknown pattern B, which is represented by the SVNUS:

$$B = \{< x_1, 0.5, -0.3, 0.2 >,< x_2, -0.6, 0.3, 0.2 >,< x_3, 0.8, 0.2, -0.1 >\}$$

Our task is to classify the pattern B in one of the classes A_1,A_2 and A_3 . According to the recognition principle of maximum similarity measure between SVNUSs, the process of assigning the pattern B to A_k (k = 1, 2, 3) is described by

$$k = \arg\max_{1\le i\le 3}\{S_R(A_i, B)\} \text{-----(15)}$$

By Eq.(3), we can get the similarity measures between B with A_i (i = 1, 2, 3) :
$S(A_1,B) = 0.7058$, $S(A_2,B) = -1.0017$, $S(A_3,B) = -0.9825$.

Then the pattern B is classified in A_3 according to the recognition rule given by Eq. (15). Some medical diagnosis problems are very complex. Physicians need to use modern medical technologies to obtain a lot of information available to physicians for the help of decision, but the information is often incomplete, indeterminate and inconsistent. The SVNUSs proposed by Wang et al. [29] can be better choice to express this kind of information than Zadeh's fuzzy sets and intuitionistic fuzzy sets. Now in Example 4 we will utilize the proposed similarity measure to solve a class of medical diagnosis problems.

Example 4:

The medical diagnosis problem is adapted from De et al. [7]. Let Q = Q_1 (Viral fever), Q_2 (Malaria), Q_3 (Typhoid), Q_4 (Stomach problem), Q_5 (Chest problem) be a set of diagnoses (diseases) and S= s_1(Temperature), s_2(Headache), s_3 (Stomach pain), s_4 (Cough), s_5(Chest pain) be a set of symptoms. Each diagnosis Q_i (i = 1, 2, 3, 4, 5) can be represented by SVNUSs as follows:

$$Q_1 = \{<s_1,-0.4,0.6,0.0>,<s_2,0.3,-0.2,0.5>,<s_3,0.1,0.2,-0.7>,<s_4,-0.4,0.3,0.3>,<s_5,0.1,-0.2,0.7>\}$$
$$Q_2 = \{<s_1,0.7,-0.3,0.0>,<s_2,0.2,0.2,-0.6>,<s_3,-0.1,0.1,0.8>,<s_4,0.7,-0.3,0.0>,<s_5,0.1,0.1,-0.8>\}$$
$$Q_3 = \{<s_1,0.3,0.4,-0.3>,<s_2,-0.6,0.3,0.1>,<s_3,0.2,-0.1,0.7>,<s_4,0.2,0.2,-0.6>,<s_5,-0.1,0.0,0.9>\}$$
$$Q_4 = \{<s_1,-0.1,0.2,0.7>,<s_2,0.2,-0.4,0.4>,<s_3,0.8,0.1,-0.1>,<s_4,-0.2,0.1,0.7>,<s_5,0.2,-0.1,0.7>\}$$
$$Q_5 = \{<s_1,0.1,-0.1,0.8>,<s_2,0.0,0.2,-0.8>,<s_3,-0.2,0.0,0.8>,<s_4,0.1,-0.1,0.8>,<s_5,0.8,0.1,-0.1>\}$$

Suppose there are two patients P_1 and P_2 , with respect to all the symptoms, can be represented by the following SVNUSs:

$$P_1 = \{<s_1,0.8,0.1,-0.1>,<s_2,-0.6,0.3,0.1>,<s_3,0.1,-0.1,0.8>,<s_4,0.6,0.3,-0.1>,<s_5,-0.1,0.3,0.6>\}$$
$$P_2 = \{<s_1,-0.1,0.1,0.8>,<s_2,0.4,-0.4,0.2>,<s_3,0.6,0.3,-0.1>,<s_4,-0.1,0.7,0.2>,<s_5,0.1,-0.8,0.1>\}$$

Our aim is to determine the patients P_1 and P_2 belong to which diagnosis of Q_j (j =1, 2, 3, 4, 5), respectively. Because the medical diagnosis problem is actually a pattern recognition problem, then we can use the diagnosis rule as follows: If $k = \arg\max_{1 \le j \le 5} \{S_R(Q_j, P_i)\}$, then we assign the patient P_1 and P_2 to the diagnosis Q_k.

