# Multi-Objective Linear Fractional Programming Based on Trapezoidal Neutrosophic Numbers

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# Multi-Objective Linear Fractional Programming Based on Trapezoidal Neutrosophic Numbers

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**Abstract:**

In this paper, a multi-objective linear fractional programming (MOLFP) problem is considered where all of its coefficients in the objective function and constraints are trapezoidal neutrosophic numbers. In the present work, by presenting the neutrosophic set concept the neutrosophic multi-objective linear fractional programming (NMOLFP) model is introduced where their parameters are characterized by a trapezoidal neutrosophic number. Two ranking functions for solving this problem by converting neutrosophic numbers to crisp numbers is presented. After converting NMOLFP problem with neutrosophic numbers into the crisp MOLFP problem, then the MOLFP problem is transformed into a single objective linear programming (LP) problem using a proposal given by Nuran Guzel. The proposed approaches are simple, efficient and capable for solving NMOLFP problem. To illustrate The proposed approaches a numerical example is presented.

**Keywords:** trapezoidal neutrosophic number, multi-objective, neutrosophic set, neutrosophic linear programming, multi-objective linear fractional programming, linear fractional programming.

## I. INTRODUCTION

In human society, Mathematical modelling of numerous existential problems are produced several objectives which are conflicted as well as inter-related to each other. Several times, they exist in the fractional or rational form of two other functions and need simultaneous optimization

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under a common set of constraints. If the numerators and denominators of the fractions objectives with constraints are all fine functions (i.e., linear plus constants), such modelled optimization problems are interpreted as a multi-objective linear fractional programming problems. Stancu-Minasian I. M. (1997) presented Some mathematical optimization problems comprise fractional objectives to be optimized which are frequently encountered in many real life situations like inventory / sales, profit / cost, risk-assets / capital, output / employee, debt / equity etc [1]. Linear Fractional programming problem which was developed by Hungarian mathematician B. Martos in 1960, has a wide range of application in several important fields such as engineering, science, economics, information theory, finance, business, management, marine transportation, water resources, health care, corporate planning and so forth [2]. MOLFPP has attracted considerable research interest since recent few years and some methods have been proposed for the determination of the optimal solutions.

In real life situations, Very often the values of coefficients of MOLFP problems are only vaguely available to the expert. The precision of data is overwhelming deceitful and this affects the optimal solution of LP problems. Probability distributions failed to transact with inaccurate and unclear information. Also fuzzy sets were introduced by Zadeh to handle vague and imprecise information [3]. But also fuzzy set does not represent vague and imprecise information efficiently, because it considers only the truthfulness function. After then, Atanassove presented the concept of intuitionistic fuzzy Set to handle vague and imprecise information, by considering both the truth and falsity function [4]. But also intuitionistic fuzzy set does not simulate the human decision making process. Because the proper decision is fundamentally a problem of arranging and explicate facts the concept of neutrosophic set theory was presented by Broumi S. et al. (2016), to handle vague, imprecise and inconsistent information [5-9]. Neutrosophic set theory simulates decision-making process of humans, by considering all aspects of decision-making process.

Single valued neutrosophic set has been developing rapidly by some researchers due to its wide range of theoretical elegance and application areas; all are illustritaed in examples [9-20].

Wang H. et al. (2005) Proposed the concept of the interval neutrosophic set (INS) which is an extension of neutrosophic set [21]. The INS can represent imprecise, uncertain, incomplete and inconsistent information which exists in the real life. Single valued neutrosophic number is an extension of fuzzy numbers and intuitionistic fuzzy numbers. Single valued fuzzy number is a special case of the single valued neutrosophic set and is of importance for decision making problems. Biswas P. et al. (2016) Studied the concept of trapezoidal neutrosophic fuzzy number as a generalized representation of trapezoidal fuzzy numbers, trapezoidal intuitionistic fuzzy numbers, triangular fuzzy numbers and triangular intuitionistic fuzzy numbers and applied them for dealing with multi-attribute decision making (MADM) problems [22]. Biswas P. et al. (2016), Deli I. and S¸ubas Y. (2017) considered the ranking of single valued neutrosophic trapezoidal numbers and applied the concept to solve MADM problems [15,23]. Liang R. et al.

(2017) presented a multi-criteria decision-making (MCDM) method based on single-valued trapezoidal neutrosophic preference relations with complete weight information [24].

Neutrosophic set, a popularization of fuzzy and intuitionistic fuzzy sets, had a truth, indeterminacy and falsity membership function in each element of the set. So, neutrosophic set can assimilate inaccurate, vague and maladjusted information efficiently and effectively according to Deli I. and S¸ubas Y. (2017) [23,25]. We now can say that neutrosophic linear programming (NLP) problem is a problem in which at least one coefficient is represented by a neutrosophic number due to vague, inconsistent and uncertain information. The NLP problems are more useful than crisp LP problems because the decision maker in his/her formulation of the problem is not forced to make a delicate formulation. The use of NLP problems is recommended to avert unrealistic modeling. In this research, it is the first time to present NMOLFP problem in a neutrosophic environment with trapezoidal neutrosophic numbers. Two ranking functions are introduced according to the problem type, for converting the NMOLFP problem to crisp problem.

The motivation of our discussion in this paper is to propose two methods to determine the optimal solution of an NMOLFP problem Based on Trapezoidal Neutrosophic Numbers.

The organization of the paper is as follows: In Section 2, some important concepts and definitions of neutrosophic set are presented. In Section 3, an MOLFP problem is discussed. In Section 4, an NMOLFP problem with Trapezoidal Neutrosophic Numbers coefficients is discussed. Section 5, proposed two approaches as a methodology for an NMOLFP problem with Trapezoidal Neutrosophic Numbers coefficients. In section 6, numerical example for illustrating the solution of proposed methods. Finally, concluding remarks are given in Section 7.

## II. NEUTROSOPHIC CONCEPTS

In this section, A review of some important concepts and definitions of neutrosophic set is presented. [26]

**Definition 1**: A single-valued neutrosophic set $\mathrm{N}$ through $\mathrm{X}$ taking the form $\mathrm{N} = \{\langle \mathrm{x}, \mathrm{T_N(x)}, \mathrm{I_N(x)}, \mathrm{F_N(x)} \rangle : \mathrm{x} \in \mathrm{X}\}$ where $\mathrm{X}$ be a universe of discourse, $\mathrm{T_N(x)}: \mathrm{X} \rightarrow [0,1]$, $\mathrm{I_N(x)}: \mathrm{X} \rightarrow [0,1]$ and $\mathrm{F_N(x)}: \mathrm{X} \rightarrow [0,1]$ with $0 \leq \mathrm{T_N(x)} + \mathrm{I_N(x)} + \mathrm{F_N(x)} \leq 3$ for all $\mathrm{x} \in \mathrm{X}$.

$\mathrm{T_N(x)}$, $\mathrm{I_N(x)}$, $\mathrm{F_N(x)}$ represent truth membership, indeterminacy membership and falsity membership degrees of x to N.

**Definition 2**: The trapezoidal neutrosophic number $\widetilde{\mathrm{A}}$ is a neutrosophic set in R with the following truth, indeterminacy and falsity membership functions:

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$$T_{\tilde{A}}(x) = \begin{cases} \alpha_{\tilde{A}}\left(\frac{X-a_1}{a_2-a_1}\right) & (a_1 \le x \le a_2) \\ \alpha_{\tilde{A}} & (a_2 \le x \le a_3) \\ \alpha_{\tilde{A}} & (a_3 \le x \le a_4) \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

$$I_{\tilde{A}}(x) = \begin{cases} \frac{a_2-X+\theta_{\tilde{A}}(x-a_1')}{(a_2-a_1')} & (a_1' \le x \le a_2) \\ \theta_{\tilde{A}} & (a_2 \le x \le a_3) \\ \frac{x-a_3+\theta_{\tilde{A}}(a_4'-x)}{(a_4'-a_3)} & (a_3 \le x \le a_4') \\ 1 & \text{otherwise} \end{cases} \qquad (2)$$

$$F_{\tilde{A}}(x) = \begin{cases} \frac{a_2-X+\beta_{\tilde{A}}(x-a_1'')}{(a_2-a_1'')} & (a_1'' \le x \le a_2) \\ \beta_{\tilde{A}} & (a_2 \le x \le a_3) \\ \frac{x-a_3+\beta_{\tilde{A}}(a_4''-x)}{(a_4''-a_3)} & (a_3 \le x \le a_4'') \\ 1 & \text{otherwise} \end{cases} \qquad (3)$$

where $\alpha_{\tilde{A}}$ , $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}}$ represent the maximum degree of truthiness, minimum degree of indeterminacy, minimum degree of falsity, respectively, $\alpha_{\tilde{A}}$ , $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}} \in [0, 1]$.

Also $a_1'' \le a_1 \le a_1' \le a_2 \le a_3 \le a_4' \le a_4 \le a_4''$.

The membership functions of trapezoidal neutrosophic number are presented in Fig. 1.

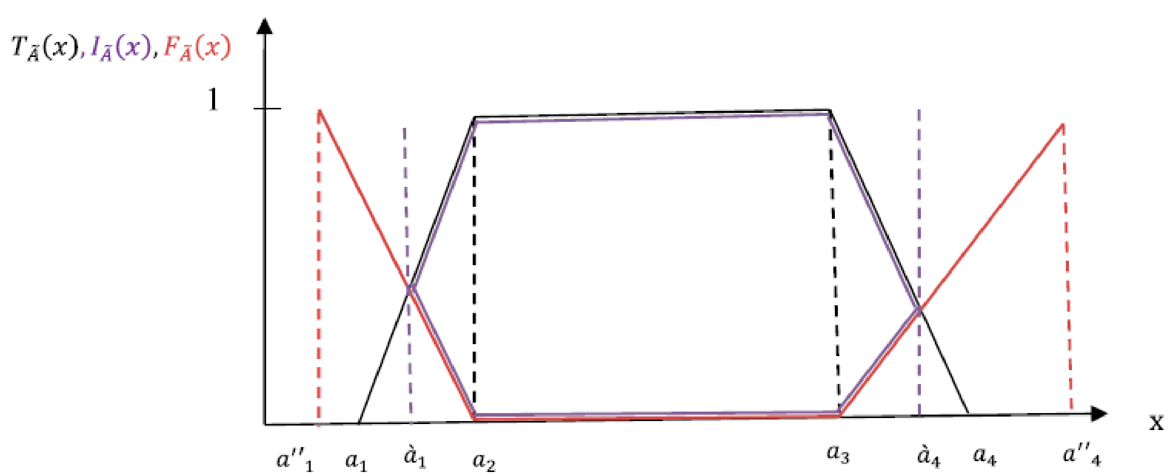


**Fig 1**: Truth membership, indeterminacy and falsity membership functions of trapezoidal neutrosophic number.

**Definition 3**: The mathematical operations on two trapezoidal neutrosophic numbers $\tilde{A} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} \rangle$ and $\tilde{B} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{B}}, \theta_{\tilde{B}}, \beta_{\tilde{B}} \rangle$ are follows:

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$$\widetilde{A}+\widetilde{B}=\langle(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4);\alpha_{\widetilde{A}}\wedge\alpha_{\widetilde{B}},\theta_{\widetilde{A}}\vee\theta_{\widetilde{B}},\beta_{\widetilde{A}}\vee\beta_{\widetilde{B}}\rangle$$

$$\widetilde{A}-\widetilde{B}=\langle(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1);\alpha_{\widetilde{A}}\wedge\alpha_{\widetilde{B}},\theta_{\widetilde{A}}\vee\theta_{\widetilde{B}},\beta_{\widetilde{A}}\vee\beta_{\widetilde{B}}\rangle$$

$$\widetilde{A}^{-1}=\langle\left(\frac{1}{a_4},\frac{1}{a_3},\frac{1}{a_2},\frac{1}{a_1}\right);\ \alpha_{\widetilde{A}},\theta_{\widetilde{A}},\beta_{\widetilde{A}}\rangle,\text{where}\left(\widetilde{A}^{-1}\neq 0\right)$$

$$c\widetilde{A}=\begin{cases}\langle(ca_1,ca_2,ca_3,ca_4);\alpha_{\widetilde{A}},\theta_{\widetilde{A}},\beta_{\widetilde{A}}\rangle & \text{if}(c>0)\\ \langle(ca_4,ca_3,ca_2,ca_1);\alpha_{\widetilde{A}},\theta_{\widetilde{A}},\beta_{\widetilde{A}}\rangle & \text{if}(c<0)\end{cases}$$

$$\frac{\widetilde{A}}{\widetilde{B}}=\begin{cases}\langle\left(\frac{a_1}{b_4},\frac{a_2}{b_3},\frac{a_3}{b_2},\frac{a_4}{b_1}\right);\ \alpha_{\widetilde{A}}\wedge\alpha_{\widetilde{B}},\theta_{\widetilde{A}}\vee\theta_{\widetilde{B}},\beta_{\widetilde{A}}\vee\beta_{\widetilde{B}}\rangle\ \text{if}\ (a_4>0,b_4>0)\\ \langle\left(\frac{a_4}{b_4},\frac{a_3}{b_3},\frac{a_2}{b_2},\frac{a_1}{b_1}\right);\ \alpha_{\widetilde{A}}\wedge\alpha_{\widetilde{B}},\theta_{\widetilde{A}}\vee\theta_{\widetilde{B}},\beta_{\widetilde{A}}\vee\beta_{\widetilde{B}}\rangle\ \text{if}\ (a_4<0,b_4>0)\\ \langle\left(\frac{a_4}{b_1},\frac{a_3}{b_2},\frac{a_2}{b_3},\frac{a_1}{b_4}\right);\ \alpha_{\widetilde{A}}\wedge\alpha_{\widetilde{B}},\theta_{\widetilde{A}}\vee\theta_{\widetilde{B}},\beta_{\widetilde{A}}\vee\beta_{\widetilde{B}}\rangle\ \text{if}\ (a_4<0,b_4<0)\end{cases}$$

$$\widetilde{A}\widetilde{B}=\begin{cases}\langle a_1b_1,a_2b_2,a_3b_3,a_4b_4;\ \alpha_{\widetilde{A}}\wedge\alpha_{\widetilde{B}},\theta_{\widetilde{A}}\vee\theta_{\widetilde{B}},\beta_{\widetilde{A}}\vee\beta_{\widetilde{B}}\rangle\ \text{if}\ (a_4>0,b_4>0)\\ \langle a_1b_4,a_2b_3,a_3b_2,a_4b_1;\ \alpha_{\widetilde{A}}\wedge\alpha_{\widetilde{B}},\theta_{\widetilde{A}}\vee\theta_{\widetilde{B}},\beta_{\widetilde{A}}\vee\beta_{\widetilde{B}}\rangle\ \text{if}\ (a_4<0,b_4>0)\\ \langle a_4b_4,a_3b_3,a_2b_2,a_1b_1;\ \alpha_{\widetilde{A}}\wedge\alpha_{\widetilde{B}},\theta_{\widetilde{A}}\vee\theta_{\widetilde{B}},\beta_{\widetilde{A}}\vee\beta_{\widetilde{B}}\rangle\ \text{if}\ (a_4<0,b_4<0)\end{cases}$$

**Definition 4**: A ranking function of neutrosophic numbers is a function $\Re: N(R)\rightarrow R$, where $N(R)$ is a set of neutrosophic numbers defined on set of real numbers, which convert each neutrosophic number into the real line.