$S(Q_1,P_1)$=-1.7693,$S(Q_2,P_1)$=-1.1158,$S(Q_3,P_1)$=0.4914,$S(Q_4,P_1)$=-1.5908 and S(Q5,P1)=-1.8019,
$S(Q_1,P_2)$=-1.1542,$S(Q_2, P_2)$=-2.2344,$S(Q_3,P_2)$=-2.3134,$S(Q_4,P_2)$=-0.0827 and $S(Q_5,P_2)$=-1.3625.

Then, By the above diagnosis rule, we can assign the patient P_1 to the diagnosis Q_3 (Typhoid), and P_2 to the diagnosis Q_4 (Stomach problem). This result is not in agreement with the one obtained in De et al. [8].

Example 5 :(Multi-Attribute Decision Making)

We consider a MADM problem adopted from Ye [33]. A manufacturing company wants to select the best global supplier form a set of four suppliers A = $\{A_1,A_2,A_3,A_4\}$ whose core competencies are evaluated according to the four attributes O = $\{O_1,O_2,O_3,O_4\}$: o_1 (the level of technology innovation) , o_2 (the control ability of flow), o_3 (the ability of management) , o_4 (the level of service). The attributes are all benefit attributes. The weight vector for the four attributes determined by decision maker is $W= (w_1,w_2,w_3,w_4)^T = (0.30, 0.25, -0.25, 0.20)^T$

Suppose that the evaluation value of the alternative A_i (i = 1, 2, 3, 4) with respect to o_j (j = 1, 2, 3, 4) is a SVNUN $a_{ij} =< T_{ij} , I_{ij} , F_{ij} >$, which is obtained from a questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to an attribute o_1 , he/she may say that the possibility in which the good statement is 0.5 and the poor statement is 0.3 and the degree in which he/she is not sure is -0.1. For the neutrosophic notation, it can be expressed as $a_{11} =< T_{11} , I_{11} , F_{11} >$. The evaluation values are listed in Table 1.

Table 1: Evaluation values of each alternative with respect to each attribute 1

Alternatives	o_1	o_2	o_3	o_4
A_1	<-0.75,0.2,0.3>	<0.7,-0.2,0.3>	<0.65,0.2,-0.25>	<-0.75,0.2,0.1>
A_2	<0.8,-0.1,0.2>	<0.75,0.2,-0.1>	<-0.75,0.2,0.1>	<0.85,-0.1,0.2>
A_3	<0.7,0.2,-0.2>	<-0.78,0.2,0.1>	<0.85,-0.15,0.1>	<0.76,0.2,-0.2>
A_4	<-0.8,0.2,0.1>	<0.85,-0.2,0.2>	<0.7,0.2,-0.2>	<-0.86,0.1,0.2>

Now, we will propose a decision making method based on the proposed similarity measure to solve this problem and the detail steps is given as follows:

Step 1: Determine the ideal solution A^* as follows:

$$A^* = \left(< T_j^*, I_j^*, F_j^* >\right)_{1\times 4} = \left(< \max_{1\le i\le 4}(T_{ij}), \max_{1\le i\le 4}(I_{ij}), \max_{1\le i\le 4}(F_{ij}) > \right)_{1\times 4}$$

Step 2: According to Eq. (4), calculate similarity measures between each alternative A_i (i = 1, 2, 3, 4) and the ideal solution A^* as follows:

$$S(A_1,A^*) = 0.3649,\ S(A_2,A^*) = 0.6957,\ S(A_3,A^*) = 1.3293 \text{ and } S(A_4,A^*) = 0.6432.$$

Step 3: According to the similarity measure values, the ranking order of the four suppliers is $A_3 \succ A_2 \succ A_4 \succ A_1$. Hence, the best supplier is A_3, which is not in agreement with the result obtained by using weighted projection similarity and weighted Dice similarity methods(Ye [33]).