Let $\widetilde{A}=\langle(a_1,a_2,a_3,a_4);\alpha_{\widetilde{A}},\theta_{\widetilde{A}},\beta_{\widetilde{A}}\rangle$ and $\widetilde{B}=\langle(b_1,b_2,b_3,b_4);\alpha_{\widetilde{B}},\theta_{\widetilde{B}},\beta_{\widetilde{B}}\rangle$ are two trapezoidal neutrosophic numbers, then

1. if $\Re(\widetilde{A})>\Re(\widetilde{B})$ then $\widetilde{A}>\widetilde{B}$,
2. if $\Re(\widetilde{A})<\Re(\widetilde{B})$ then $\widetilde{A}<\widetilde{B}$,
3. if $\Re(\widetilde{A})=\Re(\widetilde{B})$ then $\widetilde{A}=\widetilde{B}$.

The difference between neutrosophic set and fuzzy set is that neutrosophic set takes into consideration the truth, indeterminacy and falsity degree. But fuzzy set takes into consideration the truth degree only. In practical life, The decision makers he/she and problem solver (analyst) always seek to maximize the truth degree, minimize indeterminacy and falsity degree.

## III. MULTI-OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM

An MOLFP problem is defined as follows [27]

$$(\boldsymbol{MOLFP})\ \boldsymbol{Maximize}\ \{\boldsymbol{Z}(\boldsymbol{x}) = (\boldsymbol{z_1}(\boldsymbol{x}), \boldsymbol{z_2}(\boldsymbol{x}), \dots, \boldsymbol{z_k}(\boldsymbol{x}))\}$$

$$\boldsymbol{s.t}\quad \boldsymbol{Ax} \le \boldsymbol{b} \qquad (4)$$

$$\boldsymbol{x} \ge \boldsymbol{0}.$$

**Where:**

$\boldsymbol{S} = \{\boldsymbol{x}\epsilon\mathbb{R}^{\boldsymbol{n}} \mid \boldsymbol{Ax} \le \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}, \boldsymbol{b}\epsilon\mathbb{R}^{\boldsymbol{m}}\}$, is the Feasible Set in Decision Space**.**

$\boldsymbol{A\ is\ an\ m \times n\ matrix},\ \boldsymbol{x} \in \mathbb{R}^{\boldsymbol{n}}\ \boldsymbol{and}\ \boldsymbol{b} \in \mathbb{R}^{\boldsymbol{m}};\ (\boldsymbol{b} \ge \boldsymbol{0}), \boldsymbol{k} \ge \boldsymbol{2}.$

$$\boldsymbol{z_i}(\boldsymbol{x}) = \frac{\boldsymbol{c_i^T x} + \boldsymbol{\alpha_i}}{\boldsymbol{d_i^T x} + \boldsymbol{\beta_i}} = \frac{\boldsymbol{N_i}(\boldsymbol{x})}{\boldsymbol{D_i}(\boldsymbol{x})}\ ;\ \boldsymbol{c_i^T}, \boldsymbol{d_i^T}\epsilon\mathbb{R}^{\boldsymbol{n}}\ ;\ \boldsymbol{\alpha_i}, \boldsymbol{\beta_i}\ \epsilon\mathbb{R};\ for\ all\ i = 1,2,\dots,k$$

$$\boldsymbol{and}\ \boldsymbol{D_i}(\boldsymbol{x}) = \boldsymbol{d_i^T x} + \boldsymbol{\beta_i} > \boldsymbol{0}, \qquad for\ all\ i = 1,2,\dots,k, \qquad for\ all\ x\ \epsilon\ S.$$

A solution $\overline{\boldsymbol{x}}\boldsymbol{\epsilon}\ \boldsymbol{S}$ is an efficient solution of the problem (MOLFP) if and only if there is no $\boldsymbol{x}\ \boldsymbol{\epsilon}\ \boldsymbol{S}\ such\ that\ \boldsymbol{z_i}(\boldsymbol{x}) \ge \boldsymbol{z_i}(\overline{\boldsymbol{x}})\ for\ all\ i = 1,2,\dots,k\ and\ \boldsymbol{z_i}(\boldsymbol{x}) > \boldsymbol{z_i}(\overline{\boldsymbol{x}})\ for\ at\ least\ one\ i.$

Note that, for vectors $an$ ; $\boldsymbol{x} \ge \boldsymbol{y}\ implies\ \boldsymbol{x_i} \ge \boldsymbol{y_i}\ for\ each\ \boldsymbol{i}$ , $\boldsymbol{x} \ge \boldsymbol{y}\ implies\ \boldsymbol{x_i} \ge \boldsymbol{y_i}\ for\ \boldsymbol{i}\ and\ \boldsymbol{x_r} > \boldsymbol{y_r}\ for\ at\ least\ one\ \boldsymbol{i} =\ \boldsymbol{r}\ \ and\ \boldsymbol{x} > \boldsymbol{y}\ implies\ \boldsymbol{x_i} > \boldsymbol{y_i}\ for\ each\ i$ .

## IV. NMOLFP WITH TRAPEZOIDAL NEUTROSOPHIC NUMBERS COEFFICIENTS

In this section, Neutrosophic Multi-objective linear fractional programming problem with Trapezoidal Neutrosophic Numbers coefficients is considered. Formulating an NMOLFP model requires that Crisp values be selected for the model coefficients. In real word, any manager or decision maker want to obtain the crisp optimal solution of the problem, through considering vague, imprecise and inconsistent information when defining the problem. So, if we obtain the crisp value of the coefficients of decision variables, this problem can be considered as MOLFP. Then, we transform the MOLFP problem into a linear programming problem using Guzel's proposal [28].

Finally, the linear programming problem is solved by simplex method, whose optimal solution is the required efficient solution of the original problem.

Let us consider an NMOLFP with Trapezoidal Neutrosophic Numbers coefficients as:

$$max\ z_1 = \frac{\sum_{j=1}^{n} \widetilde{c_{\iota\jmath}} x_j\ +\ \widetilde{\alpha_\iota}}{\sum_{j=1}^{n} \widetilde{d_{\iota\jmath}} x_j\ + \widetilde{\beta_\iota}}$$

$$max\ z_2 = \frac{\sum_{j=1}^{n} \widetilde{c_{\iota\jmath}} x_j\ +\ \widetilde{\alpha_\iota}}{\sum_{j=1}^{n} \widetilde{d_{\iota\jmath}} x_j\ + \widetilde{\beta_\iota}}$$

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$$max\, z_k = \frac{\sum_{j=1}^{n} \widetilde{c_{ij}} x_j \; + \; \widetilde{\alpha_i}}{\sum_{j=1}^{n} \widetilde{d_{ij}} x_j \; + \widetilde{\beta_i}}$$

S.T

$$\sum_{j=1}^{n} \widetilde{a_{ij}} x_j \; \leq \widetilde{b_i} \qquad i = 1, 2, \dots, m$$

$$x_j \geq 0 \qquad j = 1, 2, \dots, n \quad (5)$$

where

$\widetilde{c_{ij}}, \widetilde{d_{ij}}, \widetilde{a_{ij}}$ *and* $\widetilde{b_i}$ are Trapezoidal Neutrosophic Numbers, $x = (x_1, x_2, \dots, x_n)^T$ denote the vector of all decision variables.

The NMOLFP problem may also be a problem with neutrosophic values of variables, coefficients in goal function and right-hand side constraints.

## V. METHODOLOGY

A recent approach suggested [4] to find the neutrosophic optimal solution of NMOLFP problems is introduced in this section.