5. Conclusion:

Neutrosophic under sets are suitable to model the indeterminate and inconsistent information occurred in many practical problems. In this paper, we have proposed a new Chi-square distance based similarity measure of SVNUSs. The new proposed similarity measure is a valid similarity measure and it can also overcome the counter-intuitive cases of the existing similarity measures by using some numerical examples. We have given the applications of the proposed similarity measure in pattern recognition and medical diagnosis. Furthermore, a multi-attribute decision making method is proposed through an example in which attribute values are expressed with SVNUVs.

As a prospect, the MADM method proposed in this paper could be applied to other MADM problems, such as the risk evaluation, credit evaluation. In the future work, we shall extend the proposed similarity to clustering analysis and image processing.

Reference:

[1] *Atanassov, K. T.(1986), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1),87–96, 1986.*

[2] *Atanassov, K., Gargov, G. (1989); Interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 31(3), 343–349, 1989.*

[3] *Bozic, M.; Ducic, N., Djordjevic, G., Slavkovic, R. (2017), Optimization of Wheg Robot Running with Simulation of Neuro-Fuzzy Control, International Journal of Simulation Modeling, 16(1), 19–30, 2017.*

[4] *Chaira, T., Panwar, A. (2014), An Atanassov's intuitionistic fuzzy kernel clustering for medical image segmentation, International Journal of Computational Intelligence Systems, 7(2), 360–370, 2014.*

[5] *Dalman H., Gazel N., Sivri M. (2016), A fuzzy set-based approach to multi-objective multi item solid transportation problem under uncertainty, International Journal of Fuzzy Systems, 18(4), 716–729, 2016.*

[6] *Das, S., Guha, D., Dutta, B. (2016), Medical diagnosis with the aid of using fuzzy logic and intuitionistic fuzzy logic, Applied Intelligence, 45(3), 850–867, 2016.*

[7] *De S. K., Biswas, R., Roy, A. R. (2001), An application of intuitionistic fuzzy sets in medical diagnosis, Fuzzy Sets & Systems, 117(2), 209–213, 2001.*

[8] *Dubois, D., Prade, H., Esteva, F. (2015), Fuzzy set modeling in case-based reasoning, International Journal of Intelligent Systems, 13(4), 345–373, 2015.*

[9] *Florentin Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics, Pro Quest Info &Learning, Ann Arbor, MI, USA, pp. 92-93, 2007.*

[10] *Gau, W. L., Buehrer, D. J.(1993), Vague sets, IEEE Transactions on Systems Man & Cybernetics, 23(2), 610–614, 1993.*

*[**11**] Huang, H. (2016), New distance measure of single-valued neutrosophic sets and its application, International Journal of Intelligent Systems, 31(10), 1021–1032, 2016.*

*[**12**] Hung, W. L., Yang, M. S. (2007), Similarity measures of intuitionistic fuzzy sets based on Lp metric, International Journal of Approximate Reasoning, 46(1), 120–136, 2007.*

[13] *H. Wang, C. M., Yang, M. S. (2013), New construction for similarity measures between intuitionistic fuzzy sets based on lower, upper and middle fuzzy sets, International Journal of Fuzzy Systems, 15(3), 371–378, 2013.*

[14] *Kar, A. K. (2015), A hybrid group decision support system for supplier selection using analytic hierarchy process, fuzzy set theory and neural network, Journal of Computational Science, 6,23–33, 2015.*

*[**15**] Karaaslan, F. (2017), Correlation coefficients of single valued neutrosophic refined soft sets and their applications in clustering analysis, Neural Computing & Applications, 28(9), 2781–2793,2017.*

*[**16**] Le, H. S., Phong, P. H. (2016), On the performance evaluation of intuitionistic vector similarity measures for medical diagnosis, Journal of Intelligent & Fuzzy Systems, 31(3), 1–12, 2016.*

[17] *Li D. F., Cheng C. T. (2012), New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions, Pattern Recognition Letters, 23(1), 221–225, 2002.*

[18] *Li D. F., Ren H. P. (2015), Multi-attribute decision making method considering the amount and reliability of intuitionistic fuzzy information, Journal of Intelligent & Fuzzy Systems, 28(4), 1877–1883, 2015.*

[19] *Liu, P. D. (2016), The aggregation operators based on archimedean t-conorm and t-norm for single-valued neutrosophic numbers and their application to decision making, International Journal of Fuzzy Systems, 18(5), 1–15, 2016.*

[20] *Meng, F., Chen, X. (2016), Entropy and similarity measure of Atanassov's intuitionistic fuzzy sets and their application to pattern recognition based on fuzzy measures, Pattern Analysis and Applications, 19(1), 11–20, 2016.*

[21] *Mousavi, M., Yap, H. J., Musa, S. N., Dawal, S. Z. M. (2017), A Fuzzy Hybrid GA-PSO Algorithm for Multi-Objective AGV Scheduling in FMS, International Journal of Simulation Modeling, 16(1), 58–71, 2017.*

[22] *Nguyen, H. (2016), A novel similarity/dissimilarity measure for intuitionistic fuzzy sets and its application in pattern recognition, Expert Systems with Applications, 45, 97–107, 2016.*

[23] *Peng, J. J., Wang, J. Q., Wang, J., Zhang, H. Y, Chen, X. H. (2016), Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems, International Journal of Systems Science, 47(10), 2342–2358, 2016.*

[24] *Perlibakas, V. (2004), Distance measures for PCA-based face recognition, Pattern Recognition Letters, 25(6), 711–724, 2004.*

[25] *Pramanik, S., Pramanik, S., Giri, B. C. (2015), TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment, Neural Computing & Applications, 27(3), 727–737, 2015.*

[26] *H.P. Ren, S.X. Xiao, H. Zhou, A chi-square distance-based similarity measure of single-valued neutrosophic set and applications, International journal of computers communications & control ISSN 1841-9836, 14(1), 78-89, February 2019.*

[27] *Smarandache, F. (1999), A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, American Research Press, 1999.*

[28] *Tavana, M., Zareinejad, M., Caprio, D. D., Kavianic M. A. (2016), An integrated intuitionistic fuzzy AHP and SWOT method for outsourcing reverse logistics, Applied Soft Computing, 40, 544–557, 2016.*

[29] *Torra V. (2010), Hesitant fuzzy sets, International Journal of Intelligent Systems, 25(6), 529–539, 2010.*

[30] *Vasavi, C., Kumar G. S., Murty, M. S. N.(2016), Generalized differentiability and integrability for fuzzy set-valued functions on time scales, Soft Computing, 20(3), 1093–1104, 2016.*

[31] *Wang, H. B.; Smarandache, F., Zhang Y. Q., Sunderraman R. (2010), Single valued neutrosophic sets, Multi-space and Multi-structure, 4, 410–413, 2010.*

[32] *Xu, Z. S., Chen, J. (2008), An overview of distance and similarity measures of intuitionistic fuzzy sets, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 16(4), 529–555, 2008.*

[33] *Ye, J.(2015), The generalized Dice measures for multiple attribute decision making under simplified neutrosophic environments, Intelligent & Fuzzy Systems, 31(1), 663–671, 2015.*

[34] *Ye J., Fu, J. (2016), Multi-period medical diagnosis method using a single valued neutrosophic similarity measure based on tangent function, Compute Methods Programs Biomed, 123, 142–149, 2016.*

[35] *Ye, J. (2017, Single-valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine, Soft Computing, 21(3), 1–9, 2017.*