Consider an NMOLFP with Trapezoidal Neutrosophic Numbers coefficients illustrated in the model (5).

**Step 1**: Let decision makers he/she insert their NMOLFP problem with trapezoidal neutrosophic numbers. Because we always want to maximize truth degree, minimize indeterminacy and the falsity degree of information, and then inform decision makers to apply this concept when entering trapezoidal neutrosophic numbers of the NMOLFP model.

**Step 2**: Regarding to definition 4, we propose a ranking function for trapezoidal neutrosophic numbers.

**Step 3**: Let $(\tilde{a} = a^l, a^{m1}, a^{m2}, a^u\ ; T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}})$ be a trapezoidal neutrosophic number, where $a^l, a^{m1}, a^{m2}, a^u$ are lower bound, first, second median value and upper bound for the trapezoidal neutrosophic number, respectively. Also $T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}$ are the truth, indeterminacy and falsity degree of trapezoidal number. If NMOLFP problem is a maximization problem, then:

Ranking function of this trapezoidal neutrosophic number is as follows:

$$R(\tilde{a}) = \left(\frac{a^l + a^u + 2(a^{m1}, a^{m2})}{2}\right) + \text{confirmation degree.}$$

Mathematically, this function can be written as follows:

$$R(\tilde{a}) = \left(\frac{a^l + a^u + 2(a^{m1}, a^{m2})}{2}\right) + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) \qquad (6)$$

If NMOLFP problem is a minimization problem, then:

Ranking function of this trapezoidal neutrosophic number is as follows:

$$R(\tilde{a}) = \left(\frac{a^l + a^u - 2(a^{m1}, a^{m2})}{2}\right) + \text{confirmation degree.}$$

Mathematically, this function can be written as follows:

$$R(\tilde{a}) = \left(\frac{a^l + a^u - 2(a^{m1}, a^{m2})}{2}\right) + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) \qquad (7)$$

**Step 4:** According to the type of NMOLFP problem, apply the suitable ranking function to convert each trapezoidal neutrosophic number to its equivalent crisp value. This leads to convert NMOLFP problem with its crisp model.

**Step 5**: Solve the crisp model using a proposal given by Nuran Guzel [28] and obtain the optimal solution of problem.

In the numerical example in the next section, we consider the truth degree $(T) = 1$, indeterminacy $(I)$ and falsity $(F)$ degree = 0, as follows (1, 0, 0) for each trapezoidal neutrosophic number and this called the confirmation degree of each trapezoidal neutrosophic number. We should also note that, according to Broumi S. and Smarandache F. (2014) each trapezoidal number is symmetric with the following form [18]:

$$\tilde{a} = (a^l, a^u, \alpha, \alpha\ )$$

Where $a^l, a^u, \alpha, \alpha$ represented the lower, upper bound and first, second median value trapezoidal number, respectively. The median values of trapezoidal numbers are with equal values (α). Now let us apply our proposed method on the same problem.

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## VI. NUMERICAL EXAMPLE

To illustrate the NMOLFP problem with trapezoidal neutrosophic numbers coefficients we present the numerical example as following. (this example is taken from [29] and transformed into trapezoidal neutrosophic numbers coefficients)

$$max\, z_1(x) = \frac{[4,8,1,1]x_1 + [6,10,2,2]x_2 + [2,6,1,1]x_3 + [2,4,0.5,0.5]}{[11,15,2,2]x_1 + [16,2,3,3]x_2 + [17,21,2,2]x_3 + [8,12,2,2]}$$

$$max\, z_2(x) = \frac{[16,,20,3,3]x_1 + [14,22,3,3]x_2 + [14,18,2,2]x_3 + [3,5,1,1]}{[10,16,2,2]x_1 + [16,20,2,2]x_2 + [14,20,2,2]x_3 + [10,14,2,2]}$$

s.t

$$[11,15,2,2]x_1 + [12,16,2,2]x_2 + [16,22,3,3]x_3 \leq [33,39,4,4]$$

$$[3,5,1,1]x_1 - [2,3,0.5,0.5]x_2 + [2,4,0.5,0.5]x_3 \leq [42,48,4,4] \qquad (8)$$

$$[4,7,1,1]x_1 + [3,7,1,1]x_2 + [7,11,2,2]x_3 \leq [29,33,3,3]$$

$$x_1, x_2, x_3 \geq 0$$

Because this NMOLFP problem is a maximization problem, then by using Equation (6) each trapezoidal number will convert to its equivalent crisp number. It is very important to remember that the confirmation degree of each trapezoidal number is (1, 0, 0) according to the decision maker, he/she opinion as we illustrated previously in the section before the example. Then, the crisp model of (8) problem will be as follows:

$$max\, z_1(x) = \frac{9x_1 + 13x_2 + 7x_3 + 5}{18x_1 + 26x_2 + 24x_3 + 15}$$

$$max\, z_2(x) = \frac{25x_1 + 25x_2 + 21x_3 + 7}{18x_1 + 23x_2 + 22x_3 + 17}$$

s.t

$$18x_1 + 19x_2 + 26x_3 \leq 45$$

$$6x_1 + 4.5x_2 + 5x_3 \leq 54 \qquad (9)$$

$$8.5x_1 + 8x_2 + 14x_3 \leq 38$$

$$x_1, x_2, x_3 \geq 0$$

To solve this MOLFP problem, we find the optimal value of each of the objective functions $Z_1, Z_2$ subject to the above constraints, using any of the methods for solving linear fractional programming problems (we used the equivalent of Charnes A. and Cooper W.W. [30]).

We get,

$max\, Z_1 = 0.467$

$max\, Z_2 = 1.121$

An LP problem, which is equivalent to the MOLFP problem, is constructed according to the proposed algorithm as follows:

$$max\, \mathrm{Z} = 9x_1 + 13x_2 + 7x_3 + 5 - (0.467)(18x_1 + 26x_2 + 24x_3 + 15) + 25x_1 + 25x_2 + 21x_3 + 7 - (1.121)(18x_1 + 23x_2 + 22x_3 + 17)$$

$$\max \mathrm{Z} = 5.416x_1 + 0.075x_2 - 7.87x_3 - 14.057$$

s.t

$$\begin{aligned} 18x_1 + 19x_2 + 26x_3 &\leq 45 \\ 6x_1 + 4.5x_2 + 5x_3 &\leq 54 \qquad (10) \\ 8.5x_1 + 8x_2 + 14x_3 &\leq 38 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Solving this LP problem by regular simplex method (by using LiPS program), we get the following optimal solution

$x_1 = 2.5\,, \quad x_2 = 0\,, \quad x_3 = 0$

Hence, an efficient solution of the FMOLFP problem is

$x_1 = 2.5\,,\ x_2 = 0\,,\ x_3 = 0$ with $\max Z_1 = 0.467\,,\ \max Z_2 = 1.121$

## VII. CONCLUSION

In this paper, A simple and efficient approaches capable for solving the MOLFP problem with trapezoidal neutrosophic numbers models are studied. By applying the neutrosophic set concept of the multi-objective linear fractional programming problems, we treated imprecise, vague and inconsistent information efficiently. A new assumption of neutrosophic MOLFP problems is presented in which all of the coefficients are trapezoidal neutrosophic numbers. In order to solve these types of problems, We presented two ranking functions (simple and efficient) for converting trapezoidal neutrosophic numbers to its equivalent crisp values. The first ranking function is for maximization problems and the second-ranking function is for minimization problems. After using the suitable ranking function and transforming the problem to its equivalent crisp MOLFP model, then we solve the MOLFP problem using a proposal given by

Nuran Guzel. A numerical example is given to illustrate our motivation for considering NMOLFP with trapezoidal neutrosophic numbers problems.

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